

Functional indefinites as conventional implicatures

Overview This paper points out that *functional indefinites* (FIs) (Kratzer, 1998; Schwarz, 2001), such as English *a certain* indefinites, exhibit notable parallels with *appositive relative clauses* (ARCs), whose content is standardly regarded as a *conventional implicature* (CI) (Potts, 2005). Although these similarities suggest analyzing FIs analogously to ARCs, existing theories of CIs have difficulty accounting for their interaction with quantifier scope. To address this challenge, we adopt the notion of *propositions as types* (see Dybjer and Palmgren (2024)), which allows us to uniformly derive the behavior of FIs/ARCs via the scopal relation between types.

Parallels between FIs and ARCs It has been observed that the meanings associated with FIs and ARCs are both **projective**, as witnessed by (1a) and (1b) (Geurts, 2010).

- (1) a. It is not true that [Sandy met *a certain student*]. $\Rightarrow$ There is a student.
 b. It is not true that [Sandy, *who is away*, will return]. $\Rightarrow$ Sandy is away.

We strengthen this parallel by focusing on **their projection behavior from quantifier scope**. Consider the FI in (2a). It allows a functional interpretation involving a function from students to areas, which results in a universally quantified implication (shown one line below). Likewise, the ARC in (2b), containing pronouns bound by *every student*, leads to a universally quantified projective content.

- (2) a. If [every student makes progress in *a certain area*], nobody will flunk the exam.
 $\Rightarrow$ For each student, there is an area. (e.g., *their weakest one*) (Schlenker, 2006)
 b. If [every student_{*i*} has bid farewell to Nate, *who had given them*_{*i*} *some great advice during their individual meetings*], then we can officially close off the session.
 $\Rightarrow$ For each student, Nate gave them some great advice. (Zhao, 2023)

Furthermore, these universal projections exhibit **an effect reminiscent of weak crossover**. In both (3a) and (3b), although the quantifier in the object position can, in principle, take scope over the subject, the universal projections observed with (2a) and (2b) are as unacceptable as standard weak crossover examples (e.g., **Their_i mother loves every student_i*).

- (3) a. *A certain technician* inspected every plane. (*functional interpretation) (Chierchia, 2001)
 b. **Nate, who had given them_i some great advice*, bid farewell to every student_i.

Theoretical Challenges To give a unified account of these parallels, it is promising to treat FIs similarly to ARCs by viewing FIs as triggers of a CI (namely, the existential implication *there is a ...*). However, existing CI theories do not seem to capture the projection behavior from quantifier scope, because theories of CIs that can handle universal projections such as (2) (Arnold, 2007; Martin, 2016) achieve this goal with a purely pragmatic mechanism called *telescoping* (Roberts, 1987), and thus do not readily account for the structural constraint at work in (3). We address this issue by employing **the type-theoretical approach to natural language semantics** (Ranta, 1995; Luo, 2012; Bekki, 2023), which uses type theory for semantic representations. As we will see, this approach enables CIs to “scopally interact” with quantifiers, leading to a principled account of the patterns of universal projections.

Propositions as Types The type-theoretical approach hinges on *the principle of propositions as types*, which identifies a proposition and its proofs with a type and its terms, respectively. For instance, a proof of $\forall x \in A. B$ can be viewed as a function that maps any element a of A into a proof of $B[x := a]$, which indicates that $\forall x \in A. B$ corresponds to a *function type* $(x : A) \rightarrow B$ (where the codomain B may depend on x). Likewise, a proof of the conditional $A \supset B$ can be regarded as a function from a proof of A to a proof of B . Given this parallel, we can treat $A \supset B$ as a universal quantification over the proofs of the proposition A . To illustrate, consider (4) and its semantic representation shown to the right.

- (4) Every student sneezed. $\rightsquigarrow (x : e) \rightarrow ((u : \text{std}(x)) \rightarrow \text{snz}(x))$ (cf. $\forall x. (\text{std}(x) \supset \text{snz}(x))$)

A similar treatment applies to $\exists$ and $\wedge$. Namely, $\exists x \in A. B$ is formalized as a *product type* $(x : A) \times B$ (i.e., a type of pairs $\langle a, b \rangle$), of which $A \wedge B$ is viewed as a special case, as exemplified in (5).

- (5) A student sneezed. $\rightsquigarrow (x : e) \times ((u : \text{std}(x)) \times \text{snz}(x))$ (cf. $\exists x. (\text{std}(x) \wedge \text{snz}(x))$)

In this way, the notion of propositions as types **allows variables to range over the proofs of a proposition**, thereby enabling propositional content to have scopal interaction with quantifiers. This feature of the type-theoretical approach will be crucial in our account that follows.

Account Our proposal is based on the type-theoretical analysis of CIs by Matsuoka et al. (2024), where a CI is translated into an intermediate representation named a *CI type* $(x \triangleleft A) \times B$. A CI type undergoes a process called **type checking** that adds the CI content A to the context Γ (formally, Γ is a list of variable declarations $x_1 : A_1, \dots, x_n : A_n$), which follows the view that CIs are *imposed* on the context (Anderbois et al., 2015). We illustrate the point with (1b) in Figure 1. Here, the content of the ARC $\text{away}(s)$ is added to the context Γ , as a result of which it projects out of the scope of negation.

$$\Gamma \mid \neg((v \triangleleft \text{away}(s)) \times \text{return}(s)) \xrightarrow{\text{type checking}} \Gamma, \boxed{v : \text{away}(s)} \mid \neg\text{return}(s)$$

projected

Figure 1: Type checking of (1b). $\Gamma \mid A$ indicates the pair of the context and the semantic representation.

To handle universal projections, we revise the behavior of the CI type. Concretely, we propose that **the context extension triggered by a CI type may be functionally dependent on the local variables**, as defined below.

Definition 1. Suppose $(x \triangleleft A) \times B$ is in the scope of variables $x_1 : A_1, \dots, x_n : A_n$. Then, its type checking may add $f : (x_1 : A_1) \rightarrow \dots \rightarrow (x_n : A_n) \rightarrow A$ to the context and replace x with $f x_1 \dots x_n$.

We explain how this definition applies to the antecedent of (2a) (the same holds for (2b)). We assume that the existential implication of the FI is lexically encoded with the CI type. Then, as shown in Figure 2, the above-defined operation abstracts the local variables (x and u) over the CI content $(y : e) \times \text{area}(y)$. Consequently, the context is extended with a universally quantified proposition (i.e., for each student, there is an area), with its proof f mapping a student to an area.

Def. 1: The CI projects with dependencies on **the local variables**

$$\Gamma \mid (x : e) \rightarrow \left((u : \text{std}(x)) \rightarrow \left((v \triangleleft \boxed{y : e, \text{area}(y)}}) \times \text{prog}(x, \pi_1 v) \right) \right)$$

$$\longrightarrow \Gamma, f : (x : e) \rightarrow \left((u : \text{std}(x)) \rightarrow \boxed{y : e, \text{area}(y)} \right) \mid (x : e) \rightarrow \left((u : \text{std}(x)) \rightarrow \text{prog}(x, \pi_1(fxu)) \right)$$

Figure 2: Type checking of the antecedent of (2a) (*Every student makes progress in a certain area*). For readability, we write $(x : A) \times B$ as $\begin{bmatrix} x : A \\ B \end{bmatrix}$. π_1 is the function that takes the first element of a pair.

Let us turn to the crossover-like effect. We show the representation of (3a) in (6), where the CI type for the FI takes scope over the function type for *every plane*. As a result, in the type checking process, the variables for the universal quantifier (y and v) are not available as local variables, so we do not obtain the universal projection, as desired. Again, the same line of reasoning applies to the ARC case (3b), too.

$$(6) \quad \left(u \triangleleft \begin{bmatrix} x : e \\ \text{technician}(x) \end{bmatrix} \right) \times \left((y : e) \rightarrow ((v : \text{plane}(y)) \rightarrow \text{inspect}(\pi_1 u, y)) \right)$$

Summary Motivated by the parallels between FIs and ARCs in terms of their projection behavior, we have proposed a unified analysis of FIs and ARCs based on the type-theoretical approach. The crucial point is that, as a consequence of the propositions-as-types principle, the projective content associated with FIs/ARCs can scopally interact with other elements in a semantic representation. Accordingly, we can distinguish the two configurations below ($>$ denotes the “scope over” relation between types), thereby uniformly accounting for how FIs/ARCs interact with quantifiers.

- (7) a. **quantifier** $\dots$ **FI/ARC** (e.g., (2)) $\rightsquigarrow (x : A) > (y \triangleleft B)$ ($\checkmark (x : A) \rightarrow B$)
- b. **FI/ARC** $\dots$ **quantifier** (e.g., (3)) $\rightsquigarrow (y \triangleleft B) > (x : A)$ ($\times (x : A) \rightarrow B$)

Selected References [1] Schwarz, B. (2001). Two kinds of long-distance indefinites. [2] Potts, C. (2005). The Logic of Conventional Implicatures. [3] Dybjer, P., & Palmgren, E. (2024). Intuitionistic Type Theory. [4] Zhao, Z. (2023). The scope of supplements. [5] Matsuoka, D., et al. (2024). Appositive projection as implicit context extension in Dependent Type Semantics.