Topic 3

Welfare and Optimal Monetary Policy

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Overview

Solve for and characterize optimal monetary policy rule

Use the utility of the representative agent as the welfare criteria

Consider optimum under discretion versus commitment

- What are the gains from commitment?
- Solution for commitment is application of Ramsey problem

Conclusions depend on scenario:

- Whether short run inflation/output tradeoff exists
- Whether steady state is efficient
- Whether economy is in a liquidity trap.

Output/Inflation Tradeoffs

Consider baseline model with both productivity and demand shocks
Let $\omega = \frac{(1-\theta)(1-\beta\theta)}{\theta}$: Inflation given by

$$\pi_t = \omega mc_t + \beta E_t \pi_{t+1}$$

Assuming NO labor market frictions (i.e., $w_t - p_t = \varphi l_t + \gamma c_t$):

$$\begin{aligned} mc_t &= -\widehat{\mu}_t = \kappa(y_t - y_t^*) = \kappa \widetilde{y}_t \\ \pi_t &= \lambda \widetilde{y}_t + \beta E_t \pi_{t+1} \\ &= E_t \sum_{i=0}^{\infty} \beta^i \lambda \widetilde{y}_{t+i} \end{aligned}$$

$\lambda = \omega\kappa, \kappa \equiv (\gamma - 1) + \frac{1+\varphi}{1-\alpha}, \widehat{\mu}_t \equiv \mu_t - \mu^*$ ($\mu^* =$ desired markup)
If CB can commit to $\widetilde{y}_{t+i} = 0 \rightarrow$ no short run tradeoff. ("divine co-incidence")

Labor Market Frictions

Adding labor market friction (simple 1st pass)

- Let $\widehat{\mu}_t^w \equiv \mu_t^w - \mu^{w*}$ be the log deviation wage markup from ss.

$$w_t - p_t = \widehat{\mu}_t^w + \varphi l_t + \gamma c_t$$

(Note: in model w/o labor market frictions, μ^{w*} and $\widehat{\mu}_t^w = 0$)
With $w_t - p_t$ sticky and l_t, c_t procyclical $\rightarrow$ $\widehat{\mu}_t^w$ is countercyclical.

For now we take $\widehat{\mu}_t^w$ as exogenous. Possible to endogenize it by introducing wage rigidity (see Gali ch. 6).

Labor Market Frictions (con't)

wage markup

$$w_t - p_t = \widehat{\mu}_t^w + \varphi l_t + \gamma c_t$$

price markup

$$y_t - l_t = \widehat{\mu}_t^p + w_t - p_t$$

with $-\widehat{\mu}_t^p = -(\mu_t^p - \mu^{p*}) = mc_t = (w_t - p_t) - (y_t - l_t)$
combine equations:

$$\begin{aligned} y_t - l_t &= \widehat{\mu}_t^p + \widehat{\mu}_t^w + \varphi l_t + \gamma c_t \\ &= \widehat{\mu}_t + \varphi l_t + \gamma c_t \end{aligned}$$

with total markup $\widehat{\mu}_t = \widehat{\mu}_t^p + \widehat{\mu}_t^w = (\mu_t^p - \mu^{p*}) + (\mu_t^w - \mu^{w*}) = \mu_t - \mu^*$

Labor Market Frictions (con't)

$\widetilde{y}_t = y_t - y_t^*$
y_t^* is solution for y_t given flexible prices and $\widehat{\mu}_t^w = 0$ (i.e., $\mu_t^w = \mu^{w*}$)

Using the same reasoning as in the baseline model of Topic 2:

$$\widehat{\mu}_t = -\kappa \widetilde{y}_t$$
$$\widehat{\mu}_t^p + \widehat{\mu}_t^w = -\kappa \widetilde{y}_t$$

$$\widehat{\mu}_t^p = -\kappa \widetilde{y}_t - \widehat{\mu}_t^w$$

given $mc_t = -\widehat{\mu}_t^p \rightarrow$

$$mc_t = \kappa \widetilde{y}_t + \widehat{\mu}_t^w$$

- With labor market frictions, mc_t no longer proportionate to $\widetilde{y}_t$

Labor Market Frictions (con't)

$$\pi_t = \omega mc_t + \beta E_t \pi_{t+1}$$
$$= \omega\kappa\widetilde{y}_t + \omega\widehat{\mu}_t^w + \beta E_t \pi_{t+1} \rightarrow$$
$$\pi_t = \lambda\widetilde{y}_t + \beta E_t \pi_{t+1} + u_t$$

with $\lambda = \omega\kappa$ and where the "cost push shock" is given by

$$u_t = \omega\widehat{\mu}_t^w$$

Iterating forward

$$\pi_t = E_t \left\{ \sum_{i=0}^{\infty} \beta^i \left[\lambda\widetilde{y}_{t+i} + u_{t+i} \right] \right\}$$

Hence π_t depends on both $\widetilde{y}_{t+i}$ and $u_{t+i} \rightarrow$ short run tradeoff.
Alternative way to motivate u_t: fluctuations in desired price markup μ^{p*} (Gali ch.5).

Model Conditional on Path of r_t^n

Given $\widetilde{y}_t = y_t - y_t^*$

$$y_t - y_t^* = -\sigma[(r_t^n - E_t\pi_{t+1}) - r_{t+1}^*] + E_t\{y_{t+1} - y_{t+1}^*\}$$

$$\pi_t = \lambda(y_t - y_t^*) + \beta E_t\pi_{t+1} + u_t$$

$$y_t^* = \tfrac{1+\varphi}{1+\varphi+(\gamma-1)(1-\alpha)} a_t$$

$$r_{t+1}^* = \rho + \tfrac{1}{\sigma}\tfrac{1+\varphi}{1+\varphi-(\gamma-1)(1-\alpha)}(E_t a_{t+1} - a_t) + (b_t - E_t b_{t+1})$$

where u_t, a_t and b_t all follow exogenous stationary first order processes:

$$\begin{aligned} a_t &= \rho_a a_{t-1} + \varepsilon_t^a \\ b_t &= \rho_b b_{t-1} + \varepsilon_t^b \\ u_t &= \rho_u u_{t-1} + \varepsilon_t^u \end{aligned}$$

Undistorted Natural (Flexible Price) Equilibrium

Deterministic steady state with production subsidy s per output unit.

$$(1-\alpha)\frac{Y_t^*}{L_t^*}(1+s) = (1+\mu^{p*})\frac{W_t^*}{P_t^*}$$
$$\frac{W_t^*}{P_t^*} = (1+\mu^{w*})\frac{L_t^{*\varphi}}{C_t^{*-\gamma}}$$

$\rightarrow$

$$(1-\alpha)\frac{Y_t^*}{L_t^*}(1+s) = (1+\mu^{p*})(1+\mu^{w*})\frac{L_t^{*\varphi}}{C_t^{*-\gamma}}$$
$$(1-\alpha)\frac{Y_t^*}{L_t^*}(1+s) = (1+\mu^*)\frac{L_t^{*\varphi}}{C_t^{*-\gamma}}$$

Let $s = \mu \rightarrow$ flexible price equilibrium is first best

$$(1-\alpha)\frac{Y_t^*}{L_t^*} = \frac{L_t^{*\varphi}}{C_t^{*-\gamma}}$$

$\rightarrow Y_t^o = Y_t^*$, efficient level of output

The policy objective

- Household preferences

$$U_t = E_t \sum_{i=0}^{\infty} \beta^i \left[\frac{1}{1-\gamma} C_{t+i}^{1-\gamma} + \frac{a_m}{1-\gamma_m} \left(\frac{M_{t+i}}{P_{t+i}} \right)^{1-\gamma_m} - \frac{1}{1+\varphi} L_{t+i}^{1+\varphi} \right]$$

- At the cashless limit (limit as $a_m \to 0$)

$$U_t = E_t \sum_{i=0}^{\infty} \beta^i \left[\frac{1}{1-\gamma} C_{t+i}^{1-\gamma} - \frac{1}{1+\varphi} L_{t+i}^{1+\varphi} \right]$$

The policy objective (con't)

- Suppose the natural eq. is undistorted (e.g. firms receive a production subsidy that offsets the steady state markup) so $\widetilde{y}_t = y_t - y_t^* = y_t - y_t^o$.
- A quadratic approximation of the HH objective function about the steady state combined with a first order approximation of the model (to replace c and n with y - see Gali ch.4) yields

$$\frac{U_t - \bar{U}}{u_c \bar{C}} \propto -\frac{1}{2} E_t \left\{ \sum_{i=0}^{\infty} \beta^i \left[\kappa (\widetilde{y}_{t+i})^2 + \frac{\epsilon}{\omega} \pi_{t+i}^2 \right] \right\} + t.i.p.$$

$$\propto -\frac{1}{2} E_t \left\{ \sum_{i=0}^{\infty} \beta^i \left[\eta (\widetilde{y}_{t+i})^2 + \pi_{t+i}^2 \right] \right\} + t.i.p.$$

where $t.i.p. \equiv$ terms independent of policy and η is given by

$$\eta = \frac{\omega\kappa}{\epsilon} = \frac{\lambda}{\epsilon}$$

- The inflation term reflects the efficiency loss from the dispersion of relative prices.

The policy objective (con't)

- Using a first order approximation of the model to replace c and n with y in the objective is valid only if linear terms are not present in the approximation. Otherwise a second order approximation is needed.
  - A linear term will be present if the optimal equilibrium $>$ flexible price equilibrium, i.e., $y_t^o > y_t^*$. (The production subsidy rules this out.)
  - With linear terms in the objective and a linear approximation of the model, errors in the linear approximation will be of the same magnitude as the second order terms in the objective
  - If the linear term in the objective is "small", a linear approximation of the model is reasonable (see Gali ch.5)

The policy problem (with an undistorted natural eq.)

$$\max_{\{r^n_{t+i},\widetilde{y}_{t+i},\pi_{t+i}\}_{i=0}^{\infty}} -\frac{1}{2}E_t\left\{\sum_{i=0}^{\infty}\beta^i\left[\eta\widetilde{y}^2_{t+i}+\pi^2_{t+i}\right]\right\} \tag{1}$$

subject to

$$\begin{aligned}
\pi_{t+i} &= \lambda\widetilde{y}_{t+i}+\beta E_{t+i}\pi_{t+i+1}+u_{t+i} &(2)\\
\widetilde{y}_{t+i} &= -\sigma(r^n_{t+i}-E_{t+i}\pi_{t+i+1}-r^*_{t+i+1})+E_t\widetilde{y}_{t+i+1} &(3)\\
u_{t+i+1} &= \rho_u u_{t+i}+\varepsilon^u_{t+i} &(4)
\end{aligned}$$

with $0\leq\rho_u<1$ and assume ε^u_{t+i} is an i.i.d. random variable with zero mean.

Note r^*_{t+i+1} incorporates the effect of both the demand b_t and productivity a_t shocks

The policy problem (con't)

Given the recursive structure, the policy problem can be solved in two stages:
First, choose $\widetilde{y}_t$ and π_t to solve:

$$\max_{\{\widetilde{y}_{t+i},\pi_{t+i}\}_{i=0}^{\infty}} -\frac{1}{2}E_t\left\{\sum_{i=0}^{\infty}\beta^i\left[\eta\widetilde{y}_{t+i}^2+\pi_{t+i}^2\right]\right\}$$

s.t.

$$\pi_{t+i}=\lambda\widetilde{y}_{t+i}+\beta E_{t+i}\pi_{t+i+1}+u_{t+i}$$

and the exogenous process for u_t.
Second, given $\widetilde{y}_t$ and π_t, find r_t^n to solve:

$$\widetilde{y}_{t+i}=-\sigma(r_{t+i}^n-E_{t+i}\pi_{t+i+1}-r_{t+i+1}^*)+E_{t+i}\widetilde{y}_{t+i+1}$$

The policy problem (con't)

- Policy problem in the tradition of the classic Tinbergen-Theil targets and instruments problem:
- The combination of a quadratic loss function and linear constraints yields a certainty equivalent decision rule for the path of the instrument.
- Important difference: Target variables depend not only on the current policy but also on expectations about future policy.

$$\begin{aligned}\pi_t &= E_t \sum_{i=0}^{\infty} \beta^i (\lambda \widetilde{y}_{t+i} + u_{t+i}) \\ \widetilde{y}_t &= -\sigma E_t \sum_{i=0}^{\infty} (r_{t+i}^n - \pi_{t+i+1} - r_{t+i+1}^*)\end{aligned}$$

- Raises issues of credibility and time consistency of policy.

Discretion vs. Commitment

- *Discretion.* A policy maker (e.g., central bank) operating under discretion chooses the current interest rate by reoptimizing in every period, without committing to future choices. As such, it is not compelled to honor any past promises.

- *Commitment.*The central bank commits to a plan for the path of interest rates, that may be a function of future state realizations, and then it sticks to it forever (Ramsey policy).Here, past promises matter.

- The key distinction between discretion and commitment is whether the policy maker can, or cannot, commit to future plans in a credible way.

Discretion

Two step process: 1. Central bank optimizes at t given beliefs about future 2. Given central bank decision rule, private sector forms beliefs (rational expectations).
In each period, the central bank chooses $\widetilde{y}_t$ and π_t to maximize

$$-\frac{1}{2}[\eta\widetilde{y}_t^2 + \pi_t^2] + F_t$$

subject to

$$\pi_t = \lambda\widetilde{y}_t + f_t$$

with

$$F_t = -\frac{1}{2}E_t[\sum_{i=1}^{\infty}\beta^i(\eta\widetilde{y}_{t+i}^2 + \pi_{t+i}^2)]$$
$$f_t = \beta E_t\pi_{t+1} + u_t$$

where the central bank takes f_t and F_t as given.

Discretion (con't)

- The FONC yields the following feedback policy:

$$\widetilde{y}_t = -\frac{\lambda}{\eta}\pi_t \tag{5}$$

- The targeting rule (5) implies that the central bank should pursue a "lean against the wind" policy: whenever inflation is above target, contract demand below capacity by raising the interest rate, and vice-versa when it is below target.

- How aggressively the central bank should reduce $\widetilde{y}_t$ depends positively on the gain in reduced inflation per unit of output loss, λ, and inversely on the relative weight placed on output loss, η.

Discretion (con't)

- To solve for the equilibrium values of π_t and $\widetilde{y}_t$ under discretion, combine the targeting rule with the AS curve to get:

$$\pi_t = -\frac{\lambda^2}{\eta}\pi_t + \beta E_t\pi_{t+1} + u_t = \frac{\eta\beta}{\eta+\lambda^2}E_t\pi_{t+1} + \frac{\eta}{\eta+\lambda^2}u_t$$

Iterating forward:

$$\pi_t = \frac{\eta}{\eta+\lambda^2}E_t\sum_{i=0}^{\infty}\left(\frac{\eta\beta}{\eta+\lambda^2}\right)^i u_{t+i}$$

Since $E_t u_{t+i} = \rho_u^i u_t$, we finally have:

$$\begin{aligned} \pi_t &= \eta q u_t \\ \widetilde{y}_t &= -\lambda q u_t \end{aligned}$$

where $q = 1/[\lambda^2 + \eta(1-\beta\rho_u)]$.

Discretion (con't)

- We can now derive the optimal feedback policy for the interest rate r_t^n by substituting the desired value of $\widetilde{y}_t$ and π_t into the IS curve:

$$r_t^n = \phi_\pi E_t \pi_{t+1} + r_{t+1}^*$$

where

$$\phi_\pi = 1 + \frac{(1-\rho_u)\lambda}{\rho_u \sigma \eta} > 1$$

- The central bank adjusts the interest rate more than one-to-one with respect to expected inflation $E_t\pi_{t+1}$ (since $\phi_\pi > 1$). Intuitively: If inflation is above target, the optimal policy requires raising real rates $(r_t^n - E_t\pi_{t+1})$ to contract demand. Note ϕ_π varies inversely with the relative weight on output, η.

The classic inflationary bias problem

A theory of high trend inflation based on lack of central bank credibility.
As we have seen, when the flexible price equilbrium is Pareto optimal the policy maker has no reason to push output y_t higher than the natural equilibrium y_t^*.
If we relax this assumption by eliminating the labor subsidy, we have a discrepancy between the natural level of output and the Pareto optimal level:

$$y_t^o = y_t^* + k$$

From the loglinear flexible price equilibrium with and without the subsidy:

$$k = \frac{1}{\kappa}\mu = \frac{1}{\gamma - 1 + \frac{1+\varphi}{1-\alpha}}\mu$$

As a result:

$$y_t - y_t^o = (y_t - y_t^*) - k = \widetilde{y}_t - k.$$

The classic inflationary bias problem(con't)

Thus the bliss point for the output gap $\widetilde{y}_t$ is positive. (Since $y_t = y_t^o$ is bliss point $\rightarrow$ bliss point for $\widetilde{y}_t = y_t - y_t^* = k > 0$.)

In this instance, under discretion steady state inflation may be inefficiently high, as originally emphasized by Kydland and Prescott (1977) and Barro and Gordon (1983) and many others.

This scenario proposed to explain persistently high inflation during the 1970s.

Welfare function: Distorted Natural Eq.

- Following Gali, if the natural eq. distortion is not too "large", we may approximate the objective function as

$$\max_{\{\widetilde{y}_{t+i},\pi_{t+i}\}_{i=0}^{\infty}} -\frac{1}{2}E_t\left\{\sum_{i=0}^{\infty}\beta^i\left[\eta(\widetilde{y}_{t+i})^2-2\eta k\widetilde{y}_{t+i}+\pi_{t+i}^2\right]\right\}+t.i.p.$$

which is equivalent to maximizing

$$\max_{\{\widetilde{y}_{t+i},\pi_{t+i}\}_{i=0}^{\infty}} -\frac{1}{2}E_t\left\{\sum_{i=0}^{\infty}\beta^i\left[\eta(\widetilde{y}_{t+i}-k)^2+\pi_{t+i}^2\right]\right\}+t.i.p.$$

- subject to the Phillips curve constraint as:

$$\pi_{t+i}=\lambda\widetilde{y}_{t+i}+\beta E_{t+i}\pi_{t+i+1}+u_{t+i}$$

Optimal Policy Under Discretion: Distorted Natural Eq.

- Under discretion, the problem becomes:

$$\max_{\widetilde{y}_t,\pi_t} -\frac{1}{2}[\eta(\widetilde{y}_t - k)^2 + \pi_t^2] + F_t$$

subject to

$$\pi_t = \lambda\widetilde{y}_t + f_t$$

where

$$\begin{aligned} F_t &= -\frac{1}{2}E_t[\sum_{i=1}^{\infty}\beta(\eta(\widetilde{y}_{t+i} - k)^2 + \pi_{t+i}^2)] \\ f_t &= \beta E_t\pi_{t+1} + u_t \end{aligned}$$

Optimal Policy Under Discretion: Distorted Natural Eq. (con't)

- The FONC is then:

$$\widetilde{y}_t - k = -\frac{\lambda}{\eta}\pi_t \tag{6}$$

- Proceeding as before we obtain the equilibrium values for π_t and $\widetilde{y}_t$ under discretion:

$$\pi_t = \eta q u_t + \frac{\lambda\eta}{\lambda^2 + \eta(1-\beta)}k \tag{7}$$

$$\widetilde{y}_t = -\lambda q u_t + \frac{(1-\beta)\eta k}{\lambda^2 + \eta(1-\beta)} \tag{8}$$

- Thus, there is a positive inflationary bias ($\pi_t > 0$ as u_t goes to zero), but as $\beta \to 1$, no difference in output with respect to the baseline case.
    - In SS, $\pi = \frac{\lambda}{1-\beta}\widetilde{y} \longleftrightarrow (1-\beta)\pi = \lambda\widetilde{y} \to \widetilde{y} \to 0$ as $\beta \to 1$.

Optimal Policy Under Discretion: Distorted Steady State (con't)

Since β is close to unity, we can gain intuition from the case where $\beta \approx 1$

- In steady state with $\beta \approx 1$

$$\begin{aligned} \pi &= \lambda \widetilde{y} + \pi \\ \widetilde{y} &= 0 \end{aligned}$$

$\rightarrow$ No long run inflation/output tradeoff

- Given targeting rule, optimal policy under discretion

$$\begin{aligned} \pi_t &= \eta q u_t + \frac{\eta}{\lambda} k \\ \widetilde{y}_t &= -\lambda q u_t \end{aligned}$$

Optimal Policy Under Discretion: Distorted Steady State (con't)

$$\pi_t = \eta q u_t + \frac{\eta}{\lambda} k$$
$$\widetilde{y}_t = -\lambda q u_t$$

$\rightarrow$ Positive steady state inflationary bias

$\rightarrow$ Cyclical behavior of π_t and $\widetilde{y}_t$ unchanged from case of undistorted steady state

- Interest rate policy that supports this bias:

$$r_t^n = \phi_\pi (E_t \pi_{t+1} - \frac{\eta}{\lambda} k) + r_{t+1}^* + \frac{\eta}{\lambda} k$$

CB effectively accepts inflation target $\frac{\eta}{\lambda} k > 0$

Institutional Solutions to the Inflationary Bias Problem

- Rogoff (1985) suggests to appoint a "conservative" central banker, who assigns little if not zero weight to the output gap ($\eta \approx 0$), in order to reduce the inflationary bias (to zero at the limit).
    - However, there will be very inefficient responses to shocks (see equation (7)).

- Blinder (1997): Alternatively, the central bank can commit to treating $k = 0$ in the objective in order to achieve the equilibrium allocation in the baseline model under discretion.
    - But, (i) k is not observable and (ii) sidesteps issue of establishing commitment.

- Inflation targeting. Commit to zero (or slightly positive) steady state inflation:
    - Support with Taylor rule that has desired inflation target $\pi^0 < \frac{\eta}{\lambda} k$

$$r_t^n = \phi_\pi (E_t \pi_{t+1} - \pi^o) + r_{t+1}^* + \pi_0$$

The gains from commitment

- By committing to a state-contingent policy (i.e. a rule), the central bank is able to influence private sector expectations.

- Optimal policy with commitment: Solve for the optimum of the welfare function (1) subject to equations (2), (3) and (4), where the choice of $\widetilde{y}_t$ and π_t potentially depends on the entire history of shocks (Ramsey).

- Form the Lagrangian (general case with $k > 0$):

$$L = -E_t \left\{ \sum_{i=0}^{\infty} \beta^i [\frac{1}{2}(\eta(\widetilde{y}_{t+i} - k)^2 + \pi_{t+i}^2) + \xi_{t+i}(\pi_{t+i} - \lambda \widetilde{y}_{t+i} - \beta \pi_{t+i+1} - u_{t+i})] \right\}$$

  where ξ_{t+i} is the Lagrange multiplier associated with constraint (2).
- Note choice of π_t at t constrained by $E_{t-1}\pi_t$, expectations of π_t at $t-1$. This constraint reflects commitment.

The gains from commitment (con't)

- The FONCs at each time $t+i$ are:

$$\begin{aligned} \frac{\partial L}{\partial \widetilde{y}_{t+i}} &= \eta(\widetilde{y}_{t+i} - k) - \lambda \xi_{t+i} = 0 \\ \frac{\partial L}{\partial \pi_{t+i}} &= \pi_{t+i} + \xi_{t+i} - \xi_{t+i-1} = 0 \qquad \forall\ i \geq 1 \\ \frac{\partial L}{\partial \pi_t} &= \pi_t + \xi_t = 0 \end{aligned}$$

- ξ_{t+i-1} reflects the constraint from commitment on currently policy choice. In the first period ($i=0$), the central bank is not constrained by past behavior.

Rules: the gains from commitment (con't)

- We obtain the following optimality conditions:

$$
\begin{aligned}
\xi_{t+i} &= \frac{\eta}{\lambda}(\widetilde{y}_{t+i} - k) \\
\widetilde{y}_{t+i} - \widetilde{y}_{t+i-1} &= -\frac{\lambda}{\eta}\pi_{t+i} \qquad \forall\ i \geq 1 \\
\widetilde{y}_t - k &= -\frac{\lambda}{\eta}\pi_t
\end{aligned}
$$

- For $i \geq 1$, difference rule for output as opposed to level rule. Exploits impact of expected persistent change in $\widetilde{y}_{t+i}$ on π_t.

$$\pi_t = E_t \sum_{i=0}^{\infty} \beta^i (\lambda \widetilde{y}_{t+i} + u_{t+i})$$

- Equivalent to price level target, given $\pi_t = p_t - p_{t\_1}$.

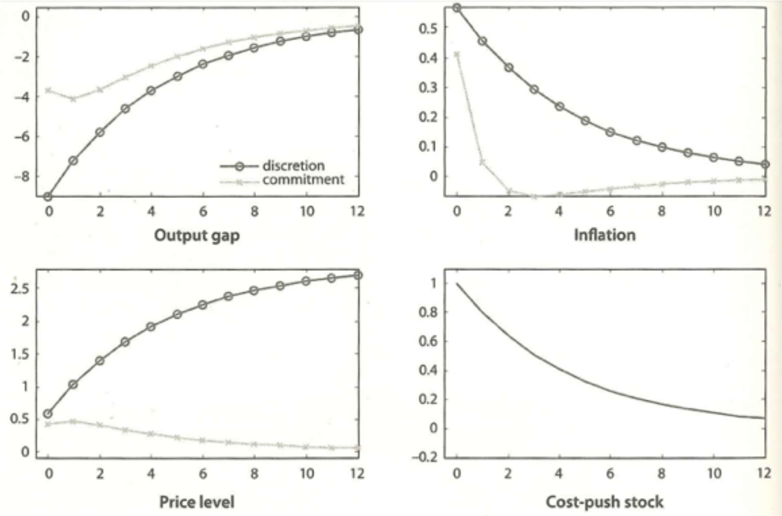
$$\widetilde{y}_{t+i} = -\frac{\lambda}{\eta} p_{t+i} \qquad \forall\ i \geq 1 \qquad \text{with } P^* = 1, \log P^* = 0$$

The gains from commitment (con't)

- Equilibrium for $i \geq 1$ given by targeting rule and Phillips curve

$$\begin{aligned} \widetilde{y}_{t+i} &= -\frac{\lambda}{\eta}\pi_{t+i} + \widetilde{y}_{t+i-1} \\ \pi_{t+i} &= \lambda\widetilde{y}_{t+i} + \beta E_{t+i}\pi_{t+1+i} + u_{t+i} \end{aligned}$$

  - Compared to discretion:
    - no inflationary bias: steady state with $\widetilde{y}_{t+i}$ and $\pi_{t+i} = 0$.
    - History dependence in output from targeting rule $\rightarrow$ smaller movements in $\widetilde{y}_{t+i}$ and π_{t+i} required in response to cost push shock.
- In first period the optimal policy is the same as under discretion (not constrained by past).
  - Woodford's timeless perspective - act as if $i \geq 1$.
  - (Makes commitment more credible.)



Discretion vs. Commitment: Responses to a Persistent Cost-Push Shock

Liquidity Trap

- ZLB: $r_t^n \geq 0$ (recall $r_t^n = \log(1 + r_t^n)$)

- Suppose $r_{t-1}^* > 0$, $r_{t+i}^* < 0$ for $i = 0$ to $k-1$, then $r_{t+i}^* > 0$ afterwards

- $\rightarrow$ the ZLB is binding for k periods:

$$\widetilde{y}_t = -\sigma E_t \sum_{i=0}^{k-1} (-\pi_{t+i+1} - r_{t+i+1}^*) - \sigma E_t \sum_{i=k}^{\infty} (r_{t+i}^n - \pi_{t+i+1} - r_{t+i+1}^*)$$

where $r_{t+i+1}^* < 0$ when the ZLB binds.

Optimal Policy in a Liquidity Trap

- ZLB constraint $r^n_{t+i} \geq 0 \Leftrightarrow$

$$\widetilde{y}_{t+i} \leq -\sigma(-E_{t+i}\pi_{t+i+1} - r^*_{t+i+1}) + E_{t+i}\widetilde{y}_{t+i+1}$$

- When the ZLB is binding, the central bank simply sets $r^n_t = 0$
- How $\widetilde{y}_t$,π_t behave depends on policy expected once outside the liquidity trap
- For simplicity, set k, $u_{t+i} = 0$
- $\rightarrow$ Policy problem:

$$\begin{aligned} L \quad = \quad & -E_t\Big\{\sum_{i=0}^{\infty}\beta^i[\frac{1}{2}(\eta(\widetilde{y}_{t+i})^2 + \pi^2_{t+i}) + \xi_{t+i}(\pi_{t+i} - \lambda\widetilde{y}_{t+i} - \beta\pi_{t+i+1}) + \\ & \Omega_{t+i}(\widetilde{y}_{t+i} - (-\sigma(-E_{t+i}\pi_{t+i+1} - r^*_{t+i+1}) + E_{t+i}\widetilde{y}_{t+i+1})]\Big\} \end{aligned}$$

Optimal Policy in a Liquidity Trap (con't)

For $i = 0$ to $k-1$, $r^n_{t+i} = 0 \rightarrow$

$$\widetilde{y}_{t+j} = -\sigma E_t \sum_{i=j}^{k-1} (-\pi_{t+i+1} - r^*_{t+i+1}) + E_t \widetilde{y}_{t+k}$$

$$\pi_{t+j} = E_t \sum_{i=j}^{k-1} \beta^{i-j} \lambda \widetilde{y}_{t+j} + \beta^{k-j} E_t \pi_{t+k}$$

Key point: $\widetilde{y}_{t+j}$ and π_{t+j} depends on $E_t \widetilde{y}_{t+k}$ and $E_t \pi_{t+k}$. The central bank has leverage over the latter two (since the liquidity trap is over at $t+k$).

Note: if $E_t \widetilde{y}_{t+k} = E_t \pi_{t+k} = 0$, $\widetilde{y}_{t+j-1} < \widetilde{y}_{t+j} < 0$ and $\pi_{t+j-1} < \pi_{t+j} < 0$.

Optimal Policy in a Liquidity Trap (con't)

Start by computing the optimal policy at $t+k$ (first period outside liquidity trap)
i. Take into account that the ZLB is binding in previous periods,i.e., $\Omega_{t+k-j} > 0$
ii. Optimal policy: promise $E_t\widetilde{y}_{t+k}$ and $E_t\pi_{t+k} > 0$ to stimulate the economy while it is in the liqudity trap.
iii. Exact amount depends on gains from stimulus versus costs of deviating from targets at $t+k$.
iv. Check whether optimal policy at violates ZLB (i.e. implies $r_t^n < 0$). If it does not, implement the policy with r_t^n.
Since $r_{t+k}^n > 0$, $\Omega_{t+k} = 0 \rightarrow$ the economy escapes the liquidity trap.
The optimal policy at $t+k+1$ reverts to the standard optimum.
iv. If the ZLB still binds at $t+k$, set $r_{t+k}^n = 0$, and re-optimize at $t+k+1$.
v. Keep going until $r_{t+k+j}^n > 0$ at the optimum.

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Optimal policy at $t+k$, first period outside the liquidity trap

FONCs:

$$\frac{\partial L_{t+k}}{\partial \widetilde{y}_{t+k}} = \eta \widetilde{y}_{t+k} - \lambda \xi_{t+k} - \frac{1}{\beta}\Omega_{t+k-1} = 0$$
$$\frac{\partial L_{t+k}}{\partial \pi_{t+k}} = \pi_{t+k} + \xi_{t+k} - \xi_{t+k-1} - \frac{\sigma}{\beta}\Omega_{t+k-1} = 0$$

Combining equations:

$$\widetilde{y}_{t+k} - \widetilde{y}_{t+k-1} = -\frac{\lambda}{\eta}\pi_{t+k} + \frac{\sigma}{\beta}\Omega_{t+k-1} + \frac{1}{\beta\lambda}(\Omega_{t+k-1} - \Omega_{t+k-2})$$

- If the ZLB did not bind in the previous two periods (i.e. $\Omega_{t+k-1} = \Omega_{t+k-2} = 0$), the policy rule reverts to the standard targeting rule under commitment.

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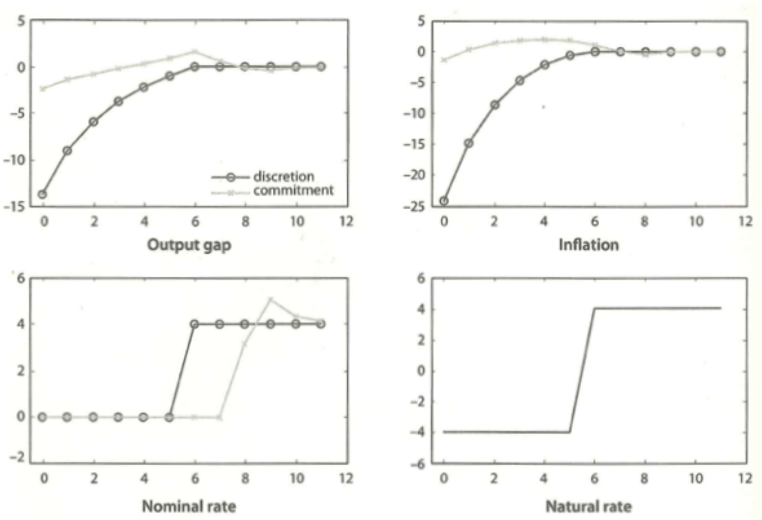
$$\widetilde{y}_{t+k} - \widetilde{y}_{t+k-1} = -\frac{\lambda}{\eta}\pi_{t+k} + \frac{\sigma}{\beta}\Omega_{t+k-1} + \frac{1}{\beta\lambda}(\Omega_{t+k-1} - \Omega_{t+k-2})$$

- If the ZLB binds in the previous period ($\Omega_{t+k-1} > 0$), $\widetilde{y}_{t+k}$ is increasing in Ω_{t+k-1}.
  - i.e. the tighter the ZLB constraint at $t+k-1$, the lower r_t^n should be to push $\widetilde{y}_{t+k}$ higher
  - If Ω_{t+k-1} is sufficiently high, the ZLB constraint could bind at $t+k$.
  - If so, re-optimize at $t+k-1$. Repeat until the ZLB is no longer binding.
- What ensures convergence back to a "normal" equilibrium?
  - $\widetilde{y}_{t+k}$ is decreasing in Ω_{t+k-2}, a factor working to offset the initial stimulus when first outside the liquidity trap at $t+k$.
- Once the ZLB has not been binding for three periods, the central bank reverts to the optimal policy under commitment with a non-binding ZLB.



Discretion vs. Commitment in the Presence of a ZLB