

# Credit Booms, Financial Crises and Macroprudential Policy

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## Abstract

We develop a model of banking panics which is consistent with two important features of the data: First, banking crises are usually preceded by credit booms. Second, credit booms often do not result in crises. That is, there are "bad booms" as well as "good booms" in the language of Gorton and Ordonez (2019). We then consider how the optimal macroprudential policy weighs the benefits of preventing a crisis against the costs of stopping a good boom. We show that countercyclical capital buffers are a critical feature of a successful macroprudential policy.

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# 1 Introduction

We develop a model of banking panics which is consistent with two important features of the data: First, banking crises are usually preceded by credit booms. Second, credit booms often do not result in crises. That is, there are "bad booms" as well as "good booms" in the language of Gorton and Ordonez (2019). We then use the model to study macroprudential policy.

Figure 1 portrays the link between credit growth and financial crises, using data from Krishnamurthy and Muir (2016). The evidence is based on a panel of annual data of industrialized countries, ranging from 1869 to 2018. The authors use the narrative based classification in Jorda, Schularick and Taylor (2011) to determine periods in which a country experienced a financial crisis. The figure then plots the average behavior of three variables, output, credit growth and credit spreads, around the time a crisis occurs. In each of the three panels, the crisis occurs at time zero. The bottom panel shows that prior to a crisis, GDP growth on average increase relative to trend by roughly two percent, but when the crisis hits it experiences a sharp and persistent decline of nearly eight percent. As a number of authors have recently emphasized, e.g. Schularick and Taylor (2012), credit growth on average steadily increases prior to the crisis before declining afterward in the upper-right panel. Finally, as support for the notion that the output contractions reflect financial crises, credit spreads increase on average prior to and during the crisis before eventually declining in the upper-left panel.

Figure 2, however, makes clear that high credit growth does not always lead to a crisis, nor is it necessary for a crisis to arise. The data in the figure plots annual demeaned credit growth in a country lagged two years before (the horizontal axis) versus one year before the crisis (the vertical axis).<sup>1</sup> The red dots are episodes where a country experienced a financial crises while the blue are instances where a crisis did not occur. If we think of a credit boom as a period in which credit growth is above average for two consecutive years, crises episodes happening after credit booms are all the red dots in the upper right hand quadrant in the figure. As the figure shows, more often than not, a credit boom does not result in a financial crisis. Conditional on a credit boom, the probability of a crisis is 4.9 percent. It is true, however, that a credit boom makes a crisis more likely: conditional on no credit boom, the

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<sup>1</sup>Credit growth data is from Jorda' Schularick and Taylor (2011). To demean the data we compute for each country separate means of credit growth for the pre-war period and post-war periods.

probability of a crisis is just 2.8 percent.

Our goal in this paper is to first develop a macroeconomic framework with banking panics that is consistent with the evidence in Figures 1 and 2, and then to use the model to study regulatory policy. The framework we develop is based on Gertler, Kiyotaki and Prestipino (2019), henceforth GKP (2019), which is a standard New Keynesian macro model modified to include banks and banking panics that disrupt real activity. Within that framework, we capture both credit booms preceding crises and the nonlinear dimension of financial crises. In the spirit of Geanakoplos (2010) and Bordalo et. al. (2018), the source of the boom is optimistic beliefs by financial intermediaries (or banks in short) about future returns to capital that are eventually disappointed.<sup>2</sup> This leads to a buildup of bank credit that is funded by an increase in bank leverage, mostly in the form of short term debt. High levels of debt, in turn, make the system vulnerable to a run by increasing the exposure of banks to negative returns on their assets, so that even small negative shocks can trigger system wide runs that result in deep contractions in economic activity.

There are several differences from our earlier work. First, while in our earlier paper we used the model to analyze a single crisis episode, here we consider recurrent credit booms that may or may not result in banking crises. This allows us to capture the statistical relationship between credit booms and financial crises described above. Moreover, the presence of good and bad credit booms sets the stage for our study of macroprudential regulation. In particular, we consider how the optimal policy weighs the benefits of preventing a crisis against the costs of stopping a good boom. We also analyze the features of optimal regulation and show, for example, that countercyclical capital buffers are a critical feature of a successfully designed macroprudential policy. On the other hand, for simplicity, we consider an endowment economy instead of a full blown macroeconomic model.

One final important modelling difference from our earlier work is that we allow for equity injections into the banking sector. In our earlier work we assumed that bank capital was only accumulated via retained earnings. What this implies is that to meet equity capital requirements, the only mar-

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<sup>2</sup>Though we use a different belief mechanism, we follow Bordalo et. al. (2018) by showing that investor forecast errors during the recent boom and bust are consistent with the evidence. See also Boz and Mendoza (2014) and Boissai, Collard and Smets (2016) for other models that try to capture the boom-bust cycle in credit associated with financial crises.

gin of adjustment is for banks to reduce assets. Allowing for new equity injections introduces a second margin of adjustment. We assume however that at the margin, equity injections are costly. If they were costless, equity finance would become the sole source of funding for banks, eliminating the possibility of runs or any other type of banking instability.<sup>3</sup> However, there is a very large literature in finance that argues that equity finance is costly for banks and stresses the important role of debt finance in contexts where agency problems affect the relationship between bank managers and outside investors.<sup>4</sup> Accordingly, in this paper we assume that equity finance comes at a cost. While we do not explicitly model the frictions that underpin this cost, we discipline its impact on banks funding choices by matching the observed average leverage ratio and equity issuance rate of financial firms. The calibrated cost also delivers an increase in equity injections after a run that is in line with that observed during the recent financial crisis. Figure 3 shows that the average annual equity issuance of financial firms was one percent of the trend equity between 1985 and 2007, and peaked at around 2.4% during 2008-2010.<sup>5</sup>

Our paper contributes to a large literature that studies the role of financial intermediaries in macroeconomic fluctuations. Much of this literature builds on the conventional financial accelerator model of Bernanke, Gertler and Gilchrist (1999), and Kiyotaki and Moore (1998). While the traditional models had been developed to study how procyclical movement in nonfinancial borrowers balance sheets work to amplify and propagate macroeconomic fluctuations, Gertler and Kiyotaki (2011) showed how the basic mechanism could be applied to study financial firms as well. One limitation of the original models was that, by studying the local behavior of the economy around a non stochastic steady state, they could not capture the non linear dimension of financial crises. To address this limitation, a series of papers have tried to capture the nonlinear dimension of financial crises by exploiting occasionally binding financial constraints, e.g. Mendoza (2010), He and Krishnamurthy (2017) and Brunnermeier and Sannikov (2014). While we also allow for occasionally binding constraints, the main source of non-linearity in our paper is the occurrence of a bank run. As in our earlier work, e.g. Gertler and Kiy-

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<sup>3</sup>With one hundred percent equity financing, the banks creditors absorb the risk, making the banking system perfectly safe.

<sup>4</sup>See, for example, Calomiris and Kahn (1991) and Diamond and Rajan (2001) .

<sup>5</sup>This data does not include the government purchase of subordinate debts and preferred stocks through the Troubled Asset Relief Program during the crisis.

otaki (2015), Gertler, Kiyotaki and Prestipino (2016) and Gertler, Kiyotaki and Prestipino (2019), we model bank runs as rollover panics following the Calvo (1988) and Cole and Kehoe (2001) models of sovereign debt crises.<sup>6</sup> The existence of a bank run equilibrium depends on the health of banks balance sheets. When banks balance sheets are weak, fears of a bank run can become self-fulfilling even in the absence of any negative fundamental shock.<sup>7</sup> Bank runs, in turn, force banks to liquidate assets at firesale prices, causing a sudden collapse in bank equity, and a deep and prolonged economic contraction.

We also contribute to the growing literature that studies the role of macroprudential regulation in preventing crises. Beginning with Lorenzoni (2008), a lengthy literature has emerged that examines bank regulation in a macroeconomic setting. This work has been both qualitative (e.g. Angeloni and Faia, 2013, Jeanne and Korinek, 2014, Chari and Kehoe, 2015) and quantitative (e.g., Bianchi and Mendoza 2018, Benigno et. al 2013, and Begenau and Landvoight 2019). We differ in two main ways. First, as we allow for endogenous nonlinear financial panics that lead to real economic disasters, the main gain from macroprudential policy in our model is reducing the likelihood of one of these disasters. In our view, avoiding such disasters is the primary objective of macroprudential policy in practice. In addition, by modeling credit booms as well as busts and making the distinction between good and bad credit booms, we are able to characterize the tradeoff between reducing the likelihood of banking crisis versus stifling good credit booms.

Section 2 develops the baseline model of banking and banking panics. Section 3 introduces beliefs and then numerically illustrates how the model can generate credit booms and busts, including good booms as well as bad booms. Section 4 then analyzes macroprudential policy. The Appendix provides a detailed development of the model and the computational algorithm

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<sup>6</sup>This is in contrast with the traditional literature on banking panics originating from Diamond and Dybvig (1983), in which sequential service constraints were key in order to generate bank runs. Our modeling of bank runs as rollover crises seems to capture well the bank runs that were at the heart of the recent financial crisis. See Bernanke (2010) and Bernanke (2018).

<sup>7</sup>Some recent examples where self-fulfilling financial crises can emerge depending on the state of the economy include Benhabib and Wang (2013), Bocola and Lorenzoni (2017), Farhi and Maggiori (2017) and Perri and Quadrini (forthcoming). For further attempts to incorporate bank runs in macro models, see Angeloni and Faia (2013), Cooper and Ross (1998), Martin, Skeie and Von Thadden (2014), Robatto (2014) and Uhlig (2010) for example.

for solving it.

## 2 Baseline Model

The framework is an endowment economy with two goods, consumption  $C_t$  and capital  $K_t$ . The latter is used to produce consumption goods. We suppose capital is fixed in supply and normalize the total to be unity. The financing of capital takes on one of two forms. First, banks may intermediate the quantity  $K_t^b$ . By "intermediate", we mean that banks issue deposits to households and then use the funds to acquire capital together with own equity. Alternatively, households directly hold the quantity  $K_t^h$ , implying that in the aggregate

$$1 = K_t^b + K_t^h. \quad (1)$$

The division of capital financing between intermediated finance versus direct holding is endogenous and determined in the general equilibrium.

We suppose that households are less efficient in evaluating and monitoring capital projects than banks. We capture this notion by assuming that household direct finance entails a management cost  $\frac{\alpha}{2} (K_t^h)^2$ , which is increasing and convex in the quantity of directly held capital,  $K_t^h$ . The increasing marginal managerial cost is meant to capture that a household has limited capacity to manage capital.<sup>8</sup>

In addition to directly holding capital and supplying deposits to banks, we suppose that households are the owners of banks. (Think of households as owning banks that are different from the ones in which they hold deposits.) Accordingly households are the recipients of bank dividend payouts and decide how much equity to inject into banks. In particular, we assume that households can costlessly inject an amount  $\bar{\xi}$  of equity in the banking system, but face a convex cost  $\frac{\alpha_\xi}{2} \left( \frac{\xi_t^N - \bar{\xi}}{\bar{\xi}} \right)^2$  when equity injections  $\xi_t^N$  exceed  $\bar{\xi}$ . We introduce costly equity injections to capture in a simple reduced form way the frictions involved for banks in raising equity.<sup>9</sup> As explained

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<sup>8</sup>We take the quadratic form for convenience since it implies that the marginal managerial cost is linear.

<sup>9</sup>Jermann and Quadrini (2012) provide an related way to model costs of equity infusion: They suppose the firm faces a quadratic cost of deviating from a positive dividend target. Equity injections are then costly since they involve negative dividend payouts. We model the costs on the household side because it simplifies the algebra within our framework.

below, though, we will pick the parameters of the cost function to match the empirical properties of equity injections in the banking sector.

As we will make precise below, we suppose that banks face constraints in borrowing funds from depositors. Bank equity helps reduce these frictions, which accounts for why households may desire to inject equity, even if it is costly at the margin. The costs of equity injections, though, work to limit the amount of equity in the banking system. This limit on bank equity in turn helps account for why banks do not intermediate the entire capital stock in equilibrium and instead households hold a fraction, even though direct household finance entails costs.

Let  $Z_t$  be a shock to the flow return on capital and  $W$  (for labor income) an endowment of consumption goods that household receives each period. The aggregate resource constraint is given by

$$\begin{aligned} C_t &= Y_t = Z_t K_t + W - \frac{\alpha}{2} (K_t^h)^2 - \frac{\alpha_\xi}{2} \left( \frac{\xi_t^N - \bar{\xi}}{\bar{\xi}} \right)^2 \\ &= Z_t + W - \frac{\alpha}{2} (K_t^h)^2 - \frac{\alpha_\xi}{2} \left( \frac{\xi_t^N - \bar{\xi}}{\bar{\xi}} \right)^2 \end{aligned} \quad (2)$$

where  $Z_t$  obeys the following first order process

$$Z_{t+1} = 1 - \rho + \rho Z_t + \varepsilon_{t+1}. \quad (3)$$

Note that the model implies that net output declines as the share of bank financing of capital falls because of the direct managerial costs  $\frac{\alpha}{2} (K_t^h)^2$ . Thus the model implies in a reduced form way that disintermediation leads to a drop in output.<sup>10</sup> A secondary factor contributing to the costs of disintermediation involves the costs of equity issuance. As the share of banking financing of capital declines due to a tightening of credit constraints, the marginal value of bank equity increases, causing equity injections and hence the costs of equity injections to rise. The quantitative effect of this second

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Another approach is Gertler, Kiyotaki and Queralto (2012) which supposes that, everything else equal, agency frictions increase as banks shift funding from short term debt to equity.

<sup>10</sup>Gertler, Kiyotaki and Prestipino (2019) provide a more realistic description of how a banking collapse leads to an output collapse. In their framework the banking panic leads to a sharp contraction in investment which reduces aggregate demand and output due to nominal rigidities.

factor on net output however is much smaller than the effect of household managerial costs.

Finally, it is instructive to compare the rates of return on bank intermediated capital,  $R_{t+1}^b$ , versus that on directly held capital  $R_{t+1}^h$ . Let  $Q_t$  denote the relative price of capital. Then

$$R_{t+1}^b = \frac{Z_{t+1} + Q_{t+1}}{Q_t}$$

$$R_{t+1}^h = \frac{R_{t+1}^b}{1 + \alpha \frac{K_t^h}{Q_t}}$$

Due to the managerial cost,  $R_{t+1}^h$  is less than  $R_{t+1}^b$ . Further, this gap widens as household directly hold a larger share of the capital stock, since the marginal managerial cost,  $\alpha \frac{K_t^h}{Q_t}$ , is increasing in  $K_t^h$ . The net effect is that in situations where banks shed assets,  $Q_t$  must drop sufficiently in order for households to absorb them. In the case of a fire sale, which will arise in the event of a run,  $Q_t$  must drop sharply.

## 2.1 Households

There is a representative household that contains a measure unity of family members. The fraction  $f$  of the members are bankers and the fraction  $1 - f$  are workers. Each worker receives an endowment (effectively labor income). Each banker manages a financial intermediary and pays dividends to the household. Within the household there is complete consumption insurance.

The household chooses consumption and saving, as well as the allocation of its portfolio between bank deposits and direct capital holdings. In addition, it can inject new equity into the banking system by providing startup equity to new banks and making additional injections into existing banks.

Further, there is turnover: Each period some bankers exit the business and become workers and an equal amount of workers become new bankers. We introduce turnover in banking to ensure that each banker has a finite expected horizon. The latter ensures that the banks use leverage to finance assets in the stationary equilibrium. In particular, with i.i.d. probability  $1 - \sigma$ , a banker exits in the subsequent period and with probability  $\sigma$  the banker survives and continues to operate, making a banker's expected horizon equal to  $\frac{1}{1-\sigma}$  periods. Each period the exiting bankers are replaced by  $(1-\sigma)f$



workers turned bankers, keeping the total populations of bankers and workers constant.

Each new banker receives a fixed startup transfer from the household,  $\frac{\bar{\xi}}{(1-\sigma)f}$ . Moreover, households can inject additional equity,  $I_t$ , into the banking system. As discussed above, we assume that these injections entail a quadratic resource cost. In particular, letting  $\xi_t^N = \bar{\xi} + I_t$ , be the total amount of equity transferred to existing and new bankers, we assume resource costs associated to  $\xi_t^N$  of the form

$$f_{\xi}(\xi_t^N) = \begin{cases} \frac{\alpha_{\xi}}{2} \left( \frac{\xi_t^N - \bar{\xi}}{\bar{\xi}} \right)^2 & \xi_t^N \geq \bar{\xi} \\ 0 & \text{otherwise} \end{cases}.$$

As we describe below, the presence of financial market frictions implies that bankers are not able to arbitrage away excess returns on their investment, so that, in equilibrium, the rate of return on their assets is above the interest rate they pay on deposits. Therefore, bankers will always prefer to keep accumulating net worth and only payout dividends when they exit. Accordingly, given households total holding of bank equity,  $X_t^N$ , and letting  $R_t^N$  denote the growth rate of bank net worth from  $t-1$  to  $t$ , the dividend payment at time  $t$  is given by the accumulated net worth of exiting bankers,  $(1-\sigma)X_{t-1}^N R_t^N$ . Households equity holdings evolve according to:

$$X_t^N = \sigma R_t^N X_{t-1}^N + \xi_t^N. \quad (4)$$

where the first term in equation (4),  $\sigma R_t^N X_{t-1}^N$ , reflects the total net worth of surviving bankers, and the second term,  $\xi_t^N$ , is the total amount of injections into both active and new banks.

Let  $C_t$  denote consumption,  $D_t$  bank deposits and  $R_t$  the return on deposits. Then the household chooses  $\{C_t, D_t, K_t^h, X_t^N, \xi_t^N\}$  to maximize

$$U_t = E_t \left( \sum_{i=0}^{\infty} \beta^i \ln C_{t+i} \right) \quad (5)$$

subject to

$$C_t + D_t + Q_t K_t^h + \frac{\alpha}{2} (K_t^h)^2 + \xi_t^N + f_{\xi}(\xi_t^N) = W + (Z_t + Q_t) K_{t-1}^h + R_t D_{t-1} + (1-\sigma) X_{t-1}^N R_t^N, \quad (6)$$

the evolution of equity in (4) and

$$\xi_t^N = 0, \text{ if there is a run at } t. \quad (7)$$

That is, when a run happens at time  $t$  households do not inject any equity in the banking system as banks do not operate during a run as explained below.

Let  $\Lambda_{t,t+1} \equiv \beta \frac{C_t}{C_{t+1}}$  denote the household stochastic discount factor. Then the household's first order conditions for deposits and direct capital holdings are given by:

$$E_t (\Lambda_{t,t+1} R_{t+1}) = 1, \quad (8)$$

$$E_t \left( \Lambda_{t,t+1} \frac{Z_{t+1} + Q_{t+1}}{Q_t + \alpha K_t^h} \right) = 1. \quad (9)$$

Note that the return on deposits  $R_{t+1}$  may be risky due to the possibility of default.<sup>11</sup>

Let  $\tilde{\psi}_t^h$  be the multiplier on (4) and  $\psi_t^h = \frac{\tilde{\psi}_t^h}{U'(C_t)}$  be the multiplier in terms of consumption goods. Then the first order conditions with respect to equity holdings  $X_t$  and equity injections  $\xi_t^N$  are given by, respectively:

$$\psi_t^h = E_t [\Lambda_{t,t+1} (1 - \sigma + \sigma \psi_{t+1}^h) R_{t+1}^N], \quad (10)$$

$$1 + f'_\xi (\xi_t^N) \geq \psi_t^h \text{ and } \xi_t^N \geq \bar{\xi}. \quad (11)$$

Note  $\psi_t^h$  is the shadow value to the household of having another unit of bank equity in its portfolio. According to equation (10) this shadow value equals the expected discounted return to bank capital, taking into account that the bankers exit with probability  $1 - \sigma$  and continue with probability  $\sigma$ . Equation 10 states that the household adds bank equity to the point where the marginal benefit equals the marginal cost of new injections.

## 2.2 Bankers

Bankers fund assets  $Q_t k_t^b$  with equity  $n_t$  and deposits  $d_t$ :

$$Q_t k_t^b = d_t + n_t. \quad (12)$$

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<sup>11</sup>See equation (18) below.

Total bank equity is the sum of retained earnings plus fresh equity injections from households:

$$n_t = \hat{n}_t + I_t(\hat{n}_t). \quad (13)$$

Retained earnings  $\hat{n}_t$  are given by the return on bank investments minus debt funding costs:

$$\hat{n}_t = \max(R_{t-1}^b Q_{t-1} k_{t-1}^b - R_t d_{t-1}, 0). \quad (14)$$

In the event of default (either due to a run or insolvency), retained earnings go to zero. We assume equity injections are distributed to bankers proportionately to their retained earnings

$$I_t(\hat{n}_t) = i_t \hat{n}_t \quad (15)$$

where  $i_t$  is the aggregate equity injection rate common across all active banks:

$$i_t = \frac{I_t}{\sigma \hat{N}_t + \bar{\xi}}. \quad (16)$$

where  $\sigma \hat{N}_t$  is the aggregate amount of retained earnings of bankers surviving from the previous period, and  $\bar{\xi}$  is the fixed startup equity of new bankers.

As we discussed earlier, the banker operates on behalf of the household and faces an exit probability  $1 - \sigma$ . The banker's objective is to maximize the expected present discounted value of dividend payouts to the household. Given the banker faces financial market frictions, which we will introduce shortly, it turns out to be optimal for the banker to delay dividend payouts until exit. Accordingly we can express the banker's objective as:

$$V_t = E_t \{ \Lambda_{t,t+1} [(1 - \sigma) \hat{n}_{t+1} + \sigma V_{t+1}] \}. \quad (17)$$

There are two additional features critical to generating banking panics. First, deposits are short term and contingent only on the possibility of default. Let  $\bar{R}_t$  be the promised deposit rate,  $p_t$  the default probability. Then the return on deposits is given by:

$$R_{t+1} = \begin{cases} \bar{R}_t, & \text{with probability (w.p.) } 1 - p_t \\ x_{t+1} \bar{R}_t, & \text{w.p., } p_t \end{cases}, \quad (18)$$

where  $x_{t+1}$  is the depositor recovery rate at  $t + 1$ , which equals the ratio of bank assets to its promised deposit obligations as

$$x_t = \frac{R_{t+1}^b Q_t k_t^b}{\bar{R}_t d_t}. \quad (19)$$

Notice that the recovery does not depend upon the place on the queue because we did not impose the sequential service constraint.

Second, we introduce an agency problem between a bank and its depositors that limits the bank's ability to obtain funds. Absent such a limit, a financial panic cannot emerge: A panic withdrawal would simply lead the bank to go to the credit market to offset the deposit loss. In particular, we introduce the following moral hazard problem: After the banker borrows funds at  $t$ , it may divert the fraction  $\theta$  of assets for personal use (specifically to pay as dividends to its owner/family). If the bank does not honor its debt, creditors can recover the residual funds and shut the bank down. Recognizing this incentive, rational depositors require that the following incentive constraint be satisfied:

$$\theta Q_t k_t^b \leq V_t. \quad (20)$$

The left side of (20) is the banker's gain from diverting funds while the right hand side is the continuation value  $V_t$  from operating honestly.

The bank's decision problem is to choose assets  $k_t^b$ , deposits  $d_t$  and  $\hat{n}_{t+1}$ , to maximize the objective (17), subject to the constraints of (12), (13), (14) and (20). We describe the solution informally and defer a detailed derivation to the Appendix.

From the bank balance sheet condition (12) and the evolution of the net worth (14), the rate of return on bank net worth is given by

$$R_{t+1}^N = (R_{t+1}^b - R_{t+1}) \frac{Q_t k_t^b}{n_t} + R_{t+1}. \quad (21)$$

The first term in the right hand side (RHS) shows how the bank can use leverage,  $\frac{Q_t k_t^b}{n_t} > 1$ , to amplify its return on net worth whenever the return on its assets exceed the deposit rate, i.e. when excess returns  $(R_{t+1}^b - R_{t+1})$  are positive. The second term is the rate of return on deposit which the bank can save by having an extra unit of net worth. The incentive constraint (20), on the other hand, limits the ability of banks to increase their assets so that, whenever excess returns on capital are positive after taking into account risks, this constraint is binding

Let  $\psi_t^b$  be the shadow value to the bank of a unit of bank net worth. This shadow value equals the discounted expected return on bank equity, given by

$$\psi_t^b = E_t [\Lambda_{t,t+1} (1 - \sigma + \sigma \psi_{t+1}^b (1 + i_{t+1})) R_{t+1}^N]. \quad (22)$$

We can then express the franchise value  $V_t$  as:

$$V_t = \psi_t^b n_t, \quad (23)$$

with

$$\psi_t^b \geq 1. \quad (24)$$

The value of an extra unit of net worth will exceed unity if the incentive constraint is binding or if there is some likelihood it will ever bind in the future. Additional net worth permits the bank to expand assets by issuing more deposits, earning the excess return  $R_{t+1}^b - R_{t+1}$ . Intuitively, substituting a unit of new worth for deposits makes it less likely the banker will divert assets, which in turn relaxes the incentive constraint.

The shadow value of net worth  $\psi_t^b$  is increasing in risk-adjusted expected excess returns, because the return on bank equity  $R_{t+1}^N$  is increasing in excess returns.<sup>12</sup> The larger are excess returns, the greater the benefit from being able to issue additional deposits. In addition, given the linear structure of the problem,  $\psi_t^b$  is independent of bank specific factors. Accordingly, combining equations (20) and (23) yields the following endogenous capital requirement,  $\kappa_t$ .

$$\kappa_t \equiv \frac{n_t}{Q_t K_t^b} \geq \frac{\theta}{\psi_t^b}. \quad (25)$$

According to (25), the required bank equity - asset ratio is increasing in the seizure rate  $\theta$  and decreasing in the shadow value of net worth  $\psi_t^b$ . A rise in  $\theta$  increases the bank's temptation to divert assets, everything else equal. To satisfy the incentive constraint the bank must reduce deposits, leading it to scale back assets relative to net worth. Conversely, an increase in  $\psi_t^b$  raises the franchise value  $V_t$  reducing the bank's temptation to divert. As a result, the bank can satisfy the incentive constraint with a smaller capital asset ratio.

There are three implications of (25) that are relevant to the analysis of runs that follows. First, the bank cannot operate with  $n_t \leq 0$ . A bank with zero or negative net worth can never satisfy the incentive constraint: It will

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<sup>12</sup>The risk adjusted expected excess return is defined as

$$E_t[\Lambda_{t,t+1}(1 - \sigma + \sigma\psi_{t+1}^b(1 + i_{t+1}))(R_{t+1}^b - R_{t+1})].$$

always want to divert the proceeds from any deposits it issues. It turns out that the inability of the bank to operate with zero net worth is critical for the existence of a bank run equilibrium, as we describe shortly.

Second, the required capital ratio  $\frac{\theta}{\psi_t^b}$  varies inversely with  $\psi_t^b$ , implying that the endogenous capital requirements are relaxed in periods when  $\psi_t^b$  rises and banks are allowed to operate with lower capital ratios. Since  $\psi_t^b$  depends positively on  $E_t(R_{t+1}^b - R_{t+1})$ , periods of high excess returns cause banks capital ratios to decline.<sup>13</sup> The significance for our purposes, is that the probability of a run equilibrium increases when banks capital ratios are low.

Finally, since  $\kappa_t$  does not depend on individual bank's characteristics, banks portfolio choices are homogeneous in bank net worth and the aggregate demand for capital by banks is simply

$$Q_t K_t^b = \frac{1}{\kappa_t} N_t, \quad (26)$$

where  $N_t$  is total bank net worth.<sup>14</sup> Hence, in what follows, we only use the portfolio choices  $K_t^b$  and  $D_t$  of a representative bank with net worth  $N_t$ .

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<sup>13</sup>In the data, net worth of our model corresponds to the mark-to-market difference between assets and liabilities of the bank balance sheet. It is different from the book value often used in the official report, which is slow in reacting to market conditions. Also bank assets here are securities and loans to the non-financial sector, which exclude those to other financial intermediaries. In the data, the net mark-to-market capital ratio of the financial intermediation sector - the ratio of net worth of the aggregate financial intermediaries to the securities and loans to the nonfinancial sector - tends to move procyclically, even though the gross capital ratio - the ratio of net worth to the book value total assets (including securities and loans to the other intermediaries) of some individual intermediaries may move procyclically. While Adrian and Shin (2010) show book leverage, i.e. the inverse of book capital ratio, is procyclical for investment bankers, He, Khang and Krishnamurthy (2010) and He, Kelly and Manela (2017) show market leverage is countercyclical, in line with our model prediction of procyclical capital ratios.

<sup>14</sup>When the constraint is binding, equation (25) holds with equality so that  $\kappa_t$  only depends on  $\psi_t^b$  and hence it is independent of individual bank's net worth  $n_t$ . When the constraint is not binding,  $\kappa_t$  will be pinned down by an arbitrage condition that expected discount excess returns equal zero (where the discount factor takes into account that the constraint might bind in the future). The arbitrage condition also depends on aggregate variables only so that it still does not depend on individual bank's net worth. See Appendix for details.

### 2.3 Bank Runs

Within our framework, a bank run is a rollover panic, similar to the Cole and Kehoe (2000) model of self-fulfilling debt crisis. In particular, a self-fulfilling bank run equilibrium (rollover crisis) exists under the following circumstances: An arbitrary depositor believes that if other households do not roll over their deposits, the depositor will lose money by rolling over. This condition is met if banks' net worth goes to zero in the event of the run. As we discussed earlier, banks with zero net worth cannot operate. Because they cannot credibly promise not to abscond with deposits, any household who lends money to banks in the wake of the run will lose money.

The timing of events is as follows: At the start of  $t + 1$ , depositors decide whether to roll over deposits. If a run equilibrium exists at  $t + 1$ , they may choose not to roll over. If the panic happens, banks liquidate capital and sell to households. Depositors get back a fraction of the promised return, depending on the recovery rate  $x_{t+1}$  as defined in equation (19). For computational simplicity as well as realism, we assume that new banks do not enter during the period of the panic: They wait until the next period when the run has stopped.

As discussed, the run equilibrium exists if bank net worth goes to zero in the event of the panic. This will be the case if the depositor recovery rate is less than unity. It follows that the run equilibrium exists at  $t + 1$  if the liquidation value of bank assets is less than the promised obligation of deposits:

$$(Q_{t+1}^* + Z_{t+1})K_t^b < \bar{R}_t D_t. \quad (27)$$

which is the same as the condition  $x_{t+1} < 1$ . The liquidation price in turn is given by the household's first order condition for capital holding,

$$Q_t^* = E_t [\Lambda_{t,t+1}(Z_{t+1} + Q_{t+1})] - \alpha K_t^h \quad (28)$$

evaluated at  $K_t^h = 1$ .

Let  $\iota_{t+1}$  be a sunspot variable that takes on a value of unity if the sunspot occurs and zero otherwise. Then a run occurs at  $t + 1$  if (i) condition (27) is met, and (ii)  $\iota_{t+1} = 1$ . In order to not introduce any exogenous cyclicity into the likelihood of a banking panic, we assume the sunspot appears with fixed probability  $\varkappa^s$ . Then, letting  $Z_{t+1}^R$  be the threshold value of  $Z_{t+1}$  below which a run is possible, the probability of a run  $p_t^R$  is given by

$$p_t^R = Pr\{Z_{t+1} < Z_{t+1}^R\} \cdot \varkappa^s \quad (29)$$

where  $Z_{t+1}^R$  is the value of productivity at which banks are just able to pay their deposit obligations even if prices drop to their liquidation value  $Q_{t+1}^*(Z_{t+1}^R)$ :

$$Q_{t+1}^*(Z_{t+1}^R) + Z_{t+1}^R = \frac{D_t \bar{R}_t}{K_t^b} \quad (30)$$

Equations (29) and (30) suggest two forces that can raise the likelihood of a run equilibrium existing. First, bad luck: a sequence of negative shocks to the productivity of capital can increase the likelihood that  $Z_{t+1}$  will fall below the threshold value  $Z_{t+1}^R$ . Second, banks financial fragility, measured by the ratio of the deposit obligation to the book value of capital,  $\frac{D_t \bar{R}_t}{K_t^b}$ . A rise in leverage increases  $Z_{t+1}^R$ , raising the likelihood that  $Z_{t+1}$  will be below  $Z_{t+1}^R$ .

## 2.4 Aggregation and Equilibrium

If there is no run at time  $t$ , aggregate net worth of active banks is given by the net worth of surviving bankers from  $t - 1$  plus new networth injected by households:

$$N_t = \sigma [(Z_t + Q_t) K_{t-1}^b - D_{t-1} R_t] + \xi_t^N \quad (31)$$

$$= \left[ \sigma \hat{N}_t + \bar{\xi} \right] (1 + i_t). \quad (32)$$

Notice that it is possible that, even without a bank run the realization of productivity is so low that the banks are forced to default. In this case, equations (18) and (31) imply that aggregate net worth is simply given by  $\xi_t^N$ <sup>15</sup>.

To derive the total return on bank equity, use (12) and (26) to substitute for  $K_{t-1}^b$  and  $D_{t-1}$  in (31) to get

$$N_t = \sigma [(R_t^b - R_t) \frac{1}{\kappa_{t-1}} + R_t] N_{t-1} + \xi_t^N, \quad (33)$$

so that in equilibrium

$$N_t = X_t^N, \quad (34)$$

$$R_t^N = (R_t^b - R_t) \frac{1}{\kappa_{t-1}} + R_t. \quad (35)$$

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<sup>15</sup>See Appendix for a characterization of the probability of insolvency without runs.



Here we see the return on bank equity  $R_t^N$  is increasing in the excess return weighted by the bank leverage multiple, the inverse of the bank equity - asset ratio.

Finally, the evolution of net worth is:

$$N_t = \begin{cases} \sigma R_t^N N_{t-1} + \xi_t^N & \text{if there is no run at } t \\ 0 & \text{if there is a run at } t \end{cases} .$$

See Appendix for a detailed description of the equilibrium equations.

### 3 Credit Booms and Busts: A Numerical Illustration

We now show via numerical simulation how the model can generate credit booms and busts consistent with the evidence presented in Figures 1 and 2. For expositional reasons, we first start with the bust phase of a crisis. That is, we consider a model where fundamental shocks are the outside force that drives the economy into a crisis zone where runs can occur. Here the idea is to illustrate how the model can generate a financial collapse which has spillover for the real economy.

We first describe how we calibrate our model. Then we illustrate how, starting with a banking system that is "safe", i.e. not susceptible to runs, a series of negative shocks can weaken bank portfolios, moving the economy to a crisis zone where a financial collapse can occur. We then introduce our belief mechanism and show how it can generate a credit boom that may or may not lead to a bust.

#### 3.1 Calibration

Table 1 shows the parameter values used in our experiments together with the calibration targets. There are eleven parameters. Four are reasonably standard: including the discount factor  $\beta$ , the serial correlation of the capital productivity shock,  $\rho_z$ , the standard deviation of this shock  $\sigma$  and the household "labor" endowment  $W$ . We set  $\beta$  at 0.99, a standard value in the literature. We choose a similarly conventional value for  $\rho = 0.95$ . We pick  $\sigma_z$  so that the model produces a standard deviation of output equal to 1.9 percent, consistent with the evidence. Finally, we set  $W$  equal to twice the

size of steady state capital income  $Z$  to capture the idea that on average the labor share is twice the capital share.

Seven parameters govern the financial sector and are nonstandard. They include: the fraction of assets banker can divert  $\theta$ , the banker survival rate  $\sigma$ , the parameter governing marginal household direct financing costs  $\alpha$ , the new bankers endowment  $\bar{\xi}$ , the parameters governing costs of equity injections,  $\alpha_\xi$ , and the sunspot probability  $\varkappa^s$ . We choose these parameters to hit the following seven targets: 1. The average bank equity - asset ratio  $\kappa$  equals 0.1; 2. An average annual spread between the return on bank assets  $R^b$  and the deposit rate  $R$  of two hundred basis points;<sup>16</sup> 3. The average household share of asset holding equals one half; 4. An average annual run probability of 3.7 percent (roughly, one every twenty-five years); 5. An output contraction during a bank run of six percent on average, consistent with the evidence from Muir and Krishnamurthy; 6. An average ratio of bank equity injections and trend financial equity of 1 percent.

### 3.2 A Run Driven by Fundamental Shocks:

Before introducing a belief mechanism that can generate credit booms and busts, we first illustrate how the model can generate a nonlinear financial crisis with fundamental shocks as the underlying driving force. Under our parametrization, a run equilibrium does not exist in the risk adjusted steady state. We accordingly suppose that at time 1, there is a negative innovation to productivity just large enough to move the economy into a crisis zone, i.e., an environment where a run equilibrium exists. Intuitively a large negative productivity shock can open up the possibility of a run by (i) reducing bank net worth and hence increasing bank leverage and (ii) reducing the liquidation price of bank assets.

The solid line in the upper left panel of Figure 4 displays the path of the productivity shock. The diamond on the vertical axis is the threshold value of the productivity shock,  $Z_{t+1}^R$ , below which a run equilibrium exists at  $t + 1$ . The threshold is almost two standard deviations below the risk adjusted steady state value of  $Z_t$ . As the panel illustrates the shock puts  $Z_{t+1}$  just below the threshold  $Z_{t+1}^R$ . Moving forward through time, the dotted line gives the crisis zone threshold for  $Z_{t+i}$  for each value of  $i > 1$  after the run has occurred.

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<sup>16</sup>See Philippon (2015).

Given the economy reaches the crisis zone in period 1, we suppose there is a run, i.e. the sunspot appears and households do not rollover deposits. The solid line in each of the remaining panels gives the response of the economy in the case of the run. For comparison the dashed line shows the response for the case where the sunspot is not observed and hence the run does not occur. The run leads to a firesale of bank assets, causing bank net worth and bank intermediation to go to zero. Because it is costly for households to absorb the assets, the spread between the expected return on bank assets and the risk free rate jumps more than three hundred basis points, causing the shadow value of bank equity to more than double. The disintermediation of bank assets leads to a sharp drop in output of more than ten percent. The figure makes clear the nonlinear aspect of the crisis. Absent the panic, output only drops less than one percent. In the wake of the run, the level of bank net worth slowly recovers as new banks enter and households increase equity injections in the financial sector in response to the sharp rise in the shadow value of bank equity. However, given that injecting equity is costly, the share of assets intermediated by banks recovers only slowly and so does output.

As discussed above, the assumption that equity injections in the financial sector are costly is key in order for financial frictions to have a bite and for banking panics to be possible. Figure 4 shows that while we calibrated our cost function to match the average level of equity injections over time, our model predictions about the increase in equity injections after a crisis captures quite well the observed market response during the recent financial crisis.

### **3.3 News Driven Optimism and Credit Booms**

One of the major weaknesses of the model of bank runs driven by fundamental shocks is that financial crises often occurs without major productivity shocks, as in recent Global Financial Crisis. To address this, we now extend the model to allow for credit booms, building on our earlier work, GKP (2019). In that framework, news that bankers receive about the possibility of improved fundamentals leads to a credit buildup. However, because the improved fundamentals do not materialize, the high leverage pushes the economy into a crisis zone where a banking panic is possible. Here we allow for the possibility the credit booms can lead to good as well as bad outcomes. Good outcomes are possible either because the improved fundamentals arise

or because, even if they don't, the run never materializes. In this latter case, the credit boom raises the share of intermediated finance, which is efficient even if the improved fundamentals do not arise. In the end, our goal is to match the data presented earlier in Figure 2, which shows that, while high credit growth makes a crisis more likely, it typically does not lead to a crisis. Conversely, crises can occur in the absence of large credit growth.

Following GKP (2019), we model beliefs by considering a variant of a "news" shock. Under the standard formulation, at time  $t$ , individuals suddenly learn with certainty that a fundamental disturbance of a given size will occur  $j$  periods in the future. We relax this assumption in two ways. First, we assume that there is a probability the shock may not occur. Second, we assume that rather than having a single date in the future when the shock can occur, there is a probability distribution over a number of possible dates. As time passes without the occurrence of the shock, individuals update their priors on these various possibilities. We also assume that only bankers, who are the experts at managing assets, have optimistic beliefs. In fact, it is the relative optimism of bankers compared to households that generates the vulnerability of the financial system.<sup>17</sup>

In particular, with some fixed probability  $\varkappa^n$ , at time  $t^N$  bankers receive news that there may be a high return on capital in the form of a large capital productivity shock. But they do not know for sure (i) whether the shock will occur and (ii) conditional on the shock arriving, when it will occur. If the shock is realized at some time  $\tau > t^N$ , it takes the form of a one time impulse to the capital productivity shock process of size  $B$ . Formally, the news bankers receive is that the capital productivity will follow the process

$$Z_\tau = 1 - \rho_\xi + \rho_\xi Z_{\tau-1} + \epsilon_\tau + \tilde{B}_\tau \text{ for } \tau > t^N$$

where  $\tilde{B}_\tau = B$  if the large shock realizes at  $\tau$ , and  $\tilde{B}_\tau = 0$  otherwise. Given the capital productivity shock is serially correlated, there will be a persistent effect of  $B$ . However, given it is a one time shock, if it occurs, there will be no subsequent realizations of this impulse. In contrast to our earlier paper,

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<sup>17</sup>As we describe in Appendix, we assume that households are aware that bankers became optimistic but do not change their beliefs about the productivity of capital, i.e. they do not believe the news. This allows us to have diverse beliefs without having households extract information from prices. A similar assumption is made for the same reason, for instance, in Cogley and Sargent (2009). Because households know bankers are more optimistic, they understand that there is less danger for bankers to divert their assets and lose their franchise. This allows bankers to raise their leverage multiple.

though, we will allow for recurrent (though infrequent) news shocks as we describe below.

When they receive the news at  $t^N$ , bankers' prior probability that a shock will eventually occur is given by  $\bar{P}$ . Conditional on the shock happening, the future date when it will happen,  $\tau \in \{t^N + 1, t^N + 2, \dots, t^N + T\}$ , is distributed according to a probability mass function  $\zeta_\tau$  which we assume to be a discrete approximation of a normal with mean  $\mu^B$  and standard deviation  $\sigma^B$  with support support  $[1, T]$ . Thus at date  $t^N$ , the probability that the shock happens at  $\tau$  is given by

$$\Pr ob_{t^N}(\tilde{B}_\tau = B) = \begin{cases} \bar{P} \cdot \zeta_\tau, & \text{for } \tau = t^N + 1, t^N + 2, \dots, t^N + T \\ 0, & \text{for } \tau > t^N + T \end{cases}.$$

As long as no shock is observed until date  $t$ , bankers update their beliefs using Bayes rule:

$$\begin{aligned} \Pr ob_t(\tilde{B}_\tau = B) &= \frac{\bar{P} \cdot \zeta_\tau}{1 - \sum_{j=t^N+1}^t \bar{P} \cdot \zeta_j} \\ &= \frac{\sum_{j=t+1}^{t^N+T} \bar{P} \cdot \zeta_j}{1 - \sum_{j=t^N+1}^t \bar{P} \cdot \zeta_j} \cdot \frac{\zeta_\tau}{\sum_{j=t+1}^{t^N+T} \zeta_j} \end{aligned}$$

for  $\tau = t + 1, \dots, t^N + T$ , and  $\Pr ob_t(\tilde{B}_\tau = B) = 0$  for  $\tau > t^N + T$ . The first term in the last line is the posterior probability of the shock ever happening, which we denote by  $\bar{P}_t$  and which is decreasing with  $t$ . The second term is the probability that the shock realizes at  $\tau$  conditional on the shock eventually happening. The latter is increasing with  $t$  until  $t = t^N + T$ , before becoming zero.

Observe that the process will generate a burst of optimism that will eventually fade if the good news is not realized. Early on, bankers will steadily raise their forecasts of the near term return on capital as they approach the date where, a priori, the shock is most likely to occur. As time passes without the realization of the shock, bankers' become less certain it will ever occur: The optimism proceeds to vanish.

We now illustrate how with the belief mechanism just described, the model generates a boom/bust scenario. Table 2 describes our calibration of the belief process. We assume that bankers receive the optimistic news ten quarters in advance of the prior on the most likely date the boom in fundamentals is likely to occur. Our empirical motivation is the housing

boom which began in early 2005 and peaked roughly ten quarters later. Accordingly we pick the mean of the conditional distribution  $\tau$ ,  $\mu^B$ , so that prior on when the shock is most likely to occur is ten quarters after receipt of the news, 2007Q2. We pick the standard deviation  $\sigma^B$  to ensure that by six quarters after the conditional mean, if the shock has not occurred, bankers' will completely give up hope that it will ever occur.<sup>18</sup> Next we set the size of the impulse  $B$  to equal a two standard deviation shock, that is, a shock which is unusually large but not beyond the realm of possibility.<sup>19</sup> Finally, we pick the prior probability that the shock will even occur  $\bar{P}$ , to ensure that economy reaches the crisis zone six quarters after the conditional mean without any fundamental shocks.

Figure 5 characterizes the dynamics of beliefs and the credit boom that can emerge absent any fundamental shocks. The top-left panel gives the prior distribution for the time the shock will happen, conditional on it happening, i.e.  $\{\zeta_{t^N+i}\}_{i=1}^T$ . The middle panel then illustrates the ingredients bankers use to forecast the shock. The blue line in the top-middle panel gives the probability the shock will eventually happen,  $\bar{P}_t$ . When the news is received at  $t = 1$ , the probability jumps to its prior value near unity. Time passing without the shock occurring leads bankers' to reduce this probability. The optimism fades rapidly as time passes the conditional mean, the most likely time the shock was expected to occur. The dashed red line then gives the probability the shock will occur in the subsequent period, conditional on it eventually happening. Notice that this conditional probability equals unity at date  $t^N + T - 1$  when the next period is the last possible date for the shock to occur. The estimate that the shock will occur in the subsequent period is then the product of the blue and red lines.

To illustrate the boom/bust nature of beliefs, the top-right panel portrays the year ahead forecast of the productivity shock (the dashed red line). After receiving the news at  $t + 1$ , optimism steadily builds, peaking just before 2007Q2. However, as time continues to pass beyond the most likely time, the optimism fades quickly, effectively vanishing by 2008Q4. Note that throughout the boom and bust in beliefs, the true fundamental shock (the blue line), is unchanged. Thus, there is serial correlation in the forecast errors of the

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<sup>18</sup>Given our discrete approximation of the normal distribution, a choice of  $\sigma^B$  translates into a maximum numbers of periods within which the shock can occur.

<sup>19</sup>Note that the prior probability that the shock will occur,  $\bar{P}_{t^N}$ , and the size of the shock when it occurs,  $B$ , only influence the expected capital productivity through their product  $\bar{P}_{t^N} \cdot B$ .

capital productivity shock.

The bottom-left panel shows the response of output to the news. The increase in bankers' optimism leads bankers to expect higher returns on assets which induces a rise in bank intermediation and, in turn, an increase in output of nearly one percent. There is however a nontrivial debt buildup of debt as bankers fund the twenty five percent increase in assets mostly by issuing deposits in the bottom-middle panel. The bank capital ratio (equity to assets) in fact declines as bankers' optimism raises their perceived shadow value of net worth  $\psi_t^b$ , relaxing the incentive constraint.<sup>20</sup> (See equation (25)). The increase in leverage raises the probability the economy moves into a crisis zone where a run is possible in the bottom-right panel. In this regard, the boom lays the seeds of the bust.

We now illustrate how a wave of optimism can generate a credit boom that leads to a banking panic. Figure 6 illustrates the experiment. The news of a possible improvement in fundamentals is received in period 1. The prior probability distribution is as described in the previous figure. The top-left panel is the forecast of capital productivity in one period ahead. Expected productivity increases as the economy approaches the prior conditional mean. However, because the productivity boom is not realized, the expected productivity begins to decline. As just described, bankers' optimism leads to an overall increase in bank assets funded by a rise in bank leverage, which moves the economy into a crisis zone. In the-top middle panel, the solid line is realized productivity, which is unchanged throughout. As before, the dotted line is the threshold value for the capital productivity shock,  $Z_{t+1}^R$ , below which a run equilibrium exists. As the panel makes clear the news shock moves the economy steadily toward a crisis zone, which it reaches roughly ten quarters later.

We assume that once the economy reaches the crisis zone, the sunspot appears and a rollover panic ensues. The difference from the earlier case is that we do not require a fundamental shock to move the economy to a crisis zone, so we do without it. Overall, the effect of the banking crisis is very similar to the case without the debt boom. The contraction in output in terms of both amplitude and persistence is similar to the case of the fundamentals driven panic. As before the spread between the expected rate of return on

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<sup>20</sup>As discussed by Gertler, Kiyotaki and Prestipino (2016), there were additional factors contributing to the leverage buildup, including financial innovation. For simplicity we abstract from these factors and note only that including them would increase the debt buildup further and the resulting degree of fragility.

bank assets and the riskfree rate increases prior to and during the crisis in the middle-middle panel, again consistent with the evidence. One important difference is that the wave of optimism generates a credit boom prior to the crisis, consistent with the evidence. Finally, as shown in the bottom-right panel, despite the increase in fragility of the banking sector households do not start injecting equity until after the crisis occurs. This is because the increase in fragility is not a consequence of bad realizations of productivity shocks, which cause expected excess returns to rise and hence induce households to increase their equity injections, but rather it is driven by excessive optimism of financial intermediaries that is not shared by households. Accordingly, households expectations of future bank excess returns do not rise as much as bankers and their subjective probability of a crisis increases by enough that, on net, their willingness to hold bank equity slightly declines before the crisis occurs.

We next illustrate that, consistent with the earlier evidence we presented, it is possible to have a credit boom that does not lead to a crisis. There are two possible reasons for why. First, the positive fundamental shock actually materializes. Second, the shock does not materialize but the panic doesn't arise even though the economy is in a crisis zone because depositors do not coordinate on the bad equilibrium (i.e. the sunspot doesn't appear). Figure 7 displays both cases. As in the previous experiment, bankers receive positive news at time 1. The solid lines portray the case where the large productivity improvement materializes as bankers expected. In this case the expected jump in productivity arises in period 10, the peak of the conditional prior mean. The runup to period ten is identical to the case where a panic occurs, as portrayed in the previous figure. However, the realization of the productivity improvement leads to an increase in output (in the middle-right panel), which moves the economy output of the crisis zone as the top-middle panel shows. The dashed lines are the case where the boom never occurs but a panic still does not arise. There is in fact a rise in output, though smaller than in the case where the productivity boom is realized. The source of the rise in output is the optimism that gives rise to an increased in the share of capital intermediated by banks.

Thus far we have characterized single episodes of credit booms and displayed circumstances where they may or may not lead to a bank run. As a prelude to analyzing macroprudential regulation, we next consider recurrent credit booms and busts. Our goal is to match the Schularick/Taylor evidence on the link between credit growth and the frequency of financial panics. We



assume that probability of receiving news  $\varkappa^n$  is equal to 2 percent per quarter, which corresponds to once every twelve and a half years on average. Further, once news is received, there is no additional news realization until the current process has played out, i.e. there is no news from  $t^N + 1$  until either  $t^N + T$  or the period in which the boom actually happens.

We suppose the true probability the boom actually happens is fifty percent conditional on bankers receiving the news. We capture the idea that bankers are optimistic by supposing that upon receiving the news, they have a strong prior probability of .999 that the boom will happen. Given that credit booms are relatively infrequent it is not unreasonable to suppose that bankers have not had enough experience to learn the true probability of good realizations. Alternatively, think of the high prior as capturing a "This Time is Different" mentality.<sup>21</sup>

We simulate the model and then record the relation between credit growth in the two years prior and the occurrence of a crisis in the current period in Figure 8. The left panel shows the data from Schularick and Taylor as in Figure 2. The right panel is the simulation result of the model. The model does a reasonable job of capturing that, as in the data, crises are more likely following a sustained period of positive credit growth. Within the model, conditional on positive credit growth in the prior two consecutive years, a crisis occurs 4.9 percent of the time just as in the data. Runs without credit booms are a bit more frequent in the model than in the data, i.e. 3.2 against 2.8, but overall the predictive power of credit booms for banking crises as captured by the odds ratio of bank runs with and without a boom is in line with the empirical counterpart, 1.5 in the model against 1.79 in the data. One difference though is that credit growth in the model is less persistent than in the data.

## 4 Macprudential Regulation

We now consider a macroprudential regulator that sets time varying bank capital requirement  $\bar{\kappa}_t$ . This implies that the relevant capital requirement for banks,  $\kappa_t$ , is now the maximum between the regulatory requirement,  $\bar{\kappa}_t$ , and the market imposed capital requirement  $\kappa_t^m$ , given by equation (25).

$$\kappa_t = \max(\bar{\kappa}_t, \kappa_t^m), \quad (36)$$

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<sup>21</sup>See Reinhart. and Rogoff (2009).

with  $\kappa_t^m = \theta/\psi_t^b$ .

We consider a simple policy rule for bank capital requirements that allows for a countercyclical buffer. Let  $\bar{N}$  be a threshold value of net worth in the banking system above which the capital requirement is set at the "normal value"  $\bar{\kappa}$ . When bank net worth falls below  $\bar{N}$ , the requirement is relaxed. We assume for simplicity the regulatory requirement goes to zero. In this instance the market requirement  $\kappa_t^m$  will apply.

We restrict policy to be determined by the simple rule

$$\bar{\kappa}_t \begin{cases} \bar{\kappa} & N_t \geq \bar{N} \\ 0 & N_t < \bar{N} \end{cases}$$

We look for  $(\bar{\kappa}, \bar{N})$  that maximize welfare, which we take to be the unconditional expected utility of the representative household. Note that the rule allows for a countercyclical capital buffer, since the capital requirement is relaxed when aggregate bank net worth drops below the threshold  $\bar{N}$ .

Figure 9 shows the market determined capital requirement in the unregulated equilibrium. At the value of equity in the risk adjusted steady state  $N_{SS}^{DE}$ , the capital requirement is ten percent. As bank net worth falls below the risk adjusted steady state the market capital requirement falls as well. With low bank net worth, credit availability is lower, implying high excess returns to bank assets. The high excess returns are associated with a high shadow value of bank net worth, which relaxes the incentive constraint permitting greater leverage and hence leads to a lower market determined capital requirement. Conversely, as net worth goes above steady state, excess returns fall which tightens capital requirements.

Figure 10 then compares the optimal regulatory capital requirements in the solid line with ones arising in the unregulated equilibrium in the dashed line. The threshold  $\bar{N}$  lies below the risk adjusted steady state value  $N_{SS}^{DE}$ . When net worth falls below  $\bar{N}$ , the regulatory requirement falls to zero. Conversely, when it goes above  $\bar{N}$ , the requirement goes to twelve percent, which is above the steady state requirement for the unregulated equilibrium. For computational reasons, we smooth out the increase as  $N$  increases above  $\bar{N}$ .

Figure 10 shows the pattern of capital requirements for the regulated equilibrium. Regulatory capital requirements are binding for intermediate levels of net worth. When bank net worth is very low,  $\bar{\kappa}_t$  drops to 0 so that market requirements become binding. When net worth is high enough, the induced

decline in excess returns causes market determined capital requirements to exceed  $\bar{\kappa}$ .

Note that as bank net worth is just below the threshold where capital requirements are binding, the market determined requirement for the regulated economy actually falls below the capital requirement for the unregulated case. This is because banks shadow value of wealth is higher in the regulated economy than in the unregulated economy. Intuitively, when regulatory requirements are binding, the shadow value of net worth in the regulated economy is higher than in the unregulated equilibrium since the run probability is lower and excess returns on bank asset is higher due to the anticipated regulation in future. This in turn has a positive impact on the shadow value of net worth when banks are close to the regulatory threshold since they will eventually move to the region where the regulatory requirements applies.

We next analyze how the optimal macroprudential policy affects behavior. In Figure 11 we consider a optimism driven credit boom of the type that lead to a banking panic. The dotted line portrays the credit boom and bust that occurs in the unregulated equilibrium. The solid line is the behavior with the macroprudential policy put in place. For comparability, we suppose the economy begins in the unregulated equilibrium, so that the initial risk adjusted steady state is the same in both cases. The macroprudential policy is then imposed at time 0. The tightening of capital requirements produces an initial drop in bank intermediation. As in the unregulated equilibrium, the optimism wave which fails to be validated by a productivity leads to an increase in the run probability. But this increase is far more modest than in the unregulated equilibrium. Absent any large negative shock to fundamentals, the economy never enters a crisis zone. The regulation avoids a panic in this case. The cost is that output growth is muted during the optimism phase.

In Figure 12 we consider a case where the credit boom is a false alarm. We consider the example where the fundamental does not materialize but the panic still does not occur (i.e., the sunspot is not turned on.). In this case the unregulated economy would produce a modest output boom. Thus, in this instance, the unregulated economy yields a better outcome. The same would be true for the case where the productivity boom is realized. Accordingly, the gain from macroprudential regulation is reducing the likelihood of a costly banking panic. This gain of course must be weighed against the cost of constraining the economy during credit booms that are false alarms.

Figure 13 shows how macroprudential policy affects the distribution of output and welfare. By preventing boom bust cycles in credit as well as good booms, macroprudential policy induces a much less variable distribution of output while having only negligible effects on average output. This stabilization properties however have non negligible effects on welfare as the policy is effective in reducing the probability of the large and persistent drops in output associated with bank runs. The overall effects of the optimal macroprudential policy on output, the run probability and welfare are reported in the middle column of Table 3, which also reports the behavior of the decentralized economy in the left column. Macroprudential policy cuts the quarterly run probability more than half, to 0.4 percent from 0.9. The capital requirements lead to a reduction in quarterly output of 0.6 percent during periods without a banking crises. However, because the likelihood of costly banking panics is reduced, average output is 0.1 percent higher. Combined with the reduction in the variance and left skewness of the output distribution, this delivers an increase in welfare of 0.25 percentage points of steady state consumption per period. Note that this is a very conservative estimate since we are using log preferences with a coefficient of relative risk aversion of unity.

The last column in Table 3 portrays the case where we eliminate the countercyclical capital buffer and instead assume that regulatory capital requirements are uniform over the cycle. This policy has the same effect on the run probability as the optimal countercyclical policy, but this reduction in the run probability comes at a much higher cost in terms of output which ends up being almost one percent below the unregulated equilibrium on average. The net effect is that the policy produces a welfare loss of about three quarters percent of steady state consumption each quarter.

Figure 14 illustrates why not relaxing the capital requirement in bad times has harmful effects. Under the optimal policy (the dotted line), relaxing capital requirements allows banks greater freedom to issue deposits to invest in high excess return assets after the crisis at date 0. This in turn allows banks to build their equity base at a faster pace, returning the economy to normal. By contrast, if capital requirements are rigid and not relaxed after the crisis (the solid line), banks build equity at a much slower pace, implying a more protracted period of low output.

## 5 Concluding Remarks

We develop a simple quantitative model of credit booms and busts. The framework is consistent with the evidence that credit booms tend to lead crises, but most of the time a boom does not lead to a bust. The model also replicates other key features of financial crises, including increasing credit spreads and sharply contracting output. Importantly, the model captures the nonlinear dimension of financial crises. Much of the time, the economy operates in a "safe zone" with a banking system that is financially strong and not susceptible to a run. However, a belief driven credit boom or a series of bad fundamental shocks can raise bank leverage ratios, making the system vulnerable to runs. These runs, further have costly effects on the real economy. We add that because the model is highly nonlinear, we use global methods to solve it numerically, as discussed in the appendix.

We then use the framework to study macroprudential policy. The particular policy we consider is a capital requirement that limits bank leverage. The primary goal of this policy is to reduce the likelihood of a disastrous financial collapse. Because in our model, as in the data, credit booms could be good as well as bad, regulators face a tradeoff between reducing the likelihood of crisis versus stifling a good credit boom. We consider a simple regulatory policy that allows for a countercyclical capital buffer. We then solve for the parameters of the rule that maximize welfare. We find that the regulatory policy indeed improves welfare mainly by reducing the frequency of costly financial panics. Further, the countercyclical buffer is important. Not relaxing capital requirements in a crisis has the effect of amplifying the downturn, thus reducing welfare.

There are several immediate directions for new research. Limits on banks' ability to raise equity capital plays a key role. It constrains their ability to raise funds and opens up the possibility that they can become vulnerable to panics. We relied on a reduced form function to capture costs of capital injections that was consistent with the evidence on new equity issuance. However, a deeper understanding of these costs would be desirable. Similarly, that banks rely heavily on short term non-contingent debt plays a key role in making them occasionally susceptible to panics. A deeper treatment of this issue is also in order. Finally, our model blurs the distinction between commercial and shadow banks. Of course, any regulation of commercial banks will affect the allocation of funds between commercial and shadow banks (e.g. Begeau and Landvoigt 2017). Adding in this consideration is

an important topic for future research.

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## 6 Appendix

### 6.1 Bankers Problem

Let  $V_t^*(n_t)$  be the optimal value of a bank with net worth  $n_t$ . This solves the Bellman equation

$$V_t^*(n_t) = \max_{k_t^b, d_t, n_{t+1}, \hat{n}_{t+1}, \bar{r}_t} E_t \left\{ \Lambda_{t,t+1} [(1 - \sigma)\hat{n}_{t+1} + \sigma V_{t+1}^*(n_{t+1})] \right\}. \quad (37)$$

subject to the flow of funds constraint

$$Q_t k_t^b = d_t + n_t, \quad (38)$$

the incentive constraint

$$\theta Q_t k_t^b \leq E_t \left\{ \Lambda_{t,t+1} [(1 - \sigma)\hat{n}_{t+1} + \sigma V_{t+1}^*(n_{t+1})] \right\}.$$

the evolution of net worth given by

$$n_{t+1} = \hat{n}_{t+1} (1 + i_{t+1}), \quad (39)$$

where

$$\hat{n}_{t+1} = \max (R_{t+1}^b Q_t k_t^b - \bar{r}_t d_t, 0) \quad (40)$$

and the promised rate satisfies the demand schedule of depositors

$$(1 - p_t^d) E_t^{ND} \{ \Lambda_{t,t+1} \bar{r}_t \} + p_t^d E_t^D \left\{ \Lambda_{t,t+1} \frac{(Q_{t+1} + Z_{t+1}) k_t^b}{d_t} \right\} = 1 \quad (41)$$

where  $p_t^d$  is the probability of default at  $t+1$  and  $E_t^{ND}$  and  $E_t^D$  are conditional expectations given default and no default. Notice that we are not explicitly capturing the dependence of  $p_t^d$  on banks' individual portfolio choices. As we explain in Gertler, Kiyotaki and Prestipino (2019), this dependence does not affect first order conditions so we will simply abstract from it here. The analysis of global optimality of this problem is the same as the one in Gertler, Kiyotaki and Prestipino (2019) so we refer the reader interested in the details to that paper.

To simplify the problem above, it is useful to introduce the leverage multiple

$$\phi_t = \frac{Q_t k_t^b}{n_t} = \frac{1}{\kappa_t} \quad (42)$$

which is the inverse of the capital ratio. We can then use (42) and (38) in (39), (40) and (41) to rewrite the evolution of network as

$$n_{t+1} = \max(n_t R_{t+1}^N(\phi_t)(1 + i_{t+1}), 0) \quad (43)$$

$$\hat{n}_{t+1} = \max(n_t R_{t+1}^N(\phi_t), 0) \quad (44)$$

where

$$R_{t+1}^N(\phi_t) = (R_{t+1}^b - \bar{r}_t(\phi_t))\phi_t + R_{t+1}. \quad (45)$$

and

$$\bar{r}_t(\phi_t) = \frac{\left[1 - \frac{\phi_t}{\phi_t - 1} p_t^d E_t^D \{\Lambda_{t,t+1} R_{t+1}^b\}\right]}{(1 - p_t^d) E_t^{ND} \{\Lambda_{t,t+1}\}}. \quad (46)$$

We can then rewrite the problem as

$$V_t^*(n_t) = \max_{\phi_t} E_t \left\{ \Lambda_{t,t+1} [(1 - \sigma)n_t R_{t+1}^N(\phi_t) + \sigma V_{t+1}^*(n_t R_{t+1}^N(\phi_t)(1 + i_{t+1}))] \right\}$$

subject to

$$\theta \phi_t n_t \leq E_t \left\{ \Lambda_{t,t+1} [(1 - \sigma)n_t R_{t+1}^N(\phi_t) + \sigma V_{t+1}^*(n_t R_{t+1}^N(\phi_t)(1 + i_{t+1}))] \right\}.$$

Now, guess that the value function  $V_t^*(n_t)$  is linear and given by

$$V_t^*(n_t) = \psi_t n_t.$$

The problem becomes

$$\psi_t n_t = \max_{\phi_t} E_t \left\{ \Lambda_{t,t+1} [n_t [(1 - \sigma) + \sigma \psi_{t+1} (1 + i_{t+1})] R_{t+1}^N(\phi_t)] \right\}$$

subject to

$$\theta \phi_t n_t \leq E_t \left\{ \Lambda_{t,t+1} [n_t [(1 - \sigma) + \sigma \psi_{t+1} (1 + i_{t+1})] R_{t+1}^N(\phi_t)] \right\}.$$

The constraint is binding when

$$\mu_t - (\phi - 1) \frac{\nu_t}{\bar{r}_t} \frac{d\bar{r}_t(\phi)}{d\phi} > 0$$

where

$$\begin{aligned} \mu_t &= (1 - p_t^d) E_t^{ND} \left\{ \Lambda_{t,t+1} [(1 - \sigma) + \sigma \psi_{t+1} (1 + i_{t+1})] [R_{t+1}^b - \bar{r}_t(\phi)] \right\} \\ \nu_t &= (1 - p_t^d) E_t^{ND} \left\{ \Lambda_{t,t+1} [(1 - \sigma) + \sigma \psi_{t+1} (1 + i_{t+1})] \bar{r}_t(\phi) \right\} \end{aligned} \quad (47)$$

$$(48)$$

In this case, the optimal elverage is given by:

$$\theta\phi_t = \psi_t.$$

Otherwise optimal leverage is given by

$$\mu_t - (\phi - 1) \frac{\nu_t}{\bar{r}_t} \frac{d\bar{r}_t(\phi)}{d\phi} = 0.$$

In either case optimal elverage does not depend on  $n_t$  and so

$$\psi_t = E_t \left\{ \Lambda_{t,t+1} \left[ (1 - \sigma) + \sigma\psi_{t+1} (1 + i_{t+1}) \right] R_{t+1}^N(\phi_t) \right\},$$

does not depend on  $n_t$  either, which verifies the guess.

## 6.2 Equilibrium equations

The state of the economy is given by  $M_t = \{N_t, Z_t, \iota_t, S_t\}$  where  $\iota_t$  is the sunspot variable and  $S_t$  is the state determining banker's and households beliefs, described below.

The equilibrium equations determining  $\{C_t, K_t^h, \xi_t^N, i_t, \psi_t^h, K_t^b, \kappa_t, \psi_t^b, \hat{N}_t, N_{t+1}, R_{t+1}^N, R_t, Q_t, \bar{R}_t, Z_{t+1}, B_{t+1}, S_{t+1}, Z_{t+1}^R, Z_{t+1}^I\}$  are given by:

Household deposit demand

$$\beta E_t^h \left\{ \left( \frac{C_t}{C_{t+1}} \right) R_{t+1} \right\} = 1. \quad (49)$$

Household demand for capital

$$\beta E_t^h \left\{ \left( \frac{C_t}{C_{t+1}} \right) \frac{Z_{t+1} + Q_{t+1}}{Q_t + \alpha K_t^h} \right\} = 1. \quad (50)$$

Household demand for bank equity

$$\begin{aligned} 1 + f'_\xi(\xi_t^N) &= \psi_t^h & \text{if no run} \\ \xi_t^N &= 0 & \text{if run} \end{aligned} \quad (51)$$

Household marginal value of bank equity

$$\psi_t^h = E_t^h \Lambda_{t,t+1} \left[ (1 - \sigma) + \sigma\psi_{t+1}^h \right] R_{t+1}^N. \quad (52)$$

Banks capital demand

$$Q_t K_t^b = \frac{1}{\kappa_t} N_t. \quad (53)$$

Banks portfolio choice

$$\kappa_t = \frac{\theta}{\psi_t^b} \quad (\text{binding IC}).^{22} \quad (54)$$

Banks marginal value of wealth

$$\psi_t^b = E_t^b \{ \Lambda_{t,t+1} [(1 - \sigma) + \sigma \psi_{t+1}^b (1 + i_{t+1})] R_{t+1}^N \}. \quad (55)$$

Equity injection rate

$$i_t = \frac{\xi_t^N - \bar{\xi}}{\sigma \hat{N}_t + \bar{\xi}}. \quad (56)$$

Total equity

$$N_t = \sigma \hat{N}_t + \xi_t^N.$$

Banker's net worth evolution

$$N_{t+1} = \begin{cases} \sigma N_t R_{t+1}^N + \xi_{t+1}^N & \text{if no run at } t+1 \\ 0 & \text{if run at } t+1 \end{cases}. \quad (57)$$

The return on net worth

$$R_t^N = \left( \frac{Z_{t+1} + Q_{t+1}}{Q_t} - R_{t+1} \right) \frac{1}{\kappa_t} + R_{t+1}. \quad (58)$$

The return on deposits

$$R_{t+1} = \min \left\{ \bar{R}_t, \frac{(Z_{t+1} + Q_{t+1})}{Q_t} \frac{1}{1 - \kappa_t} \right\}, \quad (59)$$

where we are using (53) and (12) to write the return upon default as

$$\frac{(Z_{t+1} + Q_{t+1}) K_t^b}{D_t} = \frac{(Z_{t+1} + Q_{t+1})}{Q_t} \frac{1}{1 - \kappa_t}.$$

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<sup>22</sup>In our calibration the constraint is always binding. See Gertler, Kiyotaki, and Prestipino (2019) for a formal analysis of the bank's optimal portfolio choice that allows for occasionally binding constraints.

Market clearing for assets

$$K_t^b + K_t^h = 1. \quad (60)$$

Market clearing for consumption

$$C_t = Z_t + W_h - \frac{\alpha}{2} (K_t^h)^2 - f_\xi(\xi_t^N). \quad (61)$$

The evolution of productivity

$$Z_{t+1} = \rho Z_t + B_{t+1} + \varepsilon_{t+1}, \quad (62)$$

where  $\varepsilon_{t+1} \sim N(0, \sigma^Z)$  and

$$B_{t+1}(s_t, s_{t+1}) = \begin{cases} \bar{B} & \text{if } s_t \in \{1, \dots, T\} \text{ and } s_{t+1} = T + 2 \\ 0 & \text{otherwise} \end{cases}. \quad (63)$$

$S_t \in G_S = \{1, \dots, T + 2\}$  is a finite state Markov chain with transition probability

$$TP = \begin{bmatrix} & S_{t+1} = 1 & S_{t+1} = 2 & S_{t+1} = 3 & \dots & S_{t+1} = T + 1 & S_{t+1} = T + 2 \\ S_t = 1 & 0 & 1 - \eta_1 & & \dots & \dots & \eta_1 \\ S_t = 2 & 0 & 0 & 1 - \eta_2 & \dots & \dots & \eta_2 \\ \dots & \dots & \dots & \dots & \dots & \dots & 0 \\ S_t = T & 0 & 0 & 0 & \dots & 1 - \eta_T & \eta_T \\ S_t = T + 1 & \pi^n & 0 & 0 & \dots & 1 - \pi^n & 0 \\ S_t = T + 2 & \pi^n & 0 & 0 & \dots & \dots & 1 - \pi^n \end{bmatrix} \quad (64)$$

Bankers believe that the transition probability is

$$\eta_i^b = \frac{\bar{P}\zeta_i}{1 - \bar{P}\sum_{s=1}^{i-1}\zeta_s}.$$

where  $\{\zeta_\tau\}_{\tau=1}^T$  is a discrete approximation of a normal. While households believe

$$\eta_i^h = 0.$$

Thresholds for insolvency

$$[Z_{t+1}^I + Q_{t+1}^I (Z_{t+1}^I)] K_t^b - (Q_t K_t^b - N_t) \bar{R}_t = 0, \quad (65)$$



where  $Q_{t+1}^I (Z_{t+1}^I)$  is the price of capital at  $t + 1$  if no run happens

$$Q_{t+1}^I (Z_{t+1}^I) = Q ( N_{t+1} (M_t, \varepsilon_{t+1}^{*I}, S_{t+1}) , Z_{t+1}^I, 0 , S_{t+1})$$

$$Z_{t+1}^I = \rho Z_t + B_{t+1} + \varepsilon_{t+1}^{*I}$$

and similarly thresholds for run

$$[Z_{t+1}^R + Q_{t+1}^R (Z_{t+1}^R)] K_t^b - (Q_t K_t^b - N_t) \bar{R}_t = 0, \quad (66)$$

where

$$Q_{t+1}^R (Z_{t+1}^R) = Q (0 , Z_{t+1}^R , 1 , S_{t+1}).$$

### 6.3 Computation

It is convenient for computations to let the aggregate state of the economy when there is no run be given by

$$\mathcal{M}_t = (\hat{N}_t, Z_t, \iota_t, S_t).$$

where

$$\hat{N}_t = \frac{N_t - \xi_t^N}{\sigma}$$

We can then look for equilibrium functions

$$\boldsymbol{\vartheta} = \{ \mathbf{Q}(\mathcal{M}), \mathbf{C}(\mathcal{M}), \psi^h(\mathcal{M}), \psi^b(\mathcal{M}), \mathbf{Z}_{t+1}^R(\mathcal{M}; S'), \mathbf{Z}_{t+1}^I(\mathcal{M}; S'), \mathbf{T}(\mathcal{M}; \varepsilon', \iota', S') \}$$

where  $\mathbf{T}(\mathcal{M}_t; \varepsilon', \iota', S')$  is the transition law determining the evolution of the state as a function of the state today and stochastic shocks tomorrow. All other variables can be easily recovered from variables  $\boldsymbol{\vartheta}$  by using static equilibrium conditions (see below point 5 below).

The computational algorithm to approximate the functions in  $\boldsymbol{\vartheta}$  proceeds as follows:

1. Determine a functional space to use for approximating equilibrium functions. (We use piecewise linear).
2. Fix a grid of values for the state  $G \subset [0, N^M] \times [1 - 4\sigma^Z, 1 + 4\sigma^Z] \times \{0, 1\} \times \{1, 2, \dots, T + 2\}$  and a grid of value for future of  $\varepsilon' \in G^\varepsilon \subset [1 - 4\sigma^\varepsilon, 1 + 4\sigma^\varepsilon]$

3. Set  $i = 0$  and guess initial values for the equilibrium objects of interest on the grid

$$\vartheta^0 = \left\{ Q^0(\mathcal{M}), C^0(\mathcal{M}), \psi^{h,0}(\mathcal{M}), \psi^{b,0}(\mathcal{M}), Z_{t+1}^{R,0}(\mathcal{M}; S'), Z_{t+1}^{I,0}(\mathcal{M}; S'), \mathcal{T}^0 \mathcal{M}; \varepsilon', \ell', S' \right\}_{\mathcal{M} \in G}$$

4. Assume that  $\vartheta^i$  has been found for  $i < M$  where  $M$  is set to 10000. Use  $\vartheta^i$  to find associated functions  $\boldsymbol{\vartheta}^i$  in the approximating space, e.g.  $\mathbf{Q}^i$  is the price function that satisfies  $\mathbf{Q}^i(\mathcal{M}) = Q^i(\mathcal{M})$  for each  $\mathcal{M} \in G$ .
5. Compute all time  $t + 1$  variables in the system of equilibrium equations by using the functions  $\boldsymbol{\vartheta}^i$  from the previous step, e.g. for each  $\mathcal{M} \in G$  let  $Q_{t+1}(\boldsymbol{\vartheta}^i) = \mathbf{Q}^i(\mathcal{T}^i(\mathcal{M}; \varepsilon', \ell', S'))$ , and then solve the system of equilibrium equations to get the implied  $\vartheta^{i+1}$ .

Specifically, if there is no run at time  $t$ , for any  $\mathcal{M} = \{ \hat{N}_t, Z_t, \iota_t, S_t, 1 \} \in G$  we can solve for  $\{ Q_t^{i+1}, C_t^{i+1}, \psi_t^{h,i+1}, \psi_t^{b,i+1}, K_t^{h,i+1}, K_t^{b,i+1}, \xi_t^{N,i+1}, \kappa_t^{i+1} \}$ , where we use the shorthand  $Q_t^{i+1}$  for  $Q^{i+1}(\mathcal{M})$ , by finding the root of the system

$$C_t^{i+1} \beta E_t^h \left\{ \frac{Z_{t+1} + Q_{t+1}(\boldsymbol{\vartheta}^i)}{C_{t+1}(\boldsymbol{\vartheta}^i)} \right\} = Q_t^{i+1} + \alpha K_t^{h,i+1} \quad (67)$$

$$\frac{\psi_t^{h,i+1}}{C_t^{i+1}} = E_t^h \frac{[(1 - \sigma) + \sigma \psi_{t+1}^h(\boldsymbol{\vartheta}^i)] \hat{N}_{t+1}(\boldsymbol{\vartheta}^i)}{C_{t+1}(\boldsymbol{\vartheta}^i) \hat{N}_t + \xi_t^{N,i+1}} \quad (68)$$

$$1 + f'_\xi(\xi_t^{N,i+1}) = \psi_t^{h,i+1} \quad (69)$$

$$\frac{\psi_t^{b,i+1}}{C_t^{i+1}} = E_t^b \frac{[(1 - \sigma) + \sigma \psi_{t+1}^b(\boldsymbol{\vartheta}^i)] \hat{N}_{t+1}(\boldsymbol{\vartheta}^i)}{C_{t+1}(\boldsymbol{\vartheta}^i) \sigma \hat{N}_t + \xi_t^{N,i+1}} \quad (70)$$

$$Q_t^{i+1} K_t^{b,i+1} = \frac{1}{\kappa_t^{i+1}} (\sigma \hat{N}_t + \xi_t^{N,i+1}) \quad (71)$$

$$\kappa_t^{i+1} = \frac{\theta}{\psi_t^{b,i+1}} \quad (72)$$

$$K_t^{b,i+1} + K_t^{h,i+1} = 1 \quad (73)$$

$$C_t^{i+1} = Z_t + W_h - \frac{\alpha}{2} \left( K_t^{h,i+1} \right)^2 - f_\xi \left( \xi_t^{N,i+1} \right) \quad (74)$$

We then find the new implied thresholds  $Z_{t+1}^{R,i+1}(\mathcal{M}; S')$  and  $Z_{t+1}^{I,i+1}(\mathcal{M}; S')$  by solving for any  $S' \in \{1, 2, \dots, T+2\}$

$$\left[ Z_{t+1}^{R,i+1} + \mathbf{Q}^i \left( 0, Z_t^{R,i+1}, 1, S' \right) \right] K_t^{b,i+1} - \left( Q_t^{i+1} K_t^{b,i+1} - \hat{N}_t - \xi_t^{N,i+1} \right) \bar{R}_t^{i+1} = 0 \quad (75)$$

$$\left[ Z_{t+1}^{I,i+1} + \mathbf{Q}^i \left( 0, Z_t^{R,i+1}, 0, S' \right) \right] K_t^{b,i+1} - \left( Q_t^{i+1} K_t^{b,i+1} - \hat{N}_t - \xi_t^{N,i+1} \right) \bar{R}_t^{i+1} = 0 \quad (76)$$

We then update the evolution of the state by letting for any  $\varepsilon' \in G^\varepsilon$

and any  $S' \in \{1, 2, \dots, T+2\}$

$$\begin{aligned} \hat{N}_{t+1}^{i+1}(\mathcal{M}; \varepsilon', l', S') = & \\ \left\{ \begin{array}{l} 0, \quad \text{if } Z_{t+1}(\mathcal{M}; \varepsilon') < Z_{t+1}^{I,i+1}(\mathcal{M}; S'), \text{ or} \\ \quad \quad \quad Z_{t+1}(\mathcal{M}; \varepsilon') < Z_{t+1}^{R,i+1}(\mathcal{M}; S') \text{ and } l' = 1 \\ \left( \sigma \hat{N}_t + \xi_t^{N,i+1} \right) \left[ \left( \frac{\mathbf{Q}_i(\mathcal{T}_i(\mathcal{M}; \varepsilon', 0, S')) + Z_{t+1}(\mathcal{M}; \varepsilon')}{Q_t^{i+1}} - \bar{R}_t^{i+1} \right) \frac{1}{\kappa_t^{i+1}} + \bar{R}_t^{i+1} \right], \text{ otherwise} \end{array} \right. \\ RUN_{t+1} = & \begin{cases} 1 & \text{if } Z_{t+1}(\mathcal{M}; \varepsilon') < Z_{t+1}^{R,i+1}(\mathcal{M}; S') \text{ and } l' = 1 \\ 0 & \text{otherwise} \end{cases} \end{aligned}$$

we can then collect all the values in

$$\vartheta^{i+1} = \left\{ \left[ Q^{i+1}, C^{i+1}, \psi^{h,i+1}, \psi^{b,i+1} \right] (\mathcal{M}), \left[ Z_{t+1}^{R,i+1}, Z_{t+1}^{I,i+1} \right] (\mathcal{M}; S'), \mathcal{T}^{i+1}(\mathcal{M}; \varepsilon', l', S') \right\}_{\mathcal{M} \in G}$$

6. Repeat 4 and 5 until convergence of  $|\vartheta^{i+1} - \vartheta^i| < \text{conv\_criterion}$ .

## 6.4 Impulse Response Functions

We let the risk adjusted steady state be given by  $\bar{\mathcal{M}} = (\bar{N}, 1, 0, T+1)$  which satisfies:

$$\bar{\mathcal{M}} = \mathbf{T}(\bar{\mathcal{M}}; 0, 0, T+1)$$

that is, it is a state that will remain constant in the absence of any shocks to productivity and as long as bankers do not receive any news.

We compute responses to a sequence of  $n$  shocks  $\left\{ \epsilon_t^{irfs}, \iota_t^{irfs}, S_t^{irfs} \right\}_{t=1}^n$  by starting the economy in the risk adjusted steady state,  $\mathcal{M}_0 = \bar{\mathcal{M}}$ , and computing the evolution of the state given the assumed shocks from time 1 to  $n$  and setting all future shocks to 0, i.e.  $\epsilon_t = \iota_t = 0$  for  $t \geq n + 1$  :

$$\mathcal{M}_{t+1} = \begin{cases} \mathbf{T} \left( \mathcal{M}_t; \epsilon_t^{irfs}, \iota_t^{irfs}, S_t^{irfs} \right) & \text{if } t \leq n \\ \mathbf{T} \left( \mathcal{M}_t; 0, 0, S^*(S_{t-1}) \right) & \text{if } t > n \end{cases}$$

where  $S^*(S_{t-1})$  implies no news arrival and no boom realization

$$S^*(S_{t-1}) = \begin{cases} S_{t-1} + 1 & \text{if } S_{t-1} \in \{1, 2, \dots, T\} \\ S_{t-1} & \text{if } S_{t-1} \in \{T + 1, T + 2\} \end{cases}$$

We then plot for each variable, the values of the associated policy function computed along this path for the state, e.g.  $Q_t = \mathbf{Q}(\mathcal{M}_t)$ . Notice that, given our nonlinear policy functions, these values are different from conditional expectations given the sequence of shocks  $\left\{ \epsilon_t^{irfs}, \iota_t^{irfs}, S_t^{irfs} \right\}_{t=1}^n$ .

FIGURE 1

This figure is from Krishnamurthy and Muir (2017). It plots the behavior of credit spreads, GDP, and the quantity of credit around a financial crisis with the crisis beginning at date 0. GDP and credit are expressed in deviation from (country specific) trend. Spreads are normalized by dividing by the unconditional mean.

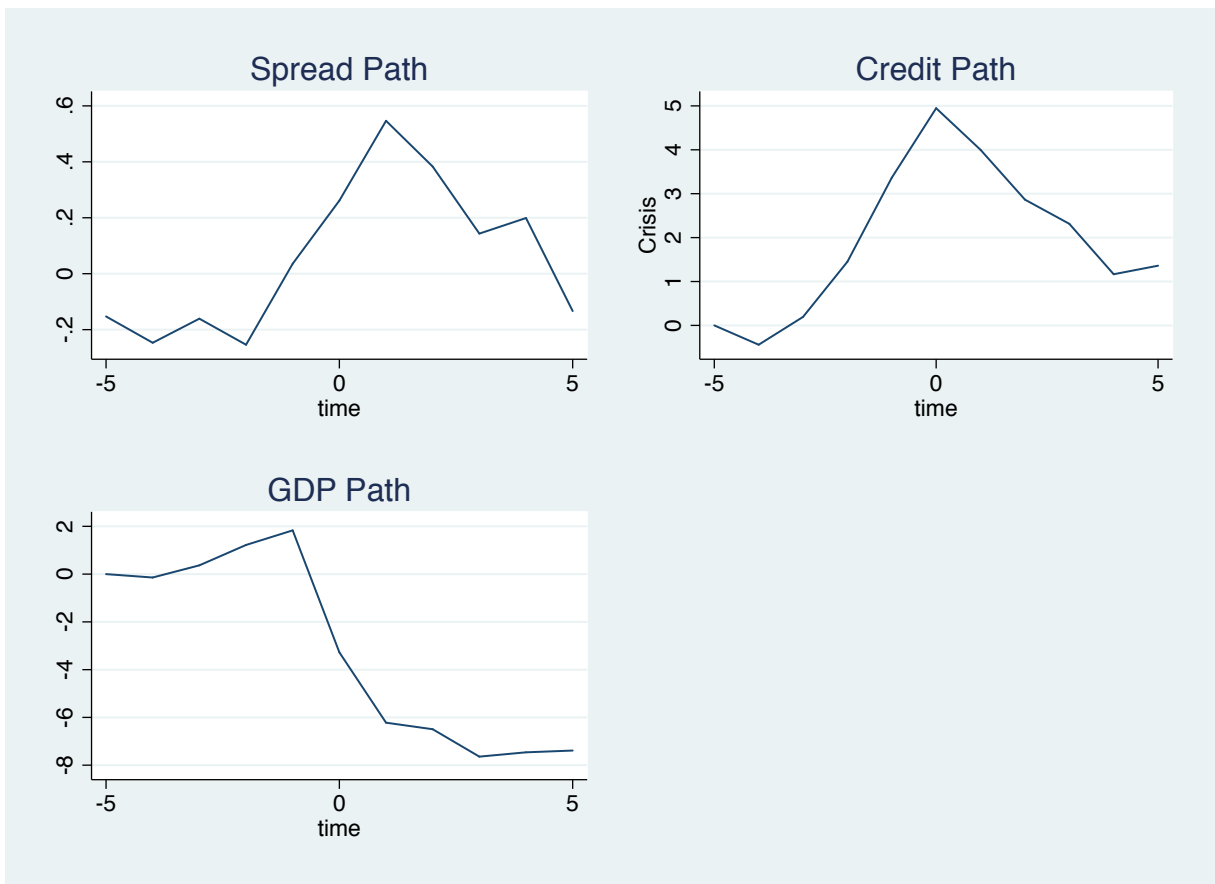
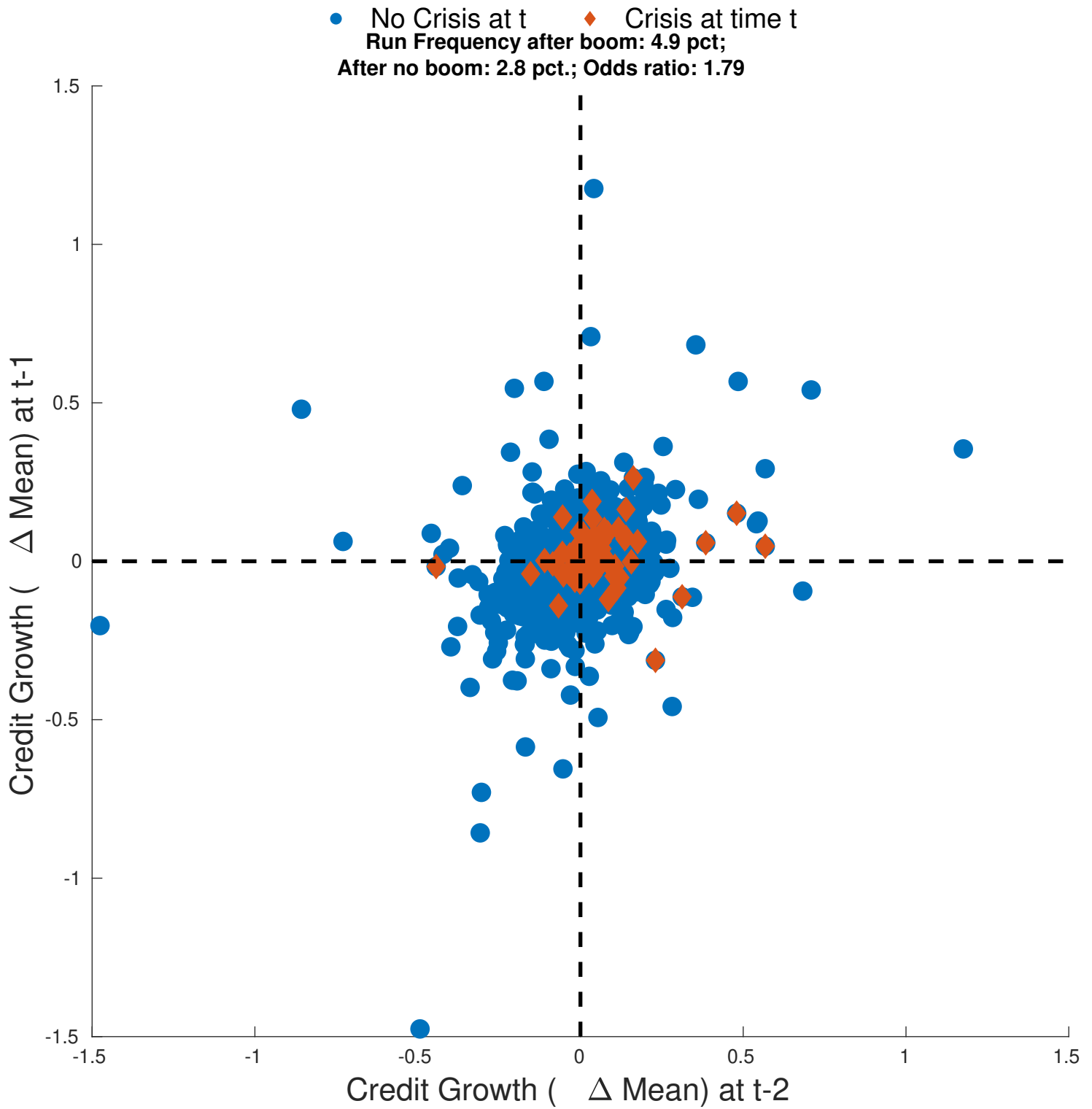


FIGURE 2



**TABLE 1**  
**Calibration**

Parameters	Description	Value	Target	Model
Calibrated Parameters				
$\theta$	Share of Divertible Assets	0.23	Capital Ratios = 10 pct	$E(\kappa) = 10$ pct
$\sigma$	Banker Survival Rate	0.935	Quarterly Spread = 50 bpts	$E(R^b - R) = 48$ bpts
$\alpha$	HH Intermediation Costs	0.00625	Output Drop During Run = 6 pct	$\frac{Y_t - Y^{SS}}{Y^{SS}} = 6.4$ pct
$\bar{\xi}$	Startup Equity	1 pct of $N^{SS}$	HH Share of Intermediation = .5 pct	$K^h = 0.49$
$\alpha_\xi$	Equity Injections Costs	0.001	Average Issuance rate = 1 pct	$E \frac{\xi^N}{N^{SS}} = 1.1$ pct
$\chi^s$	Sunspot Probability	0.125	Avg Yearly Frequency of Runs = 4 pct	$4 \cdot Ep^R = 3.6$ pct
$\sigma(\epsilon^Z)$	Std Dev of Z Innovation	0.01	Std Dev of U.S. Output = 1.9 pct	$\sigma(Y) = 1.9$ pct
Calibrated Parameters				
$\beta$	Impatience	0.99	-	-
$\rho^Z$	Serial Correlation of Z	0.95	-	-
$W$	HH Endowment	$2 \cdot Z$	-	-

**FIGURE 3**  
Run after a large negative shock

— Sunspot    - - - No Sunspot

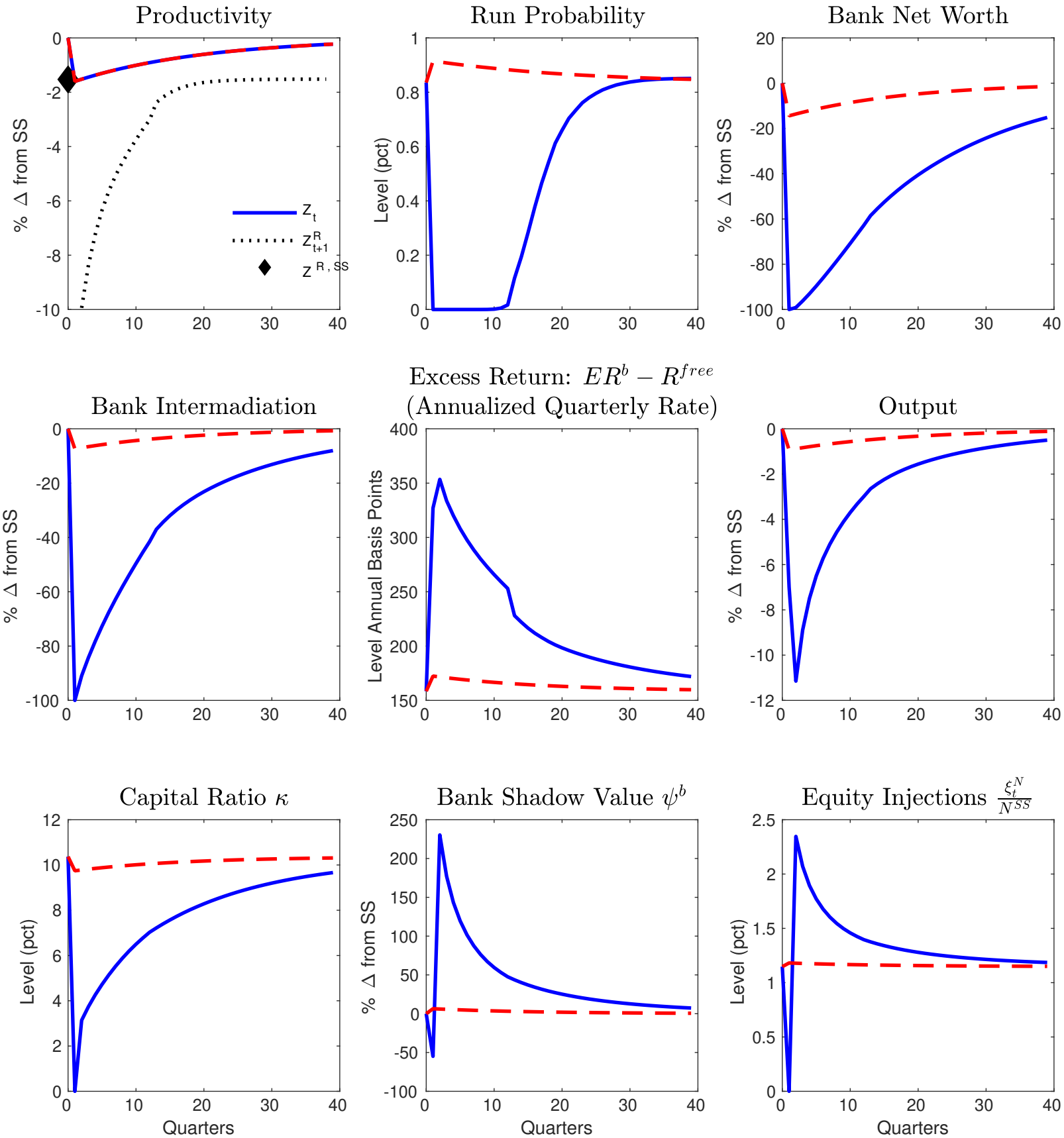
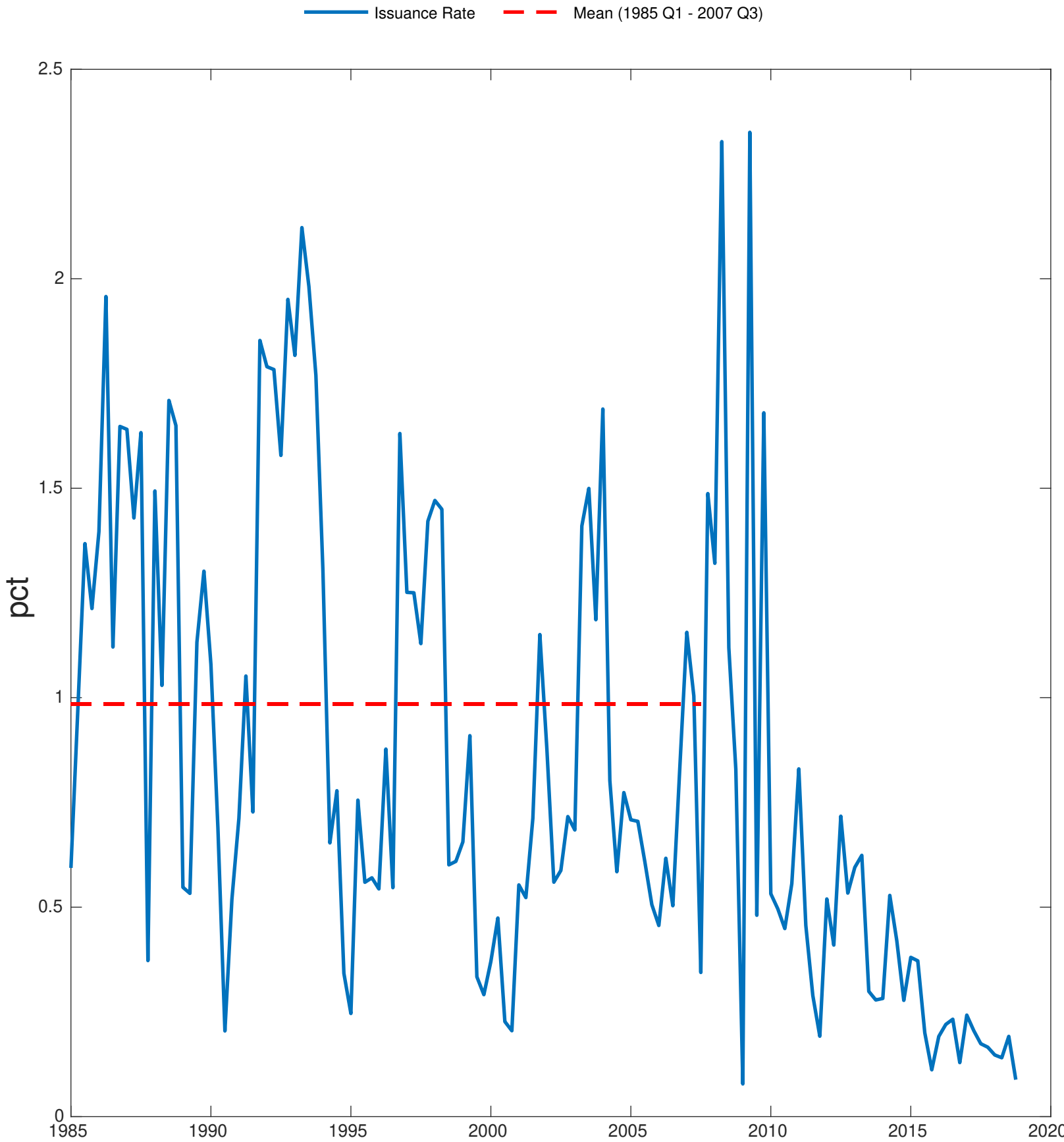




FIGURE 4

Financial firms equity issuance as a fraction of trend equity



**TABLE 2**  
**Calibration of News**

Parameters	Description	Value
$\chi^n$	Prob of Receiving News	0.02
$B$	Size of Productivity Boom	$2 \cdot \sigma(\epsilon^Z)$
$T$	News Horizon	21 Quarters
$\mu(t^B)$	Expected time of Z boom	10.5 Quarters ahead
$\sigma(t^B)$	Standard Deviation of Prior	2 Quarters
$\bar{P}_0^B$	Banker Prob. that Boom will happen	0.999
$\bar{P}_0^{TRUE}$	True Prob. that Boom will happen	0.5

FIGURE 5  
Belief Dynamics and Credit Boom

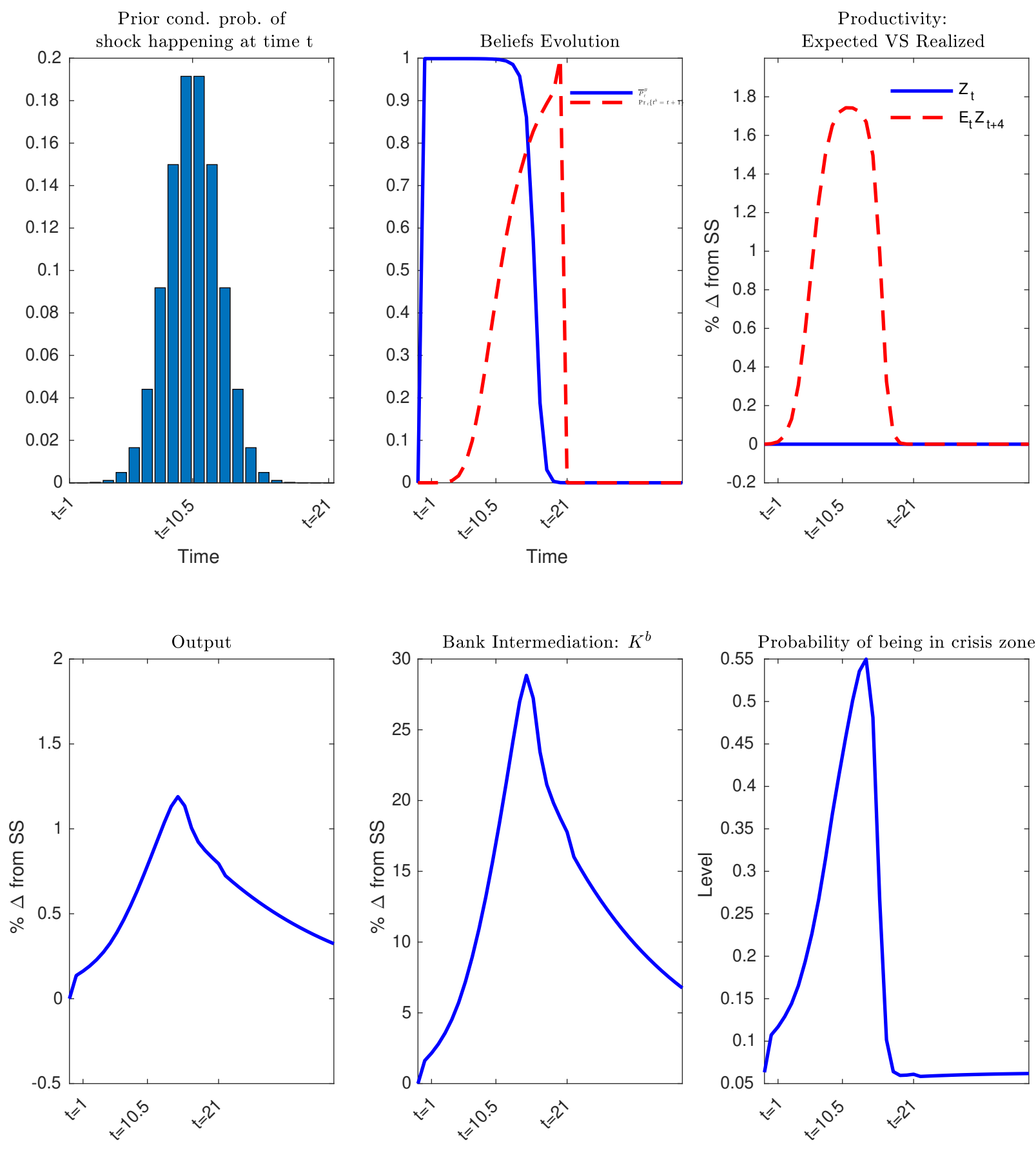
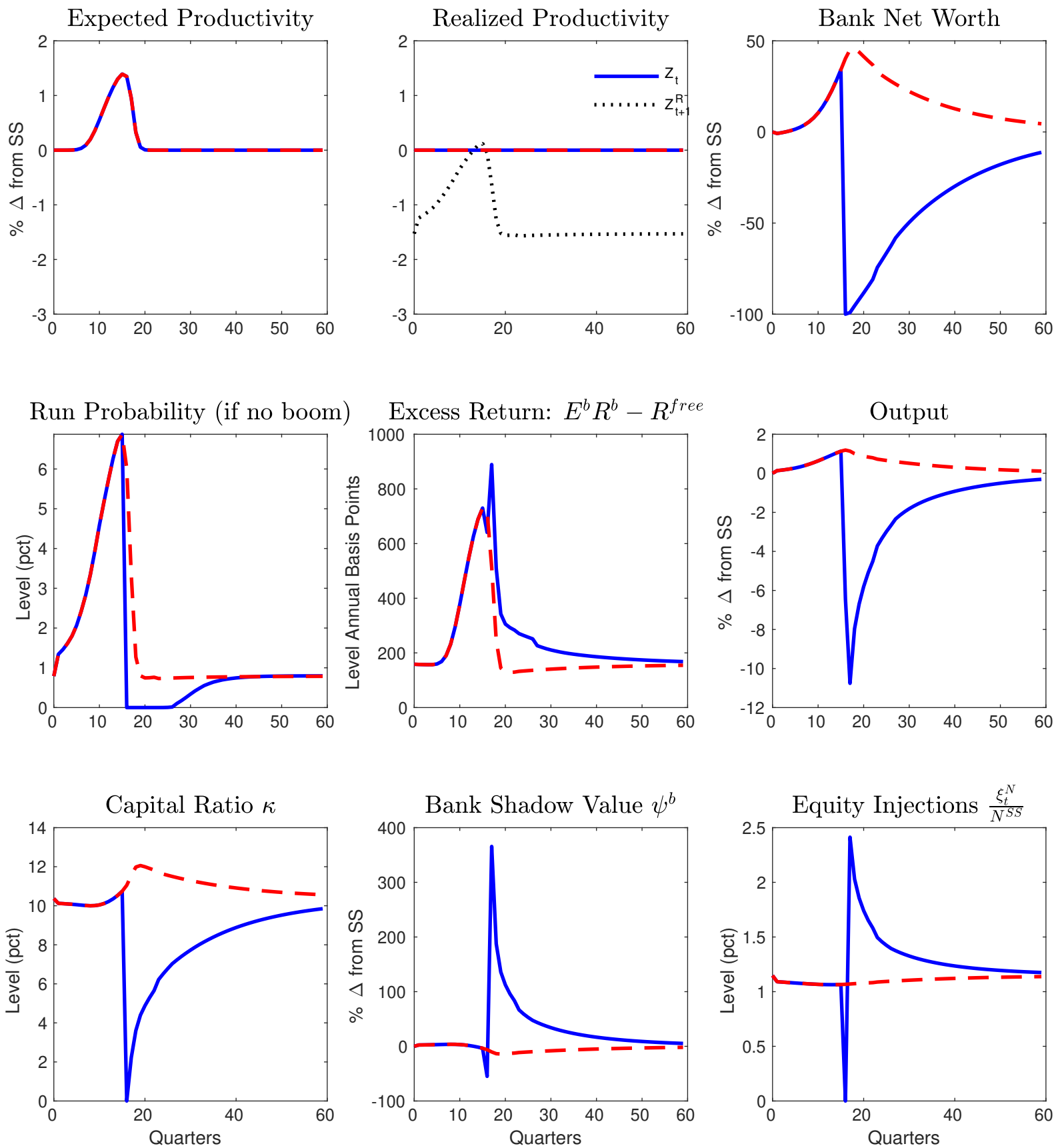


FIGURE 6  
Run After Credit Boom

— Sunspot observed    - - - No Sunspot observed



# FIGURE 7

## Run After Credit Boom

— Boom Happens    
 - - - No Sunspot is Observed

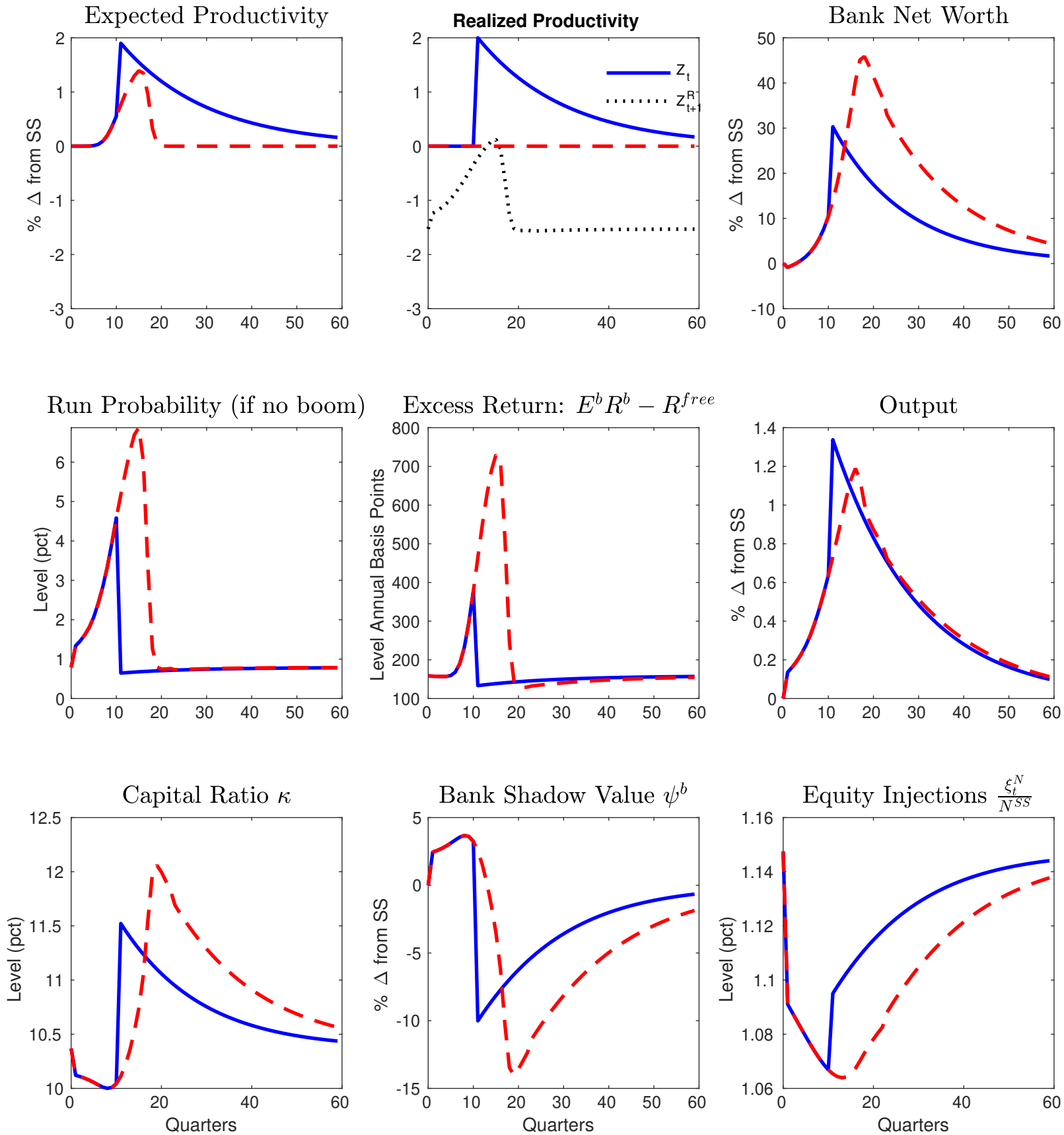
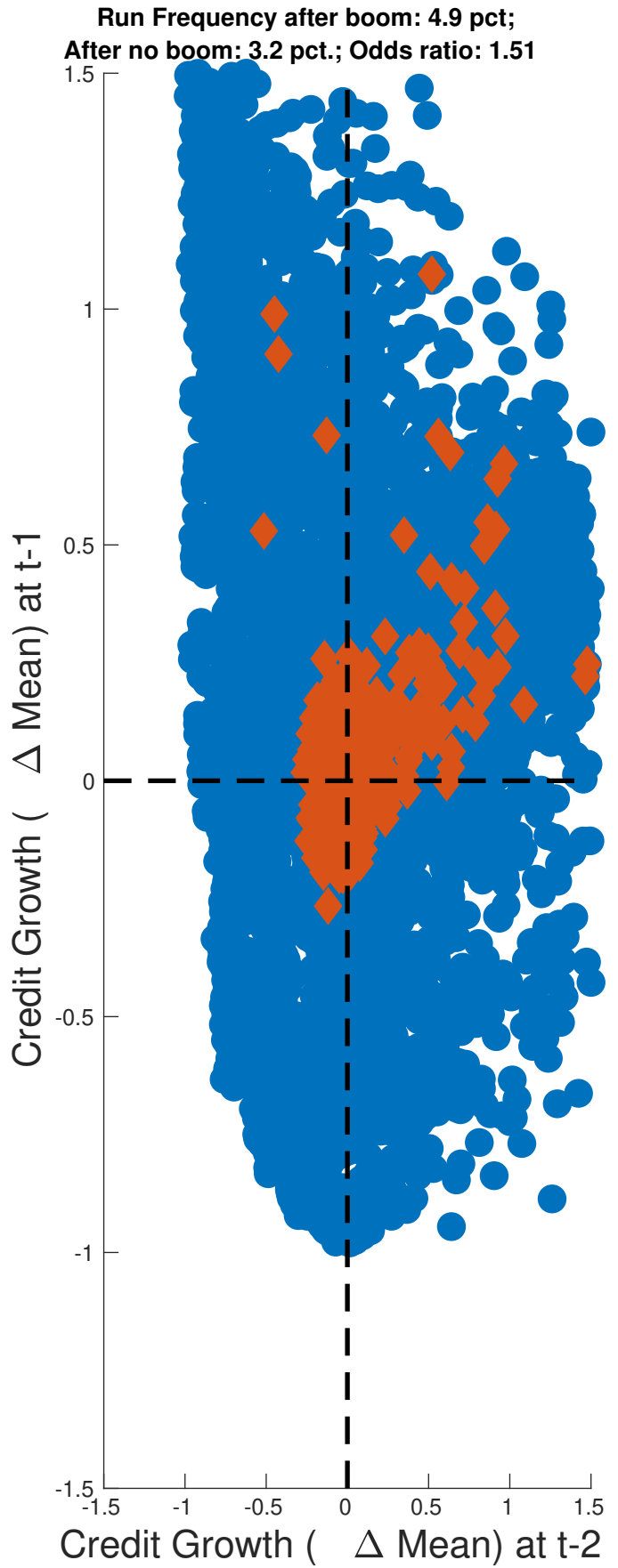
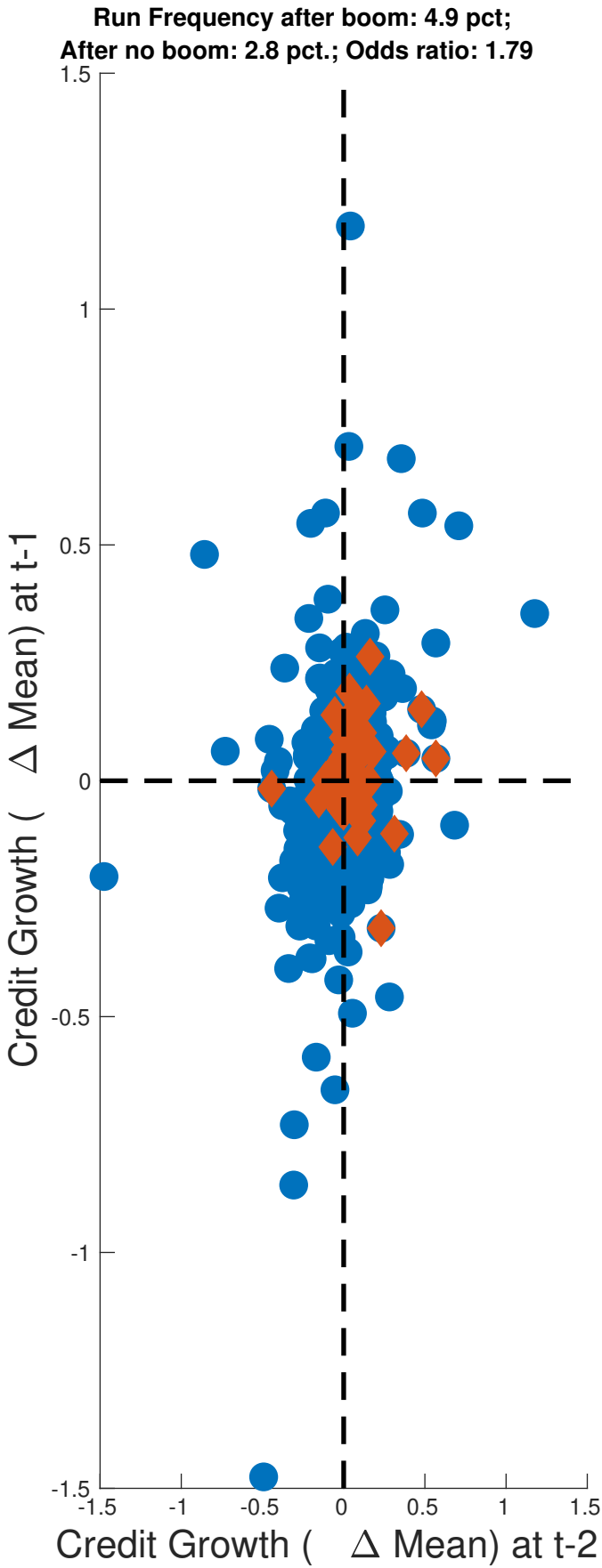
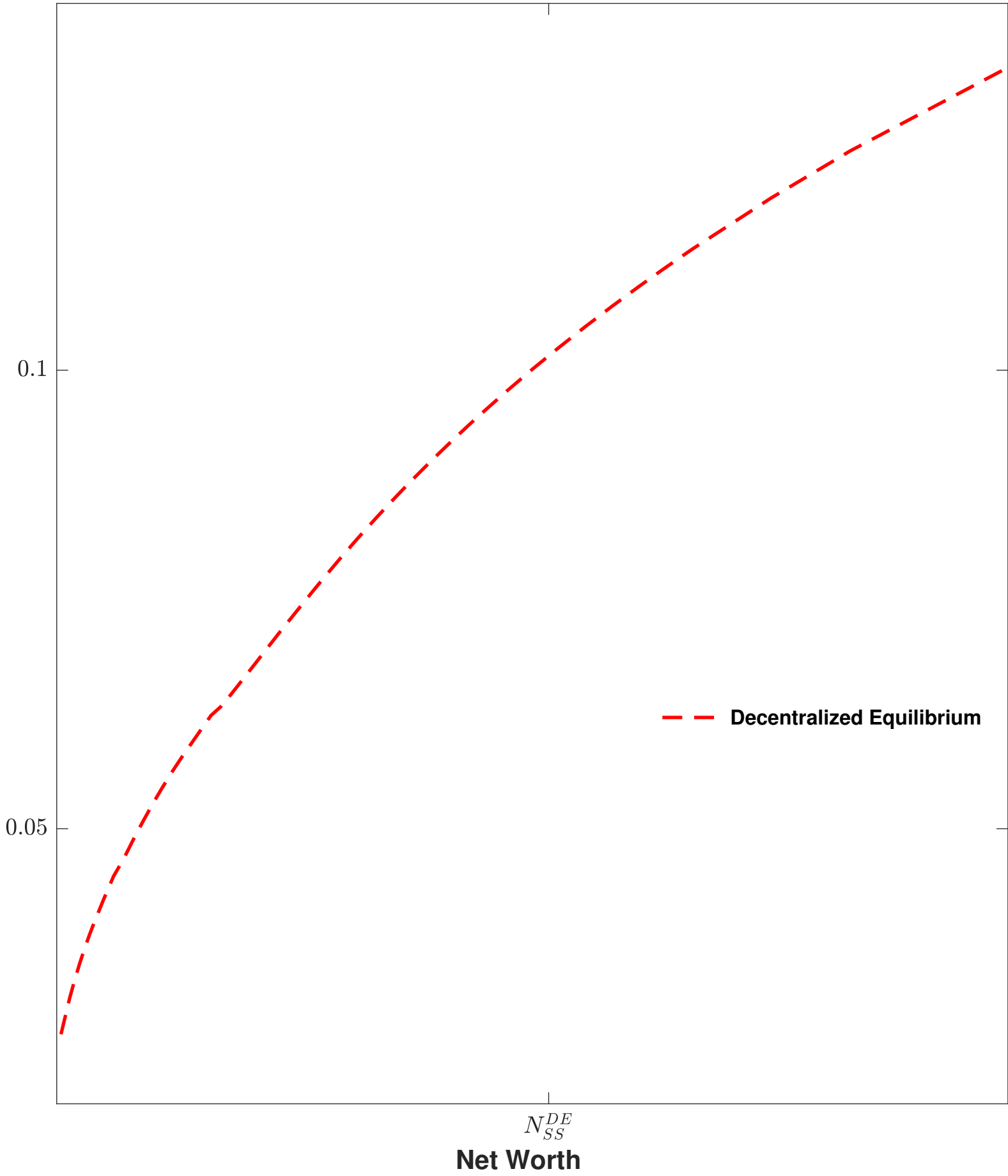


FIGURE 8

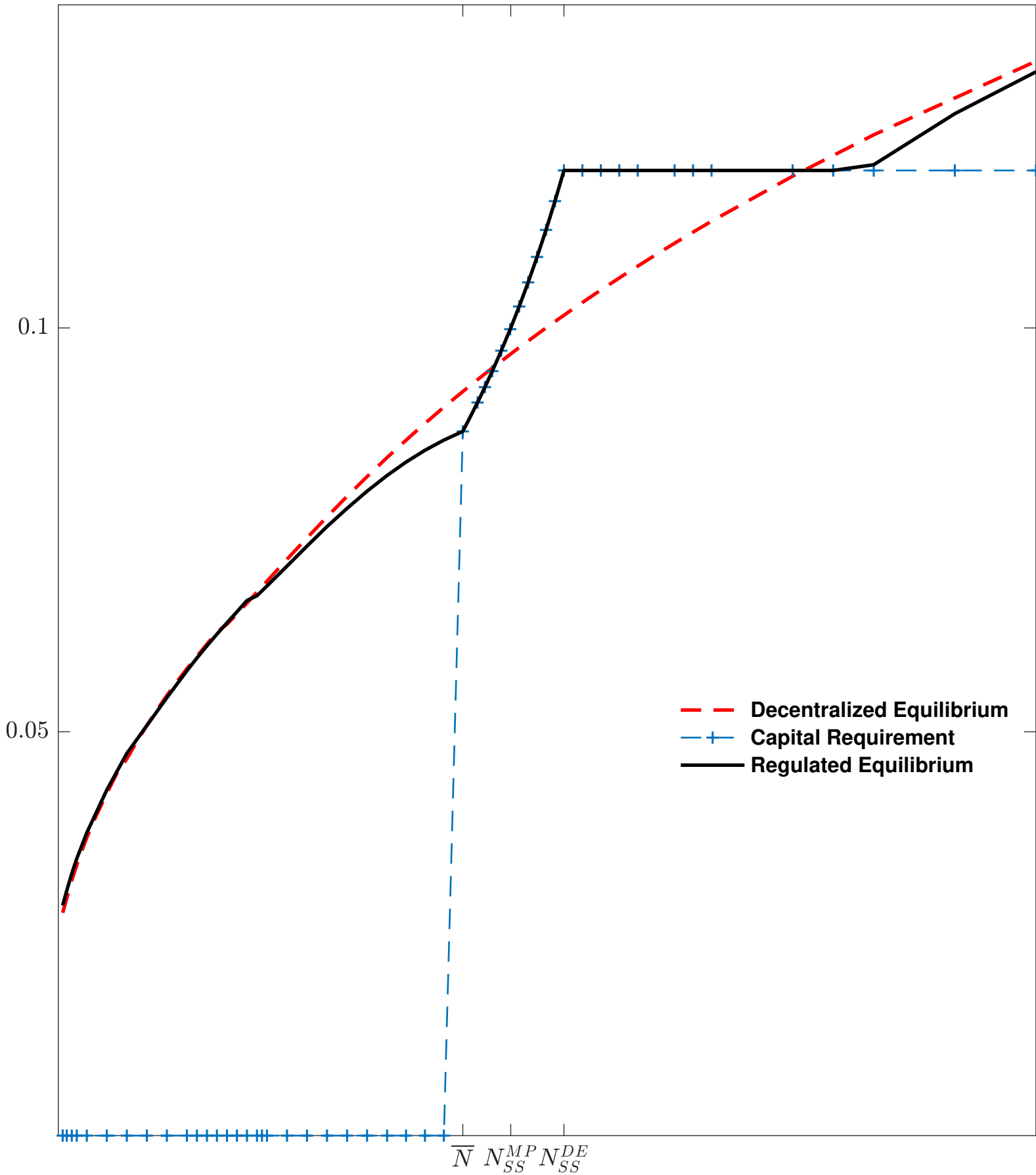
● No Crisis at t      ◆ Crisis at time t



**FIGURE 9**  
**Capital Ratio**



**FIGURE 10**  
**Capital Ratio**

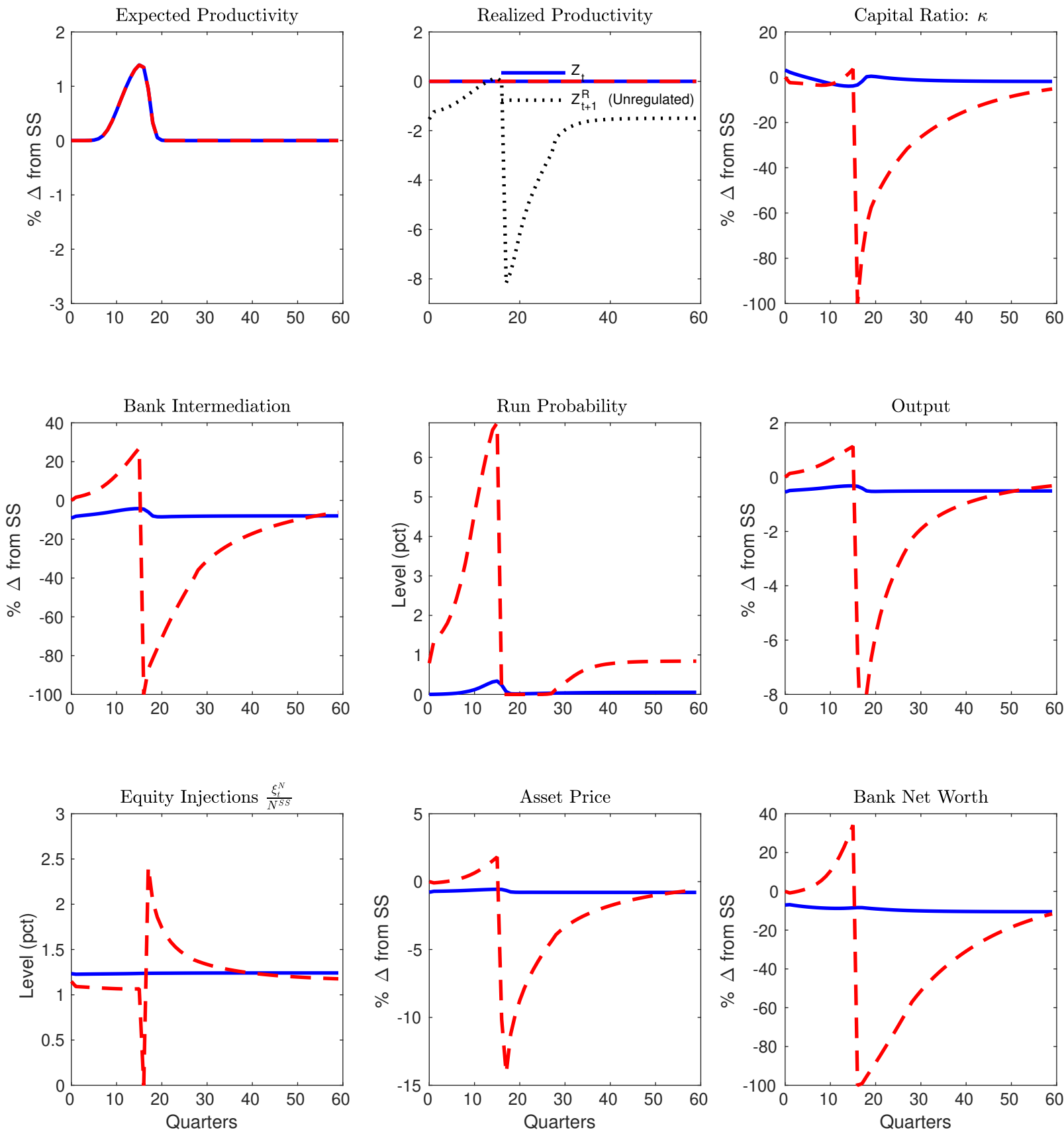




# FIGURE 11

## Avoiding Runs with Macro Pru

— Regulated    - - - Unregulated



# FIGURE 11

## Response to News: Regulated VS Unregulated economy

— Regulated    - - - Unregulated

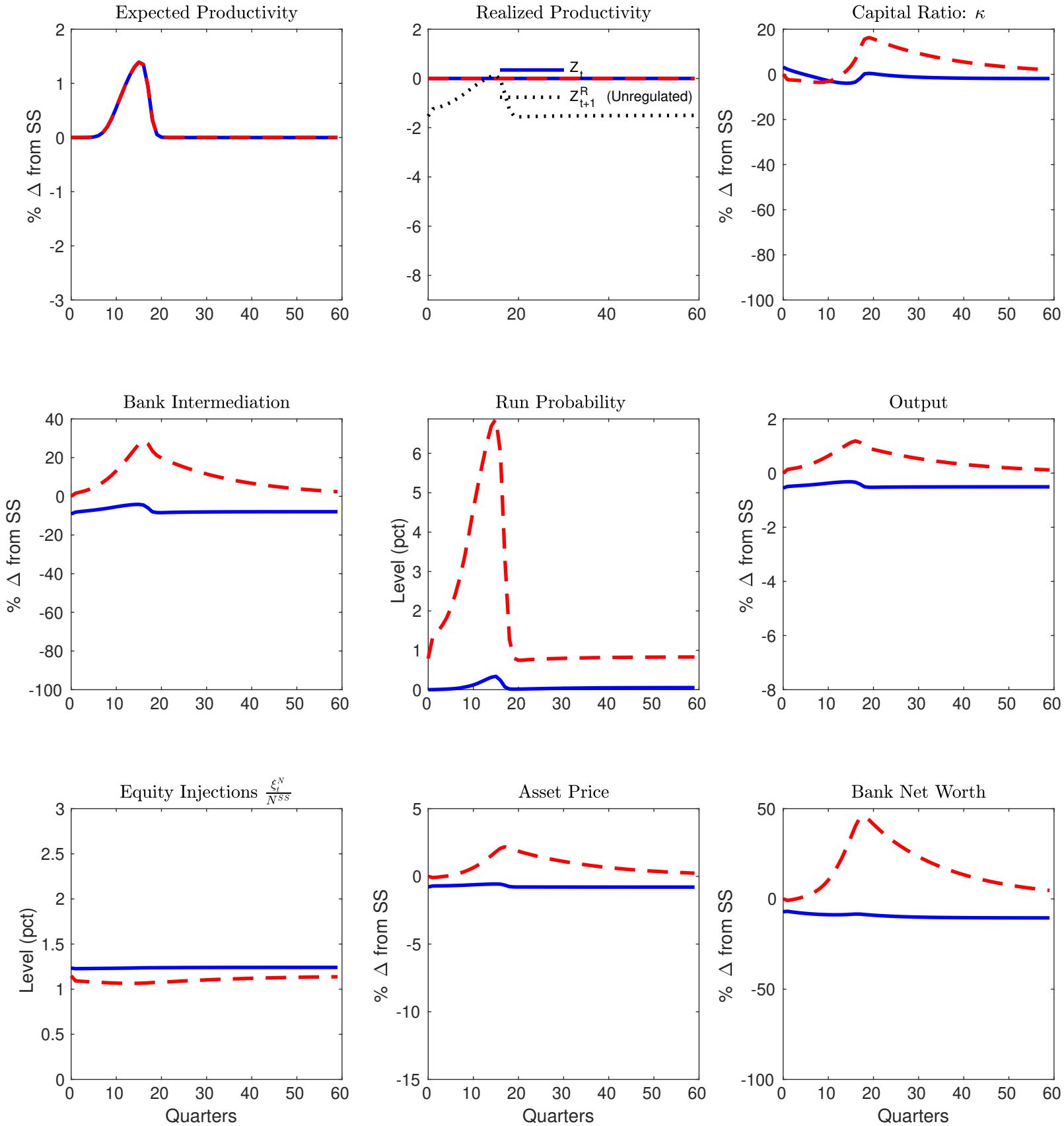
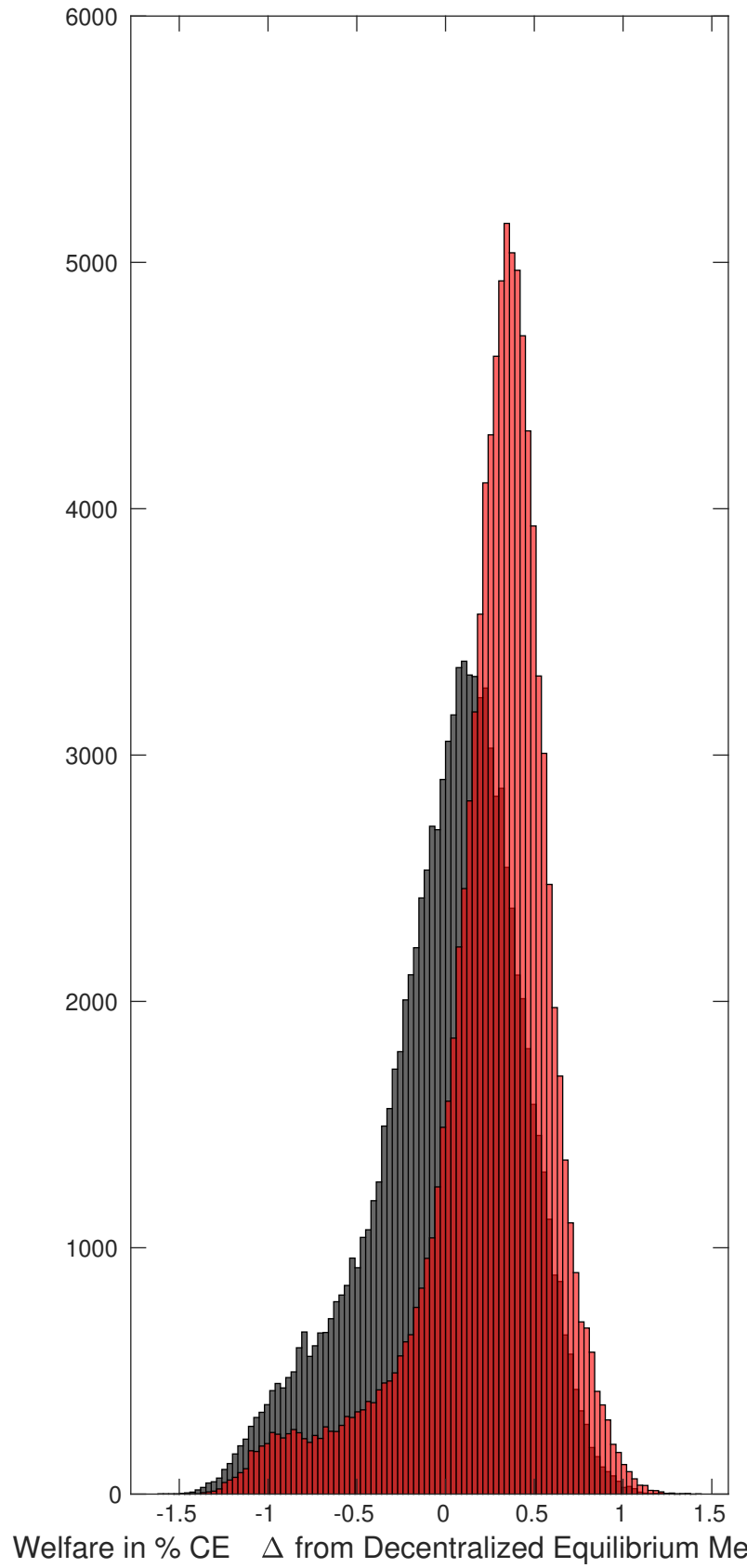
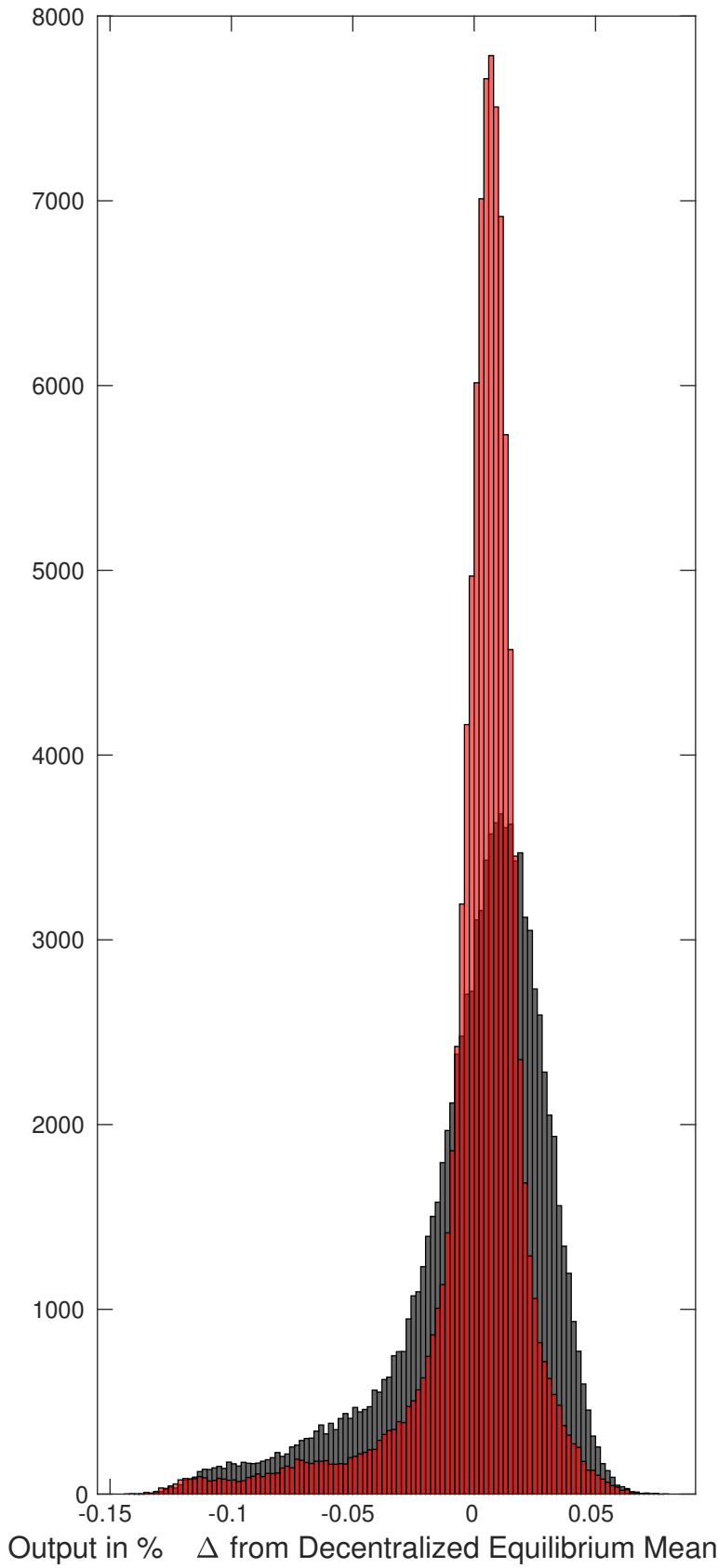


FIGURE 13  
Output and Welfare Distribution

Decentralized Equilibrium      Regulated Equilibrium



**TABLE 3**  
**Effects of Macro Pru**

	Decentralized Equilibrium	Optimal Regulation ( $\bar{\kappa}=0.12$ ; $\bar{N} = 0.8 * N_{SS}^{DE}$ )	Fixed Capital Requirements ( $\bar{\kappa}=0.12$ ; $\bar{N} = 0$ )
Run Frequency	0.9 pct	0.4 pct	0.4 pct
AVG Output Cond No Run $\Delta$ from Decentralized Economy	0 pct	-0.6 pct	-0.7 pct
AVG Output $\Delta$ from Decentralized Economy	0 pct	0.1 pct	-0.9 pct
Welfare Gain $\Delta$ Permanent Consumption	0 pct	0.25 pct	-0.77 pct

FIGURE 14

Recovery from a run: Forgiveness VS No Forgiveness

— Regulated Fixed    - - - Unregulated    ····· Regulated Countercyclical

