Topic 1

Real Business Cycle Theory: Part 2

Cyclical Dynamics: Model vs. Data

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Spring 2019

Outline

Part 1.

Stochastic growth model with variable labor supply: preferences and technology

Planning solution

Decentralized solution

Steady state

Part 2.

Loglinear approximation

Business cycle dynamics: key properties

Calibration and quantitative performance

Shortcomings

Business cycle accounting: sources of deviations from the data

Model Setup

Representative household

Household consumes C, supplies labor L,and saves capital K which it rents to firms.

Acts competitively - takes the real wage and rental rate on capital as given

Representative firm

Firm produces output using labor L and capital K

Acts competitively - takes real wage and rental rate on capital as given

Market clearing determines wages, rents and equilibrium quantities

Equivalent to planning solution

Preferences

$$E_t \left[\sum_{i=0}^{\infty} \beta^i [u(C_{t+i}) - \upsilon(L_{t+i})]\right]$$

with

$$\begin{aligned} u(C) &= \tfrac{1}{1-\gamma} C^{1-\gamma} \\ &= \log C \text{ iff } \gamma = 1 \end{aligned}$$

$$\upsilon(N) = \tfrac{1}{1+\varphi} L^{1+\varphi}$$

with $0 < \beta < 1$; $\gamma > 0; \varphi > 0$

where $C_t \equiv$ consumption, $L_t \equiv$ labor supply.

Digression on Intensive vs. Extensive Labor Supply

Standard interpretation: Household adjusts L along intensive margin (hours)

However, most hours fluctuations (2/3) are along extensive margin (bodies)

Given complete markets, can re-interpret L as adjustment along extensive margin:

Suppose continuum of measure unity members who differ according to disutility of work.

Let $j^{\varphi} \equiv$ disutility of work of member j. $L \equiv \#$ of family members working

Given complete consumption insurance within family, we can express family period utility as

$$\frac{1}{1-\gamma}C^{1-\gamma} - \int_0^L j^{\varphi} dj = \frac{1}{1-\gamma}C^{1-\gamma} - \frac{1}{1+\varphi}L^{1+\varphi}$$

Objective unchanged $\rightarrow$decision rules unchanged.

While model allows for extensive margin, it ignores search and matching and abstracts from incomplete markets (which u will study next quarter).

Technology

$$Y_t = K_t^\alpha (A_t L_t)^{1-\alpha}$$
$$= A_t^{1-\alpha} K_t^\alpha L_t^{1-\alpha}$$

where $Y_t \equiv$ output, $A_t^{1-\alpha} \equiv$ total factor productivity, $K_t \equiv$ capital, $L_t \equiv$ labor input.

Resource Constraint ($\rightarrow$ Law of Motion for Capital):

$$C_t + K_{t+1} = Y_t + (1 - \delta) K_t$$

where $0 < \delta < 1$ is the depreciation rate and where TFP obeys

$$A_t/\overline{A}_t = (A_{t-1}/\overline{A}_{t-1})^\rho e^{\epsilon_t}$$
$$\overline{A}_t/\overline{A}_{t-1} = G = 1 + g \geq 1$$

where $\overline{A}_t =$trend TFP, $0 \leq \rho < 1$ and ϵ_t is i.i.d. with mean zero.

Behavioral Relations

Labor market equilibrium

$$(1-\alpha)A_t(\frac{K_t}{A_t L_t})^{\alpha} = W_t = L_t^{\varphi}/C_t^{-\gamma}$$

Consumption/saving:

$$C_t^{-\gamma} = E_t\{\beta C_{t+1}^{-\gamma} R_{t+1}\}$$

where $R_{t+1} \equiv$ gross return on capital:

$$R_{t+1} = \alpha(\frac{K_{t+1}}{A_{t+1}L_{t+1}})^{\alpha-1} + (1-\delta)$$

Transversality condition for household budget constraint ensures non-explosive solution.

Note that the behavioral relations come from household and firm decision rules and market clearing (see Part 1).

Complete Model

Endogenous variables: (Y_t, L_t, C_t, K_{t+1})

Predetermined states: (K_t, A_t)

$$
\begin{aligned}
Y_t &= A_t^{1-\alpha} K_t^{\alpha} L_t^{1-\alpha} \\
(1-\alpha)\frac{Y_t}{L_t} &= \frac{L_t^{\varphi}}{C_t^{-\gamma}} \\
1 &= E_t\{\beta(\frac{C_{t+1}}{C_t})^{-\gamma}(\alpha\frac{Y_{t+1}}{K_{t+1}} + 1 - \delta)\} \\
K_{t+1} &= Y_t + (1-\delta)\, K_t - C_t \\
A_t/\overline{A} &= (A_{t-1}/\overline{A})^{\rho} e^{\epsilon_t}
\end{aligned}
$$

Cyclical driving force: fluctuations in A_t.

Deterministic Steady State

stationary variables: $\frac{Y}{K}, \frac{K}{AL}, \frac{C}{K}, L$

$\frac{Y}{K}, \frac{K}{AL}, \frac{C}{K}$ determined by production function, consumption/saving relation and resource constraint:

$$\frac{Y}{K} = (\frac{K}{AL})^{\alpha-1}$$
$$1 = \beta(\frac{C'}{C})^{-\gamma}[\alpha\frac{Y}{K} + 1 - \delta]$$
$$\frac{Y}{K} = \frac{C}{K} + \delta + g$$

with $\frac{C'}{C} = 1 + g$

Labor market then determines L

$$(1-\alpha)\frac{Y}{L} = L^{\varphi}/C^{-\gamma}$$

Note: if $g > 0 \rightarrow \gamma = 1$ required to have L constant along balanced growth path.

Road Ahead

- Loglinear approximation of model around deterministic steady state.
- "Calibrate" model parameters
- Evaluate business cycle dynamics versus quarterly data.

Log-linearization

- Because (i) many macroeconomic series are stationary in growth rates (e.g GDP); (ii) (for the most part) exhibit relatively small percentage changes and (iii) linear models are easy to work with; we often work with loglinear approximations:
  - Consider the following nonlinear equation

$$g(X_t) = f(Y_t)$$

  - Take a first order expansion around deterministic steady state

$$g(X) + g'(X)dX_t \approx f(Y) + f'(Y)dY_t$$

  - $\rightarrow$ linear relation for growth rates

$$g'(X)X\frac{dX_t}{X} \approx f'(Y)Y\frac{dY_t}{Y}$$

where X and Y are steady values $\Rightarrow g(X) = f(Y)$.

Log-linearization (con't)

- Let $z_t \equiv \log(Z_t/Z) = \log(Z_t) - \log(Z)$ for $z_t = x_t, y_t$.

  For small percent changes in Z_t : $\quad z_t \approx \frac{dZ_t}{Z_t}$.

- This leads to the following loglinear approximation of $g(X_t) = f(Y_t)$:

$$g'(X)X \cdot x_t = f'(Y)Y \cdot y_t$$

- Via loglinearization, a model that is nonlinear in X_t and Y_t becomes linear in the log-deviations x_t and y_t

Loglinearization of RBC Model

let $\widetilde{a}_t = (1-\alpha)a_t; \sigma = \gamma^{-1}$

Production function

$$y_t = \widetilde{a}_t + \alpha k_t + (1-\alpha)\, l_t$$

Labor market equilibrium

$$y_t - l_t = w_t = \varphi l_t + \gamma c_t$$

Consumption/Saving

$$c_t = -\sigma E_t \left\{ \alpha \frac{Y}{K} \left(y_{t+1} - k_{t+1} \right) \right\} + E_t \left\{ c_{t+1} \right\}$$

Law of motion for capital

$$k_{t+1} = \frac{Y}{KG} y_t - \frac{C}{KG} c_t + \frac{1-\delta}{G} k_t$$

with $\widetilde{a}_t = \rho \widetilde{a}_{t-1} + \varepsilon_t$.

Some Economics: Labor supply

Labor market equilibrium (after rearranging):

$$l_t = \varphi^{-1}(y_t - l_t) - (\gamma/\varphi)c_t$$
$$= \varphi^{-1}w_t - (\gamma/\varphi)c_t$$

$\varphi^{-1} =$ Frisch labor supply elasticity (percentage response of l_t to one percent change in w_t, holding c_t constant.)

Estimates of φ^{-1} depend on whether l_t reflects intensive vs. extensive margin. Low for former ($\sim$ 0.5), higher for latter ($\sim$ 1.0)

The second term reflects the "wealth" effect on labor supply: Strength of wealth effect c_t on l_t increasing in γ. $\gamma \uparrow \rightarrow$ stronger desire to smooth consumption.

Some Economics: Consumption/Saving

$r_{t+1} = \log R_{t+1} - \log R$ (approximately, deviation of net real interest rate from steady state).

Consumption/saving:

$$c_t = -\sigma E_t \left\{ \alpha \frac{Y}{K} \left(y_{t+1} - k_{t+1} \right) \right\} + E_t \left\{ c_{t+1} \right\}$$

$$c_t = -\sigma E_t \left\{ r_{t+1} \right\} + E_t \left\{ c_{t+1} \right\}$$

dependence of c_t on $E_t \{c_{t+1}\}$ reflects desire to smooth consumption

fluctatuation in $E_t \{r_{t+1}\}$ may induce intertemporal substituion of consumption across time:

$\sigma \equiv \frac{1}{\gamma}$ is the intertemporal elasticity of substitution.

$\gamma \uparrow \rightarrow \sigma \downarrow$ since greater desire to smooth consumption.

Solution

To shed some light on the mechanisms that drive output and employment, combine the production function and the labor market equilibrium to:

$$l_t = \frac{1}{\alpha+\varphi}(\tilde{a}_t + \alpha k_t) - \frac{\gamma}{\alpha+\varphi} c_t$$

$$y_t = \left(1 + \frac{1-\alpha}{\alpha+\varphi}\right)(\tilde{a}_t + \alpha k_t) - \frac{(1-\alpha)\gamma}{\alpha+\varphi} c_t \rightarrow$$
$$y_t = y(\tilde{a}_t, k_t, c_t)$$

$\tilde{a}_t + \alpha k_t$ reflects productivity which has both a direct and indirect (through labor demand) effect on y_t.

c_t reflects wealth effect on labor supply.

Three key parameters: α, φ and $\gamma(\equiv \sigma^{-1})$.

Solution (con't)

solving for c_t :

use the previous relation for y_t to eliminate y_{t+1} to obtain the following system of two first order difference equations for c_t and k_{t+1}

$$c_t = -\sigma E_t \left\{\alpha \frac{Y}{K} \left(y(\widetilde{a}_{t+1}, k_{t+1}, c_{t+1}) - k_{t+1}\right)\right\} + E_t \left\{c_{t+1}\right\}$$
$$k_{t+1} = \frac{Y}{KG} y(\widetilde{a}_t, k_t, c_t) - \frac{C}{KG} c_t - \frac{1-\delta}{G} k_t$$

with

$$\widetilde{a}_t = \rho \widetilde{a}_{t-1} + \varepsilon_t$$

with $0 \leq \rho \leq 1$and where $\widetilde{a}_t$ and k_t are predetermined.

Note that the two first order difference equations can be combined into a single second order difference in k_t, k_{t+1} and $E_t\{k_{t+2}\}$

Solution (con't)

- The system of two first order difference equation is second order with two characteristic roots. One is greater than unity (unstable) and the other is less than unity (stable).
- The unstable root is associated with the forward looking variable (consumption) and the stable root is associated with capital.
- Reduced form policy functions for c_t and k_{t+1} :

$$c_t = \pi_{ca}\widetilde{a}_t + \pi_{ck}k_t$$

$$k_{t+1} = \pi_{ka}\widetilde{a}_t + \pi_{kk}k_t$$

where the π coefficients are functions of the model parameters and can be obtained by using the method of undetermined coefficients (Campbell, JME 1994)).

Solution (con't)

From reduced form policy function for $c_t (= \pi_{ca}\widetilde{a}_t + \pi_{ck}k_t)$, possible to solve for the other variables for the model.

$$\begin{aligned} l_t &= \frac{1}{\alpha+\varphi}(\widetilde{a}_t + \alpha k_t) - \frac{\gamma}{\alpha+\varphi}c_t \\ &= \frac{1-\gamma\pi_{ca}}{\alpha+\varphi}\widetilde{a}_t + \frac{\alpha-\gamma\pi_{ck}}{\alpha+\varphi}k_t \end{aligned}$$

$$\begin{aligned} y_t &= \left(1 + \frac{1-\alpha}{\alpha+\varphi}\right)(\widetilde{a}_t + \alpha k_t) - \frac{(1-\alpha)\gamma}{\alpha+\varphi}c_t \\ &= \left(1 + (1-\alpha)\frac{1-\gamma\pi_{ca}}{\alpha+\varphi}\right)\widetilde{a}_t + (1 + (1-\alpha)\frac{\alpha-\gamma\pi_{ck}}{\alpha+\varphi})\alpha k_t \end{aligned}$$

$$k_{t+1} = \frac{Y}{KG}y_t - \frac{C}{KG}(\pi_{ca}\widetilde{a}_t + \pi_{ck}k_t) + \frac{1-\delta}{G}k_t$$

Combining the relations for y_t and $k_{t+1} \rightarrow$ policy function for $k_{t+1} (= \pi_{ka}\widetilde{a}_t + \pi_{kk}k_t)$

Solution (con't)

Observe that k_t (i.e. the log-deviation of capital stock from steady state or percent variation of capital stock) is small over the cycle. Hence we can assume:

$$c_t \approx \pi_{ca}\widetilde{a}_t \rightarrow$$

$$l_t \approx \tfrac{1-\gamma\pi_{ca}}{\alpha+\varphi}\widetilde{a}_t$$

$$y_t \approx \left(1 + (1-\alpha)\tfrac{1-\gamma\pi_{ca}}{\alpha+\varphi}\right)\widetilde{a}_t$$

Given $I_t = K_{t+1} - (1-\delta)K_t = Y_t - C_t \rightarrow \frac{I}{Y}inv_t = y_t - \frac{C}{Y}c_t \rightarrow$

$$inv_t = \tfrac{Y}{I}y_t - \tfrac{C}{I}c_t \rightarrow$$

$$inv_t \approx \tfrac{Y}{I}[\left(1 + (1-\alpha)\tfrac{1-\sigma\pi_{ca}}{\alpha+\varphi}\right) - \tfrac{C}{Y}\pi_{ca}]\widetilde{a}_t$$

Note inv_t likely more volatilite than c_t

π_{ca} not large due to consumption smooting (especially if $\widetilde{a}_t$ less persistent, i.e., ρ is low)

$\frac{Y}{I} > 1$ is large

Calibration

To pick parameter values, use information independent of the business cycle data to be explained: e.g. long run relationships in the data (average growth rate, average labor share of output), parameter estimates from micro studies (labor supply elasticity, etc.),

Detrend the data (e.g. using a Hodrick-Prescott filter). Then recover the Solow residual $\widetilde{a}_t = y_t - \alpha k_t - (1 - \alpha) l_t$

Then use filtered data to estimate the process $\widetilde{a}_t = \rho \widetilde{a}_{t-1} + \epsilon_t$.

Next, generate artificial data by feeding the estimated TFP process into the calibrated model.

From the artificial data, compute a variety of business cycle moments and compare with actual data.

Parameter Choices (example)

Parameters $(\beta, \gamma = \sigma^{-1}, \varphi, \alpha, \delta, g, \rho, \sigma_a^2)$

$\beta = 0.9375$ annually (0.984 quarterly) to match average return on capital.

$g = 0.016$ annually (0.004 quarterly) to match over growth in output per capita.

$\alpha = 0.33$ to match capital share.

$\delta = 0.10$ (0.025 quarterly) to match capital depreciation rate

$\varphi^{-1} = 1$ (to match evidence on Frisch elasticity of labor supply (extensive margin)

$\gamma = 1$

Note: calibration does not allow for parameter uncertainty making it hard to assess how confident one can be in model performance.

Properties

- A reasonably calibrated model with a_t is the sole driving force can generate a standard deviation of output equal to seventy percent of that of actual output fluctuations (for postwar data pre-1984).
- The model can produce about half the volatility of hours.
- Investment is more volatile than consumption (as in the data).
- Fluctuations are Pareto efficient - no scope for policy.

Properties (con't)

- Consumption smoothing.

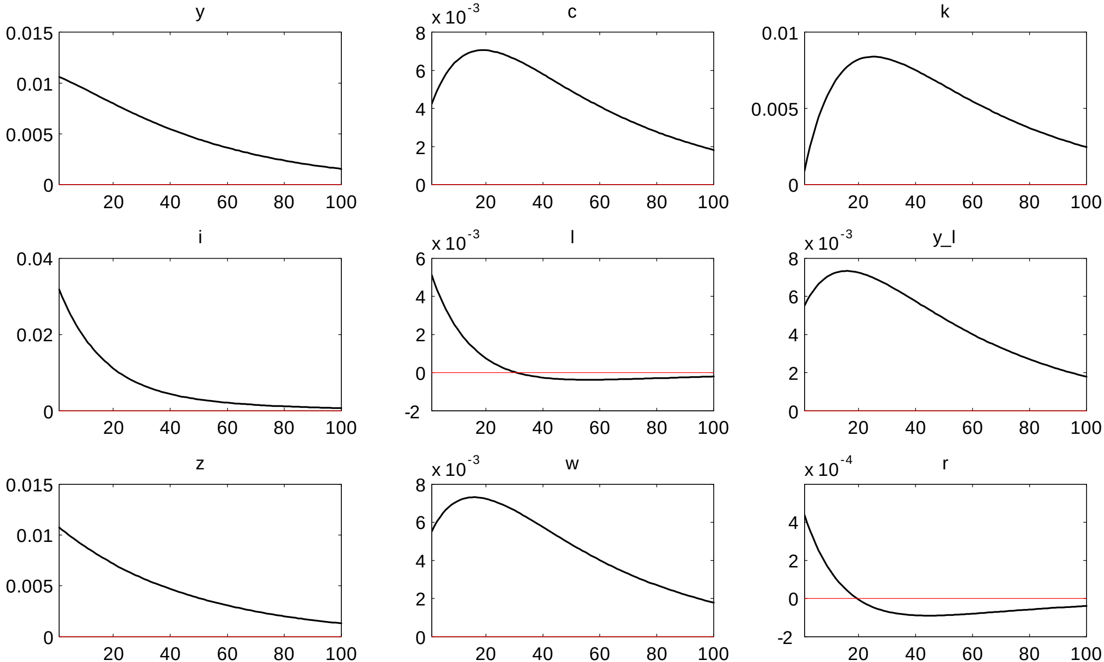
  Iterating the consumption/saving relation forward and imposing a terminal condition:

$$c_t = \sum_{i=0}^{\infty} (-\sigma)^{-1} E_t \{r_{t+1+i}\} \tag{1}$$

  with

$$r_{t+1} = \alpha \frac{Y}{K} (y_{t+1} - k_{t+1}) \tag{2}$$

- As long as long term interest rates are not too variable, consumption's behavior will be smooth.


y
c
k
i
l
y_l
z
w
r
x 10^-3
x 10^-4
0.015
0.01
0.005
0
0.04
0.02
8
6
4
2
-2
20
40
60
80
100

- See Tables 1 and 3 in the Handbook chapter of King and Rebelo. Here only some key moments are reported

	std% data	std% model	correlation with output data	correlation with output model
output	1.81	1.39	1	1
consumption	1.35	0.61	0.88	0.94
investment	5.30	4.09	0.80	0.99
hours	1.79	0.67	0.88	0.97
labor productivity	1.02	0.75	0.55	0.98

Shortcomings

1. There is no internal propagation of shocks: y_t is driven only by a_t.

2. Unlikely that high frequency variation in the Solow residual reflects true movements in TFP. Total factor productivity a_t is not observed directly but measured as a residual. Suppose $Y_t = A_t^{1-\alpha}\left(U_t^K K_t\right)^{\alpha}\left(U_t^N L_t\right)^{1-\alpha}$. The production function includes unmeasured variations of factor utilization. The log-linearized Solow residual is then $a_t^o = a_t + \alpha u_t^K + (1-\alpha)\, u_t^N$. Thus utilization may be driving the high frequency variation in the Solow residual.

3. The productivity/hours correlation at the high frequency has shifted from positive to negative, post 1984 (due mainly to "jobless" recoveries.)

4. Under a reasonable calibration the model cannot account for the magnitude of employment fluctuations..

5. Monetary/financial frictions are absent from this model.

Business Cycle Accounting

Method for evaluating deviations of key model equations from data.

Benchmark case where model holds perfectly:

Labor market

$$(1-\alpha)\frac{Y}{L} = L^{\varphi}/C^{-\gamma} \leftrightarrow$$
$$MPL = MRS$$

Capital market

$$1 = E_t\{\beta(\frac{C^{'}}{C})^{-\gamma}(\alpha\frac{Y^{'}}{K^{'}} + 1 - \delta)\}$$
$$1 = E_t\{IMRS. \cdot R_k'\}$$

Evaluating Model Residuals (Wedges)

- One way to evaluate the model is to examine the performance of the residuals (or "wedges").

1. *Labor market wedge* τ_t^L :

$$\tau_t^L = \frac{MPL_t}{MRS_t} - 1 \tag{3}$$

2. *Capital market wedge* τ_t^K :

$$\tau_t^K = \ E_t \left\{ R_{kt+1} \cdot IMRS_{t+1} \right\} - 1$$

- The RBC model presumes $\tau_t^L = \tau_t^K = 0$.

Evaluating Model Residuals (Wedges) $(con't)$

- Given restrictions on preferences and technology we can measure τ_t^L and τ_t^K.

- Significant cyclical movements in τ_t^L and τ_t^K may be regarded as evidence of some form of model mispecification.

- Accounting for the pattern of these deviations then serves a guide for reformulating the model.

Labor Market Distortions

- Hall (1999) and Shimer (2009) presents evidence that movements in τ_t^L are highly countercyclical.

  $\rightarrow$Recessions are thus associated with periods where the marginal product of labor exceeds the (measured) marginal rate of substitution.

- Mulligan (2002) and Chari, Kehoe and McGrattan (2007) show that during the Great Depression there was a sharp increase in τ_t^L.

  $\rightarrow$ The simple neoclassical labor market cannot account for the drop in employment.

- Gali, Gertler and Lopez-Salido (2007) interpret movements in τ_t^L as reflecting countercyclical markup behavior.

Labor Wedges as Markups

- Let $1+\mu_t^P$ denote the gross price markup and $1+\mu_t^W$ denote the gross wage markup

$$
\begin{aligned}
1+\mu_t^P &= \frac{P_t}{(W_t/MPL_t)} = \frac{MPL_t}{W_t/P_t} && (4)\\
1+\mu_t^W &= \frac{W_t/P_t}{MRS_t} && (5)
\end{aligned}
$$

where W_t/MPL_t is the marginal cost of producing a unit of output

It follows that

$$
\begin{aligned}
\left(1+\mu_t^P\right)\left(1+\mu_t^W\right) &= \frac{MPL_t}{W_t/P_t}\cdot\frac{W_t/P_t}{MRS_t} && (6)\\
&= \frac{MPL_t}{MRS_t} && (7)\\
&= 1+\tau_t^L && (8)
\end{aligned}
$$

Labor Wedges as Markups (con't)

- Taking logs

$$\vdots$$

$$\log MPL_t - \log MRS_t \approx \mu_t^P + \mu_t^W \tag{9}$$

We can also rewrite the log price and wage mark-ups as

$$\mu_t^P = \log MPL_t - \log(W_t/P_t) \tag{10}$$

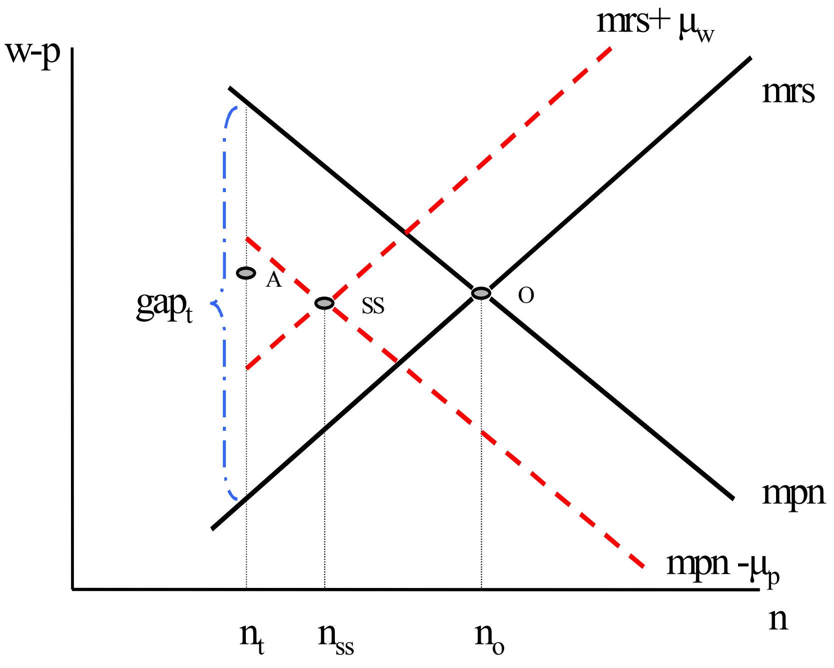
$$\mu_t^W = \log(W_t/P_t) - \log MRS_t \tag{11}$$

- The labor wedge and markups

$$\tau_t^L = \mu_t^P + \mu_t^W = \log MPL_t - \log MRS_t \tag{12}$$

- Countercyclical movements in τ_t^L reflect countercyclical movements in markups and in inefficiency of the labor market.

Figure 1. The Gap: A Diagrammatic Exposition



A Parametric Example

After a loglinear approximations around the steady state:,

Technology: Assume a constant elasticity of output with respect to hours (e.g. Cobb-Douglas).

$$
\begin{aligned}
mpl_t &= y_t - l_t \;\rightarrow & (13) \\
\mu_t^P &= (y_t - l_t) - (w_t - p_t) \equiv -ulc_t \text{ (minus log unit labor cost)} & (14)
\end{aligned}
$$

Preferences:

$$
mrs_t = \varphi l_t + \gamma c_t \;\rightarrow \tag{15}
$$

Thus

$$
\mu_t^W = (w_t - p_t) - (\varphi l_t + \gamma c_t) \tag{16}
$$

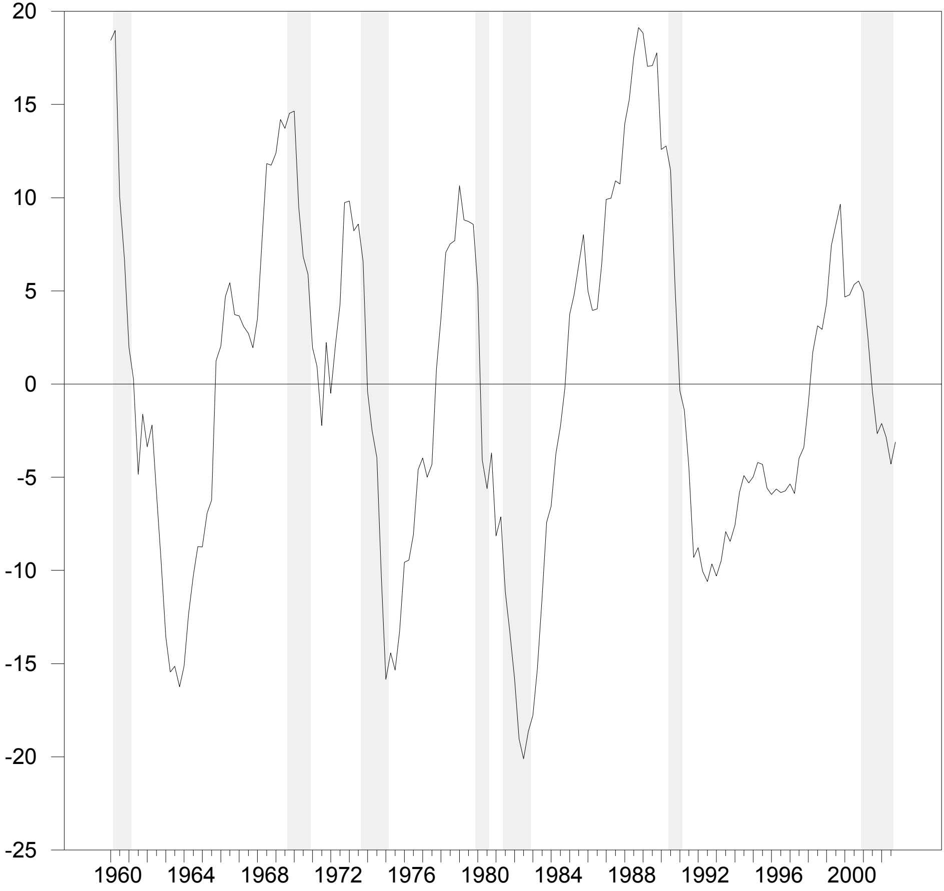
A Parametric Example (con't)

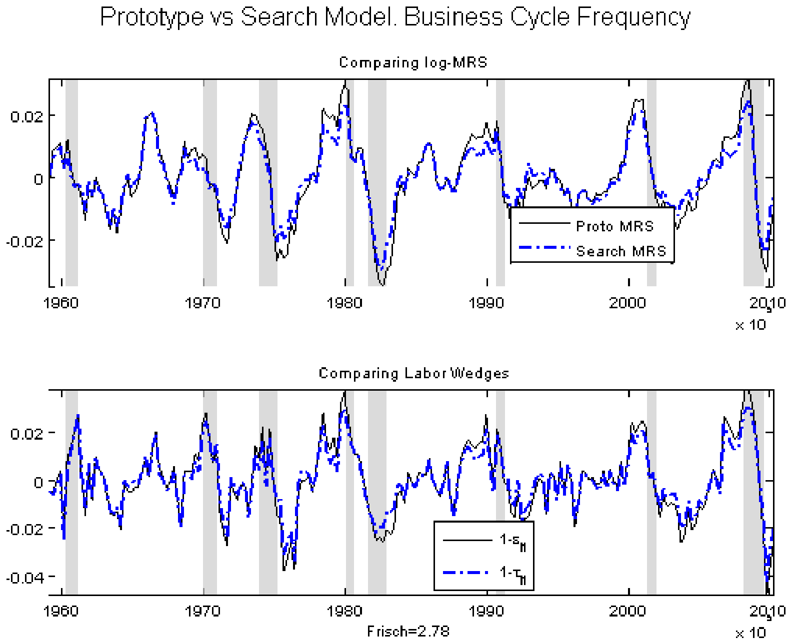
- Labor wedge:

$$
\begin{aligned}
\tau_t^L &= \mu_t^P + \mu_t^W \\
&= [(y_t - l_t) - (w_t - p_t)] + [(w_t - p_t) - (\varphi l_t + \sigma c_t)] \\
&= (y_t - l_t) - (\varphi l_t + \sigma c_t)
\end{aligned}
$$

Figure 2. The Gap

Baseline Calibration (σ=5 , φ=1)





Prototype vs Search Model. Business Cycle Frequency
Comparing log-MRS
0.02
0
-0.02
1960
1970
1980
1990
2000
2010
x 10
Proto MRS
Search MRS
Comparing Labor Wedges
0.02
0
-0.02
-0.04
1960
1970
1980
1990
2000
2010
x 10
$1-s_H$
$1-\tau_H$
Frisch=2.78

Figure 3. The Gap and the Wage Markup

Baseline Calibration

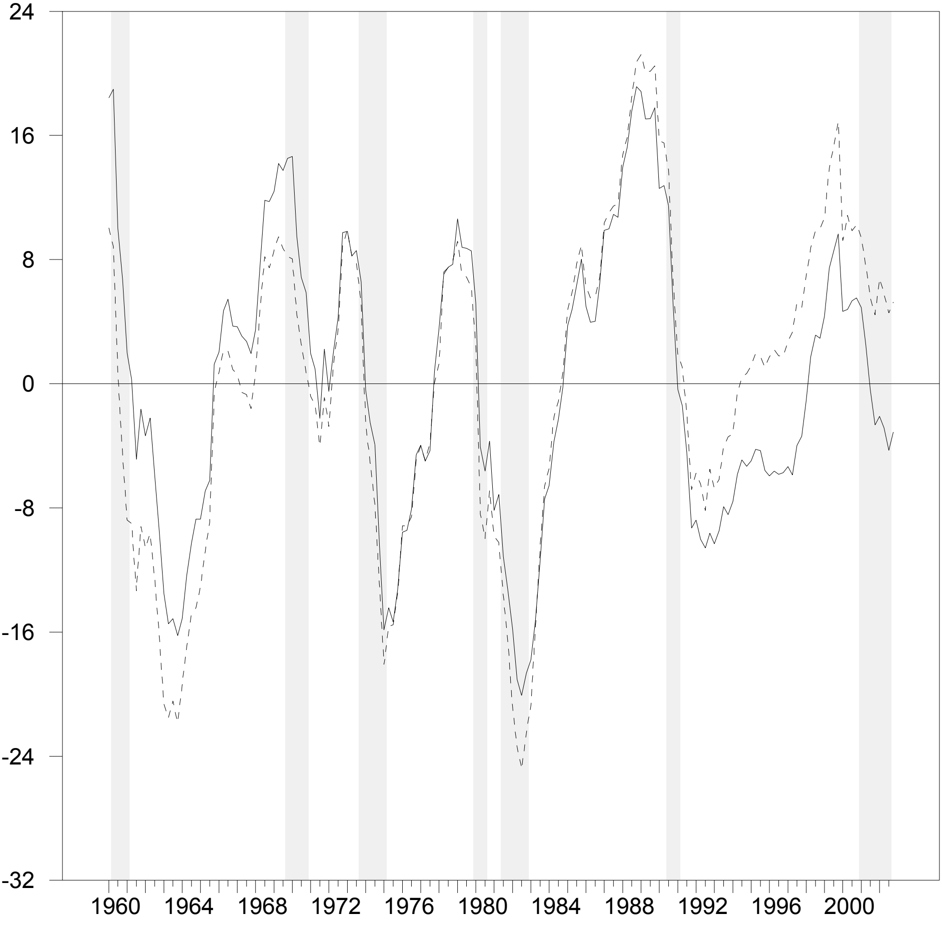
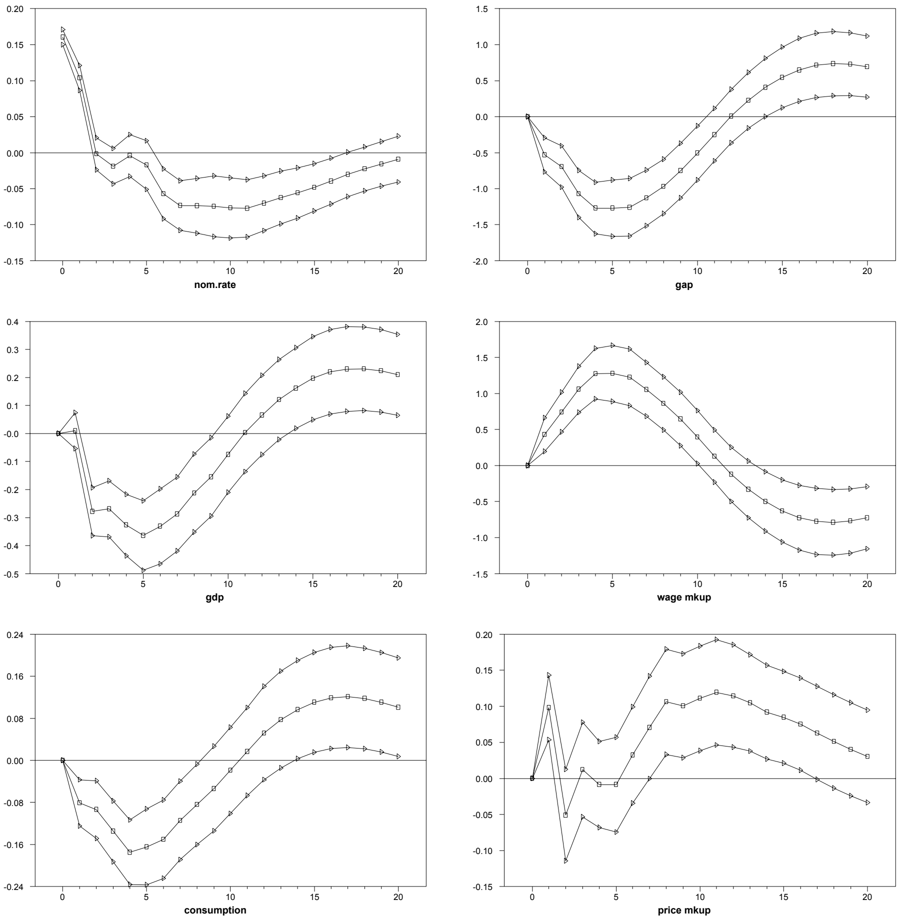


Figure 4. Dynamic Effects of Monetary Policy Shocks

Baseline Calibration



Capital Market Wedge

- Let R_{ft} $\equiv$risk free rate. With frictionless financial markets

$$E_t \{IMRS_{t+1} R_{kt+1}\} = E_t \left\{IMRS_{t+1} \cdot R_{ft+1}\right\}$$

- If

$$E_t \{IMRS_{t+1} R_{kt+1}\} > E_t \left\{IMRS_{t+1} \cdot R_{ft+1}\right\}$$

- Then

$$\tau_t^K = \; E_t \{IMRS_{t+1} \cdot R_{kt+1}\} - 1 > 0$$

since

$$E_t \left\{IMRS_{t+1} R_{ft+1}\right\} - 1 = 0$$

(From savers first order condition for the risk free rate)

- Intuitively, if $R_{kt+1} > R_{ft+1}$, (beyond what the equity premium explains) financial frictions are distorting investment demand.

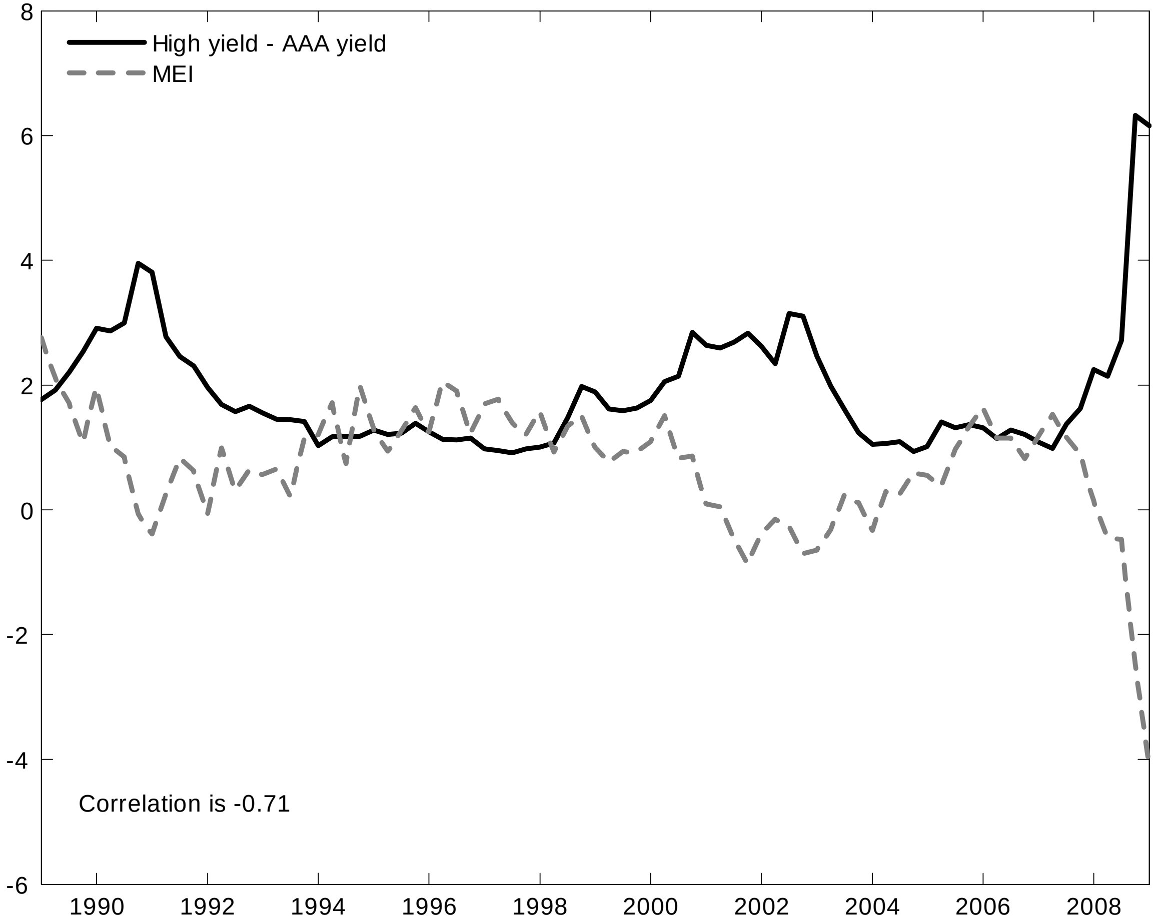


FIGURE 5. Credit spread and the marginal efficiency of investment. The credit spread (dark continuous line) is measured as the difference between the returns on high yield and AAA corporate bonds. The marginal efficiency of investment series (light dashed line) is the Kalman filter estimate of the μ_t shock at the posterior mode. Both series are standardized.