

Topic 2

The Baseline New Keynesian Model, Monetary Policy, and the Liquidity Trap: Part 2

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Outline

Part 1

Household consumption, labor supply, saving decisions, and money demand

Firm labor, capital and price setting decisions

Monetary policy: Taylor rules

Decentralized equilibrium: monetary non-neutrality and inefficient output fluctuations

Part 2

Loglinear model

Aggregate demand and the natural rate of interest

The New Keynesian Phillips curve

Monetary policy design in the basic NK model

The liquidity trap

Loglinearization: Aggregate Demand

Let $x_t = \log X_t - \log X$, except for r_t^n , π_t , p_t and m_t which are in levels

Let $\rho \equiv$ steady state net real interest rate $\beta^{-1} - 1$

Log-linearize around the steady state ($A_t = A$) with zero inflation ($\frac{P_t}{P_{t-1}} = 1$).

$$y_t = c_t \tag{1}$$

$$c_t = -\sigma [r_t^n - E_t \pi_{t+1} - \rho] + E_t \{c_{t+1}\} \tag{2}$$

$$r_t^n - E_t \pi_{t+1} - \rho = E_t \{ (1 - \nu)(m c_{t+1} + y_{t+1}) + \nu q_{t+1} - q_t \} \tag{3}$$

where $\pi_t = p_t - p_{t-1}$, $\nu = 1/[\alpha MC \frac{Y}{K} + 1]$, and $\sigma = \frac{1}{\gamma}$.

Log-linearization: Aggregate Demand (con't)

Equation (3) can be rewritten as:

$$q_t = E_t [(1 - \nu)(mc_{t+1} + y_{t+1}) + \nu q_{t+1} - (r_t^n - E_t \pi_{t+1} - \rho)] \quad (4)$$

(5)

$$= E_t \sum_{i=0}^{\infty} \nu^i [(1 - \nu)(mc_{t+1+i} + y_{t+1+i}) - (r_{t+i}^n - \pi_{t+1+i} - \rho)] \quad (6)$$

→ The log price of capital equals the loglinearized expected discounted value of earnings.

- Note that in a model with variable capital, investment will depend positively on q_t .

Log-linearization: Aggregate Supply

$$y_t = a_t + (1 - \alpha)l_t \quad (7)$$

$$y_t - l_t = -mc_t + \gamma_n l_t + \gamma_c c_t \quad (\text{note: } -mc_t = \mu_t) \quad (8)$$

$$p_t = \theta p_{t-1} + (1 - \theta)p_t^o \quad (9)$$

$$p_t^o = (1 - \theta\beta)E_t \sum_{i=0}^{\infty} (\theta\beta)^i (mc_{t+i} + p_{t+i}) \quad (10)$$

$$= (1 - \theta\beta)E_t \sum_{i=0}^{\infty} (\theta\beta)^i (mc_{t+i} + p_{t+i}) \quad (11)$$

Given $mc_t = \log MC_t - \log MC \rightarrow$

$$mc_t + p_t = \log \text{nominal marginal cost} - \log MC$$

Log-linearization: Monetary Policy

In the zero inflation steady state $r^n = r = \rho$. (from the consumption euler equation).

Monetary Policy Rule

$$r_t^n = \rho + \phi_\pi \pi_t + \phi_y (y_t - y_t^*) + v_t \quad (12)$$

Money demand

$$m_t - p_t = k + \frac{\gamma}{\gamma_m} y_t - \eta r_t^n$$

$$\text{with } k = \frac{1}{\gamma_m} \log a_m + \frac{\gamma}{\gamma_m} y, \eta = \frac{1}{\gamma_m (R^n - 1)}$$

Note again: we can ignore money demand since the central bank just adjusts m_t to support its objective for r_t^n .

Log-linearization: Flexible Price Equilibrium

$(y_t^*, c_t^*, l_t^*, r_{t+1}^*)$ determined by

$$y_t^* = c_t^*$$

$$c_t^* = -\sigma [r_{t+1}^* - \rho] + E_t\{c_{t+1}^*\}$$

$$y_t^* = a_t + (1 - \alpha)l_t^*$$

$$y_t^* - l_t^* = \gamma_n l_t^* + \gamma c_t^*$$

given $y_t^* = c_t^* \rightarrow y_t^*, l_t^*$ jointly determined by

$$y_t^* = a_t + (1 - \alpha)l_t^*$$

$$y_t^* - l_t^* = \gamma_n l_t^* + \gamma y_t^*$$

with r_{t+1}^* given by

$$y_t^* = -\sigma [r_{t+1}^* - \rho] + E_t\{y_{t+1}^*\}$$

"IS/AS" Formulation

The above system can be collapsed into two equations: an IS curve that relates output demand inversely to the real interest rate and an aggregate supply curve that relates inflation to excess demand:

$$IS : y_t = -\sigma(r_t^n - E_t\pi_{t+1} - \rho) + E_t y_{t+1} \quad (13)$$

$$AS : \pi_t = \lambda(y_t - y_t^*) + \beta E_t \pi_{t+1} \quad (14)$$

with $\lambda = \frac{(1-\theta)(1-\beta\theta)}{\theta}\kappa$, and where $\kappa \equiv$ elasticity of mc_t w.r.t. y_t

$$y_t^* = \frac{1+\varphi}{1+\varphi+(\gamma-1)(1-\alpha)} a_t$$

and where the markup (and hence the labor wedge) is countercyclical

$$mc_t = \kappa(y_t - y_t^*) \rightarrow \mu_t = -\kappa(y_t - y_t^*)$$

r_t^n is given by the Taylor rule 12

AS Curve

The Phillips curve (14) is derived from the recursive formulation of equation (10):

$$p_t^o = (1 - \beta\theta)(mc_t + p_t) + \beta\theta E_t p_{t+1}^o \quad (15)$$

From the price index equation (9), we get:

$$p_t - p_{t-1} = \pi_t = \frac{1 - \theta}{\theta}(p_t^o - p_t) \quad (16)$$

Combining (15) and (16) yields:

$$p_t^o - p_t = (1 - \beta\theta)mc_t + \beta\theta E_t [p_{t+1}^o - p_{t+1} + p_{t+1} - p_t] \quad (17)$$

$$\frac{\theta}{1 - \theta}\pi_t = (1 - \beta\theta)mc_t + \beta\theta E_t \left[\frac{\theta}{1 - \theta}\pi_{t+1} + \pi_{t+1} \right] \quad (18)$$

$$\pi_t = \frac{(1 - \theta)(1 - \beta\theta)}{\theta}mc_t + \beta E_t \pi_{t+1} \quad (19)$$

Loglinearization: Connecting mc_t to $y_t - y_t^*$

Log-linearizing equations describing the flexible price equilibrium, we get:

$$\begin{aligned}y_t^* &= a_t + (1 - \alpha)l_t^* \\y_t^* - l_t^* &= \varphi l_t^* + \gamma y_t^*\end{aligned}$$

which (given $mc = -\mu$) can be combined into

$$y_t^* = \frac{1 + \varphi}{1 + \varphi + (\gamma - 1)(1 - \alpha)} a_t \quad (20)$$

Similarly, combine (1), (7) and (8) for the sticky price eq.:

$$y_t = \frac{1 + \varphi}{1 + \varphi + (\gamma - 1)(1 - \alpha)} a_t + \frac{mc_t}{(\gamma - 1) + \frac{\varphi + 1}{1 - \alpha}} \quad (21)$$

Then

$$y_t = y_t^* + \frac{mc_t}{(\gamma - 1) + \frac{\varphi + 1}{1 - \alpha}} \quad (22)$$

Connecting mc_t to $y_t - y_t^*$ (con't)

- marginal cost and the output gap.

$$mc_t = \kappa(y_t - y_t^*) \quad (23)$$

with $\kappa = (\gamma - 1) + \frac{\varphi+1}{1-\alpha}$.

- - note: $mc_t = -\mu_t \rightarrow$ countercyclical markup \rightarrow countercyclical labor wedge
- Combining (19) and (23) yields the New Keynesian Phillips curve (14):

$$\pi_t = \lambda(y_t - y_t^*) + \beta E_t \pi_{t+1} \quad (24)$$

with $\lambda = \frac{(1-\theta)(1-\beta\theta)}{\theta} \kappa$.

Captures short run positive relation between $y_t - y_t^*$ and π_t .

Forward looking in contrast to traditional PC: $E_t \pi_{t+1}$ enters, not π_{t-1} .

Baseline New Keynesian Model

Standard representation

$$IS : y_t = -\sigma(r_t^n - E_t\pi_{t+1} - \rho) + E_t y_{t+1}$$

$$AS : \pi_t = \lambda(y_t - y_t^*) + \beta E_t \pi_{t+1}$$

$$MP : r_t^n = \rho + \phi_\pi \pi_t + \phi_y (y_t - y_t^*) + v_t$$

with

$$y_t^* = \frac{1+\varphi}{1+\varphi+(\gamma-1)(1-\alpha)} a_t$$

$$a_t = \rho_a a_{t-1} + \varepsilon_{at}$$

$$v_t = \rho_m v_{t-1} + \varepsilon_{mt}$$

Monetary policy non-neutral. $v_t \uparrow \rightarrow r_t^n \uparrow \rightarrow y_t \downarrow \rightarrow \pi_t \downarrow$.

Nominal price stickiness key.

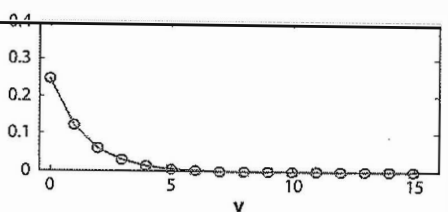
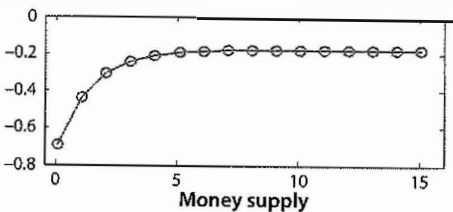
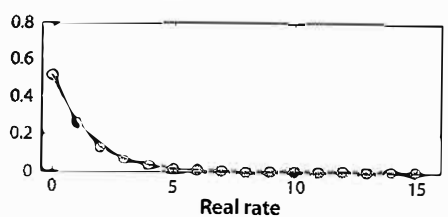
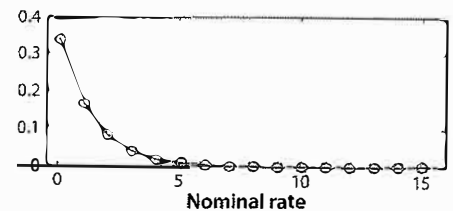
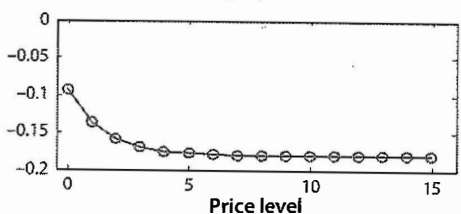
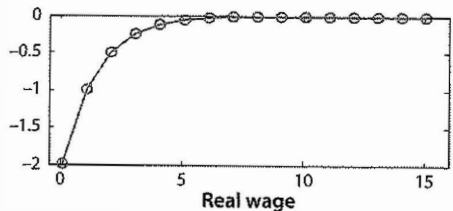
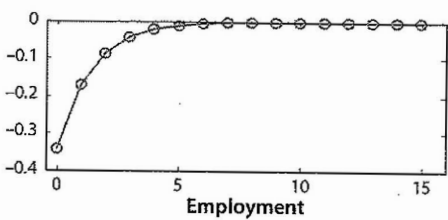
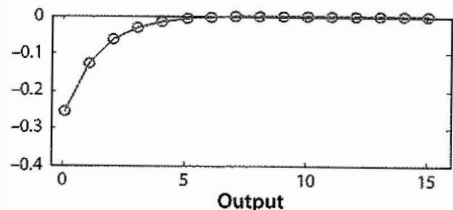
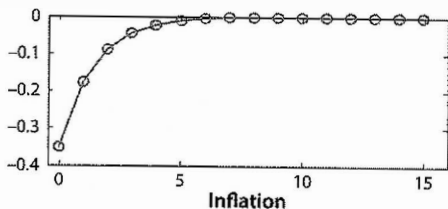
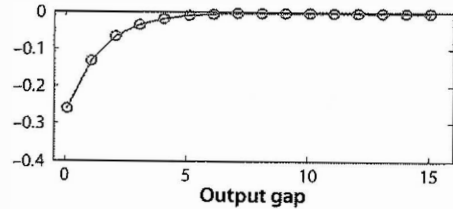


Figure 3.1. Dynamic Responses to a Monetary Policy Shock: Interest Rate Rule.

Output Gap and the Natural Rate of Interest

Output gap: $\tilde{y}_t = y_t - y_t^*$; Natural rate of interest $\equiv r_{t+1}^*$

y_t^* and r_{t+1}^* determined in flexible price equilibrium:

$$y_t^* = -\sigma(r_{t+1}^* - \rho) + E_t y_{t+1}^*$$

$$y_t^* = \frac{1+\varphi}{1+\varphi+(\gamma-1)(1-\alpha)} a_t$$

→

$$\begin{aligned} r_{t+1}^* &= \rho + \frac{1}{\sigma} \frac{1+\varphi}{1+\varphi-(1-\gamma)(1-\alpha)} (E_t a_{t+1} - a_t) \\ &= \rho + \frac{1}{\sigma} \frac{1+\varphi}{1+\varphi-(1-\gamma)(1-\alpha)} (\rho_a - 1) a_t \end{aligned}$$

r_{t+1}^* depends on expected productivity growth

The NK Model in Terms of \tilde{y}_t and π_t

Combining sticky and flex price equilibria \rightarrow

$$\tilde{y}_t = -\sigma[(r_t^n - E_t\pi_{t+1}) - r_{t+1}^*] + E_t\tilde{y}_{t+1}$$

$$\pi_t = \lambda(\tilde{y}_t) + \beta E_t\pi_{t+1}$$

$$r_t^n = \rho + \phi_\pi\pi_t + \phi_\pi\tilde{y}_t + v_t$$

with

$$r_{t+1}^* = \rho + \frac{1}{\sigma} \frac{1+\gamma_n}{1+\gamma_n - (1-\gamma)(1-\alpha)} (\rho_a - 1) a_t$$

$\rightarrow \tilde{y}_t$ depends inversely on "interest rate" gap $(r_t^n - E_t\pi_{t+1}) - r_{t+1}^*$

The Role of Expectations

We can represent the IS and AS curves as a system of simultaneous first order difference equations in \tilde{y}_t and π_t conditional on the path of the policy instrument r_t^n .

$$\begin{aligned}\tilde{y}_t &= -\sigma[(r_t^n - E_t\pi_{t+1}) - r_{t+1}^*] + E_t\tilde{y}_{t+1} \\ \pi_t &= \lambda(\tilde{y}_t) + \beta E_t\pi_{t+1}\end{aligned}$$

There are no endogenous predetermined states. Both \tilde{y}_t and π_t are endogenous at t and depend on beliefs about the future. \rightarrow To solve iterate forward

$$\begin{aligned}\tilde{y}_t &= E_t \sum_i -\sigma[(r_{t+i}^n - E_t\pi_{t+1+i}) - r_{t+1+i}^*] \\ \pi_t &= E_t \left\{ \sum_{i=0}^{\infty} \beta^i \lambda(\tilde{y}_{t+i}) \right\}\end{aligned}$$

\tilde{y}_t depends inversely on expected path of interest rate gap. (forward guidance matters!)

π_t depends positively on expected path of \tilde{y}_t (forward looking price setting).

Monetary Policy Design: The "Taylor" Principle

$$\tilde{y}_t = \sum_i -\sigma[(r_{t+i}^n - E_t \pi_{t+1+i}) - r_{t+1+i}^*]$$

$$\pi_t = E_t \left\{ \sum_{i=0}^{\infty} \beta^i \lambda(\tilde{y}_{t+i}) \right\}$$

$$r_t^n = \rho + \phi_\pi \pi_t + \phi_y \tilde{y}_t + v_t$$

Suppose the objective of policy is $\tilde{y}_t, \pi_t = 0$.

For a unique solution for (y_t, π_t) to exist with $\lim_{i \rightarrow \infty} E_t \{\tilde{y}_{t+i}\} = 0$ and $\lim_{i \rightarrow \infty} E_t \{\pi_{t+i}\} = 0$, it must be the case that

$$\lim_{i \rightarrow \infty} E_t \{(r_{t+i}^n - E_t \pi_{t+1+i}) - r_{t+1+i}^*\} = 0.$$

A sufficient condition to ensure convergence is that $\phi_\pi > 1$. ("Taylor" principle": see Gali).

The Taylor Principle and Macroeconomic Stability: Intuition

$$\tilde{y}_t = \sum_i -\sigma[(r_{t+i}^n - E_t\pi_{t+1+i}) - r_{t+1+i}^*]$$

$$\pi_t = E_t \left\{ \sum_{i=0}^{\infty} \beta^i \lambda(\tilde{y}_{t+i}) \right\}; \quad r_t^n = \rho + \phi_\pi \pi_t + \phi_y \tilde{y}_t + v_t$$

Intuitively, suppose $r_{t+1+i}^* \uparrow$ (due e.g. to a drop in a_t) $\rightarrow \tilde{y}_t \uparrow$ (given r_t^n) $\rightarrow \pi_t \uparrow$.

Then $\phi_\pi > 1 \rightarrow r_{t+i}^n \uparrow$ enough to raise real rates $r_{t+i}^n - E_t\pi_{t+1+i} \rightarrow r_{t+i}^n - E_t\pi_{t+1+i}$ converge to $r_{t+1+i}^* \rightarrow \tilde{y}_{t+i}$ and $\pi_{t+i} \rightarrow 0$

$\phi_\pi > 1$ also eliminates self-fulfilling movements in inflation.

Suppose $E_t\pi_{t+1} \uparrow \rightarrow (r_t^n - E_t\pi_{t+1}) \downarrow$ (given r_t^n) $\rightarrow \tilde{y}_t \uparrow \rightarrow \pi_t \uparrow$

With $\phi_\pi > 1 \rightarrow r_t^n \uparrow$ enough to raise real rates, choking off self-fulfilling inflation

Evidence: $\phi_\pi < 1$ from mid 60s to late 70s, a period of volatile inflation and output

Conversely, $\phi_\pi > 1$ from early 1980s to 2007, the Great Moderation.

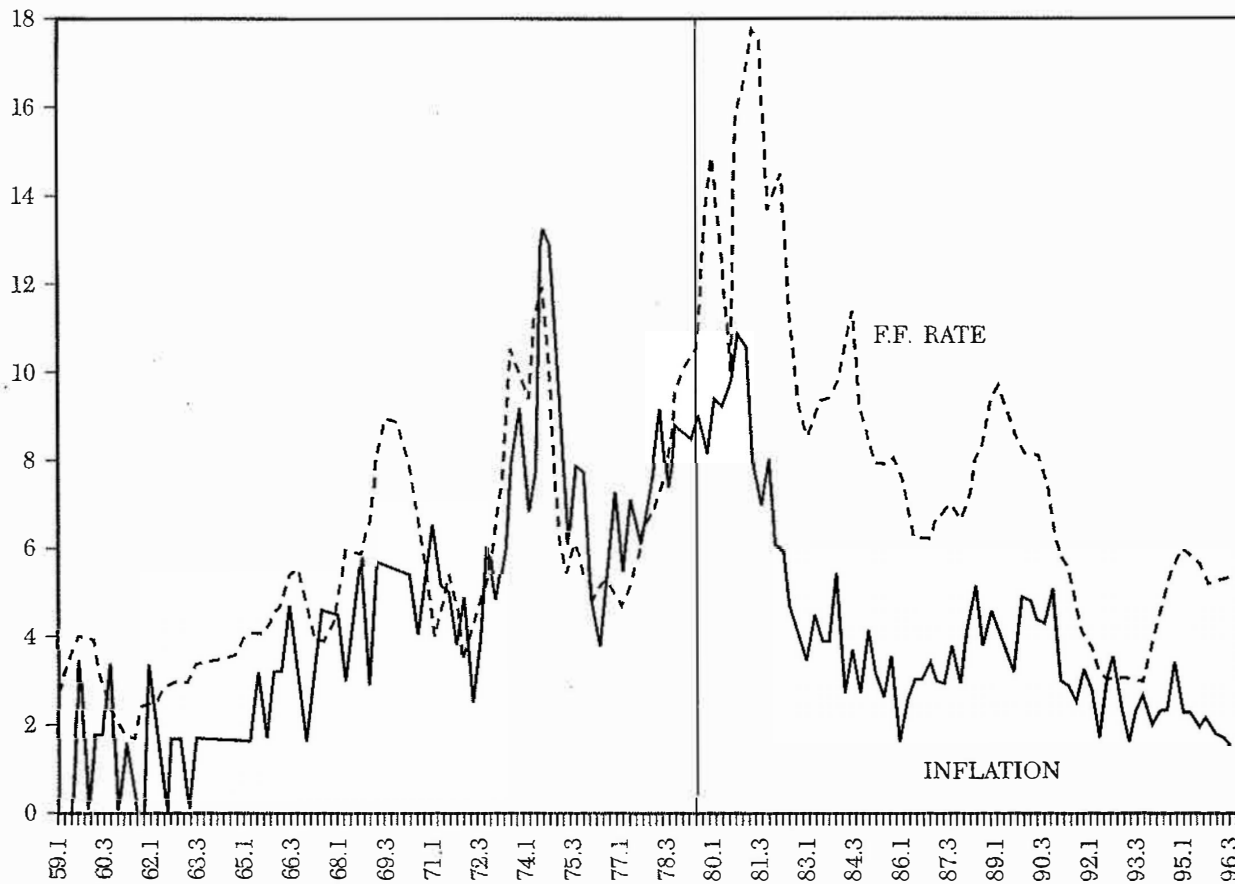


Figure 4. The Federal Funds Rate and the Inflation Rate

TABLE I
ESTIMATES OF POLICY REACTION FUNCTION

	γ_{π}	γ_x	ρ
Pre-Volcker	0.83 (0.07)	0.27 (0.08)	0.68 (0.05)
Volcker-Greenspan	2.15 (0.40)	0.93 (0.42)	0.79 (0.04)

The Taylor Principle and Macroeconomic Stability: Formalities

$$\tilde{y}_t = -\sigma[(r_t^n - E_t\pi_{t+1}) - r_{t+1}^*] + E_t\tilde{y}_{t+1}$$

$$\pi_t = \lambda(\tilde{y}_t) + \beta E_t\pi_{t+1}$$

$$r_t^n = \rho + \phi_\pi\pi_t + \phi_y\tilde{y}_t + v_t$$

Use the policy rule to eliminate r_t^n in the IS equation \rightarrow

$$\begin{bmatrix} y_t \\ \pi_t \end{bmatrix} = A \begin{bmatrix} E_t y_{t+1} \\ E_t \pi_{t+1} \end{bmatrix} + B \cdot u_t$$

where A is 2×2 and B is 2×1 .

Unique solution exists if the two roots of A lie within the unit circle.

\rightarrow unique solution can be obtained through forward iteration.

Sufficient condition for the roots of A in the unit circle: $\phi_\pi > 1$. (Gali p.65)

Optimal Policy Rule: Given objective $\tilde{y}_t, \pi_t = 0$

$$\tilde{y}_t = \sum_i -\sigma[(r_{t+i}^n - E_t \pi_{t+1+i}) - r_{t+1+i}^*]$$

$$\pi_t = E_t \left\{ \sum_{i=0}^{\infty} \beta^i \lambda(\tilde{y}_{t+i}) \right\}$$

Preferable policy rule (ignoring issues of commitment for now):

$$r_{t+i}^n = r_{t+1+i}^* \forall i \geq 0 \rightarrow \tilde{y}_t, \pi_t = 0$$

To ensure $\pi_t \rightarrow 0$, need to specify that policy will adjust if π_t deviates from 0 :

A rule that accomplishes this is

$$r_t^n = r_{t+1}^* + \phi_\pi \pi_t \text{ with } \phi_\pi > 1$$

As in the previous case, $\phi_\pi > 1$ ensures a determinate solution for \tilde{y}_t and π_t (thus ruling out self-fulfilling solutions).

The difference in this case is that \tilde{y}_t and π_t go right to 0

Demand Shocks

Standard approach: preference shifter to induce fluctuations in consumption demand:

Modify utility function as follows:

$$E_t \left\{ \sum_{i=0}^{\infty} \beta^i e^{b_{t+i}} \left[\frac{1}{1-\gamma} C_{t+i}^{1-\gamma} - \frac{1}{1+\varphi} L_{t+i}^{1+\varphi} \right] \right\}$$

where the preference shock b_t obeys

$$b_t = \rho_b b_{t-1} + \varepsilon_{bt}$$

→ Consumption euler equation:

$$e^{b_t} C_t^{-\gamma} = E_t \left\{ \beta e^{b_{t+1}} C_{t+1}^{-\gamma} R_t^n \frac{P_t}{P_{t+1}} \right\}$$

Demand Shocks (con')

In loglinear form (given $\sigma = 1/\gamma$)

$$\begin{aligned}c_t &= -\sigma[(r_t^n - E_t\pi_{t+1}) - \rho] + E_t\{c_{t+1}\} + \sigma(b_t - E_t\{b_{t+1}\}) \\ &= -\sigma[(r_t^n - E_t\pi_{t+1}) - \rho] + E_t\{c_{t+1}\} + \sigma(1 - \rho_b)b_t\end{aligned}$$

since $y_t = c_t$:

$$y_t = -\sigma[(r_t^n - E_t\pi_{t+1}) - \rho] + E_t\{y_{t+1}\} + \sigma(1 - \rho_b)b_t$$

natural rate of output:

$$y_t^* = -\sigma[r_{t+1}^* - \rho] + E_t\{y_{t+1}^*\} + \sigma(1 - \rho_b)b_t$$

$\rightarrow r_{t+1}^*$ depends on b_t

IS/AS Model with Demand Shocks

Given $\tilde{y}_t = y_t - y_t^*$

$$\tilde{y}_t = -\sigma[(r_t^n - E_t\pi_{t+1}) - r_{t+1}^*] + E_t\tilde{y}_{t+1}$$

$$\pi_t = \lambda(\tilde{y}_t) + \beta E_t\pi_{t+1}$$

with

$$y_t^* = \frac{1+\varphi}{1+\varphi+(\gamma-1)(1-\alpha)} a_t$$

$$r_{t+1}^* = \rho + \frac{1}{\sigma} \frac{1+\gamma_n}{1+\gamma_n-(1-\gamma)(1-\alpha)} (\rho_a - 1)a_t + (1 - \rho_b)b_t$$

r_{t+1}^* summarizes the effect of b_t and a_t relevant to monetary policy

Optimal to continue to set $r_t^n = r_{t+1}^*$.

Complication: r_{t+1}^* not directly observable (though π_t provides information).

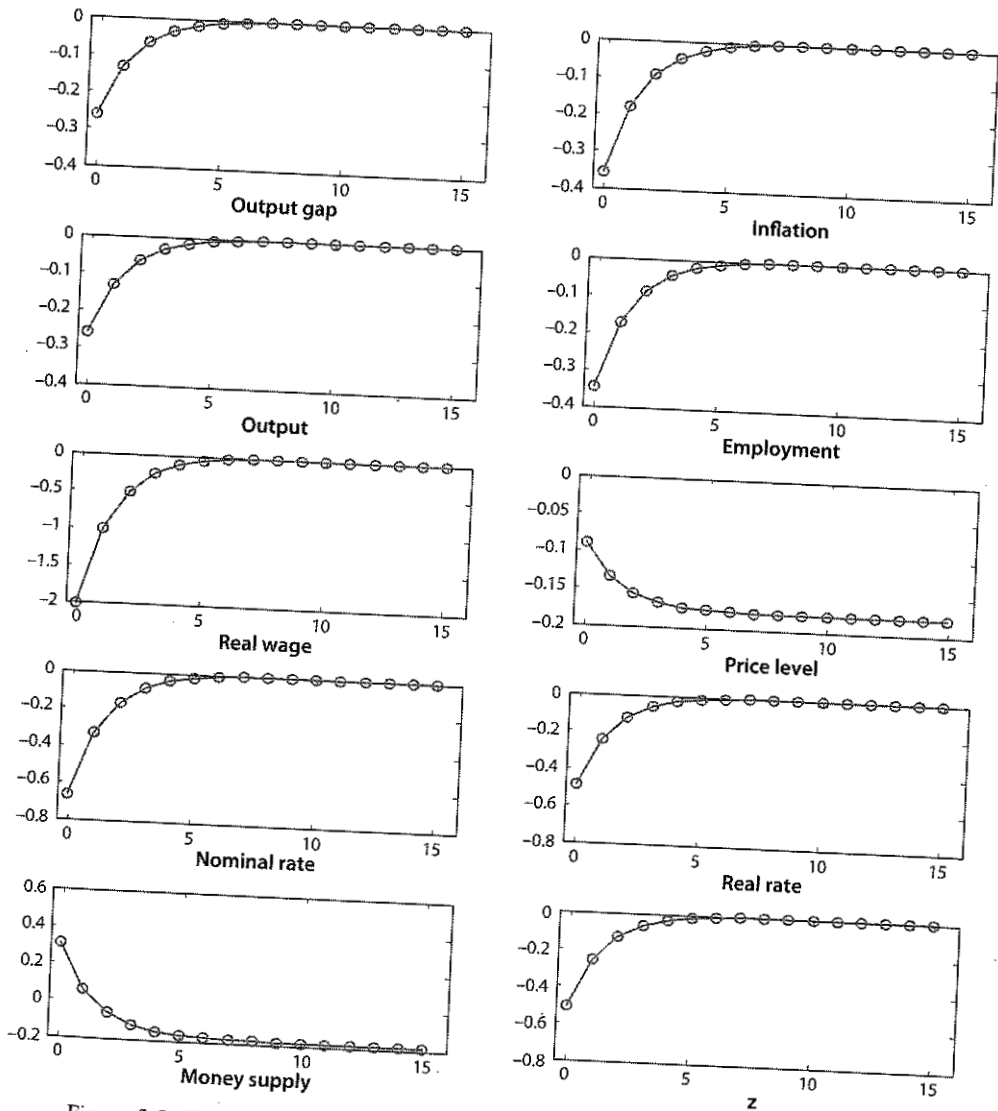


Figure 3.2. Dynamic Responses to a Discount Rate Shock: Interest Rate Rule.

Baseline New Keynesian Model: Properties

- \tilde{y}_t depends inversely on current and expected future movements of $(r_{t+i}^n - E_t\pi_{t+1+i})$ relative to r_{t+1+i}^* .
- π_t depends positively on current and expected future movements of \tilde{y}_t .
- No short run trade-off between π_t and \tilde{y}_t for a **credible** central bank (i.e. a central bank that can commit to keeping $\tilde{y}_{t+i} = 0 \forall i > 0$.
 - Requires committing to adjust path of r_{t+i}^n so $(r_{t+i}^n - E_t\pi_{t+1+i}) - r_{t+1+i}^* = 0 \forall i$.
 - Result depends on absence of labor market frictions (otherwise mc_t not simply proportionate to \tilde{y}_t).
 - If steady state output is inefficiently low, the central might be tempted to inflate.
 - If zero lower bound on the nominal rate binds, the economy is susceptible to deflation and output losses.

Liquidity Trap and the Zero Lower Bound (ZLB)

- Liquidity trap: a situation where the central bank cannot stimulate the economy by reducing the short term interest rate.
- Emerges when ZLB constraint on net nominal interest rate binds
 - ZLB: $R_t^n - 1 \geq 0 \Leftrightarrow R_t^n \geq 1 \Leftrightarrow \log R_t^n = r_t^n \geq 0$
 - From earlier: desirable to set $r_t^n = r_{t+1}^*$ (natural interest rate) \rightarrow
- ZLB binds if natural real rate $R_{t+1}^* < 1 \Leftrightarrow r_{t+1}^* < 0$ where $r_{t+1}^* = \log R_{t+1}^*$
- Deflationary spiral can emerge, with $\tilde{y}_t < 0$ and $\pi_t < 0$.

Liquidity Trap and the Zero Lower Bound (con't)

- Suppose:

- for k periods $r_{t+1+i}^* < 0$

- central bank pushes r_{t+i}^n to ZLB over this period $\rightarrow r_{t+i}^n = 0$

$$\tilde{y}_t = E_t \left\{ \sum_{i=0}^{k-1} -\sigma [(-E_t \pi_{t+1+i}) - r_{t+1+i}^*] + \sum_{i=k}^{\infty} -\sigma [(r_{t+i}^n - E_t \pi_{t+1+i}) - r_{t+1+i}^*] \right\}$$

- If for $i \geq k + 1$, $(r_{t+i}^n - E_t \pi_{t+1+i}) = r_{t+1+i}^*$:

$$\tilde{y}_t = E_t \left\{ \sum_{i=0}^{k-1} -\sigma [(-E_t \pi_{t+1+i}) - r_{t+1+i}^*] \right\}$$

- $r_{t+1+i}^* < 0 \rightarrow$ a liquidity trap emerges with $\tilde{y}_{t+i}, \pi_{t+i} < 0$ until $i \geq k + 1$.

Escaping A Liquidity Trap

- Way out - commit to inflation after r_{t+1+i}^* becomes positive.

$$\tilde{y}_t = \sum_{i=0}^{k-1} -\sigma [(-E_t \pi_{t+1+i}) - r_{t+1+i}^*] + \sum_{i=k}^{\infty} -\sigma [(r_{t+i}^n - E_t \pi_{t+1+i}) - r_{t+1+i}^*]$$

- That is commit to $[(r_{t+1+i}^n - E_t \pi_{t+1+i}) - r_{t+1+i}^*] < 0$ for $i \geq k + 1$.
- Note that this implies $\pi_{t+i} > 0$ if this commitment is kept \Rightarrow credibility problem: Incentive to renege when out of liquidity trap.
- Fiscal policy may be an alternative (to raise r_{t+1+i}^*)
- In an economy with financial market frictions, credit policy may also be an alternative.