Topic 2: Part 3
Introducing Heterogeneity and Borrowing Constraints: Implications for Output Dynamics and the Liquidity Trap

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Debt, Deleveraging and the Liquidity Trap (Eggertsson/Krugman)

- Objective: introduce heterogeneity and lending and borrowing in simple NK model
  - Allow for financial constraints that impede credit flow
  - Illustrate how tightening of financial constraints may reduce aggregate demand
  - By doing, may reduce the natural rate of interest, possibly moving the economy into a liquidity trap and recession
  - Illustrate how the deleveraging process (drawing down of debt) can cause the downturn to persist
- Motivation: tightening of borrowing constraints on households played an important role in Great Recession
  - Decline in house prices limited ability to obtain credit
  - Financial institutions that experienced losses also tightened lending terms

Setup

- Baseline: NK model with consumption goods only
- Two types of agents:
  - Saver: consumes C_t^s and lends the amount D_t in capital market
    - Discount factor of β
  - Borrower: consumes C_t^b and borrows D_t
    - Discount factor of $\gamma < \beta$ (motive for borrowing)
    - Faces borrowing constraint $R_{t+1}D_t \leq \overline{D}_t$
- For simplicity we assume borrowers get the fraction ν of output Y_t as income and savers the fraction $1 - \nu$
  - Goal is to derive IS curve, not complete model
- We also restrict attention to real debt, but discuss the implications of nominal debt and deflation (which raises real debt burdens).

Borrower Behavior

No uncertainty, abstract from labor supply - deterministic problem

- Objective

$$\max_{\{C^b_{t+i}, D_{t+i}\}_{i\geq 0}} E_t \sum_{i=0}^{\infty} \gamma^i \log C^b_{t+i}$$

- Budget constraint

$$C^b_t = \nu Y_t - R_t D_{t-1} + D_t$$

- Borrowing constraint

$$R_{t+1} D_t \leq \overline{D}_t$$

Borrower's Decision Problem

- Bellman equation

$$V_t(R_t D_{t-1}) = \max_{C_t^b, D_t} \log C_t^b + E_t\{\gamma V(R_{t+1} D_t)\}$$

subject to

$$\begin{aligned} C_t^b &= \nu Y_t - R_t D_{t-1} + D_t \\ R_{t+1} D_t &\leq \overline{D}_t \end{aligned}$$

$\Omega_t \equiv$ Lagrange multiplier on borrowing constraint (i.e. the shadow value of increasing the debt limit)

- First order necessary condition for consumption/saving

$$\frac{1}{C_t^b} = R_{t+1}[E_t\{\gamma \frac{1}{C_{t+1}^b}\} + \Omega_t]$$

Solution

- If borrowing constraint does not bind (i.e. $\Omega_t = 0$)

$$\frac{1}{C_t^b} = R_{t+1} E_t \{ \gamma \frac{1}{C_{t+1}^b} \}$$

- If constraint binds (i.e. $\Omega_t > 0$)

$$C_t^b = \nu Y_t - \overline{D}_{t-1} + \overline{D}_t / R_{t+1}$$

- Note:
    - Constraint more likely to bind, the lower the discount factor γ
    - Tightening the borrowing limit $\overline{D}_t$ reduces C_t^b
    - Conversely, lower inherited debt $\overline{D}_{t-1}$ raises C_t^b

Saver Behavior

- Objective

$$\max_{\{C^s_{t+i}, D_{t+i}\}_{i\geq 0}} E_t \sum_{i=0}^{\infty} \beta^i \log C^s_{t+i}$$

- Budget constraint

$$C^s_t = (1-\nu)Y_t + R_t D_{t-1} - D_t \rightarrow$$

First order necessary condition for consumption/saving

$$\frac{1}{C^s_t} = R_{t+1} E_t \{\beta \frac{1}{C^s_{t+1}}\}$$

Note $\beta > \gamma \rightarrow$ stronger incentive to save than for borrower

Equilibrium (taking output as given for now)

- Resource constraint

$$
\begin{aligned}
Y_t &= C_t \\
&= C_t^s + C_t^b
\end{aligned}
$$

- Saver behavior

$$
\frac{1}{C_t^s} = R_{t+1} E_t \{\beta \frac{1}{C_{t+1}^s}\}
$$

- Borrower behavior (assuming borrowing constraint is binding)

$$
C_t^b = \nu Y_t - \overline{D}_{t-1} + \overline{D}_t / R_{t+1}
$$

Deterministic Steady State

- From saver behavior

$$\frac{1}{C^s} = R\beta\frac{1}{C^s} \rightarrow$$
$$1 = R\beta$$

- From borrower

$$C^b = \nu Y - \overline{D} + \overline{D}/R \rightarrow$$
$$C^b = \nu Y - \frac{R-1}{R}\overline{D}$$

- From resource constraint and borrower

$$C^s = Y - C^b$$
$$= (1-\nu)Y + \frac{R-1}{R}\overline{D}$$

Given Y, C^b varies inversely with $\overline{D}$ and C^s positively.

The Short Run, Deleveraging and the Liquidity Trap

- Derive IS curve (a relation for Y conditional on R) from saver's Euler equation

$$C_t^s = (\beta R_{t+1})^{-1} E_t C_{t+1}^s \rightarrow$$

$$Y_t - C_t^b = (\beta R_{t+1})^{-1} E_t (Y_{t+1} - C_{t+1}^b) \rightarrow$$

$$Y_t = (\beta R_{t+1})^{-1} E_t Y_{t+1} + C_t^b - (\beta R_{t+1})^{-1} E_t C_{t+1}^b$$

$$\begin{aligned} Y_t &= (\beta R_{t+1})^{-1} E_t Y_{t+1} \\ &\quad + \nu Y_t - \overline{D}_{t-1} + \overline{D}_t / R_{t+1} - (\beta R_{t+1})^{-1} E_t (\nu Y_{t+1} - \overline{D}_t + \overline{D}_{t+1} / R_{t+2}) \end{aligned}$$

$$\begin{aligned} Y_t &= (\beta R_{t+1})^{-1} E_t Y_{t+1} \\ &\quad + \frac{1}{1-\nu} [-\overline{D}_{t-1} + (1 + \beta^{-1}) \overline{D}_t / R_{t+1} - (\beta R_{t+1})^{-1} E_t (\overline{D}_{t+1} / R_{t+2})] \end{aligned}$$

IS Curve with Debt Constraints

$$
\begin{aligned}
Y_t &= (\beta R_{t+1})^{-1} E_t Y_{t+1} \\
&\quad + \frac{1}{1-\nu}[-\overline{D}_{t-1} + (1+\beta^{-1})\overline{D}_t / R_{t+1} - (\beta R_{t+1})^{-1} E_t(\overline{D}_{t+1}/R_{t+2})]
\end{aligned}
$$

$\frac{1}{1-\nu}$ is multiplier effect, arises because C_t^b depends on Y_t

- Debt constraint affects position of IS curve. Given R_{t+1}, R_{t+2}
  - Increased debt overhang reduces output $\overline{D}_{t-1} \uparrow \rightarrow C_t^b \downarrow \rightarrow Y_t \downarrow$
  - Tightening of borrowing limit reduces output $\overline{D}_t \downarrow \rightarrow C_t^b \downarrow$ and $C_t^s \downarrow$ (the latter because $C_{t+1}^s \downarrow$) $\rightarrow Y_t \downarrow$

"Deleveraging" Shock and the Liquidity Trap

- Determination of natural rate of interest R^*_{t+1} :

$$Y^*_t = (\beta R^*_{t+1})^{-1} E_t Y^*_{t+1} + \frac{1}{1-\nu}[-\overline{D}_{t-1} + (1+\beta^{-1})\overline{D}_t / R^*_{t+1} - E_t(\overline{D}_{t+1}/\beta R^*_{t+1} R^*_{t+2})]$$

  where $Y^*_t \equiv$ natural rate of output
- Deleveraging shock $\equiv$ tightening of borrowing limit which forces a reduction in leverage: $\rightarrow$ drop in $\overline{D}_t$
- Drop in $\overline{D}_t$ induces drop in R^*_{t+1}
  - Intuitively: $\overline{D}_t \downarrow$ induces drop in spending. R^*_{t+1} must fall to induce an increase in saver spending to make $Y_t = Y^*_t$.
- If the drop is large enough, R^*_{t+1} goes below unity $\rightarrow$ ZLB binds.
- With nominal debt, a fall in the price level raises the inherited real debt burden $\overline{D}_{t-1} \rightarrow$ spiral of output contraction and deflation.

Some Issues

- Borrowing constraint exogenous
- Debt and debt dynamics exogenous (driven by exogenous variation in debt constraint)
  - Except when debt is in nominal terms, i.e. $D_t = \frac{D_t^n}{P_t}$ where D_t^n is the nominal value of the debt. As the economy weakens, the price level falls, raising real debt burdens. This induces a further decline in output, and so on.
- An MPC of unity for constrained borrowers seems unrealistic. Borrowers may use some of extra income to pay down debt.
  - Will happen in an environment with uncertainty as to whether the constraint will be binding.
  - Can have "precautionary" saving (i.e. building up buffer of liquid assets) to limit impact of constraint if it becomes binding. (Will also have precautionary saving with transitory income uncertainty.)
- If alternative saving vehicles are available, tightening of household borrowing constraint will not push natural rate to zero
  - With borrowers constrained, savers will substitute to these alternative assets with modest declines in real rates.
  - Unless there are frictions in supplying funds to these sectors.