Topic 2

The Baseline New Keynesian Model,

Monetary Policy, and the Liquidity Trap: Part 1

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Outline

Part 1

Household consumption, labor supply, saving decisions, and money demand

Firm labor, capital and price setting decisions

Monetary policy: Taylor rules

Decentralized equilibrium: monetary non-neutrality and inefficient output fluctuations

Part 2

Loglinear model

Aggregate demand and the natural rate of interest

The New Keynesian Phillips curve

Monetary policy design in the basic NK model

The liquidity trap

Motivation

1. Evidence that monetary policy matters:

    Experience from Volcker's tightening of monetary policy in late 1979-80 that induced a recession

    Evidence from vector autoregressions that monetary policy has real effects

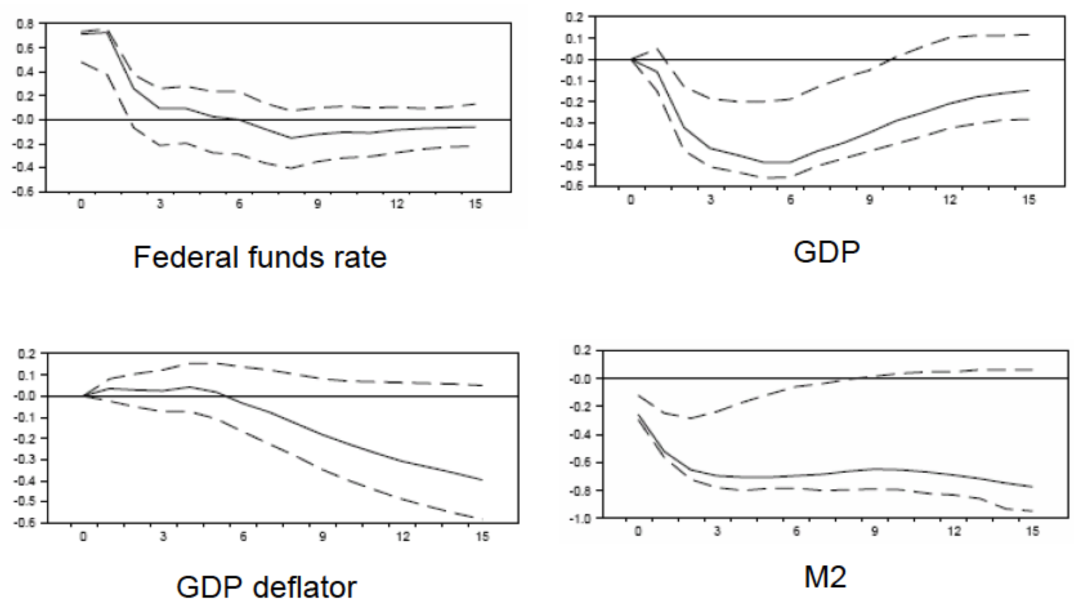
2. Panel data evidence on price stickiness

    Correcting for sales, prices fixed for roughly 9 months on average (Bils/Klenow, Nakamura/Steinsson and Kehoe/Midrigan)

3. Evidence that gap between MPN and MRS (the labor wedge) is countercyclical$\longrightarrow$

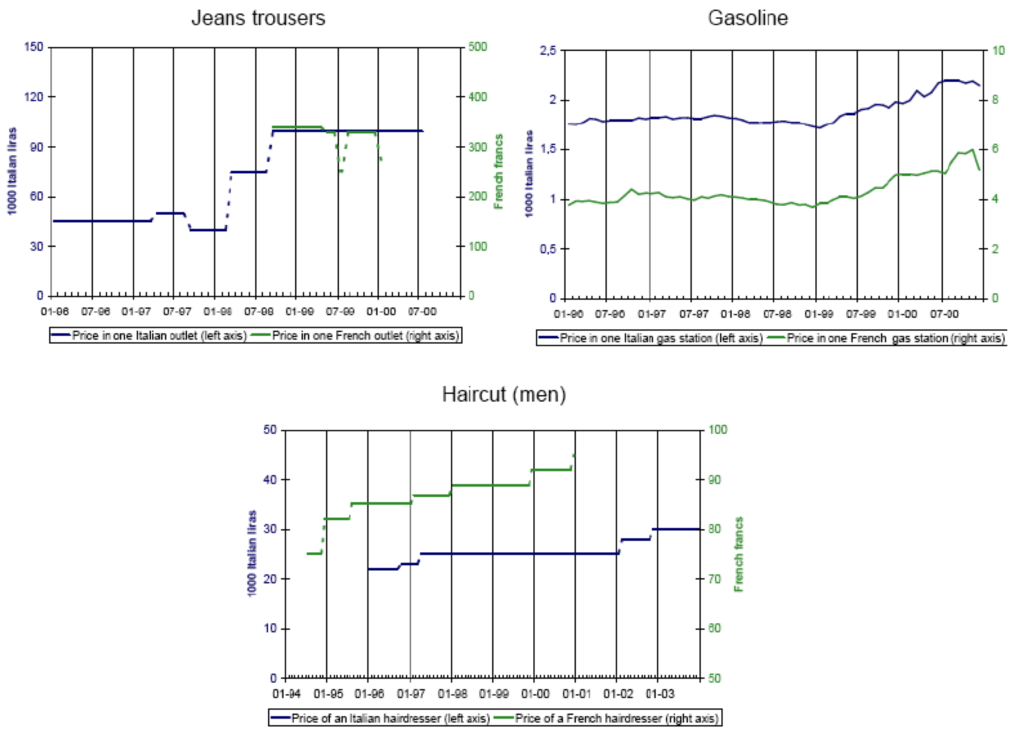
    Recessions reflect inefficient contractions in employment relative to first best.

Figure 1. Estimated Dynamic Response to a Monetary Policy Shock



Source: Christiano, Eichenbaum and Evans (1999)

Figure 1 - Examples of individual price trajectories (French and Italian CPI data)



Note : Actual examples of trajectories, extracted from the French and Italian CPI databases. The databases are described in Baudry *et al*. (2004) and Veronese *et al*. (2005). Prices are in levels, denominated in French Francs and Italian Lira respectively. The dotted lines indicate events of price changes.

Source: Dhyne et al. WP 05

Basic Ingredients

Starting point: A dynamic stochastic general equilibrium (DSGE) model (effectively on RBC model)

Three modifications:

Money:

Needed to model nominal variables and monetary policy

Imperfect competition

Needed to model price setting

Also helps motivate fluctuations in output above the natural level.

Nominal price rigidities

Needed for monetary non-neutrality and fluctuations in the labor wedge

Also helps explain short/run inflation output dynamics

Environment

Representative household:

Consumes final good C_t, supplies labor L_t,

Saves in form of real money balances M_t/P_t, capital K_t (which it rents to firms)

and private one period discount bonds B_t that pay 1$ in subsequent period.

(in zero net supply.)

Two types of firms.

Final good producers: competitors

Produce output Y_t using intermediate goods $Y_t\,(f)$.

Intermediate good firms: monopolistic competitors

Produce a differentiated product $Y_t\,(f)$ using capital $K_t\,(f)$ and labor $N_t\,(f)$.

These firms set prices $P_t\,(f)$ on a staggered basis.

Environment (con't)

The government conducts monetary and fiscal policy.

For simplicity: Capital is fixed in aggregate supply, though perfectly mobile across firms:

Capital price Q_t is endogenous

Straightforward to add investment: will vary positively with Q_t

For convenience we also assume zero growth in steady state

We also restrict attention to productivity shocks and shocks to monetary policy

Though later we add preferences shocks in the form of demand shocks.

Decentralized equilibrium no longer equivalent to planning solution, given frictions (imperfect competition, nominal rigidities)$\rightarrow$

Need to construct decentralized equilibrium

Digression on "Money"

Conventional definition: "Anything generally acceptable in exchange."

Link between liquidity (ability to trade w/o loss of value) and acceptability in exchange

Common forms of money: outside (currency); inside (bank deposits)

Outside is gov't liability; inside is private liability.

In addition to exchange role, money provides unit of account and store of value.

Money typically pays a lower nominal yield than non-monetary assets (often zero)

Classic question: why do individuals hold money when it pays a lower yield.

Challenging issue in neoclassical models, since exchange usually not modeled.

For NK model, key feature of money is unit-of-account role: Nominal prices are in units of money. (Typically study "cashless limit" of a monetary economy).

Household Preferences

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^i \left[\frac{1}{1-\gamma} C_t^{1-\gamma} + \frac{a_m}{1-\gamma_m} \left(\frac{M_t}{P_t} \right)^{1-\gamma_m} - \frac{1}{1+\varphi} L_t^{1+\varphi} \right] \right\}$$

with $\gamma, \gamma_{m,}\varphi > 0$

Money in the utility function reflects convenience yield from holding money

Yield is increasing $\frac{M_t}{P_t}$ but decreasing at the margin.

M_t is outside money.

Loosely motivated by inventory-theoretic models of money demand (Baumol/Tobin) where money holdings reflect trade-off between need for transactions versus fixed cost of obtaining money. Yields a similar reduced form money demand function

Alternative approaches:

"cash-in-advance" (Lucas): $C_t \leq \frac{M_t}{P_t}$.

Search and matching (Kiyotaki/Wright, Lagos/Wright)

The household's problem

$W_t/P_t \equiv$ real wage, $Z_t \equiv$ rental cost of capital, $\Pi_t \equiv$ profits from monop. competitive firms, $TR_t \equiv$ government transfers, $R_t^n \equiv$ gross nominal interest rate, and $Q_t \equiv$ price of capital.

Choose $\left\{C_t, L_t, \frac{M_t}{P_t}, \frac{B_t}{P_t}, K_{t+1}\right\}_{t\geq 0}$ to maximize

$$E_0\left\{\sum_{t=0}^{\infty}\beta^t\left[\frac{1}{1-\gamma}C_t^{1-\gamma}+\frac{a_m}{1-\gamma_m}\left(\frac{M_t}{P_t}\right)^{1-\gamma_m}-\frac{1}{1+\varphi}L_t^{1+\varphi}\right]\right\} \quad \text{s.t.}$$

$$C_t=\frac{W_t}{P_t}L_t+Z_tK_t+\Pi_t+TR_t-\frac{M_t-M_{t-1}}{P_t}-\frac{\left(\frac{1}{R_t^n}\right)B_{t+1}-B_t}{P_t}-Q_t\left(K_{t+1}-K_t\right)$$

and assuming no "Ponzi schemes", i.e the intertemporal budget constraint is satisfied.

$1/R_t^n \equiv$price of one period discount bond earning the gross nominal return R_t^n , since $R_t^n = 1/(1/R_t^n)$

Since households are identical, there will be no trade in bonds $\rightarrow B_t = 0$ in equilibrium.

Useful to include bonds to obtain relation for nominal rate.

FONCs

labor supply $$\frac{W_t}{P_t} = \frac{L_t^{\varphi}}{C_t^{-\gamma}}$$

consumption/saving (capital) $$1 = E_t \left\{ \beta (\frac{C_{t+1}}{C_t})^{-\gamma} \frac{Z_{t+1} + Q_{t+1}}{Q_t} \right\}$$

consumption/saving (bonds) $$1 = E_t \left\{ \beta (\frac{C_{t+1}}{C_t})^{-\gamma} R_t^n \frac{P_t}{P_{t+1}} \right\}$$

consumption/saving (money) $$1 = E_t \left\{ \beta (\frac{C_{t+1}}{C_t})^{-\gamma} \frac{P_t}{P_{t+1}} \right\} + \frac{a_m \left(\frac{M_t}{P_t} \right)^{-\gamma_m}}{C_t^{-\gamma}}$$

where $R_t^n \frac{P_t}{P_{t+1}} = R_{t+1} \equiv$ real interest rate:

(time subscript based on when return known with certainty)

Zero lower bound on net nominal rate $R_t^n - 1 \geq 0 \rightarrow R_t^n \geq 1$

If $R_t^n > 1$, household will hold money only if $a_m \left(\frac{M_t}{P_t} \right)^{-\gamma_m} / C_t^{-\gamma} > 0$.

Money Demand

Household's problem is completely standard, except for the inclusion of money in the utility function. The latter offers a convenient way to motivate a money demand function.

Combining the money demand and bond pricing foncs yields the money demand relation:

$$\frac{M_t}{P_t} = a_m^{\frac{1}{\gamma_m}} \left(1 - \frac{1}{R_t^n}\right)^{\frac{-1}{\gamma_m}} C_t^{\frac{\gamma}{\gamma_m}}$$

In the case of log utility $(\gamma = 1, \gamma_m = 1)$:

$$\frac{M_t}{P_t} = a_m \left(1 - \frac{1}{R_t^n}\right)^{-1} C_t$$

Money demand is reasonably standard: $\frac{M_t}{P_t}$ varies inversely with R_t^n, the opportunity cost of holding money, and positively with C_t, which (loosely speaking) can be viewed as a measure of transactions needs.

Money Demand (con't)

$$\frac{M_t}{P_t} = a_m^{\frac{1}{\gamma_m}} \left(1 - \frac{1}{R_t^n}\right)^{\frac{-1}{\gamma_m}} C_t^{\frac{\gamma}{\gamma_m}}$$

Satiation level of real money balances:

Typically it is assumed that there is a satiation level of real money balances $(\overline{M_t/P_t})$ above which the marginalconvenience yield goes to zero.

So when $R_t^n \to 1$, $M_t/P_t \to (\overline{M_t/P_t})$.

Cashless limit:

Empirically, M_t/P_t only small fraction of total wealth$\rightarrow$

NK model typically considers limit of the economy as it becomes cashless (i.e. as $a_m \longrightarrow 0$)

Final good firms

Final good firms are competitive producers of a homogeneous good, Y_t, using intermediate goods, $Y_t(f)$.

There is a continuum of intermediate goods of unit measure, indexed by f.

The production function that transforms intermediate goods into final output is given by:

$$Y_t = \left[\int_0^1 Y_t(f)^{\frac{\varepsilon-1}{\varepsilon}} df \right]^{\frac{\varepsilon}{\varepsilon-1}}$$

where $\varepsilon > 1$ is the (constant) elasticity of substitution between intermediate goods.

Note that this production function also exhibits constant return to scale and diminishing marginal product for each input ($\frac{\varepsilon-1}{\varepsilon} < 1$).

Final goods firms (con't)

Each chooses $Y_t(f)\ \forall\ f$ to minimize costs $\int_0^1 P_t(f)\,Y_t(f)df$ for a given level of output $Y_t = \left[\int_0^1 Y_t(f)^{\frac{\varepsilon-1}{\varepsilon}}\,df\right]^{\frac{\varepsilon}{\varepsilon-1}}$ and given $P_t(f)\ \forall\ f$. The result is the following demand function for each intermediate good f (after integrating across the demands of the measure unity of final goods firms):

$$Y_t(f) = \left[\frac{P_t(f)}{P_t}\right]^{-\varepsilon} Y_t$$

Combining with the production function yields the following (nominal) price index for the final good:

$$P_t = \left[\int_0^1 P_t(f)^{1-\varepsilon}\,df\right]^{\frac{1}{1-\varepsilon}}$$

The real price of the final good is normalized at unity.

Intermediate good firms

There is a continuum of intermediate good firms owned by consumers, indexed by $f \in [0,1]$

Each produces a differentiated good and is a monopolistic competitor

Each firm uses both labor $L_t(f)$ and capital $K_t(f)$ to produce output according to:

$$Y_t(f) = A_t K_t(f)^{\alpha} L_t(f)^{1-\alpha}$$

where A_t is a technology parameter and $0 > \alpha > 1$ is the capital share.

Capital is freely mobile across firms.

Firms rent capital from households in a competitive market on a period by period basis.

Intermediate good firms: Price Setting

Micro-evidence: firms keep nominal price fixed on average for roughly nine months (Steinsson/Nakamura).

$\rightarrow$Need model where firms adjust prices infrequently.

Two approaches: state-dependent vs. time dependent pricing.

State-dependent: adjustment frequency endogenous.

Time-dependent: adjustment frequency exogenous

Classic state-dep. model: fixed costs of price adjustment ("menu" costs).

For firm to adjust price, gap between desired price and current price must be large enough to justify fixed cost.

$\rightarrow$endogenous "Ss" bands. Firms adjust only when desired price moves outside bands.

$\rightarrow$firms only adjust prices periodically.

Intermediate good firms: Price Setting (con't)

Unfortunately, with Ss pricing it is difficult to aggregate across firms: Firms can differ according to how far current price is away from Ss boundaries and thus according to the likelihood of adjusting price.

Baseline NK approach: tractable time-dependent pricing that approximates state-dependent model for economy with low inflation, as follows:

Intermediate good firms set prices on a staggered basis.

Following Calvo (1983), each period a firm adjusts its price with probability $1-\theta$ and keeps it fixed with probability θ. (All firms have same likelihood of adjustment).

The adjustment probability is independent over time and across firms (i.e. it does not depend on how long a firm's price has been fixed.) Facilitates aggregation.

The average time a price remains fixed is given by

$$(1-\theta)\,\Sigma_{i=1}^{\infty}\theta^{i-1}i = \Sigma_{i=0}^{\infty}\theta^{i} = \tfrac{1}{1-\theta}.$$

Intermediate good firms: Price Setting (cont')

Firms that are able to adjust their prices choose $P_t(f), Y_t(f), K_t(f)$ and $L_t(f)$.

These firms maximize expected discounted profits given the production technology and the demand curve.

Firms that do not adjust their prices choose output to meet demand as long as $P_t(f)/MC_t^n(f) \geq 1$ where $MC_t^n(f) \equiv$ nominal marginal cost

Countercyclical markups: while $MC_t^n(f)$ increases in booms, $P_t(f)$ is sticky, so that the price mark-up $P_t(f)/MC_t^n(f)$ decreases.

Countercyclical markups $\rightarrow$ countercyclical labor wedge

Both types of firms choose inputs $K_t(f)$ and $N_t(f)$ to minimize costs, given output demand.

Cost minimization problem

Firm f chooses $N_t\left(f\right)$ and $K_t\left(f\right)$ to minimize total cost, given by

$$\frac{W_t}{P_t} L_t\left(f\right) + Z_t K_t\left(f\right)$$

subject to

$$A_t K_t\left(f\right)^{\alpha} L_t\left(f\right)^{1-\alpha} - \bar{Y} \geq 0$$

$MC_t(f) \equiv$ Lagrange multiplier, interpretable as the marginal cost of producing output.$\longrightarrow$

- FONC

$$\frac{W_t/P_t}{(1-\alpha)Y_t(f)/L_t(f)} = MC_t(f)$$
$$\frac{Z_t}{\alpha Y_t(f)/K_t(f)} = MC_t(f)$$

Marginal Cost

By combining the FONC:

$$\frac{L_t(f)}{K_t(f)} = \frac{1-\alpha}{\alpha}\frac{Z_t}{(W_t/P_t)} = \frac{L_t}{K_t}$$

which is constant across firms.

Marginal cost (after combining FONCs) is given by

$$MC_t(f) = \frac{1}{A_t}\left(\frac{W_t/P_t}{1-\alpha}\right)^{1-\alpha}\left(\frac{Z_t}{\alpha}\right)^{\alpha} \equiv MC_t$$

$MC_t(f)$ depends on $W_t/P_t, Z_t$, and A_t.

Marginal Cost, the Markup and the Labor Wedge

Marginal cost MC_t vs. the average markup $1 + \mu_t$.

$$(1 + \mu_t(f)) = \frac{P_t(f)/P_t}{MC_t}$$

Since $P_t(f)/P_t = 1$ on average

$$1 + \mu_t \equiv \frac{1}{MC_t}$$

Observe that $1 + \mu_t = \frac{1}{MC_t} = \frac{(1-\alpha)Y_t/N_t}{W_t/P_t} \Rightarrow$

$$\mu_t = \frac{(1-\alpha)Y_t/N_t}{W_t/P_t} - 1$$

$$= \text{labor wedge}$$

The NK model can produce countercyclical μ_t as in data.

Optimal price setting

Price setting firms choose the optimal reset price $P_t^o(f)$ to maximize

$$E_t \sum_{i=0}^{\infty} \theta^i \left[\Lambda_{t,i} \left(\frac{P_t^o(f)}{P_{t+i}} - MC_{t+i} \right) Y_{t,t+i}(f) \right]$$

subject to

$$Y_{t,t+i}(f) = \left(\frac{P_t^o(f)}{P_{t+i}} \right)^{-\varepsilon} Y_{t+i}$$

where $\Lambda_{t,i} = \beta^i C_{t+i}^{-\gamma} / C_t^{-\gamma}$ is the stochastic discount factor.

Optimal price setting (con't)

Substitute for $Y_{t,t+i}(f)$ in the objective function to rewrite the problem as:

Choose $P_t^o(f)$ to maximize

$$E_t \sum_{i=0}^{\infty} \theta^i \left[\Lambda_{t,i} \left(\frac{P_t^o(f)}{P_{t+i}} - MC_{t+i}\right) \left(\frac{P_t^o(f)}{P_{t+i}}\right)^{-\varepsilon} Y_{t+i} \right]$$

Any firm that is resetting price at time t faces the same decision problem and then chooses the same optimal price P_t^o.

FONC:

$$E_t \sum_{i=0}^{\infty} \theta^i \Lambda_{t,i} Y_{t,t+i}(f) \left[\frac{P_t^o}{P_{t+i}} - (1+\mu)\, MC_{t+i} \right] = 0$$

where $1 + \mu = \varepsilon/(\varepsilon - 1)$ is the steady-state gross mark-up.

Optimal price setting: Flexible Price Benchmark

When $\theta = 0$ (perfect price flexibility), this condition implies

$$\frac{P^o}{P_t} = (1 + \mu)\, MC_t$$

Averaging across firms $\longrightarrow$

$$1 = (1 + \mu)\, MC_t \iff MC = \frac{1}{1+\mu}$$

Hence the desired mark-up is constant and depends only on the elasticity of demand ε.

When $\theta > 0$, the firm sets P^o so that the expected present value of future marginal revenues equal to the expected present value of future marginal cost. .

Price Index

Given:

(i) all firms that adjust in period t choose the same price P_t^o and

(ii) the average price of firms that do not adjust is simply last period's price level P_{t-1} (since firms adjusting are a random draw from the total population)

$\rightarrow$We can rewrite the price index as:

$$P_t = \left[\theta P_{t-1}^{1-\varepsilon} + (1-\theta) P_t^{o\,1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}$$

The entrance of P_{t-1} in P_t introduces nominal inertia.

Aggregation

$$Y_t(f) = A_t \left(\frac{K_t(f)}{L_t(f)}\right)^{\alpha} L_t(f)$$

$$\frac{K_t(f)}{L_t(f)} = \frac{K_t}{L_t} \ \rightarrow$$

$$\int_0^1 Y_t(f)df = A_t \left(\frac{K_t}{L_t}\right)^{\alpha} \int_0^1 L_t(f)df = A_t K_t^{\alpha} L_t^{1-\alpha}$$

Given $Y_t(f) = (P_t(f)/P_t)^{-\varepsilon} Y_t$:

$$Y_t \int_0^1 \left(\frac{P_t(f)}{P_t}\right)^{-\varepsilon} df = A_t K_t^{\alpha} L_t^{1-\alpha} \ \rightarrow$$

Aggregate production:

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha} \cdot V_t$$

with:

$$V_t = \left[\int_0^1 \left(\frac{P_t(f)}{P_t}\right)^{-\varepsilon} df\right]^{-1}$$

Aggregation (con't)

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha} \cdot V_t$$

$$V_t = \left[\int_0^1 \left(\frac{P_t(f)}{P_t} \right)^{-\varepsilon} df \right]^{-1} \leq 1$$

V_t reflect misallocation of intermediate inputs due to relative price dispersion.

$V_t = 1$ in the zero inflation steady state (since $P_t(f)/P_t = 1$ in this case)

It vanishes in a first-order log-linearization around a zero inflation steady state since deviations of $\log P_t(f)/P_t$ must average to zero.

It will however remain in a second-order approximation and thus matters for welfare analysis.

V_t is decreasing in the variance of $P_t(f)/P_t$ (see Gali p.85).

Aggregation (con't)

Intuitively: relative price dispersion increases employment dispersion, reducing efficiency.

Due to staggered price setting, inflation creates relative price dispersion.

Further, dispersion increasing in inflation rate (or deflation rate)

It is through V_t that inflation creates a welfare cost: Due to staggered price adjustment, inflation creates movement in relative prices and thus relative output.)

Resource Constraints

Given there is no investment since capital is in fixed supply:

$$Y_t = C_t$$

and

$$K_t = K$$

Monetary Policy

$Y_t^* \equiv$ natural (i.e. flexible price equilibrium) level of output

$\bar{R}^n$ is the steady state nominal interest rate

Monetary policy: Central bank sets the nominal interest rate according to the following simple feedback rue:

$$R_t^n = R^n \left(\frac{P_t}{P_{t-1}}\right)^{\phi_\pi} \left(\frac{Y_t}{Y_t^*}\right)^{\phi_y} e^{\upsilon_t}$$

with $\phi_\pi > 1$ and $\phi_y > 0$ and υ_t is a "monetary policy" shock.

The rule is known as a "Taylor rule", after John Taylor who first argued that it (i) provided a good description of U.S. monetary policy and (ii) that is had desirable stabilization features.

Later we will make clear why the rule may have desirable futures.

We will also compare with the optimal monetary policy.

Monetary Policy (con't)

To implement its target for R_t^n, the central bank adjusts M_t to equate money demand and money supply:

$$\frac{M_t}{P_t} = a_m^{\frac{1}{\gamma_m}} \left(1 - \frac{1}{R_{nt}}\right)^{\frac{-1}{\gamma_m}} (Y_t)^{\frac{\gamma}{\gamma_m}}$$

In principle, the central bank could use either R_t^n or M_t as the policy instrument. In practice they have opted for the latter, since disturbances to money demand induce variation in R_t^n that may affect real activity (with nominal rigidities present.)

e.g. suppose a_m is random. Then fixing M_t will lead to variation in R_{nt}.

In practice R_{nt} is the rate in the interbank market for reserves - "The Federal Funds Rate"- also (by arbitrage) equal to the interest on bank reserves. .

The relevant "money demand" is the demand for reserves. Since we assume the central bank accommodates money demand, capturing this detail is not critical.

Fiscal Policy

Fiscal policy: Any seigniorage revenue is rebated lump-sum to the households $\Rightarrow$ government budget constraint:

$$\frac{M_t - M_{t-1}}{P_t} = TR_t$$

For the time being we ignore goverment expenditures.

Equilibrium

An equilibrium is defined as an allocation (Y_t, C_t, L_t) and a price system

- $(Z_t, W_t, P_t, P_t^o, R_t^n, Q_t, MC_t)$ such that all agents are maximizing subject to their respective constraints, all markets clear, and all resource constraints are satisfied, given the endogenous predetermined state state P_{t-1} and and the exogenous predetermined states A_t, υ_t.

In practice, it is convenient to express the equilibrium as the vector

- $(Y_t, C_t, N_t, P_t, P_t^o, R_t^n, Q_t, MC_t)$ that satisfies a system of 8 equations, given P_{t-1}, A_t and $\mu = \frac{1}{1-1/\varepsilon}$ (8 unknowns, 8 equations).

It is useful to group the equations into aggregate demand, aggregate supply and policy blocks.

Digression on Consumption in Partial vs. General Equilibrium

Partial equilibrium:

Individual takes as given $(Z_t, W_t, P_t, R_t^n, \Pi_t\ Q_t)$

Chooses consumption to satisfy euler equation and budget constraint:

$$C_t^{-\gamma} = E_t \left\{ R_t^n \frac{P_t}{P_{t+1}} \beta C_{t+1}^{-\gamma} \right\}$$

$$C_t = \frac{W_t}{P_t} L_t + Z_t K_t + \Pi_t + TR_t - \frac{M_t - M_{t-1}}{P_t} - \frac{\left(\frac{1}{R_t^n}\right) B_{t+1} - B_t}{P_t} - Q_t \left(K_{t+1} - K_t\right)$$

Solution: Smooth consumption over time according to euler equation and satisfy the sequence of current and future budget constraints

$\longrightarrow$

C_t will depend on wealth equal to the sum of financial wealth and human wealth (discounted labor income) - as you will show on a problem set.

Digression on Consumption (con't)

General equilibrium

Since in equilibrium: : $B_t = 0$, $TR_t - \frac{M_t - M_{t-1}}{P_t} = 0$, $K_{t+1} - K_t = 0$,

$\frac{W_t}{P_t} L_t = \frac{1-\alpha}{1+\mu_t} Y_t, Z_t K_t = \frac{\alpha}{1+\mu_t} Y_t, \Pi_t = \mu_t (\frac{W_t}{P_t} L_t + Z_t K_t) = \frac{\mu_t}{1+\mu_t} Y_t \rightarrow$

The budget constraint becomes

$$C_t = Y_t$$

Individual budget constraints satisfied $\Longleftrightarrow$ resource constraint satisfied

Combine with Euler eq. to obtain

$$Y_t^{-\gamma} = E_t \left\{ R_t^n \frac{P_t}{P_{t+1}} \beta Y_{t+1}^{-\gamma} \right\}$$

$\rightarrow$ Restriction on the relation between Y_t (and thus C_t)and $R_t^n \frac{P_t}{P_{t+1}}$

Need complete model to say more

Equilibrium: Aggregate Demand

- Aggregate Demand

$$Y_t = C_t \tag{1}$$

$$C_t = E_t \left\{ R_t^n \frac{P_t}{P_{t+1}} \beta C_{t+1}^{-\gamma} \right\}^{-1/\gamma} \tag{2}$$

$$E_t \left\{ \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} R_t^n \frac{P_t}{P_{t+1}} \right\} = E_t \left\{ \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} \frac{MC_{t+1}\alpha \frac{Y_{t+1}}{K} + Q_{t+1}}{Q_t} \right\} \tag{3}$$

with $Z_t = MC_t \alpha \frac{Y_t}{K_t}$ and $MC_t = 1/(1+\mu_t)$

Equation (2) relates aggregate demand (which is equal only to consumption in this model) to the inverse of the interest rate. The asset price Q_t also varies inversely with R_t^n.

Equilibrium: Aggregate Supply

- Aggregate Supply

$$Y_t = A_t K^\alpha L_t^{1-\alpha} V_t \tag{4}$$

$$(1-\alpha)\frac{Y_t}{L_t} = \frac{1}{MC_t}\frac{N_t^\varphi}{C_t^{-\gamma}} \tag{5}$$

$$P_t = \left[\theta P_{t-1}^{1-\varepsilon} + (1-\theta) P_t^{o\,1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}} \tag{6}$$

$$E_t \sum_{i=0}^{\infty} \theta^i \Lambda_{t,i} \left[\frac{P_t^o(f)}{P_{t+i}}\right]^{-\varepsilon} Y_{t+i} \left[\frac{P_t^o}{P_{t+i}} - (1+\mu)\, MC_{t+i}\right] = 0 \tag{7}$$

with $\frac{1}{MC_t} = 1 + \mu_t$;

Equilibrium: Monetary Sector

- Monetary Policy Rule

$$R_t^n = R\left(\frac{P_t}{P_{t-1}}\right)^{\gamma_\pi}\left(\frac{Y_t}{Y_t^*}\right)^{\gamma_y} e^{\upsilon_t} \tag{8}$$

- Money Demand

$$\frac{M_t}{P_t} = a_m^{\frac{1}{\gamma_m}}\left(1 - \frac{1}{R_{nt}}\right)^{\frac{-1}{\gamma_m}} (Y_t)^{\frac{\gamma}{\gamma_m}} \tag{9}$$

Note that under the interest rule, we can ignore money demand. The money demand equation only determines the nominal stock M_t required to support a given nominal interest rate R_t^n (Critical to this argument is that there is no direct effect of $\frac{M_t}{P_t}$ on C_t).

Flexible price equilibrium

There are two distortions that can keep the economy from the first best.

The first is the distortion from imperfect competition which reduces output below the first best, even if prices are flexible.

The second is nominal rigidity due to staggered price setting. This makes money non-neutral and, relatedly, opens up the possibility of output fluctuations around the natural (flexible price equilibrium) output level.

It useful to characterize the flexible price equilibrium for two reasons:

It is where the economy converges as prices adjust

It provides as a benchmark for policy.

Flexible price equilibrium: Real Variables

Flexible prices $\longrightarrow$ price mark-up $1+\mu$ is constant:

Let $R^*_{t+1} = R^n_t E_t\{\frac{P_t}{P_{t+1}}\} \longrightarrow$

$$Y^*_t = C^*_t$$

$$Y^*_t = A_t K^{*\alpha} L^{*1-\alpha}_t$$

$$(1-\alpha)\frac{Y^*_t}{L^*_t} = (1+\mu)\frac{L^{*\varphi}_t}{C^{*-\gamma}_t}$$

$$1 = E_t\left\{\beta(\frac{C^*_{t+1}}{C^*_t})^{-\gamma} R^*_{t+1}\right\}$$

$$E_t\left\{\beta(\frac{C^*_{t+1}}{C^*_t})^{-\gamma} R^*_{t+1}\right\} = E_t\left\{\beta(\frac{C^*_{t+1}}{C^*_t})^{-\gamma} \frac{\frac{1}{1+\mu}\alpha\frac{Y^*_{t+1}}{K}+Q^*_{t+1}}{Q^*_t}\right\}$$

This system is similar to the one obtained in the benchmark RBC model (without capital), except for the mark-up $1+\mu = 1/MC$, which affects the steady state. Out-of-steady state, local dynamics are the same as RBC. Money is neutral.

Flexible price equilibrium: Nominal Variables

The price level is determined by

$$\frac{M_t}{P_t} = a_m^{\frac{1}{\gamma_m}} \left(1 - \frac{1}{R_t^n}\right)^{\frac{-1}{\gamma_m}} (Y_t^*)^{\frac{\gamma}{\gamma_m}}$$

with

$$Y_t^* = E_t \left\{ R_{t+1}^* \beta {Y_{t+1}^*}^{-\gamma} \right\}^{-\frac{1}{\gamma}}$$

$$R_{t+1}^* = R_t^n E_t\{\frac{P_t}{P_{t+1}}\} \rightarrow R_t^n \approx R_{t+1}^* E_t\{\frac{P_{t+1}}{P_t}\}$$

Holding constant R_t^n, P_t varies positively with M_t. (Quantity theory of money).

Given $R_t^n \approx E_t\{R_{t+1}^*\}E_t\{\frac{P_{t+1}}{P_t}\}$, P_t depends on expected future M_{t+i} as well as current M_t.(follows from money demand since P_t depends on $E_t\{P_{t+1}\}$).

Consistent with German hyperinflation, where anticipation of higher money growth caused prices to increase faster than current money growth

Deterministic Steady State with Zero Inflation:

Corresponds to flexible price equilibrium with zero inflation and A_t at ss. value:

$$Y = C$$

$$Y = AK^{\alpha}L^{1-\alpha}$$

$$(1-\alpha)\frac{Y}{L} = (1+\mu)\frac{L^{\varphi}}{C^{-\gamma}}$$

$$1 = \beta R^{*} = \beta R^{n}$$

$$R^{n} = \frac{\frac{1}{1+\mu}\alpha\frac{Y}{K} + Q}{Q}$$

$$\frac{M}{P} = a_m^{\frac{1}{\gamma_m}}\left(1 - \frac{1}{R^n}\right)^{\frac{-1}{\gamma_m}}(Y)^{\frac{\gamma}{\gamma_m}}$$

ss $R^n = R = \beta^{-1}$, since $\frac{P'}{P} = 1$.

M fixed at some arbitrary level. P is proportionate to M (Quantity Theory of Money).

Road Ahead

- Loglinear approximation around the steady
- Local dynamics and the interaction between the real and monetary scctors
- The New Keynesian Phillips curve: joint dynamics of inflation and output
- Monetary Policy and the Liquidity Trap