Topic 4
Investment and the Complete NK DSGE

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Investment

Motivation: Durable goods central to business cycles and transmission of monetary policy, but missing in simple NK model.

We now add investment in the form of producer durables (capital)

Straightforward to add residential investment and consumer durables

Working hypothesis: Dynamics of producer durables similar to aggregate investment, including housing and consumer durables.

Three steps:

1. Partial equilibrium model of investment: Tobin's "Q" Theory
2. Integrate "Q" theory of investment into NK model
3. Explore implications for fluctuations and monetary policy, etc.
4. Describe additional modifications needed to match data.

Tobin's Q Theory

Relates investment rate to its "Q" value: i.e. the ratio of the shadow value of a unit of capital within the firm to its replacement cost.
Key assumption: convex costs to a firm of adjusting its capital stock.
$\rightarrow$The value of a unit of installed capital can differ from its replacement cost.
$\rightarrow K_t$ is a predetermined state of the firm
Let $p_t^I \equiv$ replacement cost of capital.
Then the cost of acquiring I_t units of capital is given by

$$p_t^I(I_t + \frac{1}{2}c(\frac{I_t}{K_t})^2 K_t)$$

Where $p_t^I \frac{1}{2}c(\frac{I_t}{K_t})^2 K_t \equiv$ adjustment costs (which we assume are in units of output).
Note adjustment costs are constant returns to scale.

Tobin's Q Theory: Optimization Problem

Production

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha}$$

Capital

$$K_{t+1} = I_t + (1-\delta)K_t$$

Profits

$$\Pi_t = Y_t - W_t L_t - p_t^I I_t - \frac{1}{2} p_t^I c (\frac{I_t}{K_t})^2 K_t$$

Objective: Let $\Lambda_{t,t+i} \equiv$ stochastic discount factor

$$\max_{Y_t, L_t, I_t K_{t+1}} E_t \{ {}_{i=0}^{\infty} \Lambda_{t,t+i} \Pi_{t+i} \}$$

Optimization Problem (con't)

Bellman's equation.

$$V(K_t, A_t) = \max_{Y_t, L_t, I_t K_{t+1}} \Pi_t + E_t\{\Lambda_{t,t+1} V(K_{t+1}, A_{t+1})\}$$

subject to

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha}$$

$$K_{t+1} = I_t + (1-\delta)K_t$$

$$\Pi_t = Y_t - W_t L_t - p_t^I I_t - \frac{1}{2} p_t^I c\left(\frac{I_t}{K_t}\right)^2 K_t$$

Optimization Problem (con't)

Eliminating the constraints:

$$\begin{aligned} V(K_t, A_t) &= \max_{L_t, I_t} A_t K_t^{\alpha} L_t^{1-\alpha} - W_t L_t - p_t^I (I_t + \frac{1}{2} c (\frac{I_t}{K_t})^2 K_t) \\ &\quad + E_t\{\Lambda_{t,t+1} V(I_t + (1-\delta) K_t, A_{t+1})\} \end{aligned}$$

fonc

$$(1-\alpha)\frac{Y_t}{L_t} = W_t$$

$$p_t^I (1 + c\frac{I_t}{K_t}) = E_t\{\Lambda_{t,t+1} V_1(K_{t+1}, A_{t+1})\}$$

$\Lambda_{t,t+1} V_1(K_{t+1}, A_{t+1}) \equiv$ discounted marginal value of a unit capital.
Note: $\frac{\partial K_{t+1}}{\partial I_t} = 1$

Tobin's Q Theory of Investment

Tobin's Q: Ratio of discounted marginal value of installed capital to its replacement cost

$$Q_t = \frac{E_t\{\Lambda_{t,t+1} V_1(K_{t+1}, A_{t+1})\}}{p_t^I}$$

Rearranging fonc for investment

$$\frac{I_t}{K_t} = \frac{1}{c}(Q_t - 1)$$

$\rightarrow$Investment depends positively on the difference between Q_t and unity

Sensitivity depends inversely on adjustment cost parameter c

Note it is the marginal adjustment cost that yields a wedge between Q_t and unity.

Otherwise firms would invest to the point where $Q_t = 1$

Need to find an expression for shadow value $E_t\{\Lambda_{t,t+1} V_1(K_{t+1}, A_{t+1})\}$

Tobin's Q Theory of Investment (con't)

Substituting for Q_t :

$$\frac{I_t}{K_t} = \frac{1}{c}\left(\frac{E_t\{\Lambda_{t,t+1}V_1(K_{t+1},A_{t+1})\}}{p_t^I} - 1\right)$$

Need expression for $V_1(K_{t+1}, A_{t+1})$
Envelope condition (Note $\frac{\partial K_{t+1}}{\partial K_t} = 1 - \delta$)

$$V_1(K_t, A_t) = \alpha\frac{Y_t}{K_t} + \frac{1}{2}p_t^I c\left(\frac{I_t}{K_t}\right)^2 + (1-\delta)E_t\{\Lambda_{t,t+1}V_1(K_{t+1}, A_{t+1})\}$$

$$V_1(K_t, A_t) = E_t\{\sum_{i=0}^{\infty}\Lambda_{t,t+i}(1-\delta)^i[\alpha\frac{Y_{t+i}}{K_{t+i}} + \frac{1}{2}p_{t+i}^I c\left(\frac{I_{t+i}}{K_{t+i}}\right)^2]\}$$

$\rightarrow$

$$E_t\{\Lambda_{t,t+1}V_1(K_{t+1}, A_{t+1})\} = E_t\{\sum_{i=1}^{\infty}\Lambda_{t,t+i}(1-\delta)^{i-1}[\alpha\frac{Y_{t+i}}{K_{t+i}} + \frac{1}{2}p_{t+i}^I c\left(\frac{I_{t+i}}{K_{t+i}}\right)^2]\}$$

$=$ discounted sum of rental earnings and savings on adjustment costs.

Marginal vs. Average Q:

Marginal Q is not directly observable

Need series on expected rental earnings and adjustment costs

However, with constant returns, marginal Q = average Q (Hayashi)

Average Q measured as ratio of market value to book value of capital.

Let $q_t = V_1(K_t, A_t)$. Then

$$q_t = \alpha \frac{Y_t}{K_t} + \frac{1}{2} p_t^I c(\frac{I_t}{K_t})^2 + (1-\delta) E_t\{\Lambda_{t,t+1} q_{t+1}\}$$

Multiply each side by K_t :

$$q_t K_t = \alpha Y_t + \frac{1}{2} p_t^I c(\frac{I_t}{K_t})^2 K_t + (1-\delta) E_t\{\Lambda_{t,t+1} q_{t+1} K_t\}$$

Since $(1-\delta)K_t = K_{t+1} - I_t$:

$$q_t K_t = \alpha Y_t + \frac{1}{2} p_t^I c(\frac{I_t}{K_t})^2 K_t - E_t\{\Lambda_{t,t+1} q_{t+1} I_t\} + E_t\{\Lambda_{t,t+1} q_{t+1} K_{t+1}\}$$

Marginal vs. Average Q (con't)

Given optimality condition $p_t^I(1 + c\frac{I_t}{K_t}) = E_t\{\Lambda_{t,t+1}q_{t+1}\} \rightarrow$

$$p_t^I(I_t + c(\tfrac{I_t}{K_t})^2 K_t) = E_t\{\Lambda_{t,t+1}q_{t+1}I_t\} \rightarrow$$

$$\begin{aligned} q_t K_t &= \alpha Y_t - p_t^I(I_t + \frac{1}{2}c(\frac{I_t}{K_t})^2 K_t) + E_t\{\Lambda_{t,t+1}q_{t+1}K_{t+1}\} \\ &= \Pi_t + E_t\{\Lambda_{t,t+1}q_{t+1}K_{t+1}\} \\ &= E_t\{\sum_{i=0}^{\infty}\Lambda_{t,t+i}\Pi_{t+i}\} \end{aligned}$$

given $Y_t - W_t L_t = Y_t - (1-\alpha)(Y_t/L_t)L_t = \alpha Y_t$
Dividing thru by $K_t \rightarrow$ marginal value of capital equals average value

$$q_t = E_t\{\textstyle\sum_{i=0}^{\infty}\Lambda_{t,t+i}\Pi_{t+i}\}/K_t$$

Investment and Average Q

$$\frac{I_t}{K_t} = \frac{1}{c}(Q_t - 1)$$

$$Q_t = \frac{E_t\{\Lambda_{t,t+1} q_{t+1}\}}{p_t^I}$$

$$E_t\{\Lambda_{t,t+1} q_{t+1}\} = E_t\{\sum_{i=1}^{\infty} \Lambda_{t,t+i} \Pi_{t+i}/K_{t+1}\}$$

$$\rightarrow \text{marginal } Q = \text{average } Q$$

$$Q_t = \frac{E_t\{\sum_{i=1}^{\infty} \Lambda_{t,t+i} \Pi_{t+i}\}}{p_t^I K_{t+1}}$$

Q is thus measured by the ratio of market value to replacement cost
(Requires constant returns, including CRS in adjustment costs)

Investment and Average Q (con't)

$$\frac{I_t}{K_t} = \frac{1}{c}(Q_t - 1)$$

with

$$Q_t = \frac{E_t\{\sum_{i=1}^{\infty} \Lambda_{t,t+i}\Pi_{t+i}\}}{p_t^I K_{t+1}}$$

Note: Q_{t+1} summarizes the effect on investment of both expected future cash flows and discount rates (via $\Lambda_{t,t+i}$)

Strong implication: Q_{t+1} should be a "sufficient statistic" for investment.

- In practice, stock market measures of Q_t work poorly in explaining I_t
- "Fundamental" measures based on forecasts of future earnings work better
- Liquidity variables (cash flow, credit spreads) add explanatory power.

Adding Investment to the NK model

We now incorporate investment based on Tobin's Q theory

We do so in a way that facilitates aggregation

Internal versus external adjustment costs

- Internal: at the firm level (as in the previous partial equilibrium model)
- External: increasing marginal costs in the production of new capital goods

Disadvantage of internal: capital stock is a predetermined state for each firm - complicates aggregation.
$\rightarrow$External adjustment costs typically standard
$\rightarrow$Generates "Q" behavior of investment.

Environment

- Representative household:
  - Consumes final good C_t, supplies labor L_t at wage W_t/P_t
  - Saves in the form of:
    - Capital K_t, which it rents to firms at rate Z_t
    - One period private nominal bonds B_t earning net nominal rate r_t^n.
    - Real money balances M_t/P_t,
- Three types of firms:
  - Final good producers: competitors
    - Produce output Y_t using intermediate goods $Y_t(f)$.
  - Intermediate good firms: monopolistic competitors
    - Produce a differentiated product $Y_t(f)$ using capital $K_t(f)$ and labor $L_t(f)$. Set prices $P_t(f)$ on staggered basis.

Environment (con't)

- Three Types of firms (con't)
  - Capital producers: competitors
    - Use final output to make new capital which they sell at price Q_t
    - Investment involves adjustment costs $\rightarrow$ "Tobin's Q" relation for investment along with variable price of capital.
- Central bank and government conducts monetary policy and fiscal policy.
- Frictionless financial markets
  - Monopolistic competition and price rigidities only distortions from first best.

Decision problems of households, final good firms and intermediate goods firms are essentially the same as in the baseline model with consumption goods only.
Hence we turn to the problem of capital producers

Capital Producers

- A representative capital producer invests I_t units of final output and rents K_t units of capital (after use in output production) to produce J_t units of new capital
- The technology for producing new capital goods is given by

$$J_t = I_t - \frac{1}{2}c(\frac{I_t}{K_t} - \delta)^2 \cdot K_t$$

$\frac{1}{2}c(\frac{I_t}{K_t} - \delta)^2 \cdot K_t$ reflects increasing marginal costs of producing new capital goods (after depreciation δ). Adj. costs in unit of capital.

- The capital producer sells new capital to households at the market price Q_t
- Capital producer's objective:

$$\max Q_t J_t - I_t - Z_{kt} K_t$$

$Z_{kt} \equiv$ rental for capital used to produce new capital. $Z_{kt} \approx 0$ near steady state

Capital Producers (con't)

- Capital producer's optimization problem (given $Z_{kt} \approx 0$ near steady state)

$$\max_{I_t} Q_t J_t - I_t$$

s.t.

$$J_t = I_t - \frac{1}{2} c (\frac{I_t}{K_t} - \delta)^2 \cdot K$$

- Will verify shortly $Z_{kt} \approx 0$ near steady state.

Investment and Q_t

- FONC conditions for $I_t \rightarrow$

$$\frac{I_t}{K_t} = \delta + \frac{1}{c}(1 - \frac{1}{Q_t})$$

  - In steady state with zero growth, net investment, $\frac{I_t}{K_t} - \delta = 0; \rightarrow Q_t = 1$
  - Outside ss, $\frac{I_t}{K_t} - \delta$ varies positively with Q_t (as with standard Q theory)
  - Sensitivity of $\frac{I_t}{K_t}$ to Q_t depends inversely on c.
- Marginal product of capital (for producing investment goods).

$$\frac{\partial J_t}{\partial K_t} = \frac{1}{2}c(\frac{I_t}{K_t} + \delta)(\frac{I_t}{K_t} - \delta) \approx 0 \text{ near steady state}$$

  - $\rightarrow$ rental rate on capital ≈ 0 near steady state.
  - Can ignore rental of old capital for producing new capital.

Resource constraints

- Income and expenditure

$$Y_t = C_t + I_t + G_t$$

- Evolution of capital

$$K_{t+1} = I_t - \frac{1}{2}c(\frac{I_t}{K_t} - \delta)^2 \cdot K_t + (1-\delta)K_t$$

Monetary and Fiscal Policy

- The central bank sets the nominal interest rate according to the following simple feedback rule:

$$1 + r_t^n = \max\{(1+r)\left(\frac{P_t}{P_{t-1}}\right)^{\phi_\pi}\left(\frac{Y_t}{Y_t^*}\right)^{\phi_y} e^{\epsilon_t^r}, 1\} \quad (1)$$

  where $Y_t^* \equiv$ natural (i.e. flexible price equilibrium) level of output, with $\phi_\pi > 1$ and $\phi_y > 0$ and $1 + r$ is the zero inflation steady state real interest rate, equal to the nominal rate

- Fiscal policy

$$G_t = G$$

- Government budget constraint:

$$G_t = T_t + \frac{M_t - M_{t-1}}{P_t} \quad (2)$$

Equilibrium

- An equilibrium is defined as an allocation $(Y_t, L_t, C_t, I_t, K_{t+1})$ and a price system $(Z_t, W_t, P_t, P_t^o, r_t^n, Q_t, \mu_t)$ such that all agents are maximizing subject to their respective constraints, all markets clear, and all resource constraints are satisfied, given P_{t-1}, A_t, and K_t..
- In practice, it is convenient to express the equilibrium as the vector $(Y_t, C_t, I_t, L_t, P_t, P_t^o, R_t^n, Q_t, \mu_t, K_{t+1})$ that satisfies a system of 10 equations, given the predetermined states P_{t-1}, A_t, and K_t and $\mu = \frac{1}{1-1/\varepsilon}$ (10 unknowns, 10 equations).
- It is useful to group the equations into aggregate demand, aggregate supply and policy blocks.

Aggregate Demand

output, consumption, investment, arbitrage:

$$Y_t = C_t + I_t + G$$
$$C_t = E_t \left\{ [(1 + r_t^n) \frac{P_t}{P_{t+1}} \beta]^{-\sigma} C_{t+1} \right\}$$
$$\frac{I_t}{K_t} = \delta + \frac{1}{c}(1 - \frac{1}{Q_t})$$
$$E_t \left\{ \Lambda_{t,+1} (1 + r_t^n) \frac{P_t}{P_{t+1}} \right\} = E_t \left\{ \Lambda_{t,+1} (\frac{Z_t + (1-\delta) Q_{t+1}}{Q_t}) \right\}$$

with $Z_t = \frac{1}{1+\mu_t} \alpha \frac{Y_t}{K_t}$, $\Lambda_{t,+1} = \beta^i C_{t+i}^{-\gamma} / C_t^{-\gamma}, \sigma = 1/\gamma$.
- These equations define an "IS curve" that relates spending inversely to the real rate $(1 + r_t^n) \frac{P_t}{P_{t+1}}$ and expectations of the future
- Interest rates affect investment inversely via Q_t

Aggregate Supply

eqs. for output, labor market, price index, price setting, capital. ($\frac{1}{1+\mu_{t+i}} = MC_{t+i}$.)

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha} V_t$$

$$(1-\alpha)\frac{Y_t}{L_t} = (1+\mu_t) a_n \frac{L_t^{\varphi}}{C_t^{-\gamma}}$$

$$P_t = \left[\theta P_{t-1}^{1-\varepsilon} + (1-\theta) P_t^{o\,1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}$$

$$E_t \sum_{i=0}^{\infty} \theta^i \Lambda_{t,i} \left[\frac{P_t^o}{P_{t+i}}\right]^{-\varepsilon} Y_{t+i} \left[\frac{P_t^o}{P_{t+i}} - \frac{1+\mu}{1+\mu_{t+i}}\right] = 0$$

$$K_{t+1} = I_t - \frac{1}{2} c \left(\frac{I_t}{K_t} - \delta\right)^2 \cdot K_t + (1-\delta) K_t$$

Policy Block

Interest Rate Rule

$$1 + r_t^n = \max\{(1 + r)\left(\frac{P_t}{P_{t-1}}\right)^{\phi_\pi}\left(\frac{Y_t}{Y_t^*}\right)^{\phi_y} e^{\epsilon_t^r}, 1\}$$

Given exogenous processes for productivity A_t and the money shock ν_t, this completes the system of 10 unkowns in 10 equations.

Flexible price equilibrium

$$(Y_t^*, C_t^*, I_t^*, Q_t^*, L_t^*, R_{t+1}^*, K_{t+1}^*\}$$

$\rightarrow$ mark-up $1+\mu$ is constant; $R_{t+1}^* \equiv [(1+i_t)P_t/P_{t+1}]^*$; $\Lambda_{t,+1}^* \equiv \beta C_{t+1}^{*-\gamma}/C_t^{*-\gamma}$

$$\begin{aligned}
Y_t^* &= C_t^* + I_t^* + G \\
C_t^* &= E_t\left\{(R_{t+1}^*\beta)^{-\sigma} C_{t+1}^*\right\} \\
\frac{I_t^*}{K_t^*} &= \delta + \frac{1}{c}\left(1 - \frac{1}{Q_t^*}\right) \\
E_t\left\{\Lambda_{t,+1}^* R_{t+1}^*\right\} &= E_t\left\{\Lambda_{t,+1}^*\left[\frac{1}{1+\mu}\alpha\frac{Y_{t+1}^*}{K_{t+1}^*} + (1-\delta)Q_{t+1}^*\right]/Q_t^*\right\} \\
Y_t^* &= A_t K_t^{*\alpha} L_t^{*1-\alpha} \\
(1-\alpha)\frac{Y_t^*}{L_t^*} &= (1+\mu)a\frac{L_t^{*\varphi}}{C_t^{*-\gamma}} \\
K_{t+1}^* &= I_t^* - \frac{1}{2}c\left(\frac{I_t^*}{K_t^*} - \delta\right)^2 \cdot K_t^* + (1-\delta)K_t^*
\end{aligned}$$

RBC with steady state distortion ($\mu > 0$) and investment adjustment costs.

Loglinear System: Aggregate Demand

Log-linearize around the flexible price steady state with zero inflation:

$$y_t = \frac{C}{Y} c_t + \frac{I}{Y} inv_t$$

$$c_t = -\sigma \left[r_t^n - E_t \pi_{t+1} - \rho \right] + E_t \{ c_{t+1} \}$$

$$inv_t - k_t = \frac{1}{\delta c} q_t$$

$$r_t^n - E_t \pi_{t+1} - \rho = E_t \left\{ (1 - \nu) z_{t+1} + \nu q_{t+1} - q_t \right\}$$

with $z_t = y_t - k_t - \widehat{\mu}_t$;
$\pi_t = p_t - p_{t-1}$, $\nu = (1 - \delta) / [\frac{\alpha Y}{(1+\mu) K} + 1 - \delta]$,

Interest Rates, Asset Prices and Aggregate Demand

Consumption:

$$c_t = E_t\{\sum_{i=0}^{\infty} -\sigma(r^n_{t+i} - \pi_{t+1+i} - \rho)\}$$

Asset price

$$q_t = E_t\{(1-\nu)z_{t+1} - (r^n_{t+i} - \pi_{t+1+i} - \rho) + \nu q_{t+1}\}$$

$$= E_t\{\sum_{i=0}^{\infty} \nu^i[(1-\nu)z_{t+1} - (r^n_{t+i} - \pi_{t+1+i} - \rho)]\}$$

Investment

$$inv_t - k_t = \frac{1}{\delta c} E_t\{\sum_{i=0}^{\infty} \nu^i[(1-\nu)z_{t+1} - (r^n_{t+i} - E_t\pi_{t+1+i} - \rho)]\}$$

c_t and inv_t depend on expected path of $r^n_{t+i} - E_t\pi_{t+1+i}$

Log-linear System: Aggregate Supply

output, labor market eq., NK Phillips Curve and capital

$$y_t = a_t + \alpha k_t + (1-\alpha) l_t$$
$$y_t - l_t = \widehat{\mu} + \varphi l_t + \gamma c_t$$
$$\pi_t = \lambda(-\widehat{\mu}_t) + \beta E_t\{\pi_{t+1}\}$$
$$k_{t+1} = \delta inv_t + (1-\delta) k_t$$

$\pi_t = p_t - p_{t-1}; \lambda = \frac{(1-\theta)(1-\beta\theta)}{\theta}$
Note: with capital $-\widehat{\mu} = mc_t \neq y_t - y_t^*$ (though it's close)
NK Phillips Curve derived from

$$p_t = \theta p_t + (1-\theta) p_t^o$$
$$p_t^o = (1-\theta\beta) E_t \sum_{i=0}^{\infty} (\theta\beta)^i (-\mu_t + p_{t+i}).$$

"Compact" Loglinear System

AD

$$y_t = \frac{C}{Y} c_t + \frac{I}{Y} inv_t$$
$$c_t = E_t \sum_{i=0}^{\infty} -\sigma(r^n_{t+i} - \pi_{t+1+i} - \rho)$$
$$inv_t - k_t = \frac{1}{\delta c} E_t \{\sum_{i=0}^{\infty} \nu^i [(1-\nu) z_{t+1} - (r^n_{t+i} - E_t \pi_{t+1+i} - \rho)]\}$$

AS

$$\pi_t = \lambda(-\mu_t) + \beta E_t\{\pi_{t+1}\}$$
$$\mu_t = -\kappa y_t - \gamma c_t + (1+\kappa)(a_t + \alpha k_t); \quad (\kappa \equiv (1+\varphi)/(1-\alpha))$$
$$k_{t+1} = \delta inv_t + (1-\delta) k_t$$

MP

$$r^n_t = \rho + \phi_\pi \pi_t + \phi_y (y_t - y^*_t)$$

Model Expressed in Deviations from Flexible Price Equilibrium

$\widetilde{x}_t \equiv x_t - x_t^*$ and since k_t is predetermined $\widetilde{k}_t = 0$

$$\widetilde{y}_t = \frac{C}{Y}\widetilde{c}_t + \frac{I}{Y}\widetilde{inv}_t$$
$$\widetilde{c}_t = E_t \sum_{i=0}^{\infty} -\sigma(r_{t+i}^n - \pi_{t+1+i} - r_{t+1+i}^*)$$
$$\widetilde{inv}_t = \frac{1}{\delta c} E_t\{\sum_{i=0}^{\infty} \nu^i[(1-\nu)\widehat{z}_{t+1} - (r_{t+i}^n - E_t\pi_{t+1+i} - r_{t+1+i}^*)]\}$$
$$\pi_t = \lambda(-\widehat{\mu}_t) + \beta E_t\{\pi_{t+1}\}$$
$$\widehat{\mu}_t = -\kappa\widetilde{y}_t - \gamma\widetilde{c}_t$$
$$\widetilde{k}_{t+1} = \delta\widetilde{inv}_t$$

$\widetilde{z}_t = \widetilde{y}_t - \widehat{\mu}_t$; $r_{t+1}^* \equiv$ natural flexible price equilibrium real rate.

- Both $\widetilde{c}_t$ and $\widetilde{inv}_t$, and hence $\widetilde{y}_t$ vary inversely with expected path on interest rate gap $E\{r_t^n - \pi_{t+1} - r_{t+1}^*\}$
- $\widetilde{\mu}_t$ varies inversely with $\widetilde{y}_t$ and $\widetilde{c}_t \rightarrow \pi_t$ varies positively with $\widetilde{y}_t$ and $\widetilde{c}_t$.

Taking the Model to Data

- Seminal papers:.Christiano/Eichenbaum/Evans (2005), Smets/Wouters (2007)
  - CEE use methods of moments: match model to IRFs from monetary shock
  - SW estimate full system using Bayesian methods: one shock for each variable.
- Variables the same as in simple baseline presented earlier
- Modifications of structural equations to address following issues
  - Hump-shaped dynamics of output, investment and consumption
    - As opposed to simple mean reversion
  - Smooth behavior of inflation with (relatively) volatile output behavior.
  - The effect of monetary policy on measured productivity.

Key Modifications

1. Habit formation in consumption

$$E_t \left\{ \sum_{i=0}^{\infty} \beta^i e^{b_{t+i}} \left[\log \left(C_{t+s} - hC_{t+s-1} \right) - \frac{1}{1+\varphi} L_{t+i}^{1+\varphi} \right] \right\}$$

Induces dependency of C_t on C_{t-1} as well as expectations of future, consistent with data.

Body Math 2. Flow investment adjustment costs

$$K_{t+1} = (1 - \delta_t) K_t + \zeta_t \left(1 - S \left(\frac{I_t}{I_{t-1}} \right) \right) I_t$$

with $S' > 0, S'' > 0$ and $\zeta_t \equiv$ investment shocks - literally a shock to the inverse of the replacement price of capital goods.

Induces dependency of I_t on I_{t-1} as well expectations of future, consistent with data.

Key Modifications (con't)

3. Variable utilization of capital

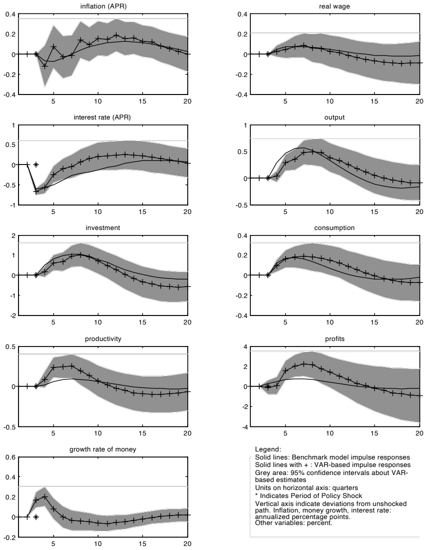
$$Y_t = A_t(U_t K_t)^\alpha L_t{}^{1-\alpha}$$
$$\delta_t = \delta(U_t)$$

with $\delta' > 0, \delta'' > 0$. MB of increasing utilization:$\alpha \frac{Y_t}{U_t}$; MC: $\delta' K_t$.
True TFP$= A_t$; Measured $TFP = A_t U_t{}^\alpha \rightarrow$ procyclical movements in U_t can induce procyclical movements in *measured* TFP,
4. Nominal wage rigidity: Households are monopsonistic competitors competitors that supply differentiated labor. Set nominal wages on staggered basis (Calvo style).

$$\frac{1}{1+\mu_t} = MC_t = \frac{W_t/P_t}{(1-\alpha)Y_t/L_t}$$

Nominal wage and price rigidity $\rightarrow$ stickiness in real wages $\rightarrow$ stickiness in MC_t and $\frac{1}{1+\mu_t} \rightarrow$ stickiness in inflation.

Figure 1: Model- and VAR-Based Impulse Responses



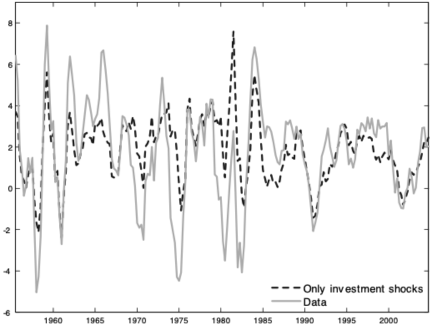
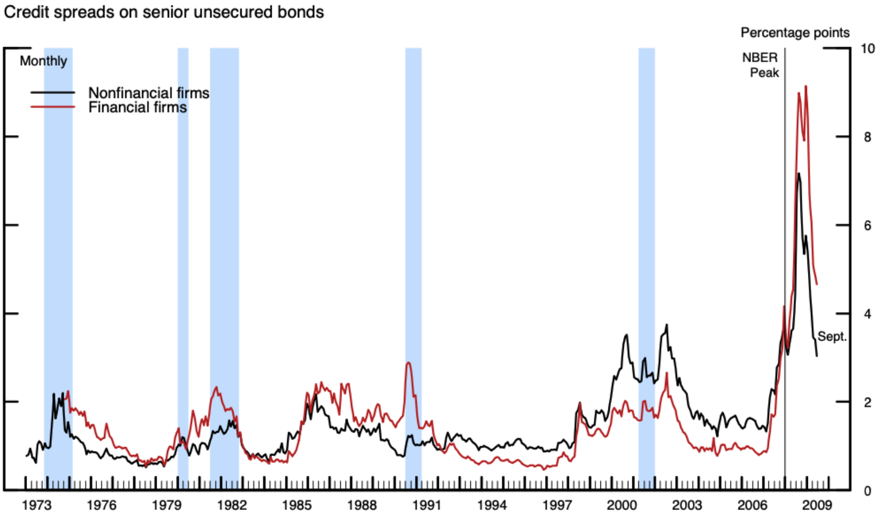


Figure 1: Year-over-year output growth in the data and in the model with only investment shocks.

Post-Mortem

- Model accounts reasonably well for post-war date up to 2007.
- "Investment shock" is most important driving shock
  - Though interpretation of this shock as well as the others is open since they are not directly observed
- However; model breaks down during Great Recession
  - Using data through 2008:Q3 fails to forecast both subsequent downturn and slow recovery
  - Not useful for interpreting unconventional policy interventions
- But not all is lost: Investment shock highly correlated with credit spread, a simple measure of financial stress (Justinano/Primiceri/Tambalotti 2011).
  - Suggests value of more explicit modeling of the shock, allowing for financial frictions.


Credit spreads on senior unsecured bonds
Percentage points
Monthly
Nonfinancial firms
Financial firms
NBER
Peak
Sept.
10
8
6
4
2
0
1973
1976
1979
1982
1985
1988
1991
1994
1997
2000
2003
2006
2009

Financial Accelerator (Very Brief Overview)

Three Key Ingredients

1. External finance premium
   1. With frictions in the credit market, the cost of uncollateralized borrowing exceeds the opportunity cost of internal funds, controlling for risk.
   2. Sources: asymmetric information and costly contract enforcement
2. External finance premium varies inversely with borrowers' financial position
   1. Financial position measured by (i) balance sheet strength (e.g equity/assets) (ii) cash flows (iii) tangible collateral, etc.
   2. Stronger financial position $\rightarrow$ greater collateralization of the loan (implicit or explicit) $\rightarrow$ reducing expected losses to the lender $\rightarrow$ lender charges borrower a lower external finance premium.

Financial Accelerator (con't)

3. Borrowers' financial positions are procyclical $\rightarrow$ external finance premium countercyclical
   (a)e.g. Weakening of real economy $\rightarrow$ weakening of borrower financial positions $\rightarrow$ EFP increases, raising the cost of credit $\rightarrow$ reducing demand for investment and consumer durables, and so on.
   (b)$\rightarrow$ Mutual feedback between financial and real sectors.

Note that theory applies to households and banks, as well as non-financial firms

Financial Accelerator (con't)

Let $\Psi_t - 1 \geq 0 \equiv$ external finance premium $\rightarrow$

$$E_t \left\{ \Lambda_{t,+1}(1 + r_t^n)\frac{P_t}{P_{t+1}} \right\} = E_t \left\{ (\Lambda_{t,+1}/\Psi_t)(\frac{Z_t + (1-\delta)Q_{t+1}}{Q_t}) \right\}$$

with $Z_t = \frac{1}{1+\mu_t}\alpha\frac{Y_t}{K_t}$, $\Lambda_{t,+1} = \beta^i C_{t+i}^{-\gamma}/C_t^{-\gamma}, \sigma = 1/\gamma$
Loglinear approximation:

$$r_t^n - E_t\pi_{t+1} - r = E_t\left\{(1-\nu)z_{t+1} + \nu q_{t+1} - q_t\right\} - \psi_t\} \rightarrow$$
$$\psi_t + r_t^n - E_t\pi_{t+1} - r = E_t\left\{(1-\nu)z_{t+1} + \nu q_{t+1} - q_t\right\}$$

where $\psi_t = \log \Psi_t \approx \Psi_t - 1 = \text{EFP} \rightarrow$

$$q_t = E_t\{\sum_{i=0}^{\infty} \nu^i[(1-\nu)z_{t+1} - (\psi_{t+i} + r_{t+i}^n - \pi_{t+1+i} - r)]\}$$

$\rightarrow$Variation in ψ_{t+i} affects q_t and investment demand (and thus aggregate demand).
Financial crisis: sharp increase in ψ_{t+i} (reflected in credit spreads) $\rightarrow q_t, inv_t \downarrow$

Figure 1: DSGE Model Forecasts of the Great Recession