NUMBERS INTO NOTES: CAST YOUR MIND BACK 200 YEARS

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ABSTRACT

Supposing Charles Babbage had built the Analytical Engine and Ada Lovelace had pursued its potential for musical composition. Inspired by composer Emily Howard and mathematician Lasse Rempe-Gillen, our online tool explores what might have come next, based on the mathematics of the time.

1. INTRODUCTION

Charles Babbage gave a seminar about his proposed steam-powered computer, the Analytical Engine, at the University of Turin in 1840. Originally published in French, in 1842 Ada Lovelace translated it into English and added extensive notes. Famously these notes contain the first algorithm designed for execution by a general purpose computing machine, which is regarded as the first computer program. They also contain an important quotation about computers and music:

> Supposing, for instance, that the fundamental relations of pitched sounds in the science of harmony and of musical composition were susceptible of such expression and adaptations, the engine might compose elaborate and scientific pieces of music of any degree of complexity or extent. (Note A in [1]).

December 2015 saw the 200th anniversary of Lovelace's birth, and a major symposium was held to mark the occasion. This included the discussion of a thought experiment: had Lovelace lived longer, and had Babbage successfully built the Analytical Engine, what might have happened in pursuit of Lovelace's musical observation? [2]

As part of this we developed an interactive tool for people to generate music from integer sequences. The workflow of the tool mirrors our hypothesized workflow involving the Analytical Engine: the machine runs a parameterized program to generate a number sequence, and parts of this sequence are then given to different instruments. Inspired by the use of punched cards in the Jacquard loom and the proposed Analytical Engine, we generate virtual 'piano rolls'.

Hence this is an exercise in algorithmic music generation but with points for creative human intervention. The programmer and operator (or 'attendant') were not allowed to change the numbers generated by the machine, but had full control of the mapping from numbers to notes and then from notes to instruments.

2. THE NUMBERS INTO NOTES TOOL

The interactive tool, a single page web app[1], was developed to generate Fibonacci numbers and reduce them with clock arithmetic, generating a periodic sequence which is then mapped to notes, and the music explored interactively by visualizing and selecting fragments to play. Additional algorithms have been added to stimulate discussion of mathematical and musical calculations on the Analytical Engine (see Section 3).

The final stage of this workflow is to export the musical fragment in various formats. The tool generates music notation (using Lilypond), as well as standard MIDI files. We also export metadata, with an automatically generated natural language description of the algorithm parameters, mapping, and selection; another of the output formats is W3C PROV-N, from which an SVG visualization is generated (see Figure 1). These outputs are designed to enable someone to understand how the fragment was generated, or indeed to regenerate it using different tooling; *i.e.* to reproduce the results of the experiment.

3. THE MATHEMATICS

The online tool uses mathematical concepts that were established at the time of Lovelace, including Powers and Fibonacci which occur in the libretto of the *Ada sketches* operatic work composed by Emily Howard. While some of the mathematics is of course familiar, the exercise also reminds us what we take for granted today that was not established two centuries ago. The number sequences used in the tool include:

Fibonacci Numbers – Lagrange noted periodic functions in Fibonacci numbers in 1774, and the period is known as the *Pisano period*.

Pi — Calculated by hand to 100 decimal places in 1706 using Machin's algorithm. Pi is believed to be a 'normal

[1] http://demeter.oerc.ox.ac.uk/NumbersIntoNotes/

number' which means any possible musical sequence appears somewhere in its digits.

Prime numbers — Calculated using the Sieve of Eratosthenes. The size of the sieve is estimated using a 1798 result by Legendre, and in those days 1 was considered to be a prime.

Golden Ratio — This is the number that the ratio of successive Fibonacci numbers converges to (1.618...) and is calculated using a square root function. The square root algorithm we use today is Newton's method published in 1711, though it can be traced through first-century C.E. Greece and sixth-century B.C.E. Babylon.

Random — How could random numbers have been generated on the Analytical Engine? Musical dice games (*Musikalisches Würfelspiel*) were popular in the 1700s. Today we analyse and generate music using a mathematical approach due to Markov— but he was not born until 1856.

Factorials — The notation for factorial, *e.g.* 20!, was introduced in 1808. 20! can be stored in a 64 bit integer before we run out of bits, but the Analytical Engine might have gone up to 41! with its proposed support for 50 decimal digits.

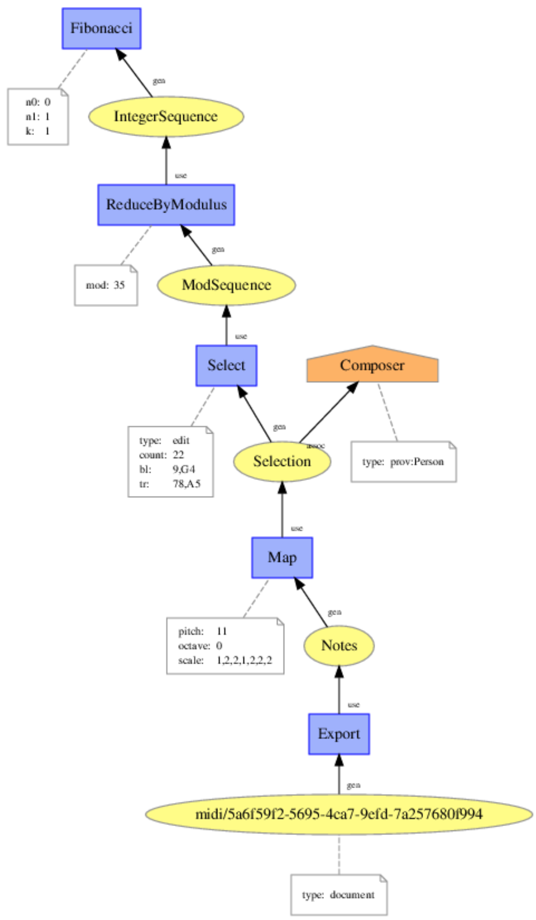

Figure 1. A Provenance graph generated by the tool, describing a MIDI file generated from a Fibonacci sequence *mod* 35.

4. REPRODUCING EULER

Today we often refer to the consonance curve from Helmholtz's 1895 *On the sensations of tone* [3]. Before Lovelace, the *suavitatis gradus* function for calculating consonance was defined by Leonhard Euler in his 1739 book *Tentamen novae theoriae mvsicae ex certissimis harmoniae principiis dilvcide expositae* [4].

We have reproduced the gradus table on page 41 of *Tentamen* and submitted the sequence to the On-Line Encylopedia of Integer Sequences as sequence A275314 [5], where it is defined as in Eqn (1).

$$\Gamma(n) = 1 + \sum k_j\,(p_j - 1) \quad \text{where} \quad n = \prod p_j^{k_j} \qquad (1)$$

We have also reproduced the table on page 61 of *Tentamen*, which identifies whole number ratios with gradus from 2 to 10. Using Mathematica, we found that the gradus 10 ratio 2:21 is missing from the published table.

5. FUTURE WORK

Next we consider what Ada Lovelace might do, were she transported to 2016. Using arduinos with sensors and the audience's co-creation with them, we take inspiration from the mathematics of Lovelace's day to produce interactive music, 'numbers in places' [6].

6. ACKNOWLEDGEMENTS

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7. REFERENCES

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