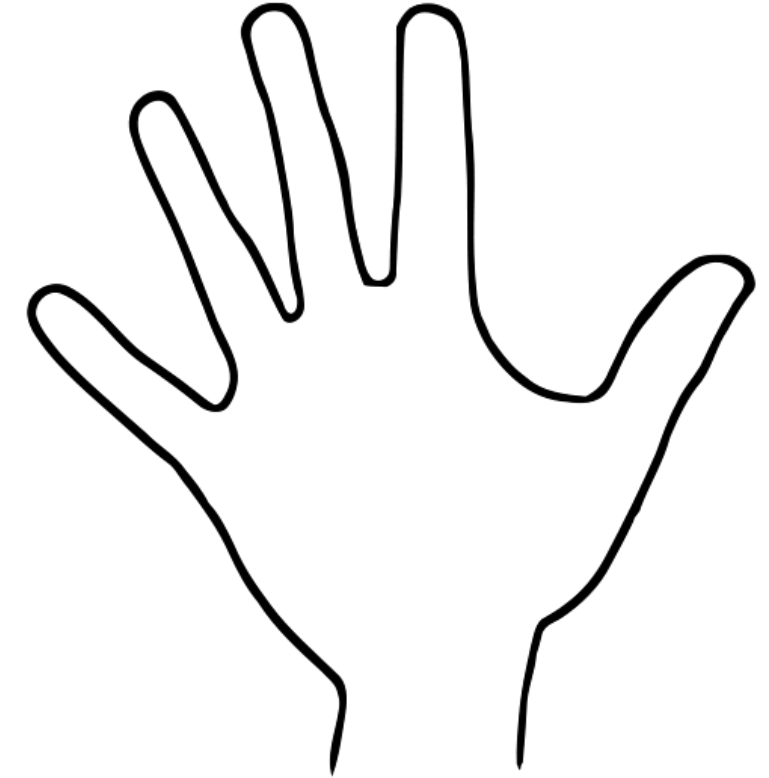
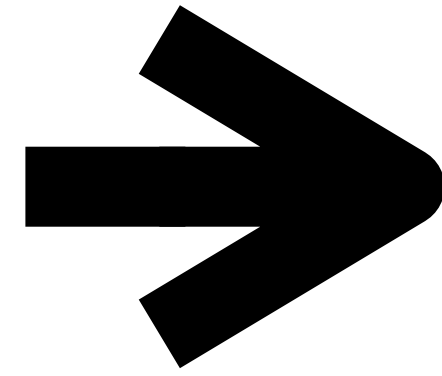
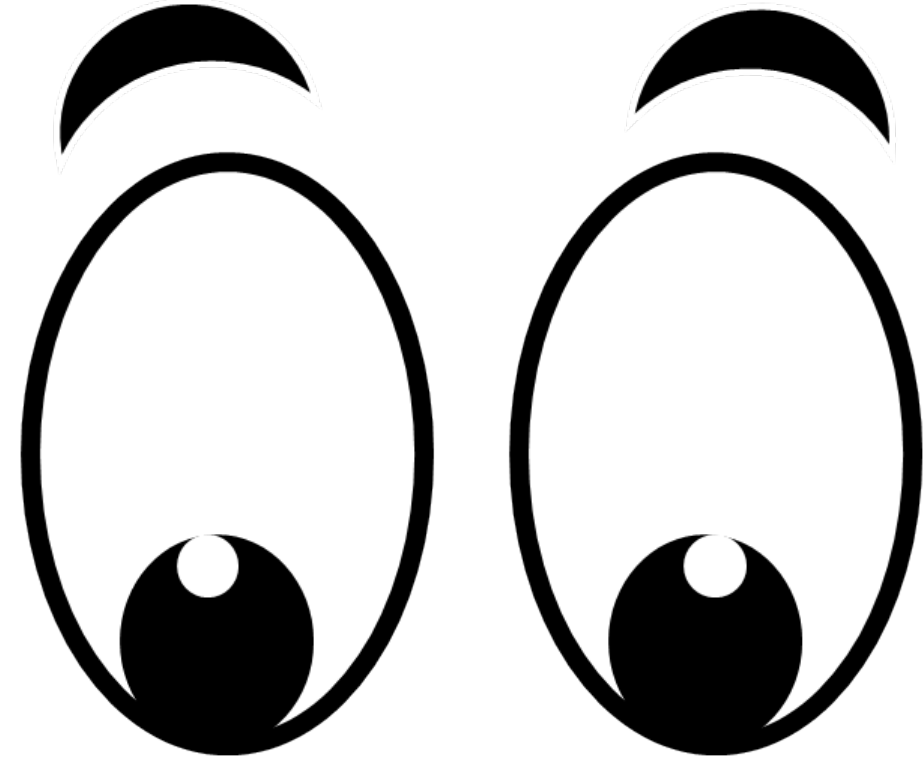


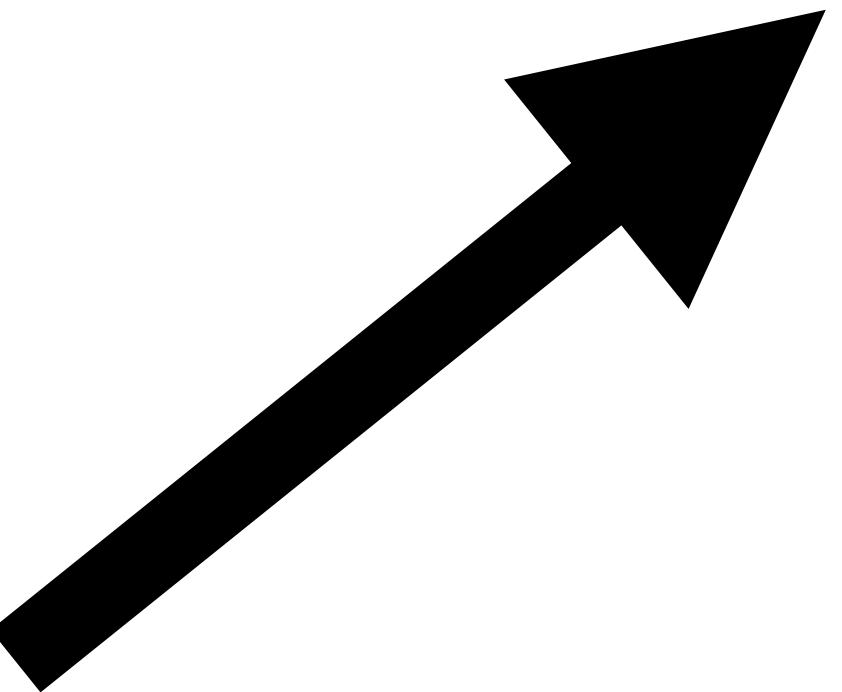
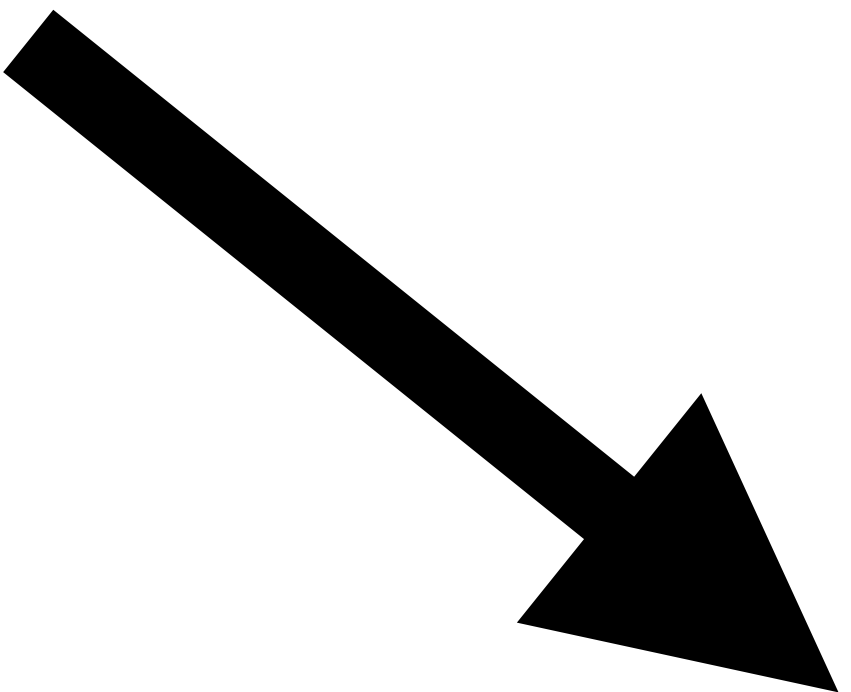
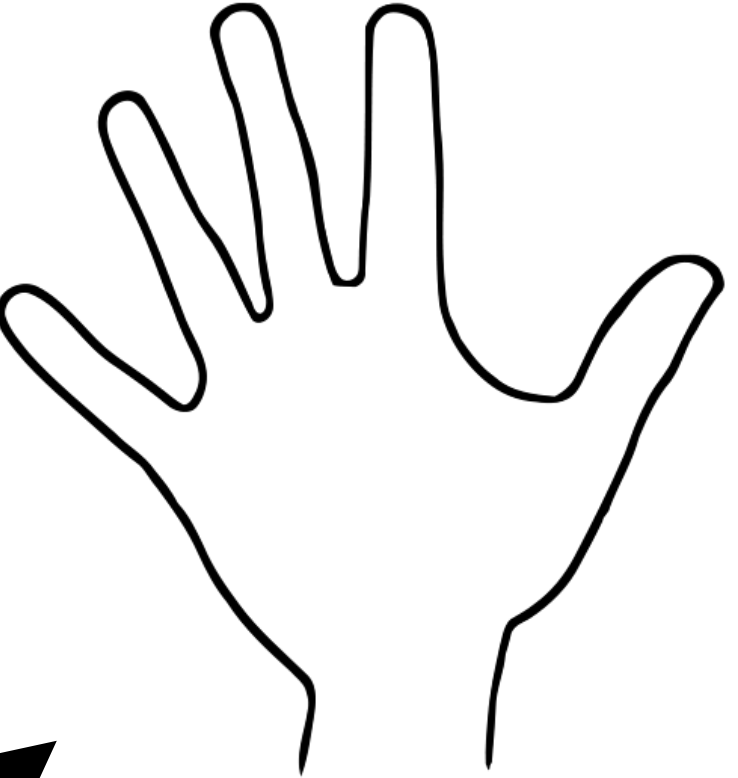
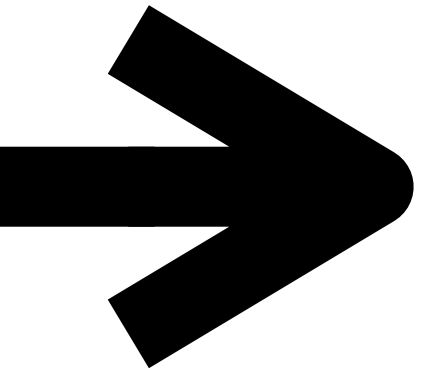
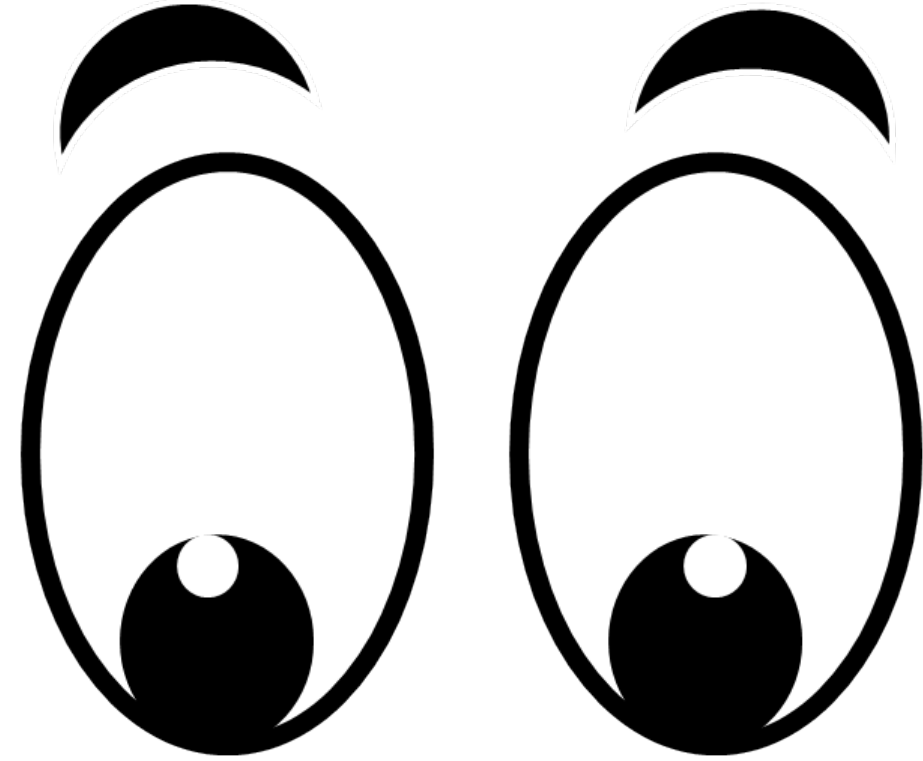
# Attention Allocation in Decision Making as Optimal Sequential Sampling

Fred Callaway  
Princeton University

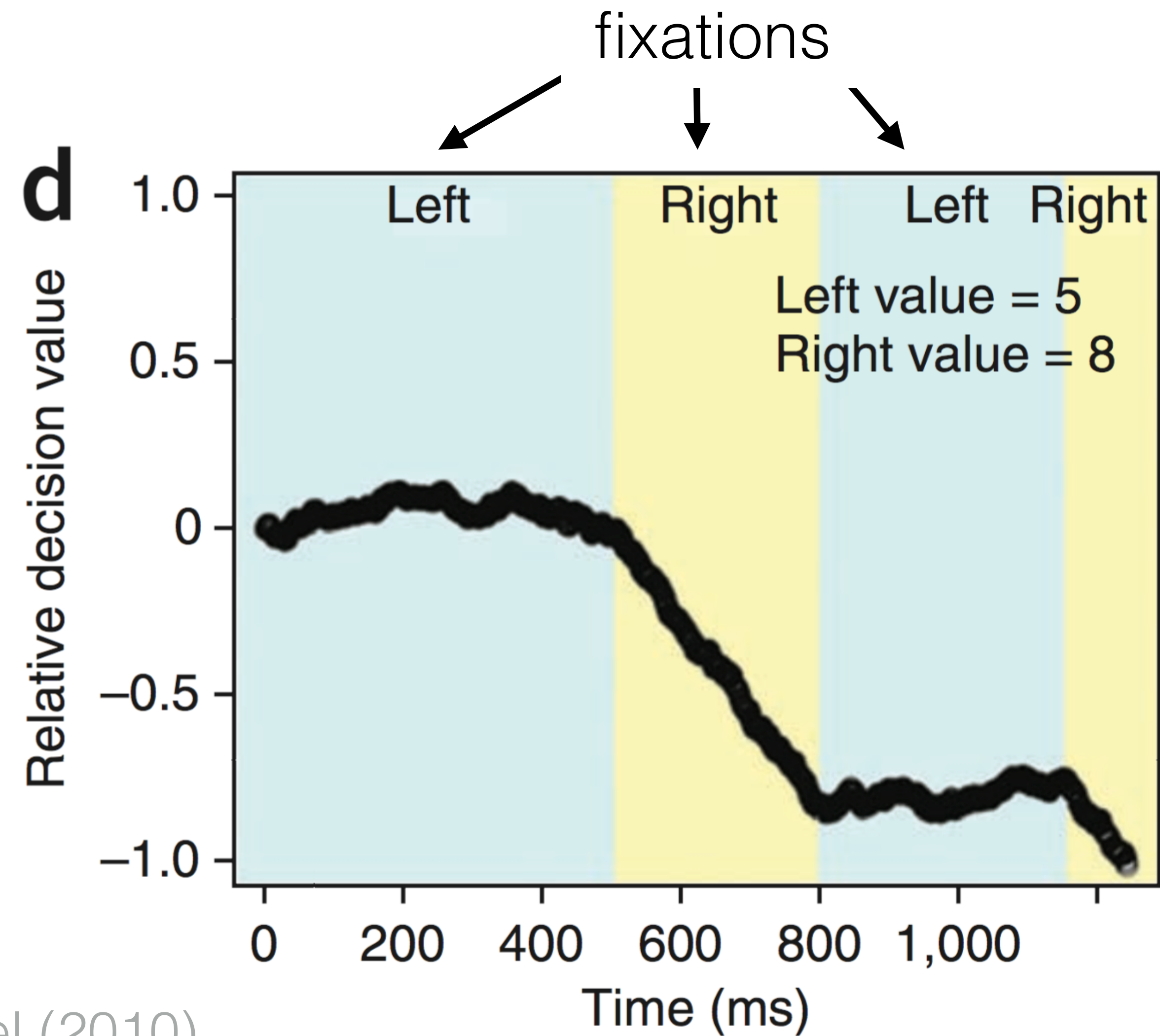
The 2019 Sloan-Nomis Workshop on the Cognitive Foundations of Economic Behavior

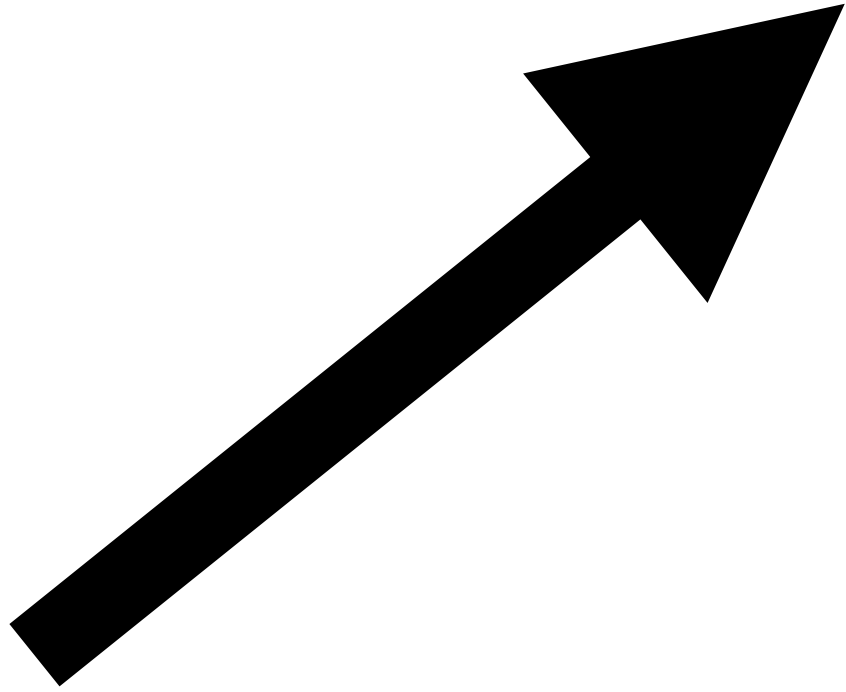
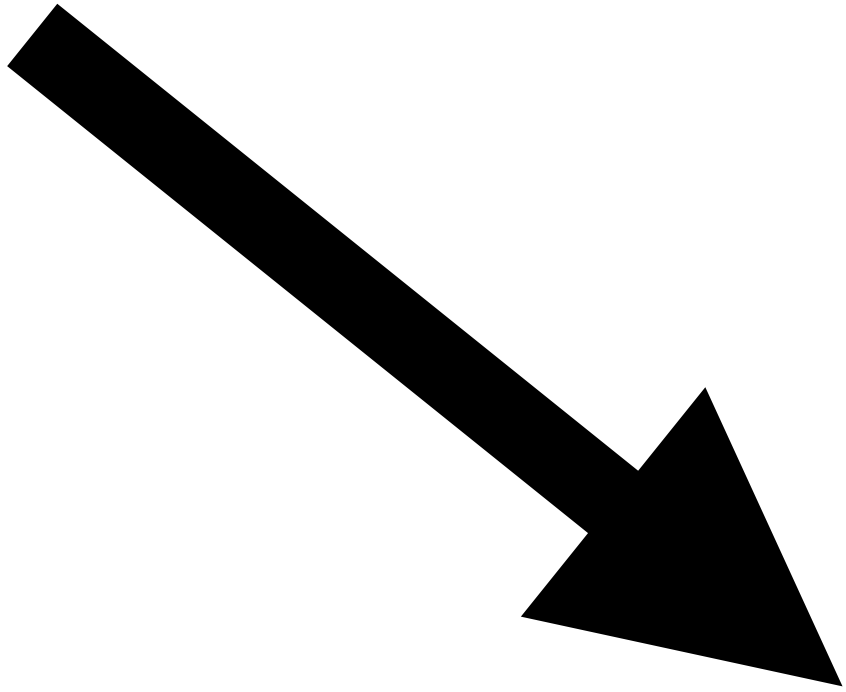
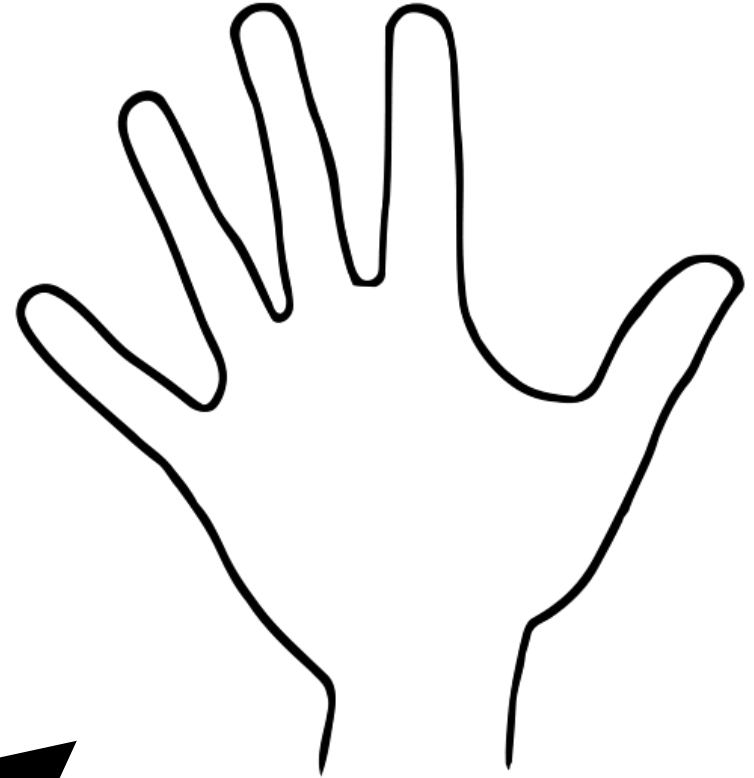
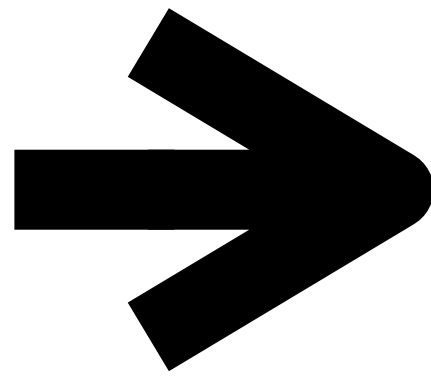
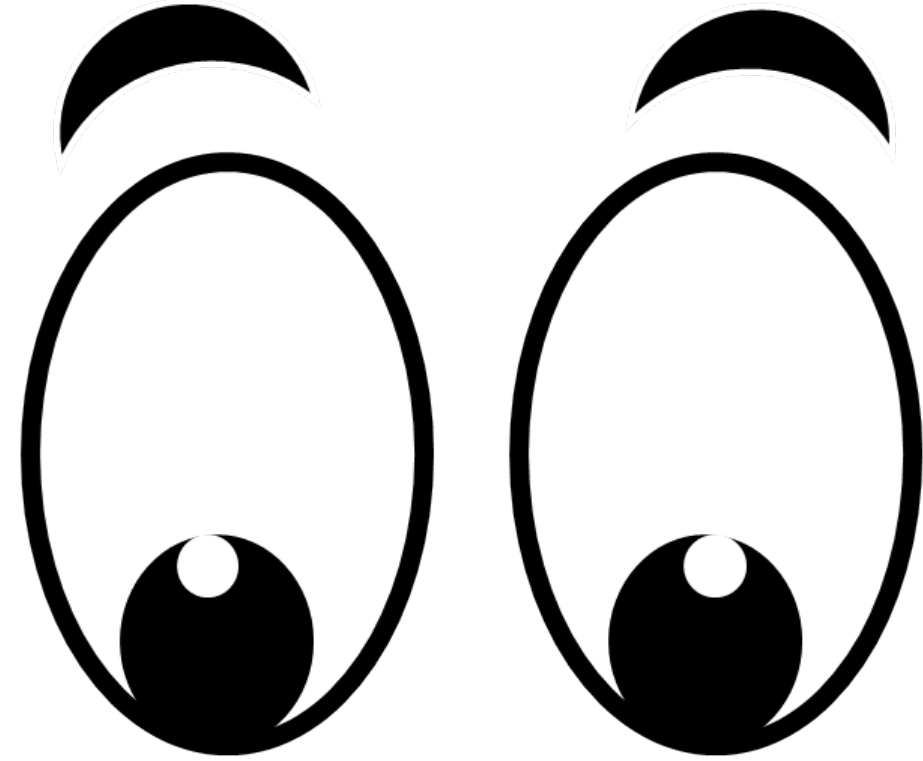


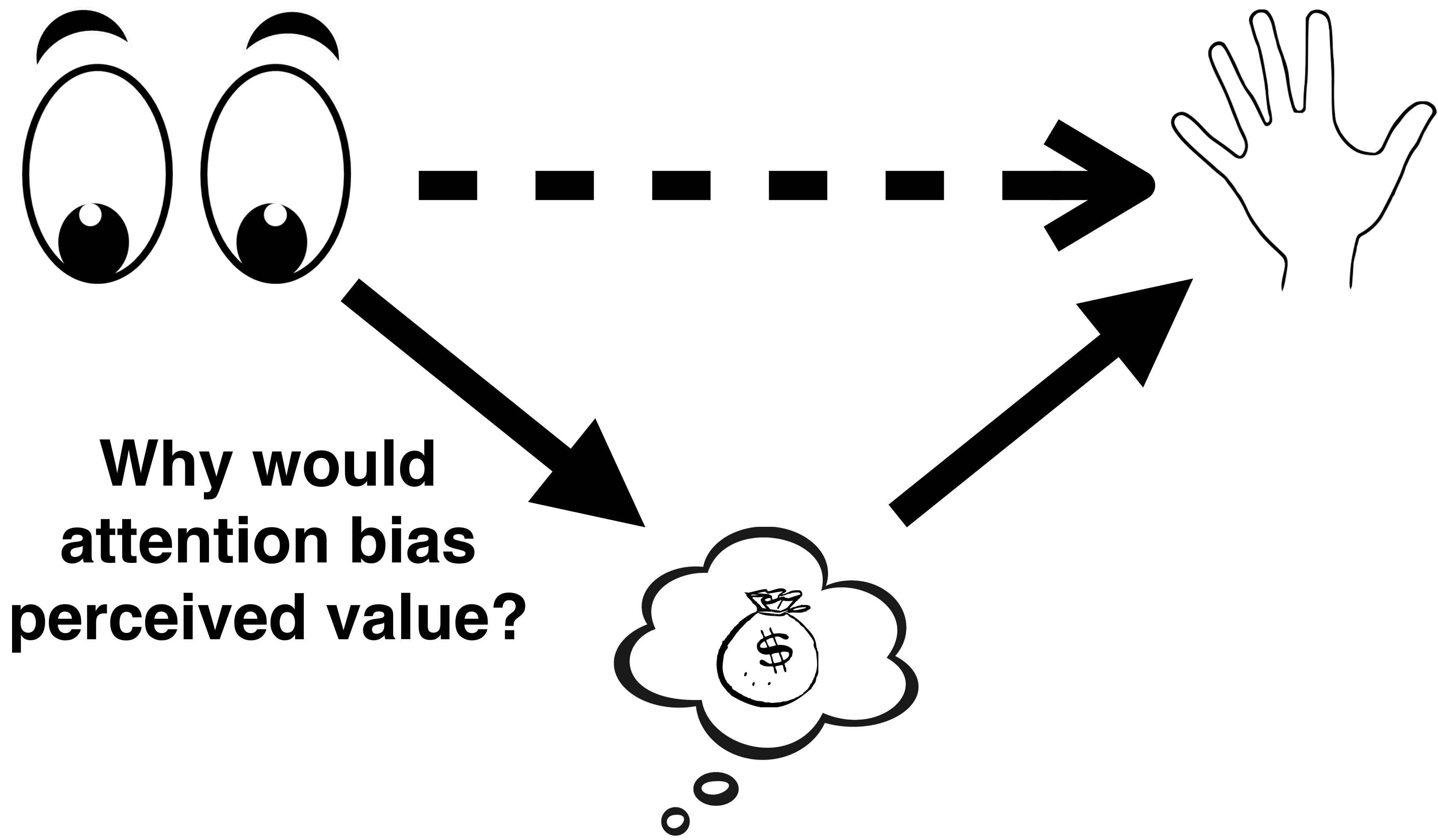




# Attentional drift diffusion model

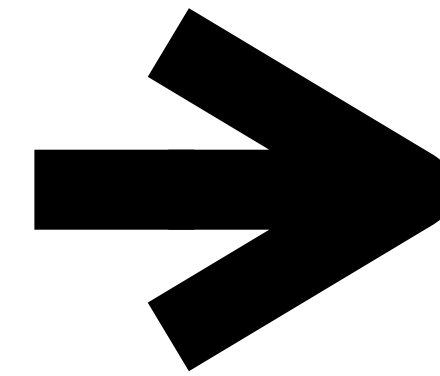
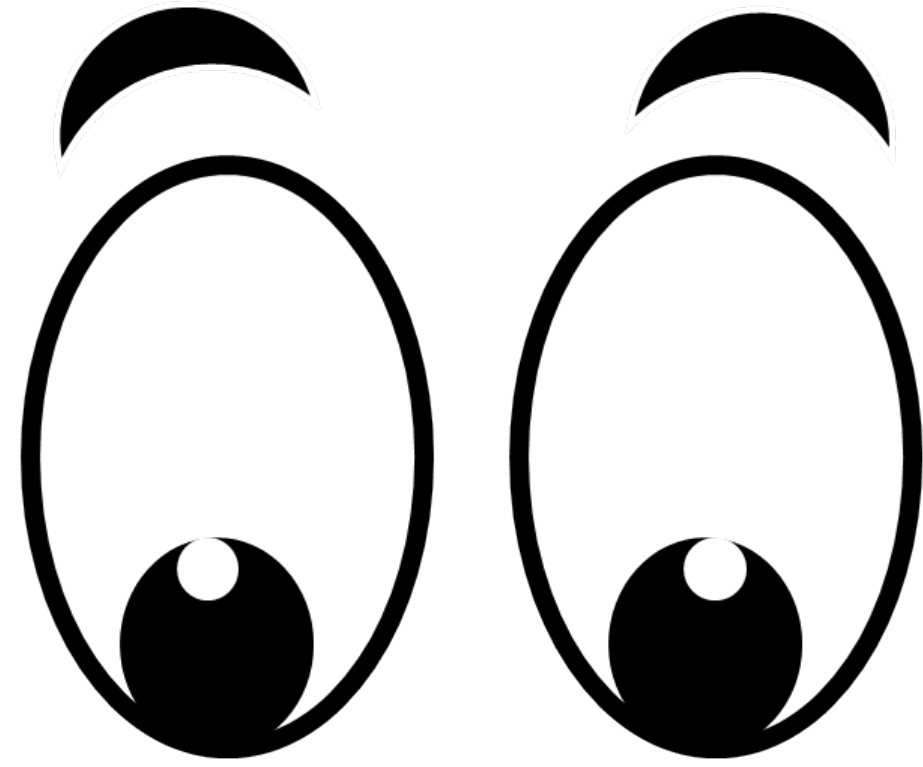




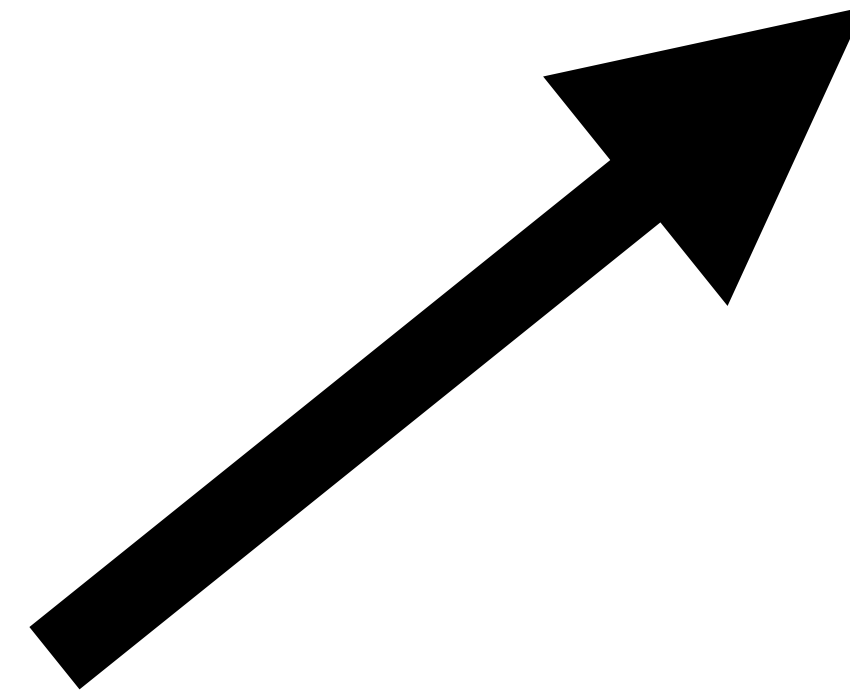
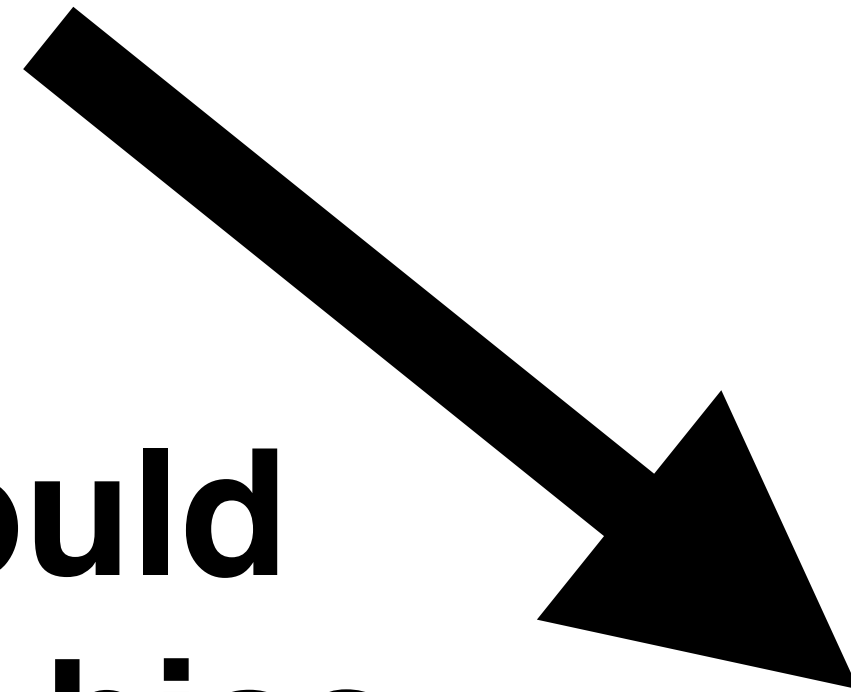


**Why would  
attention bias  
perceived value?**

**How do we choose  
what to look at?**

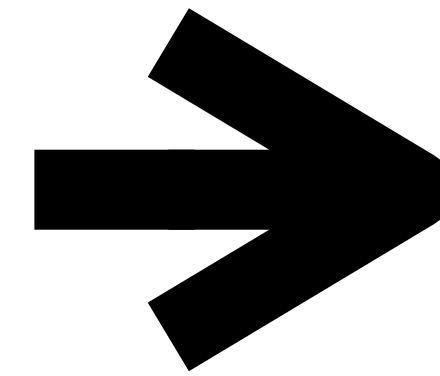
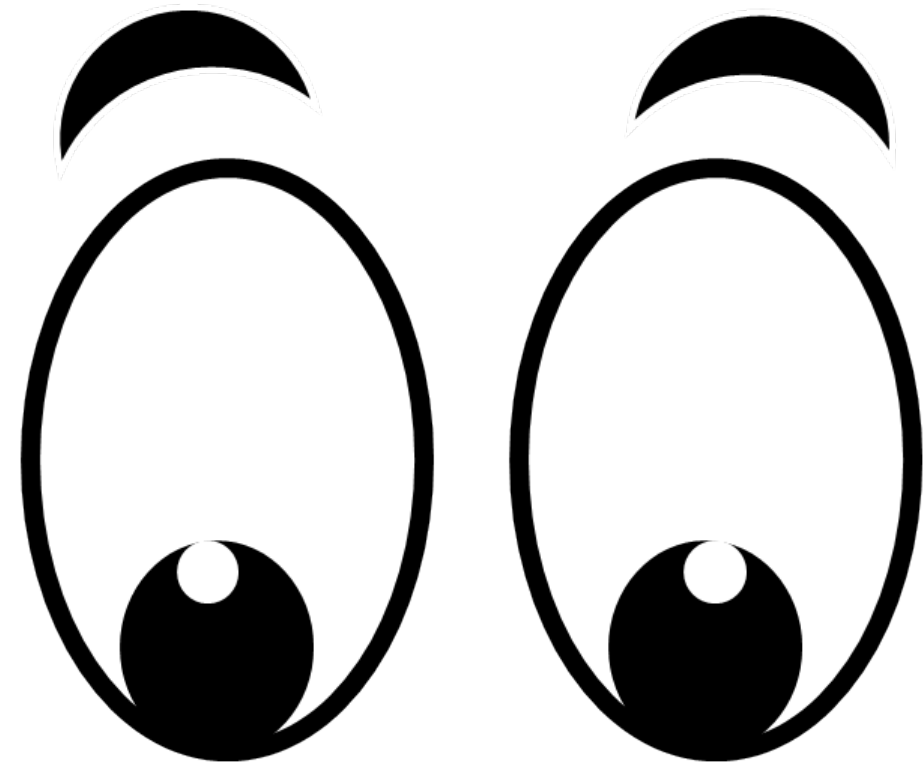


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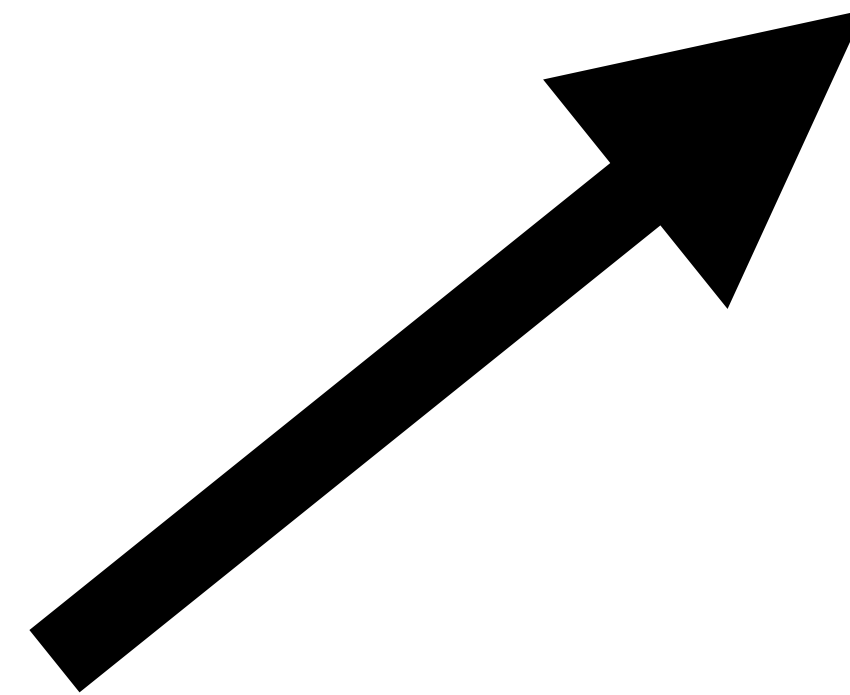
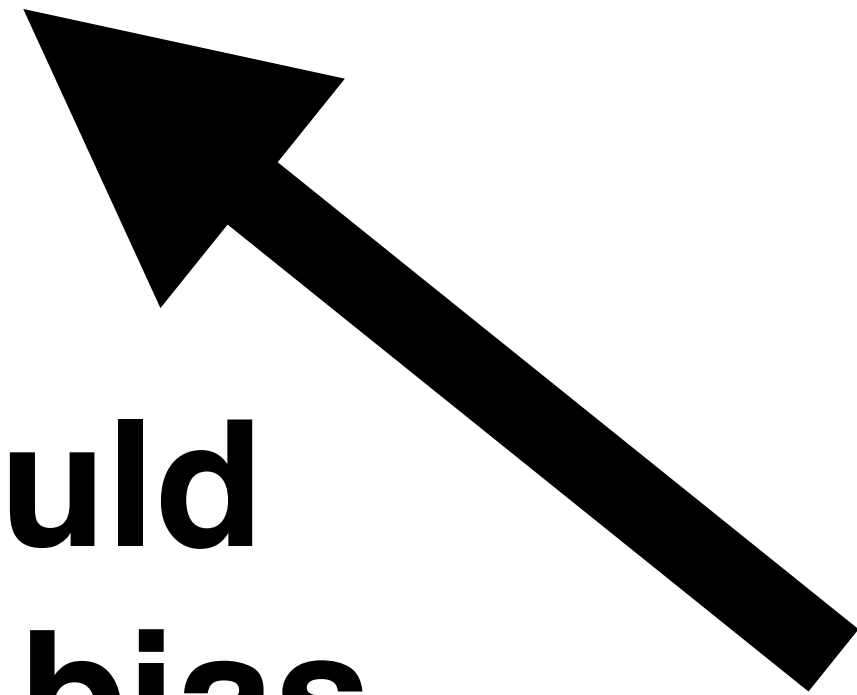




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- ▶ Take the action with maximal expected utility.

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“Do the right *thinking*.”

How can we model the right thinking?

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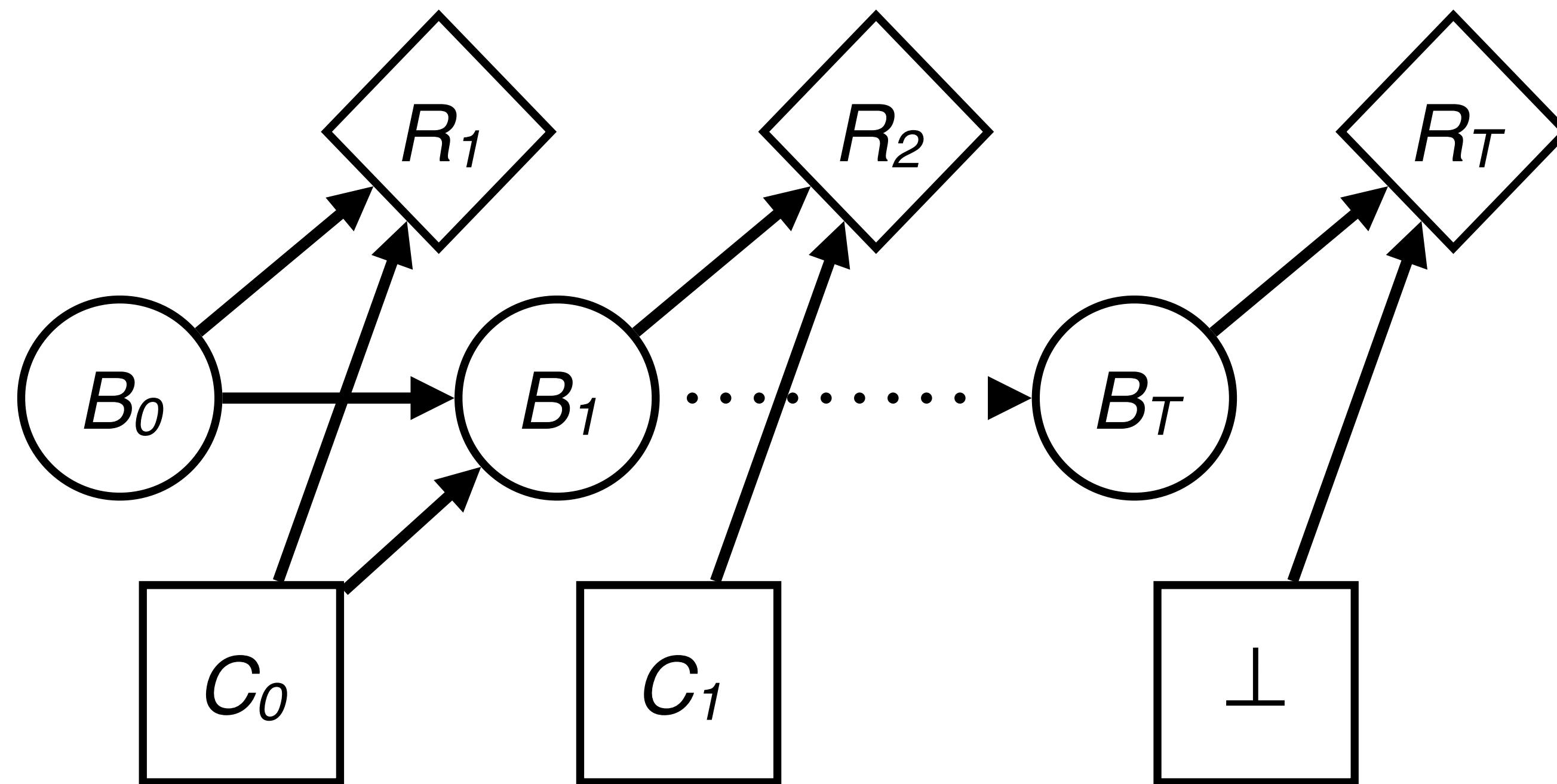
**Key Intuition**

*Thinking is a sequential  
decision problem.*



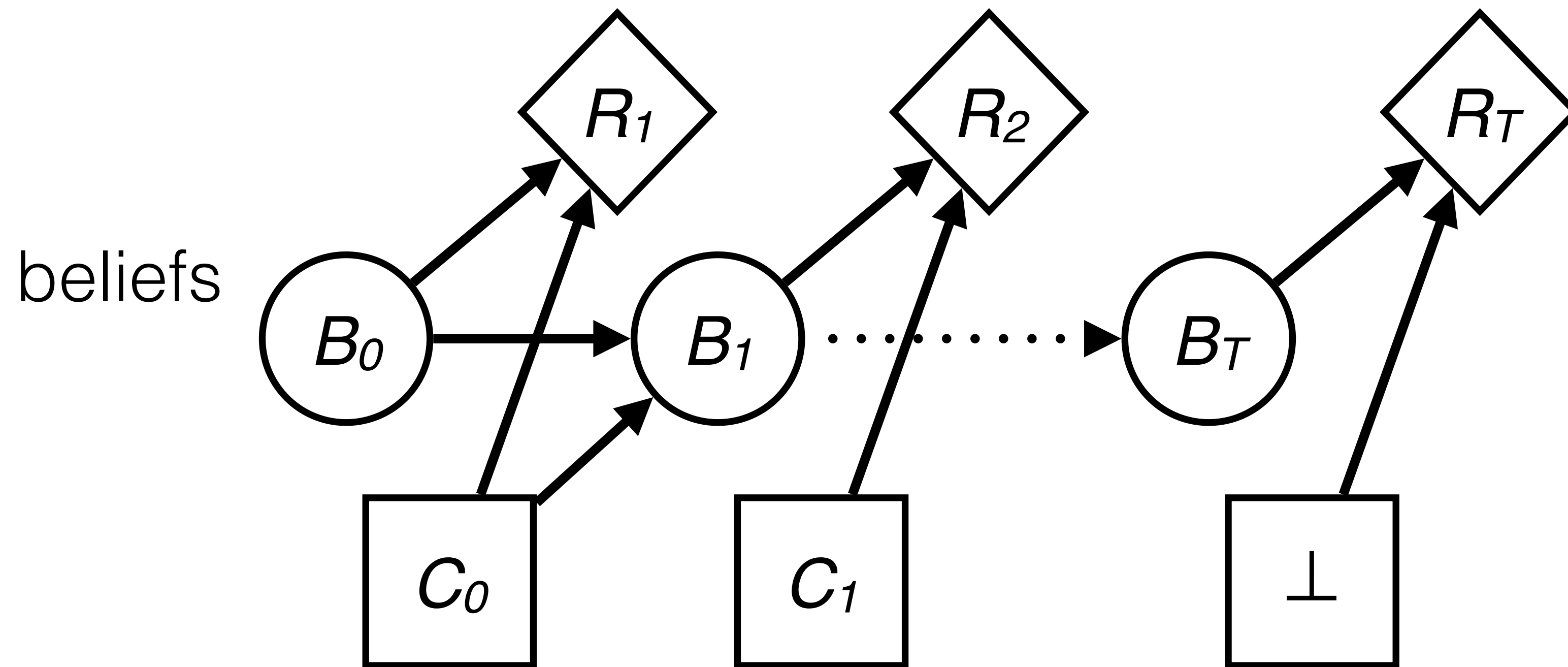
# How can we model the right thinking?

## ***Metalevel* Markov decision process**



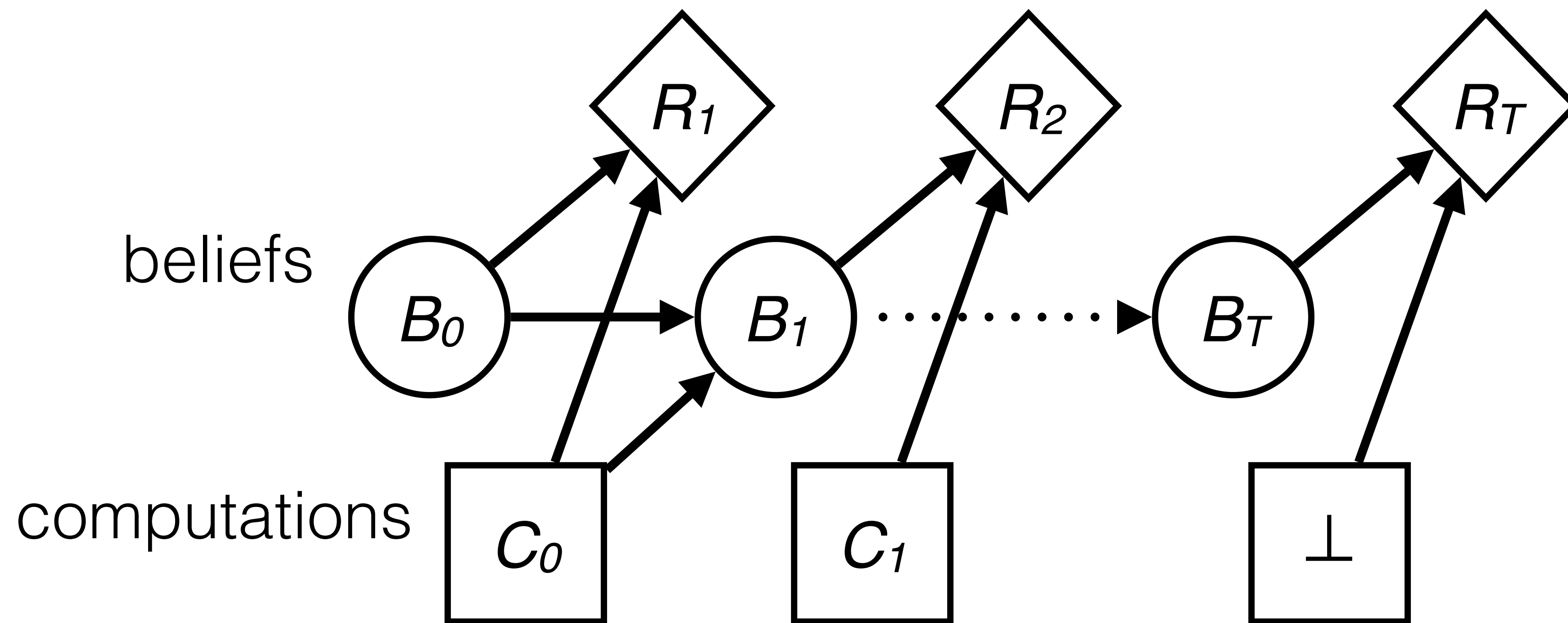
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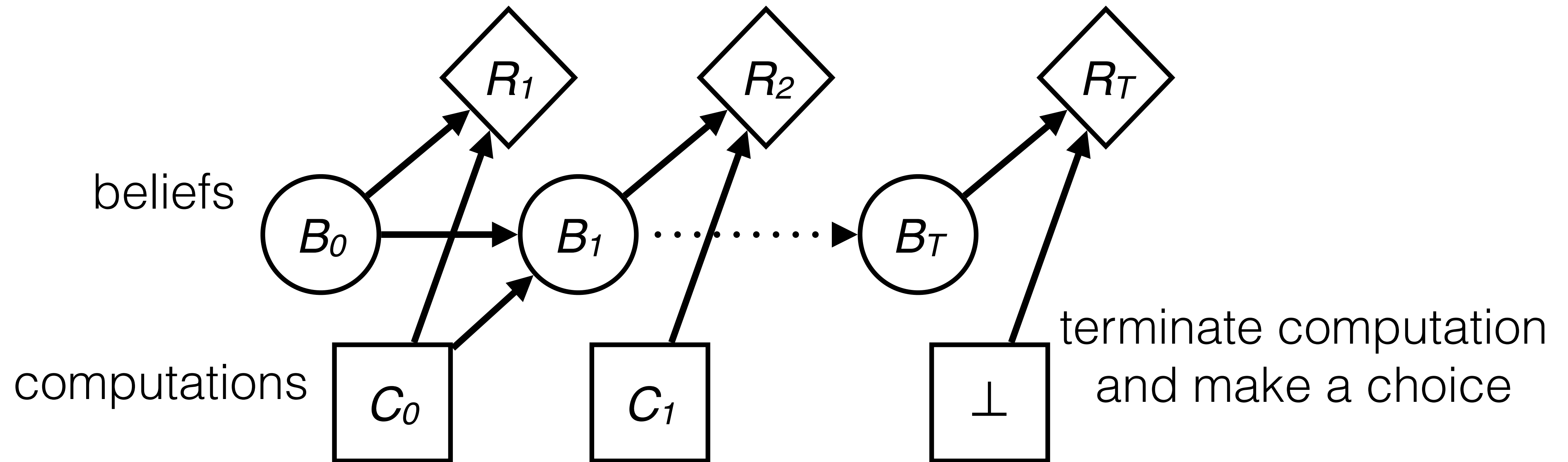
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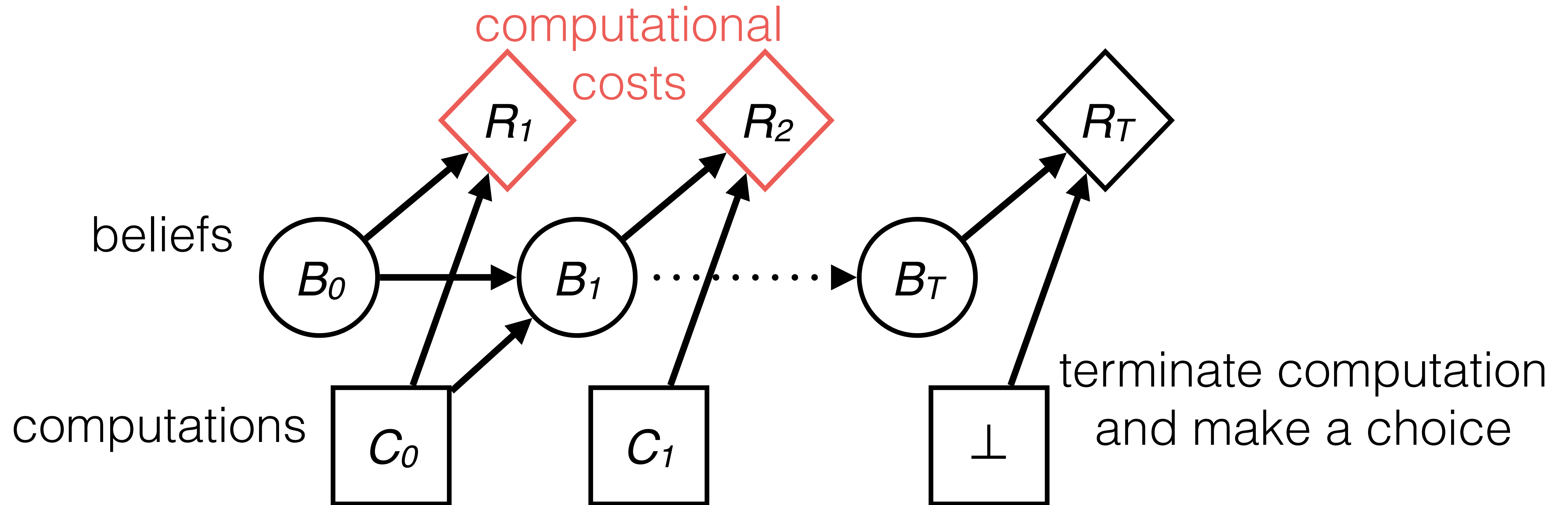
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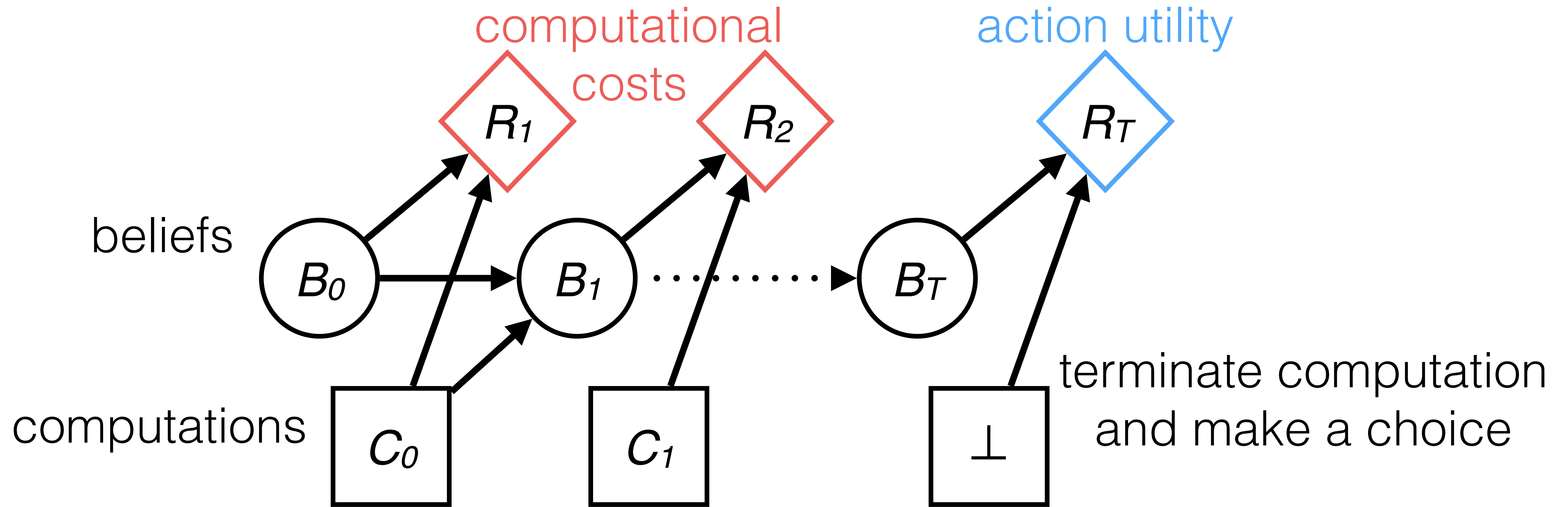
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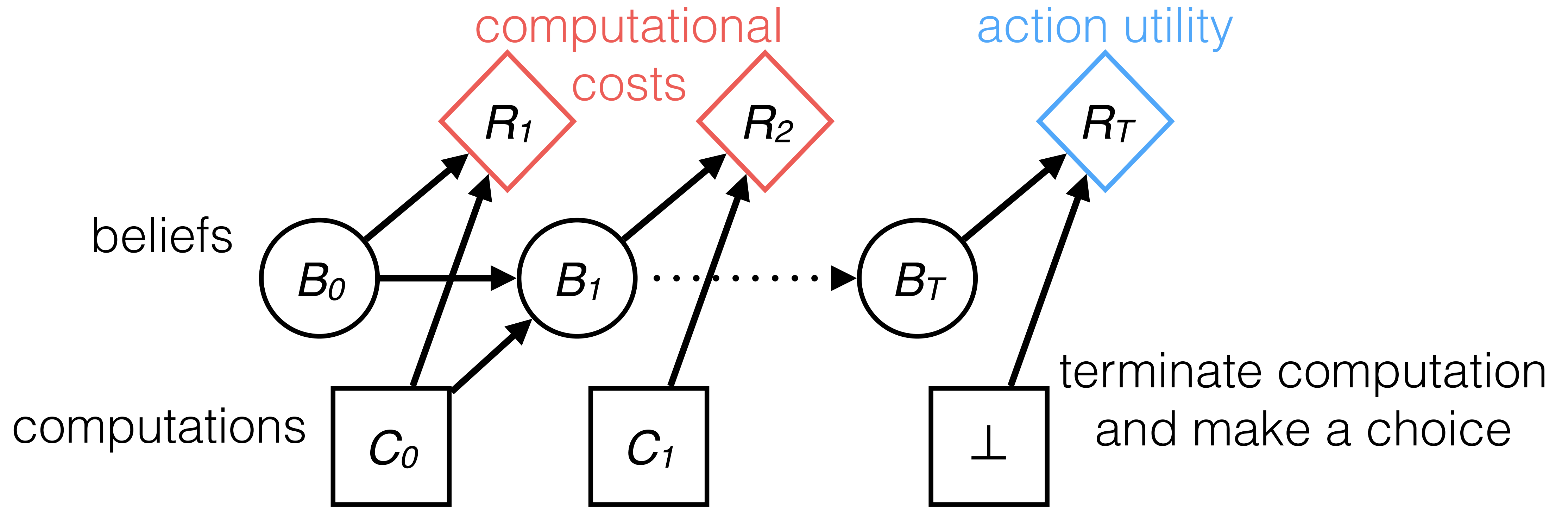
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Hay et al. (2012)

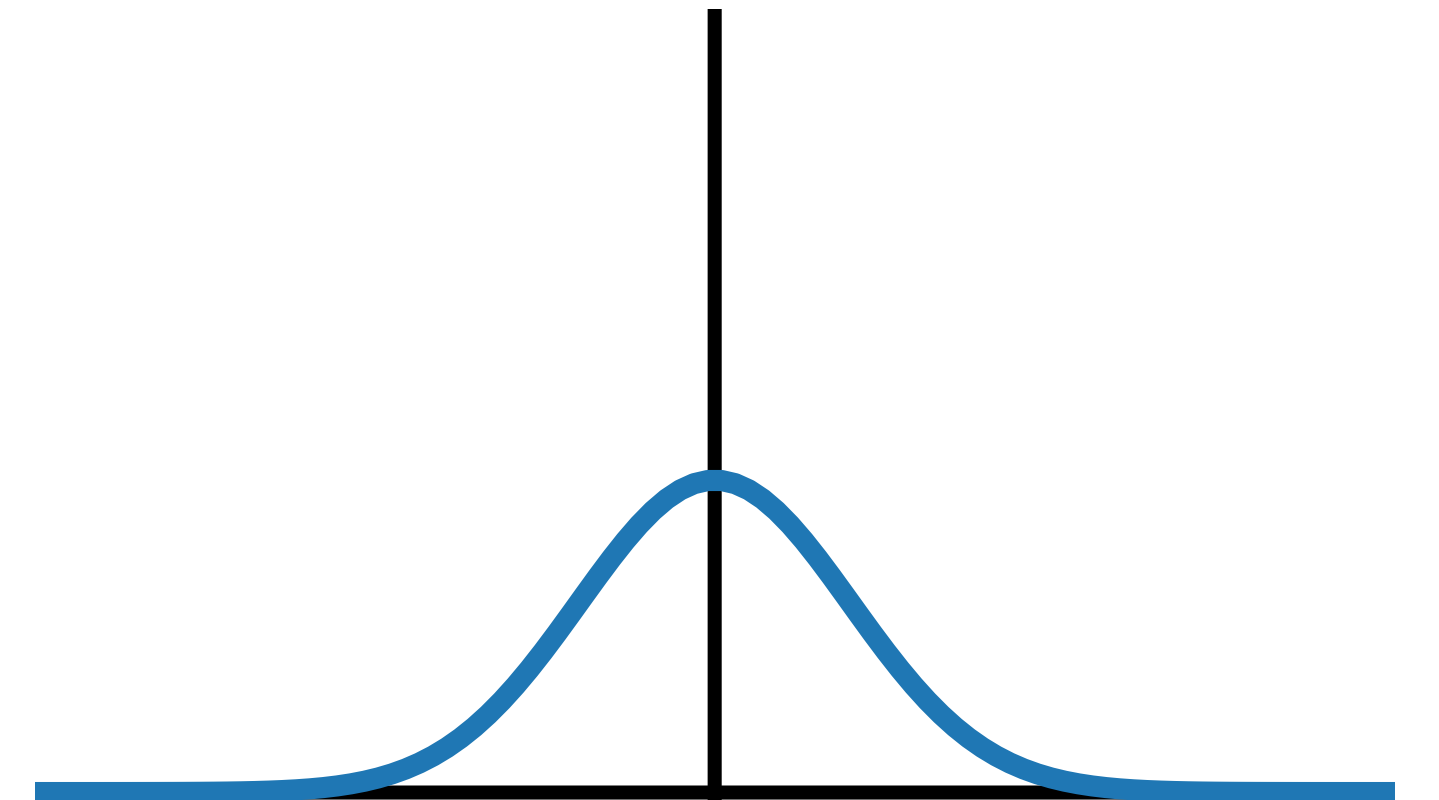
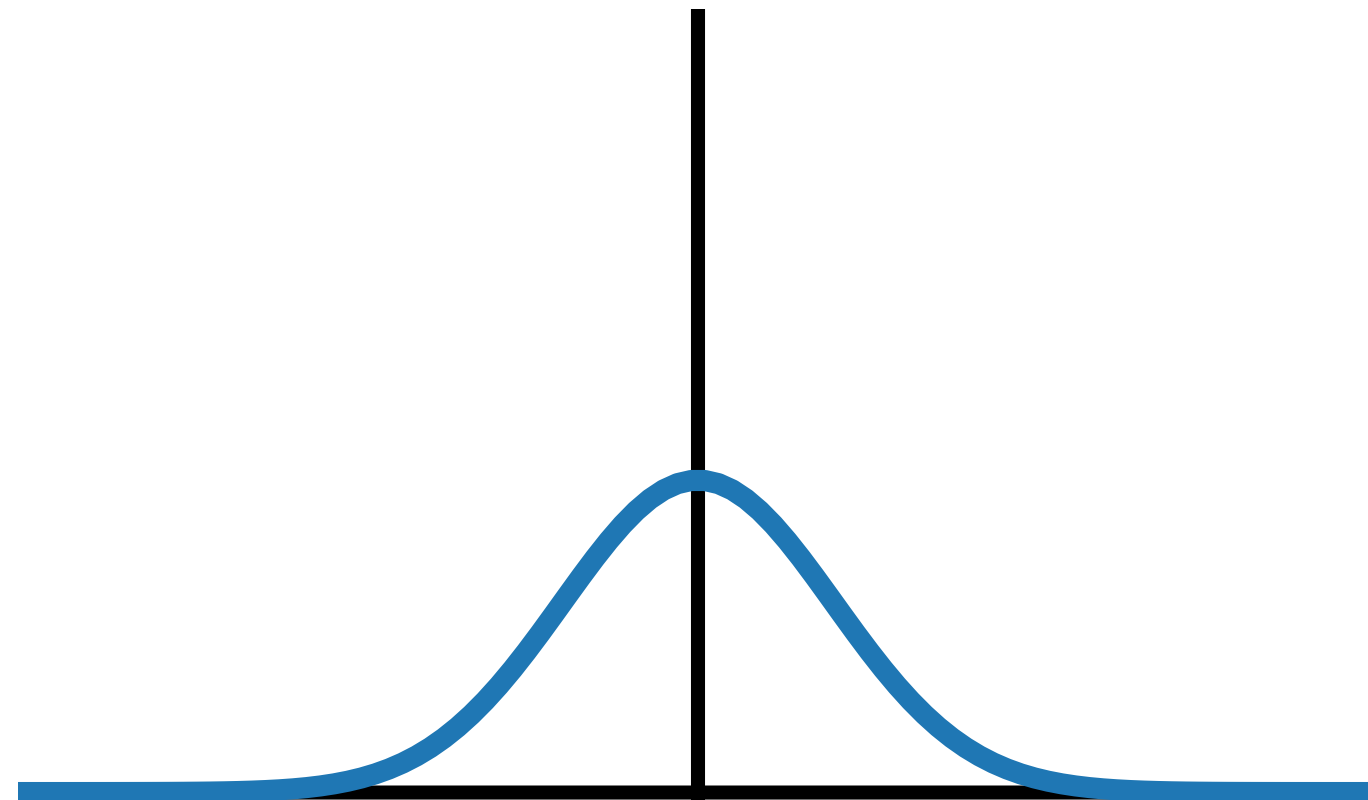
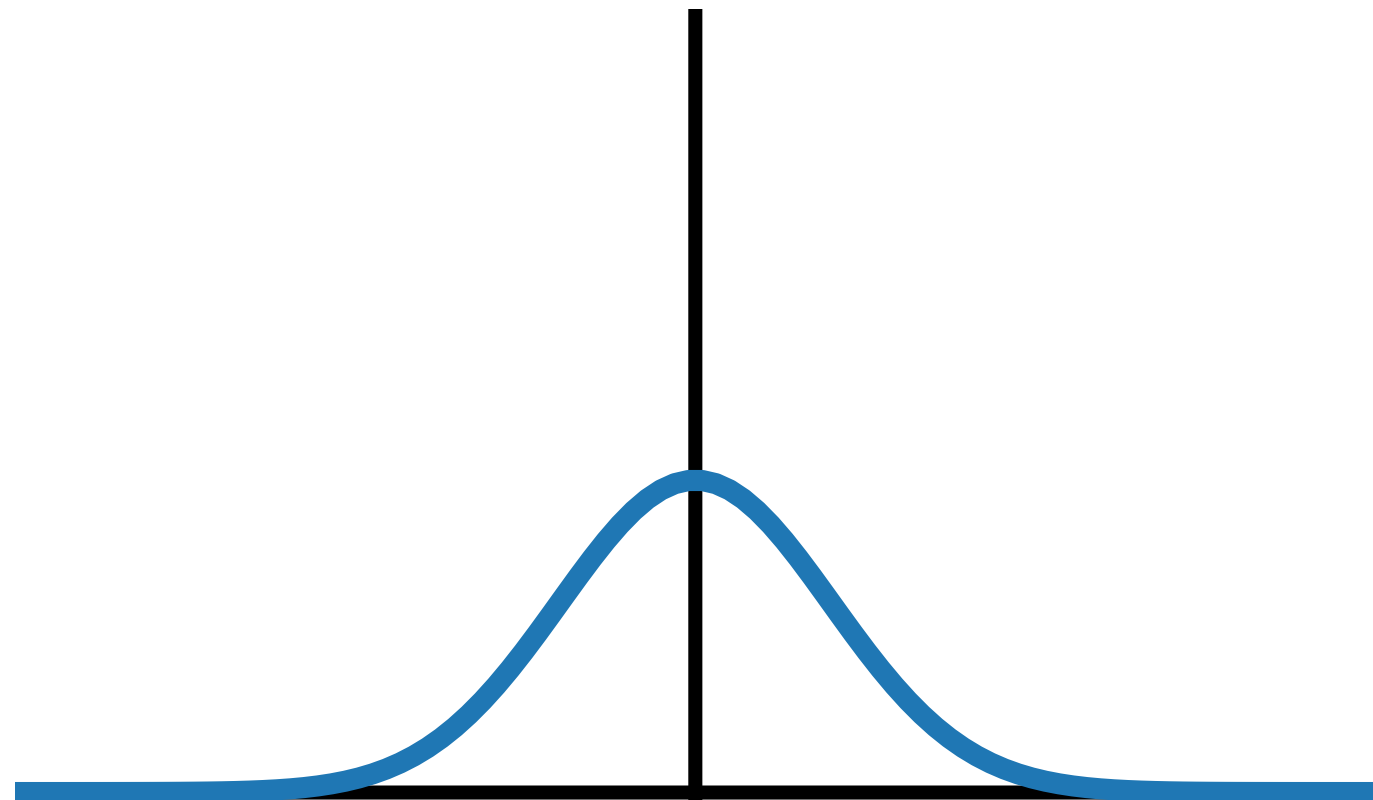
$$\arg \max_{\pi} \mathbb{E} \left[ \underbrace{\max_a \mathbb{E} [U(a) | B_T]}_{\text{action utility}} - \underbrace{\sum_{t=0}^{T-1} \text{cost}(B_t, C_t)}_{\text{computational costs}} \mid C_t \sim \pi(B_t) \right]$$





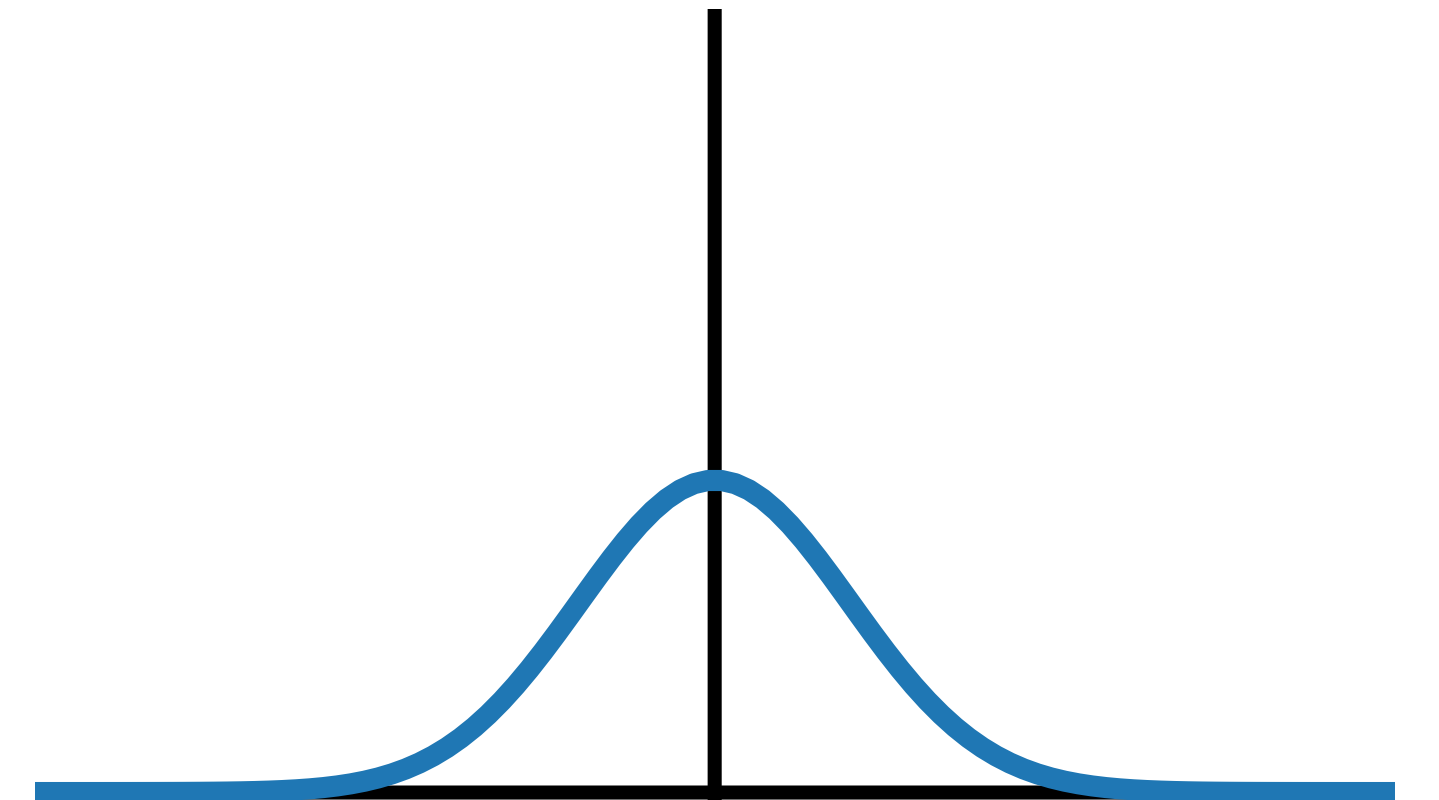
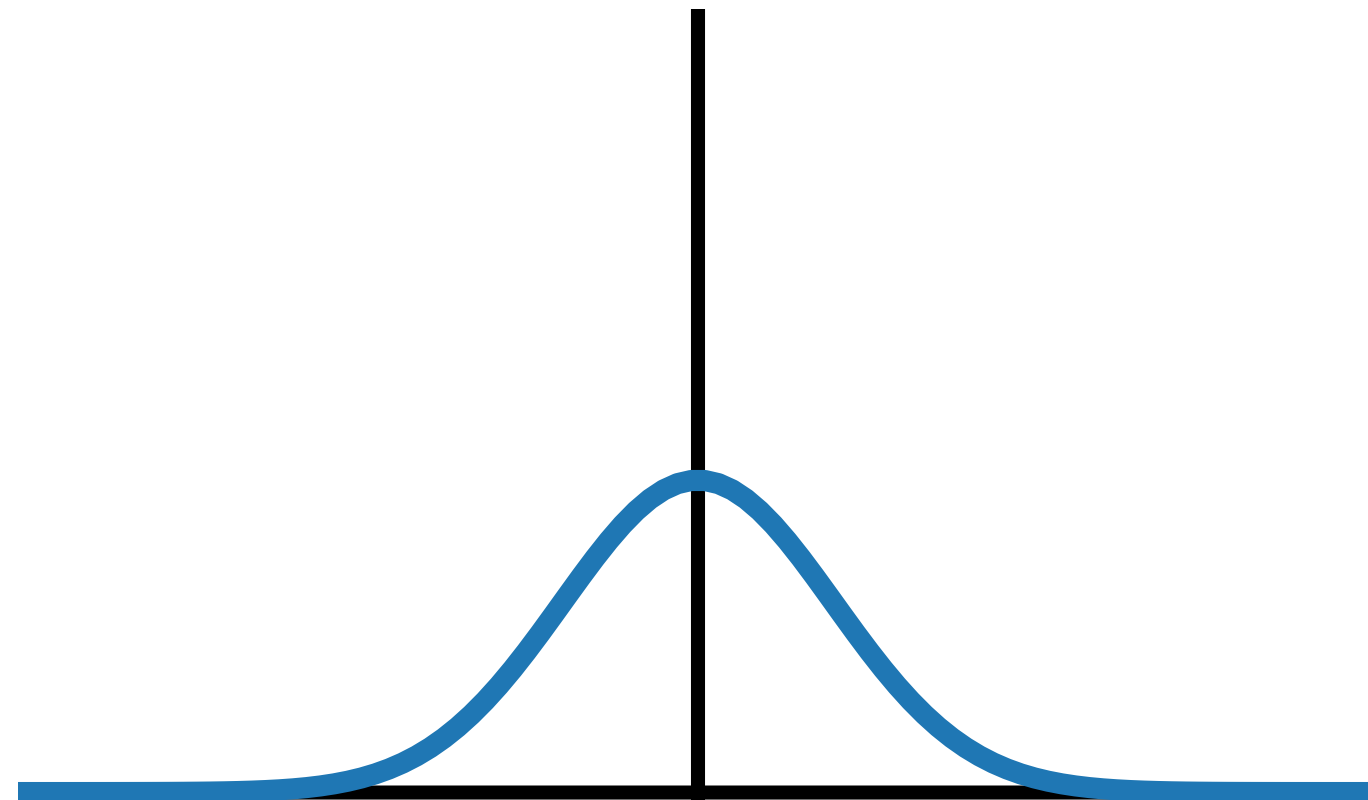
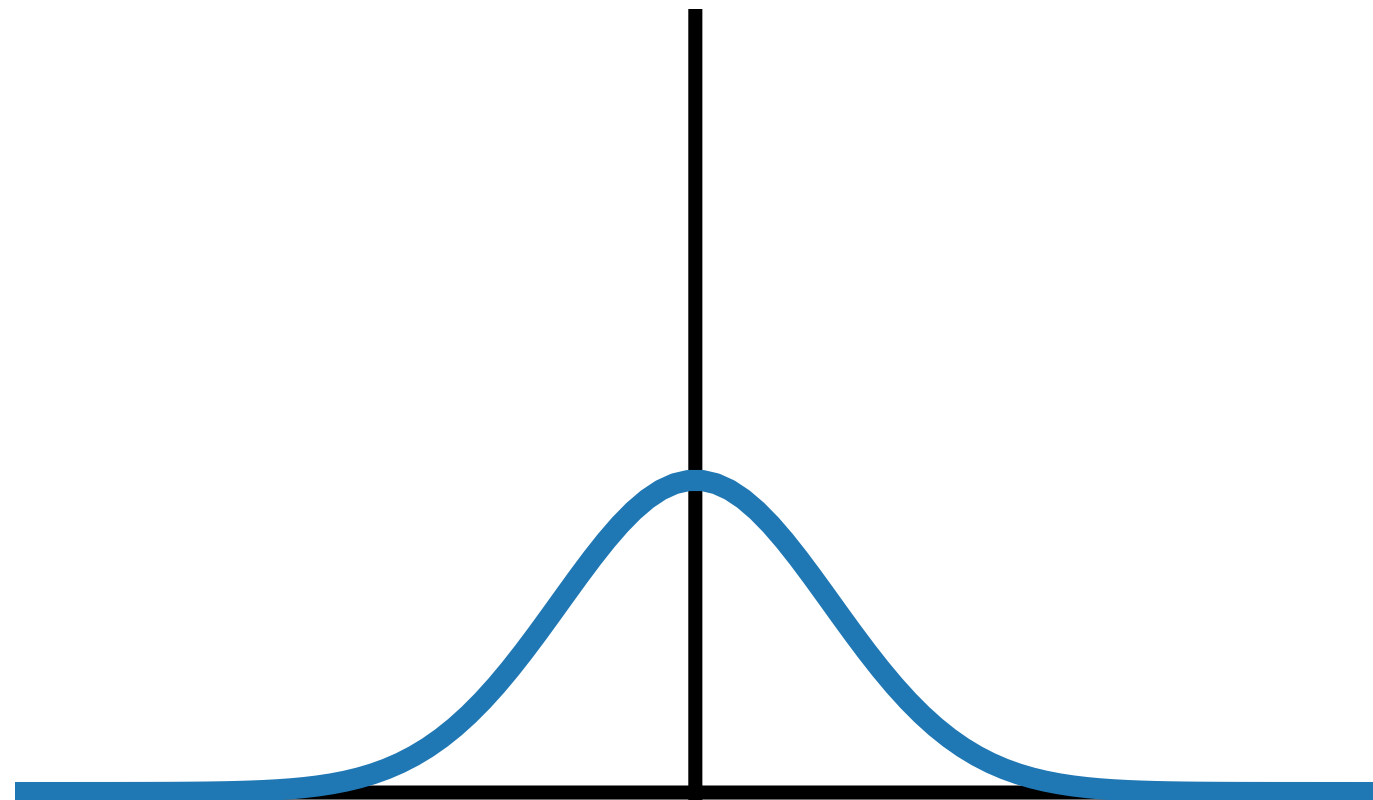
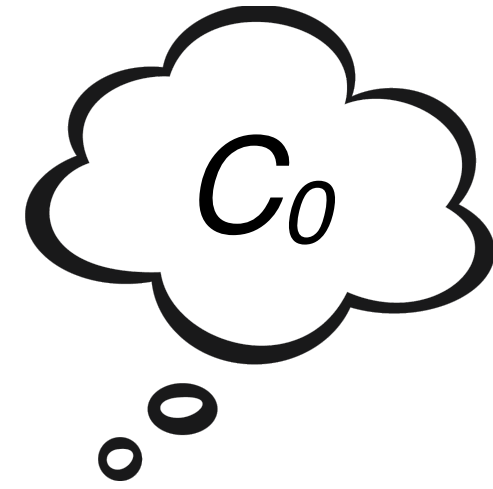


$B_0$



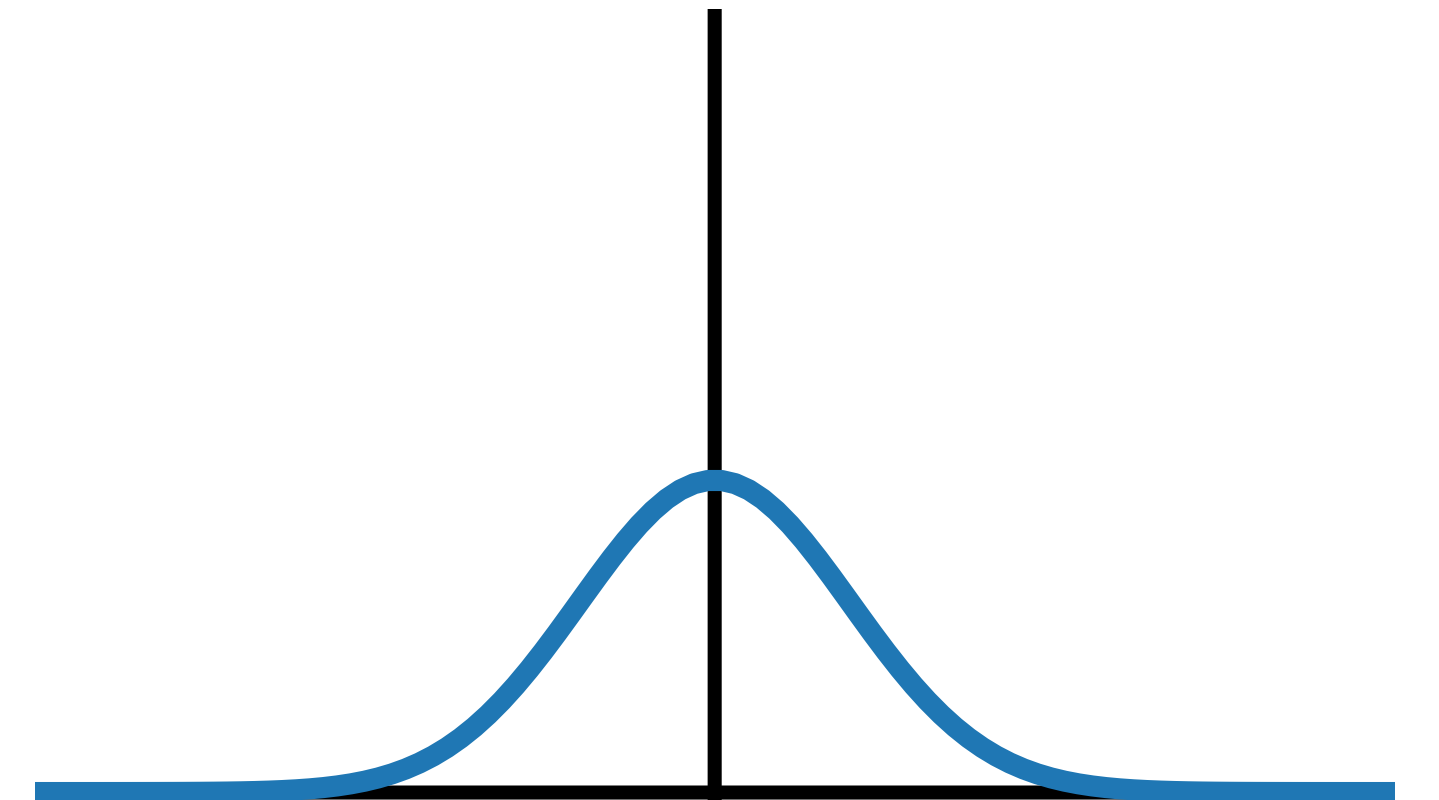
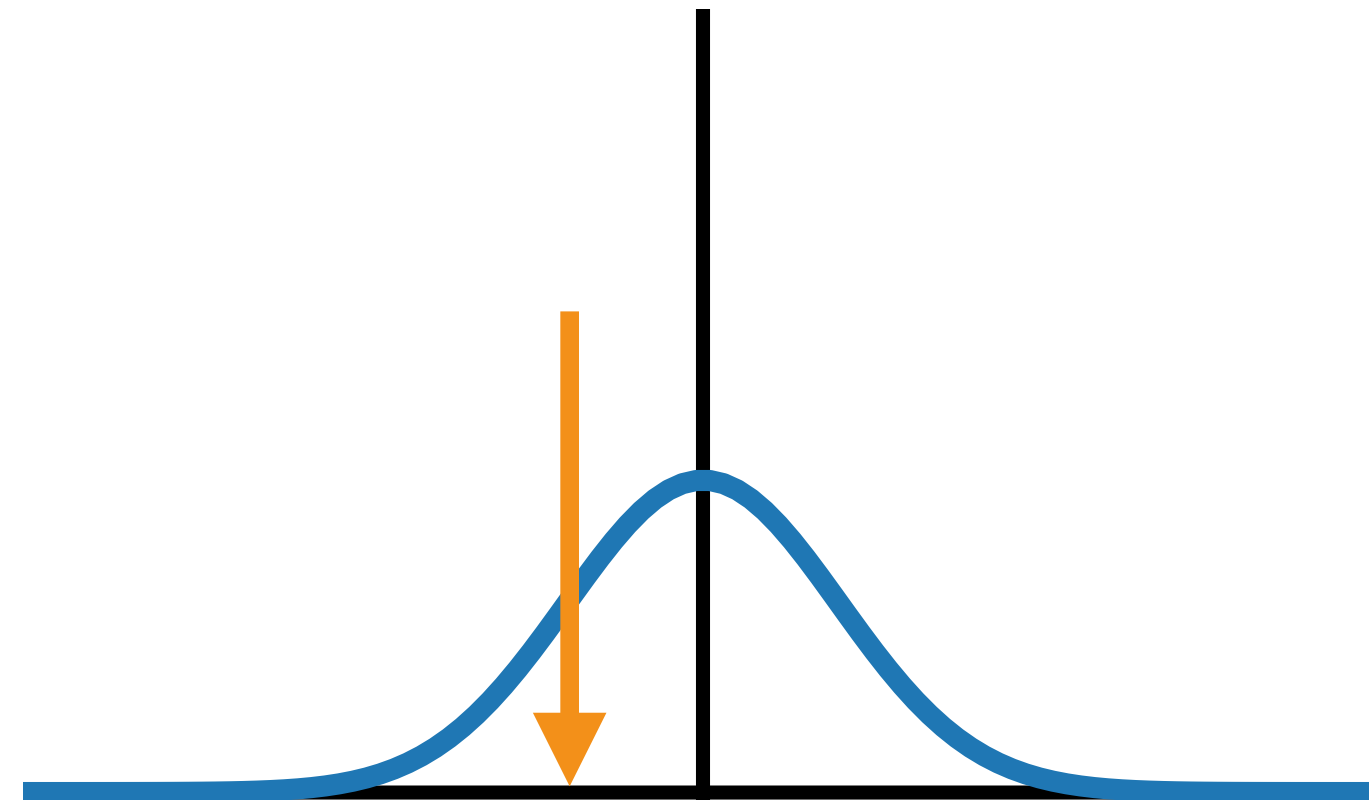
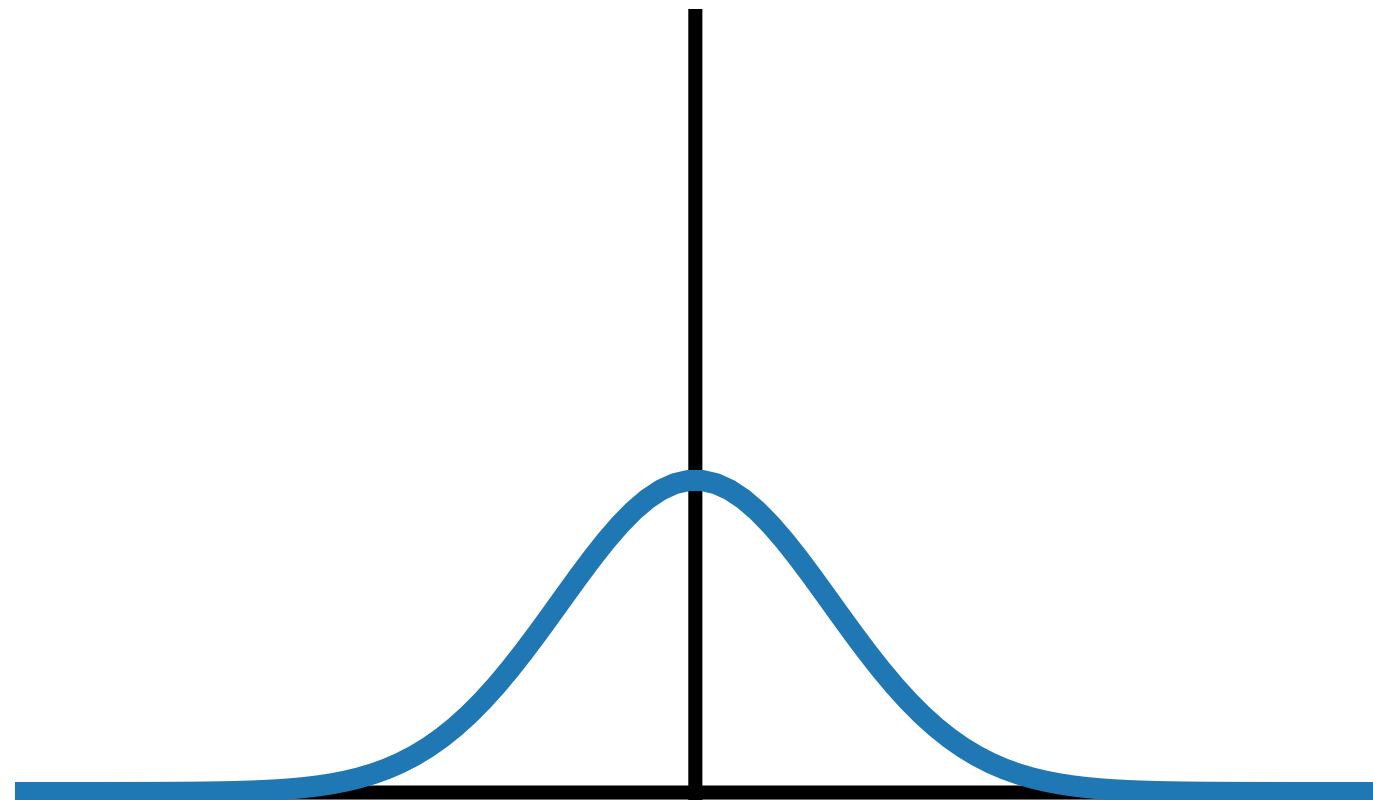
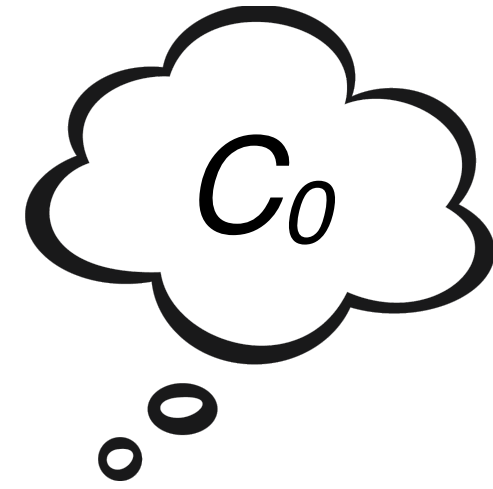


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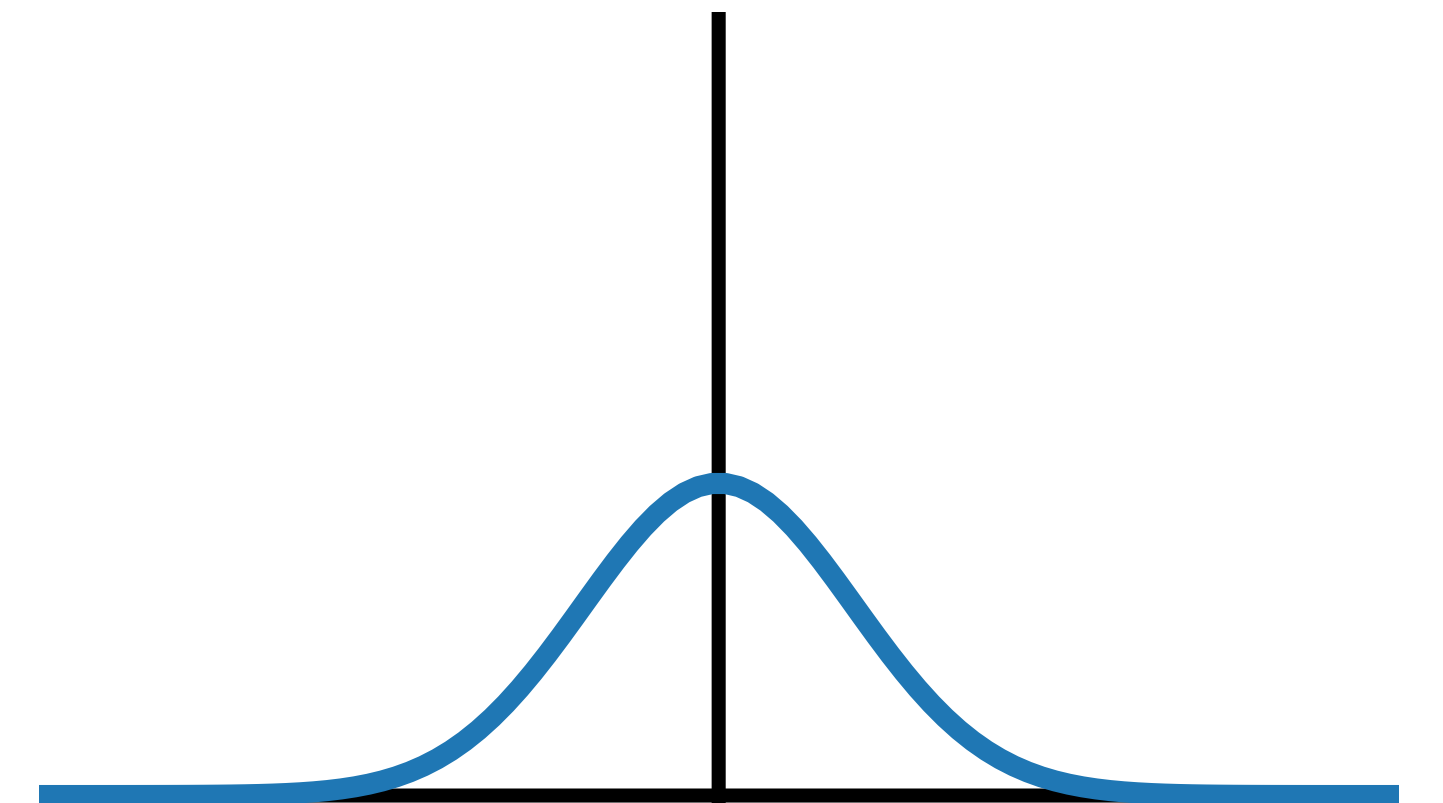
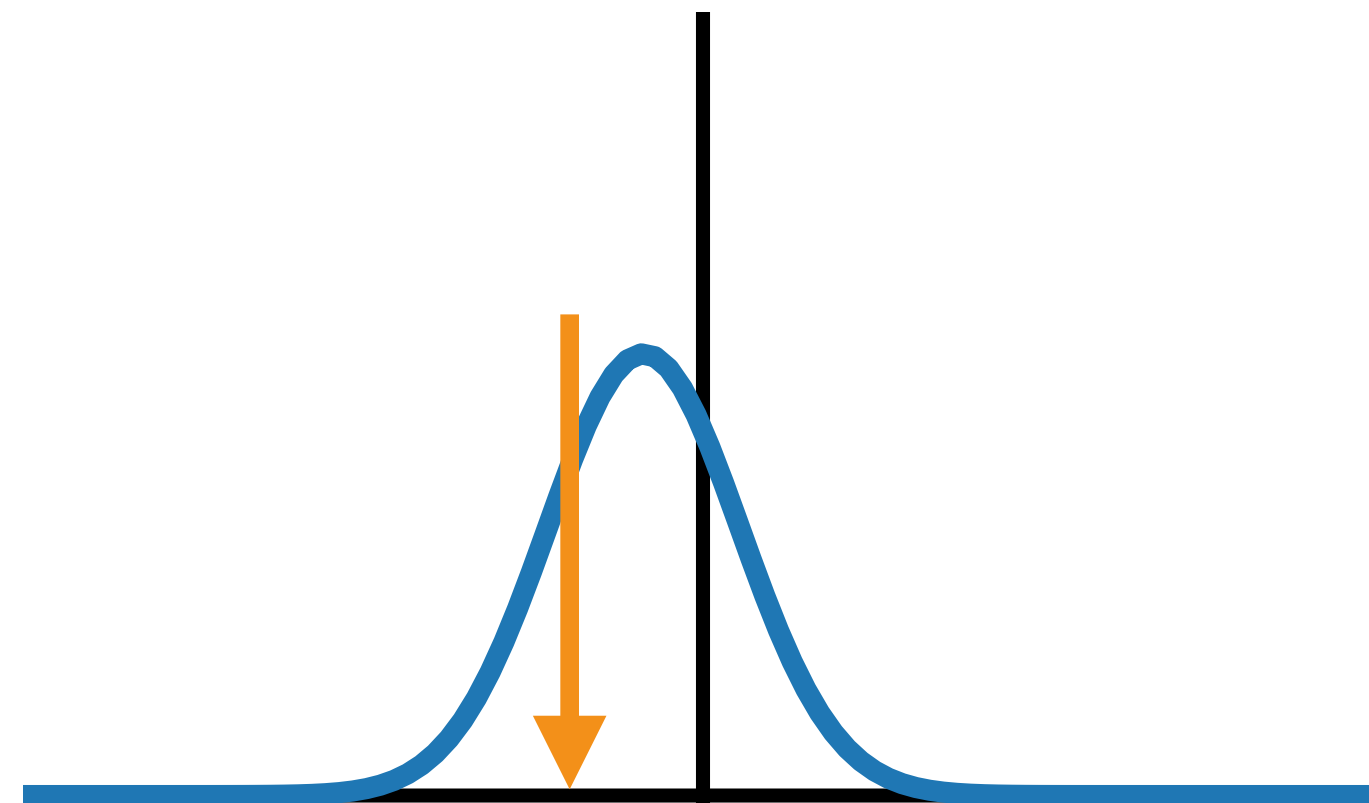
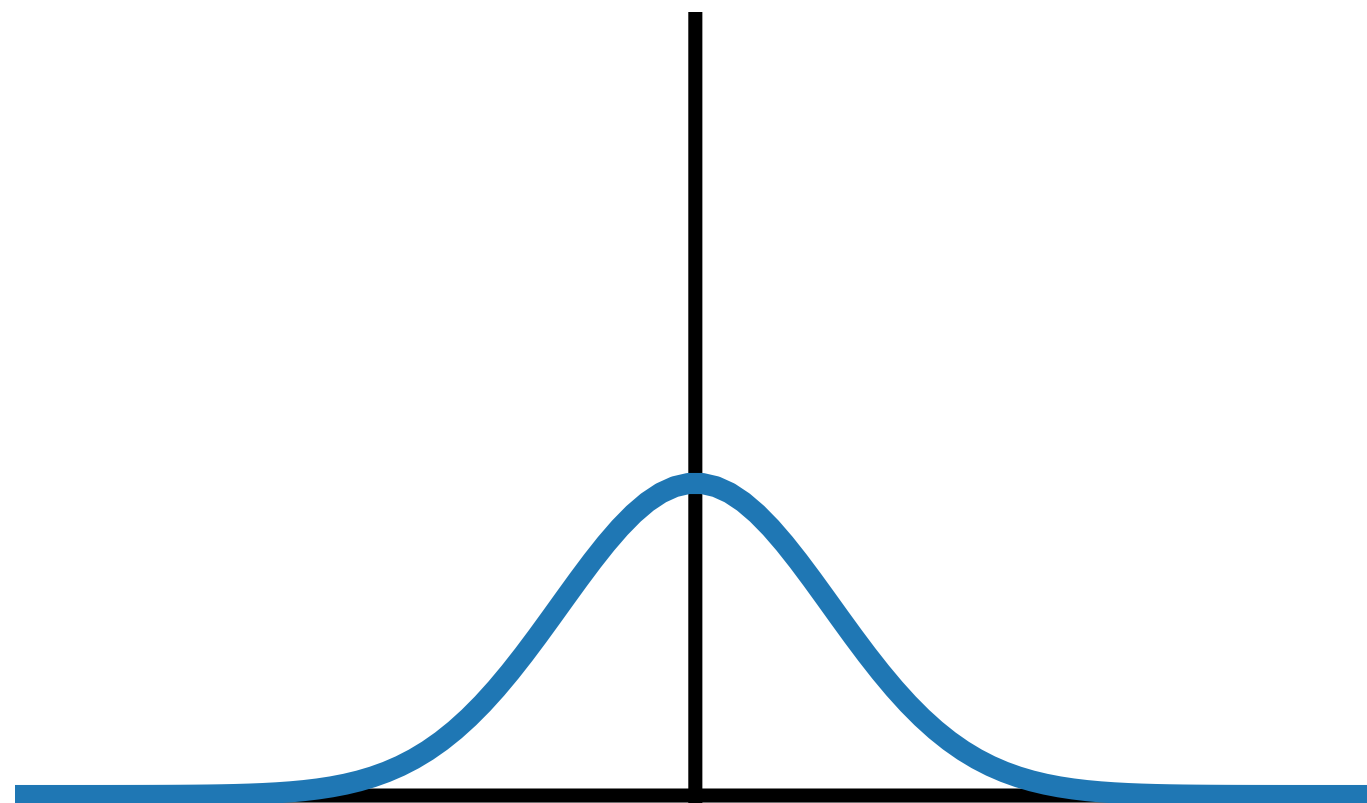
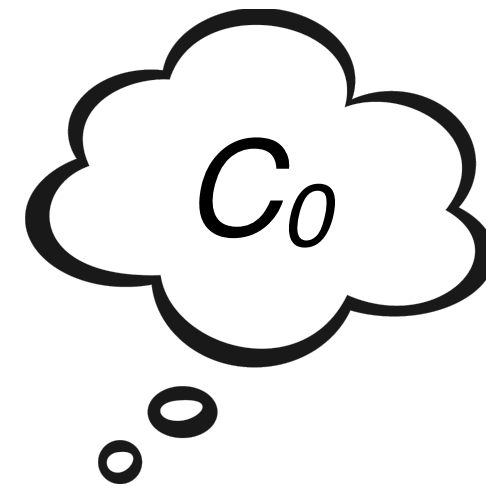


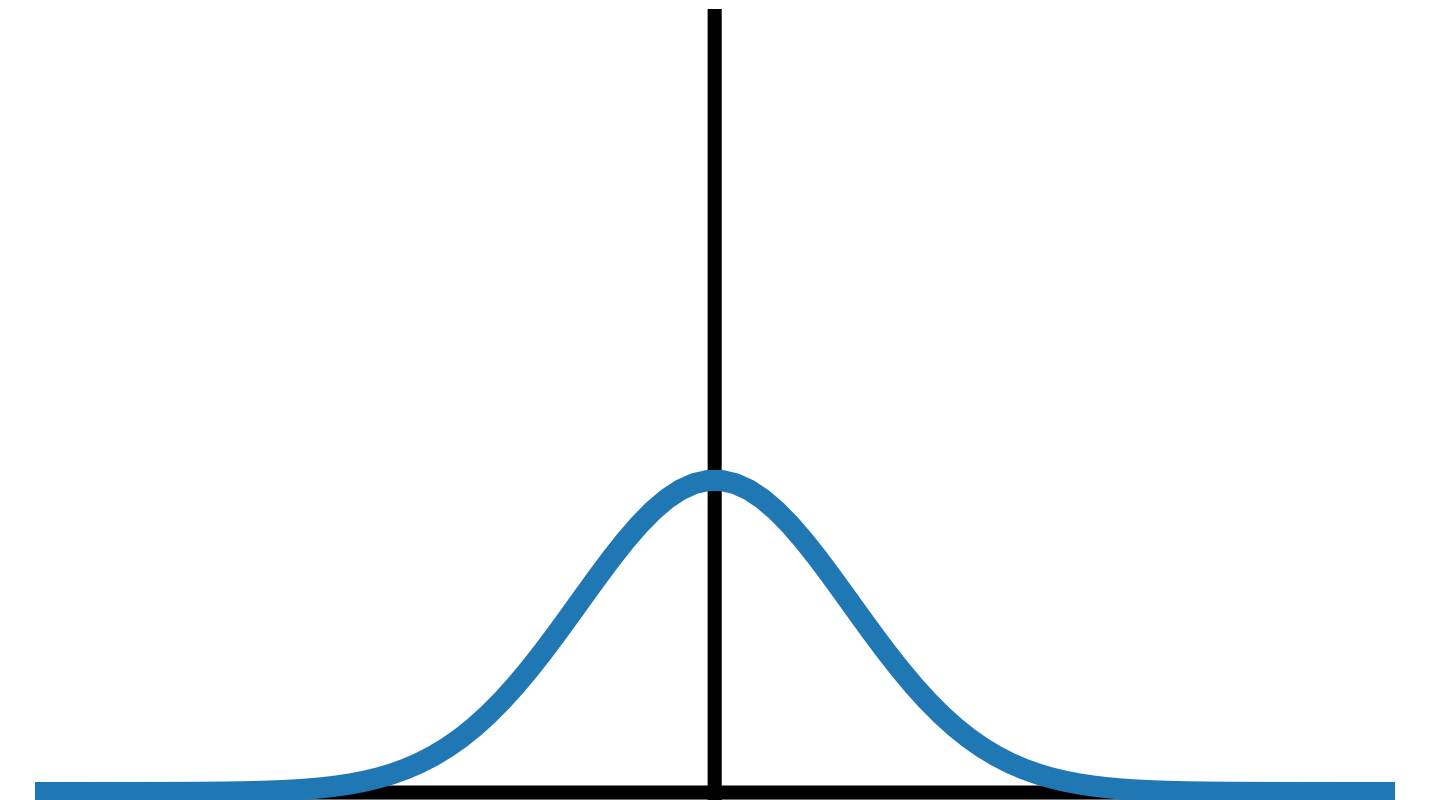
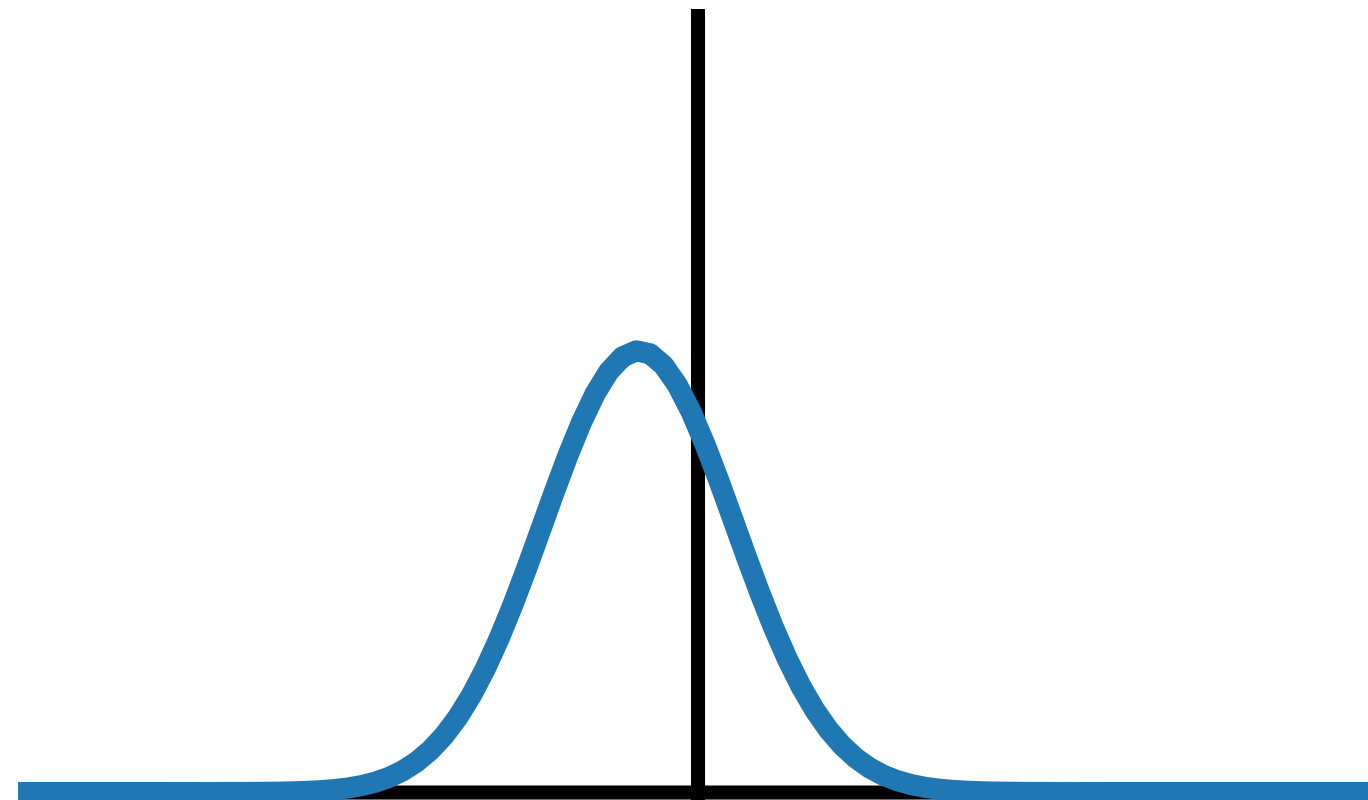
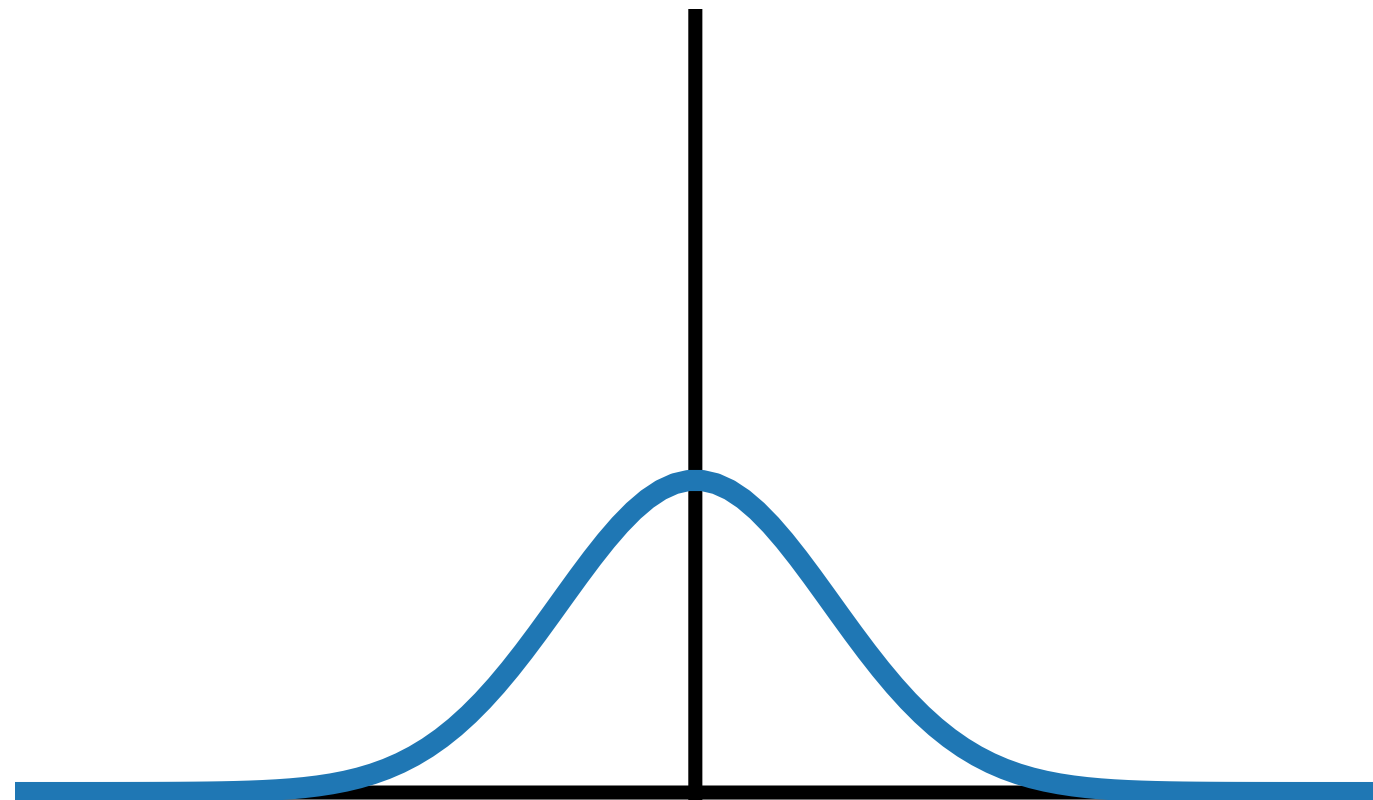
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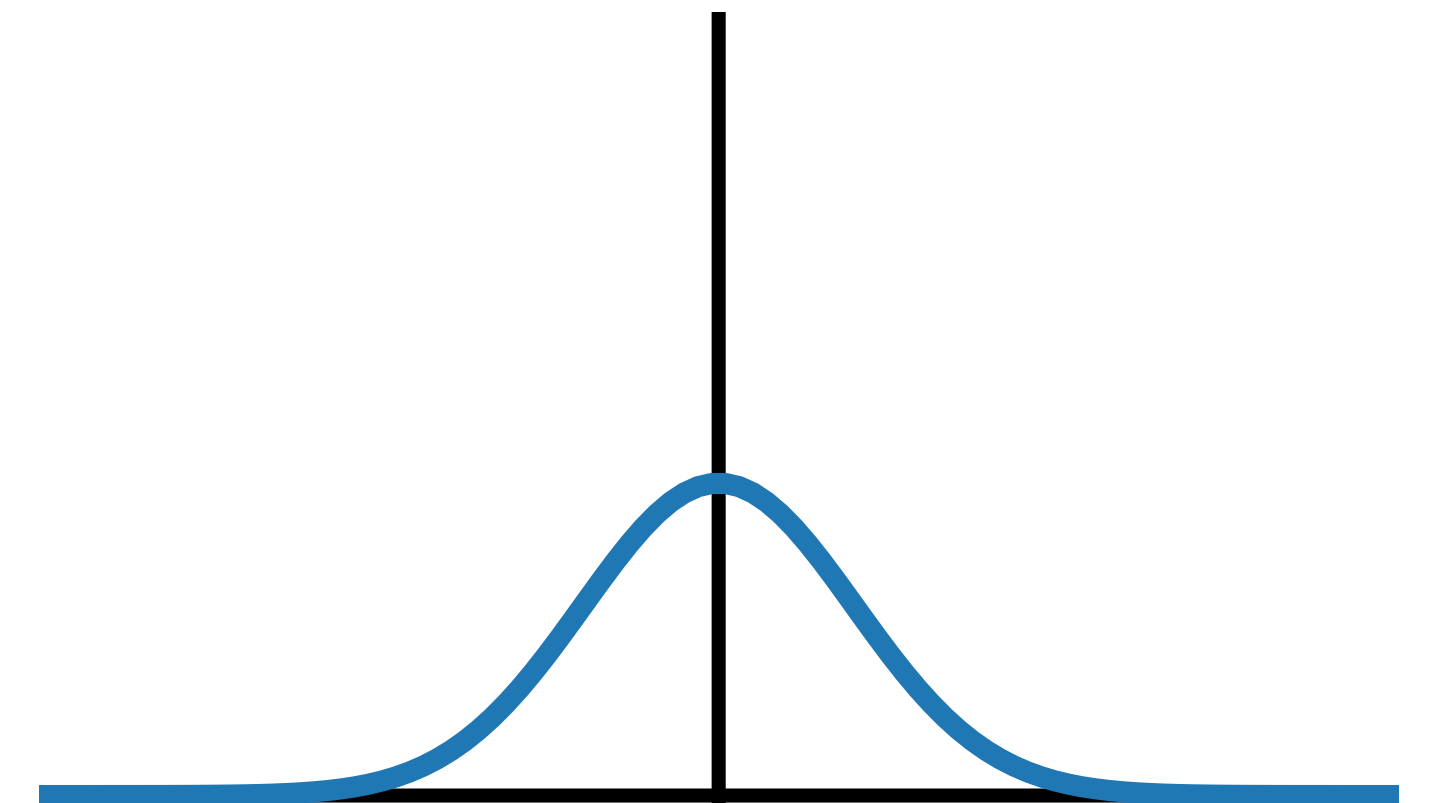
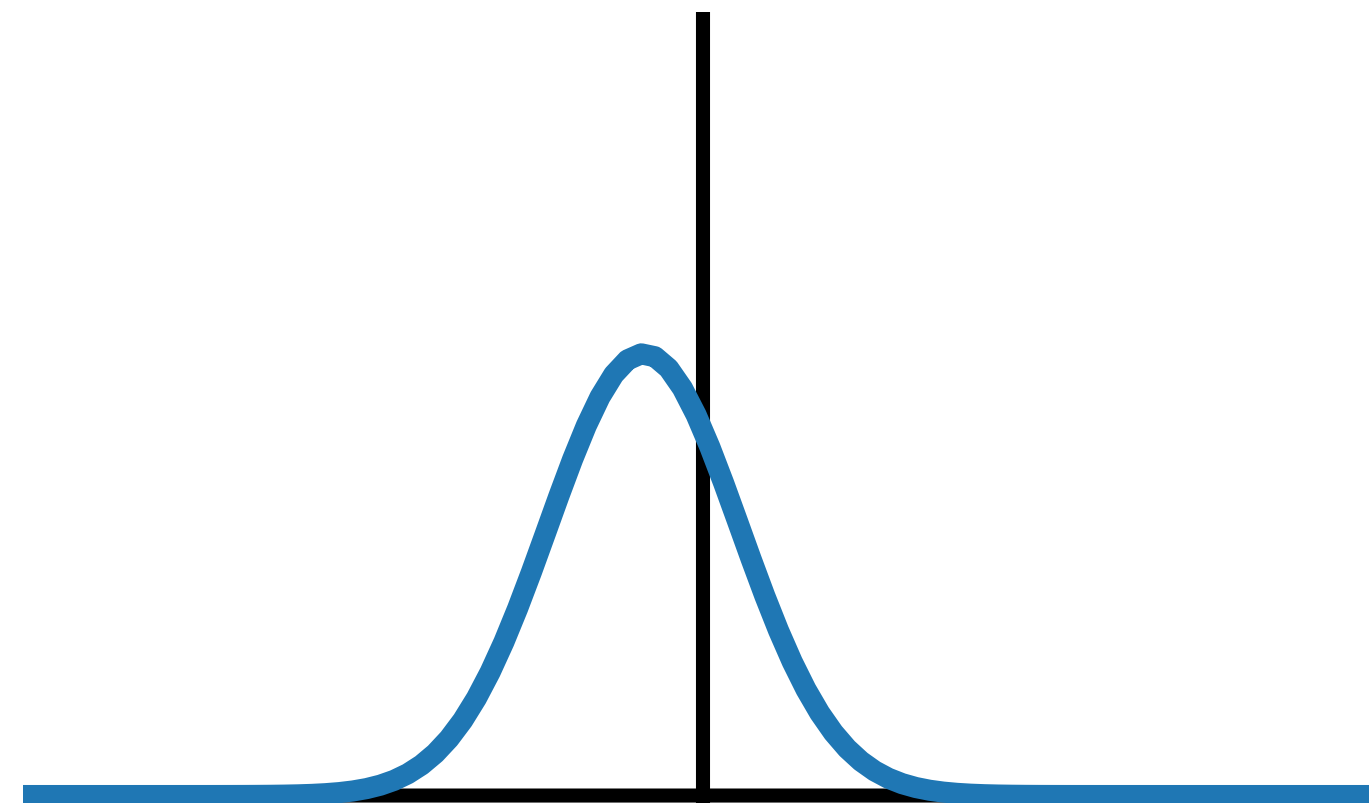
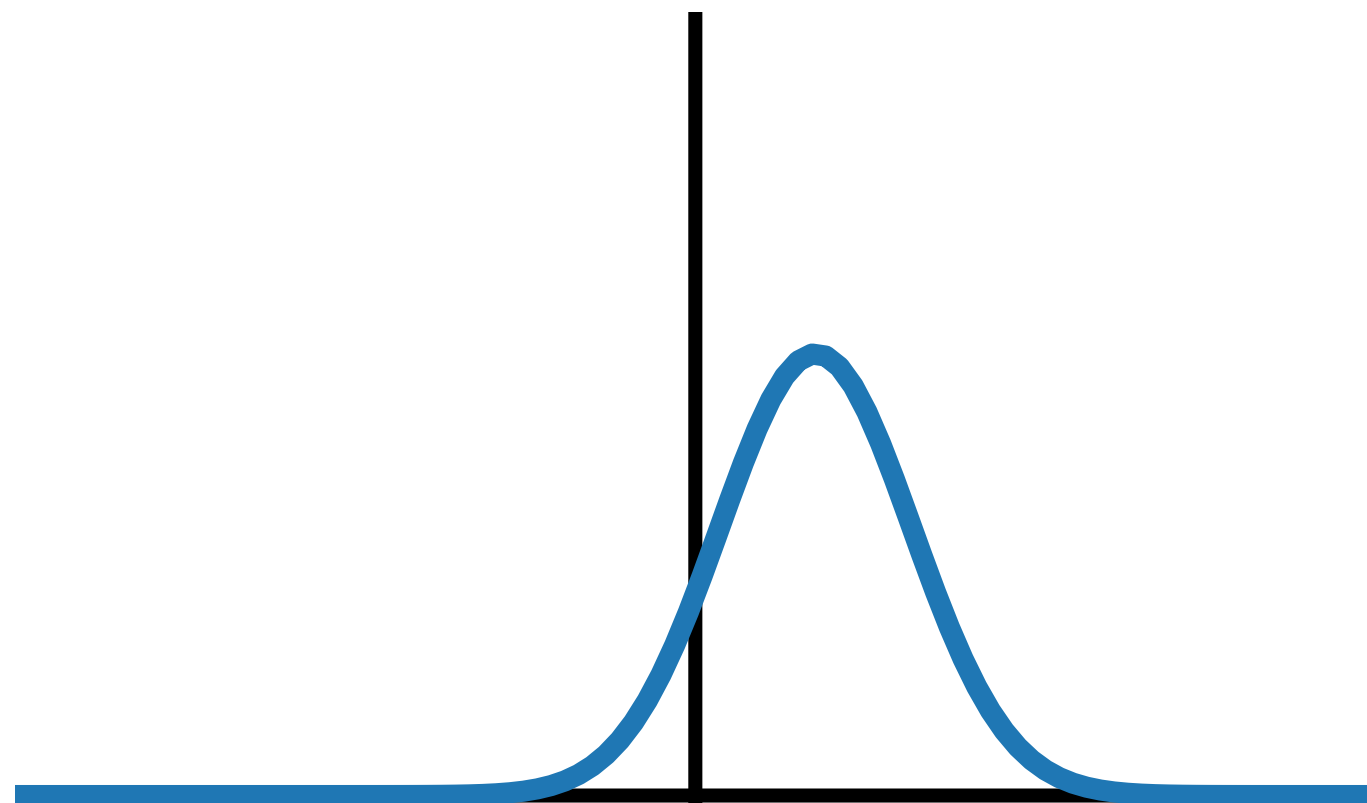


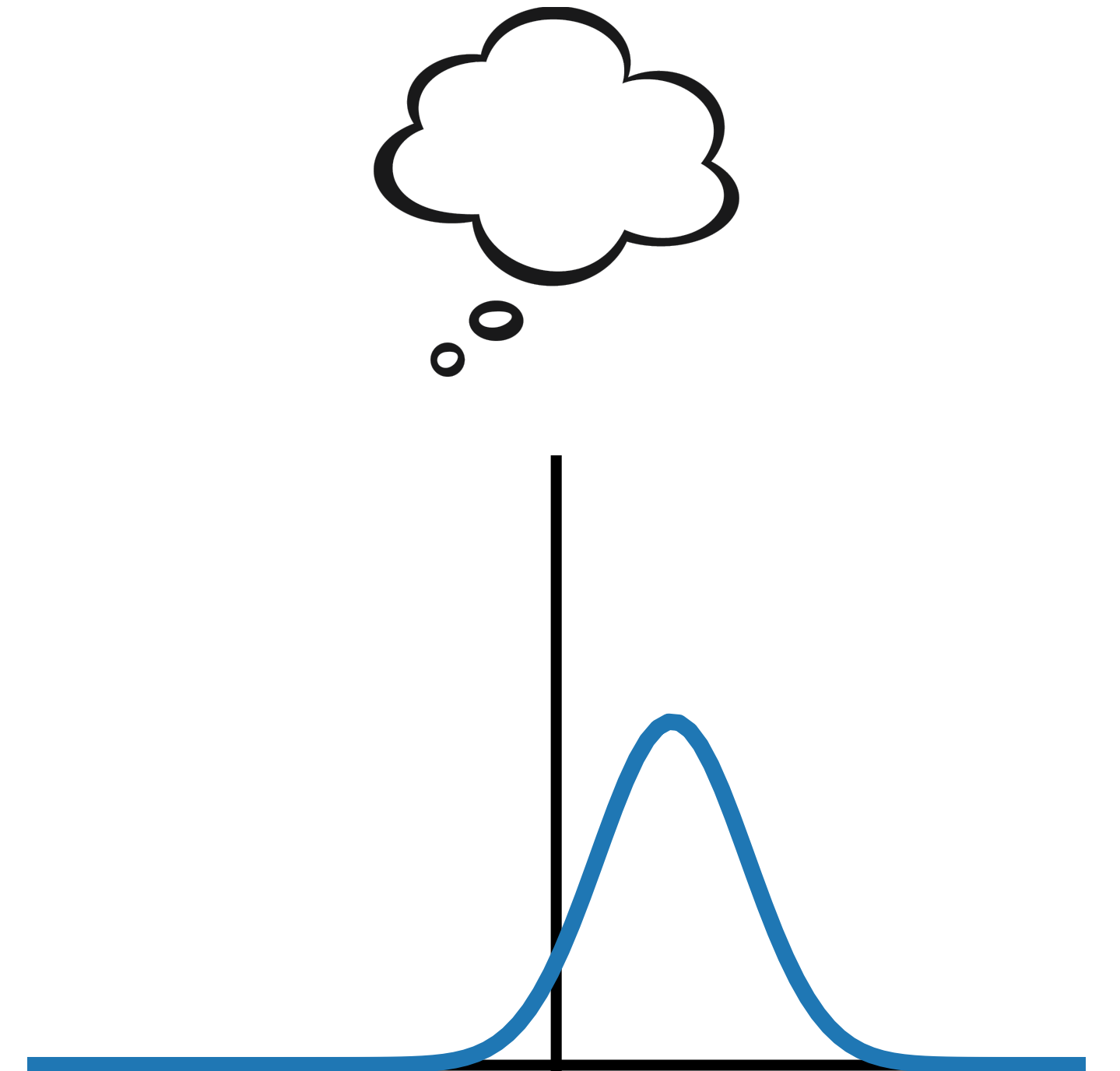
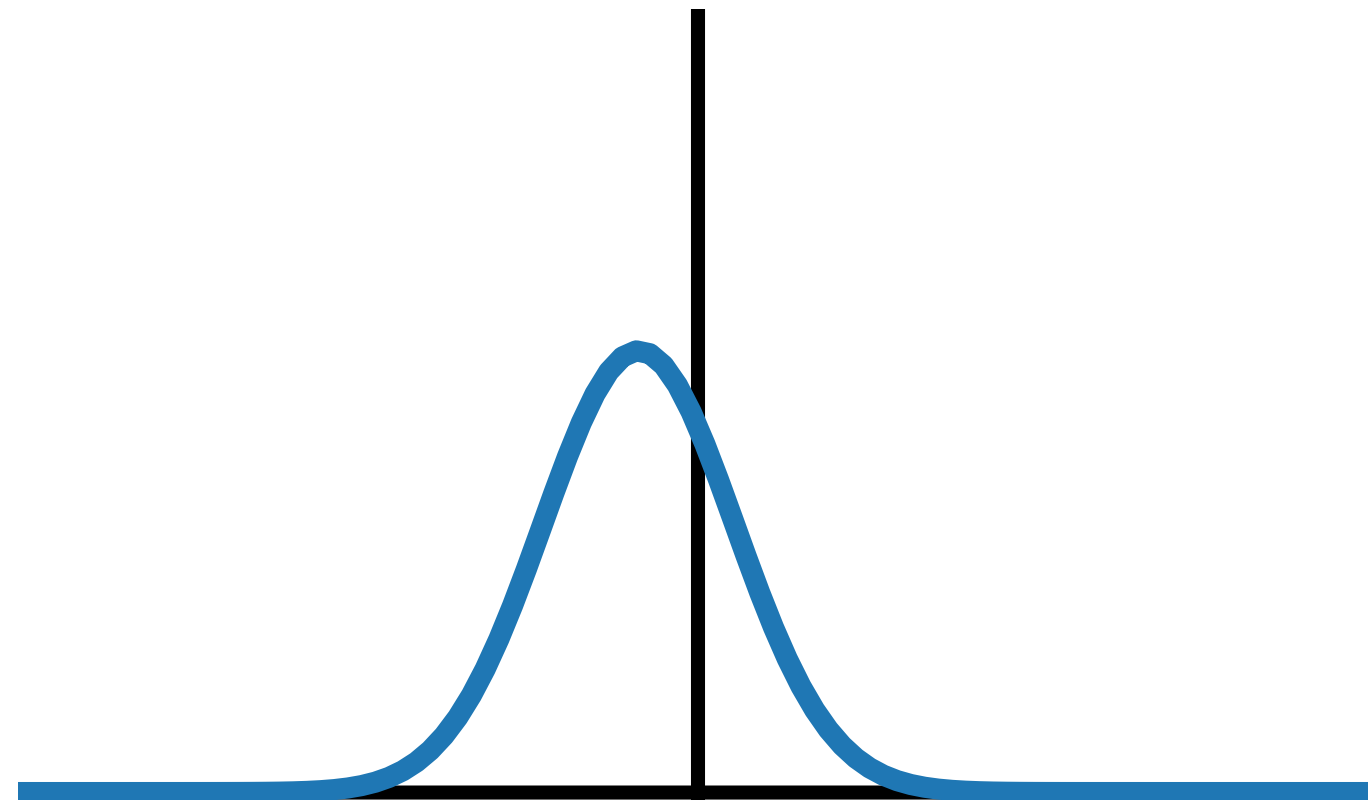
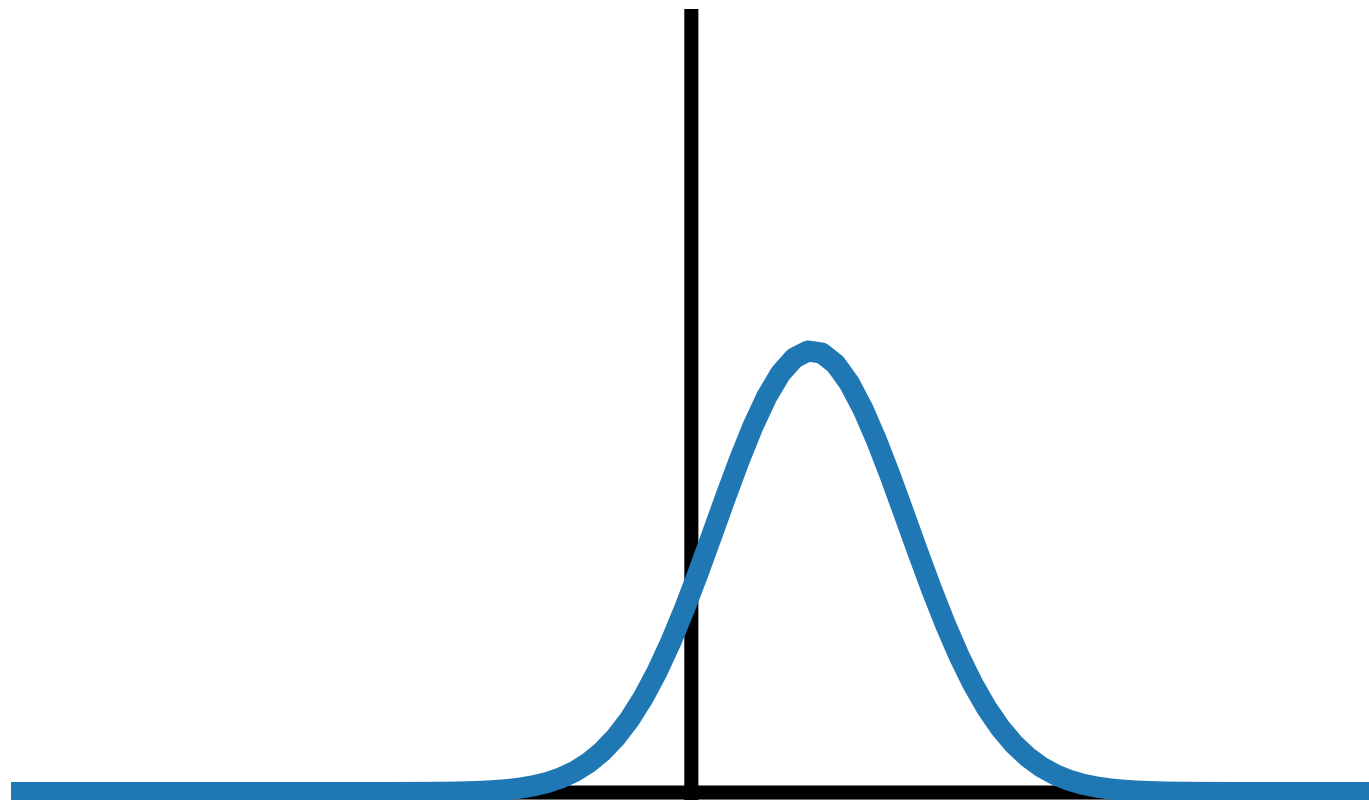


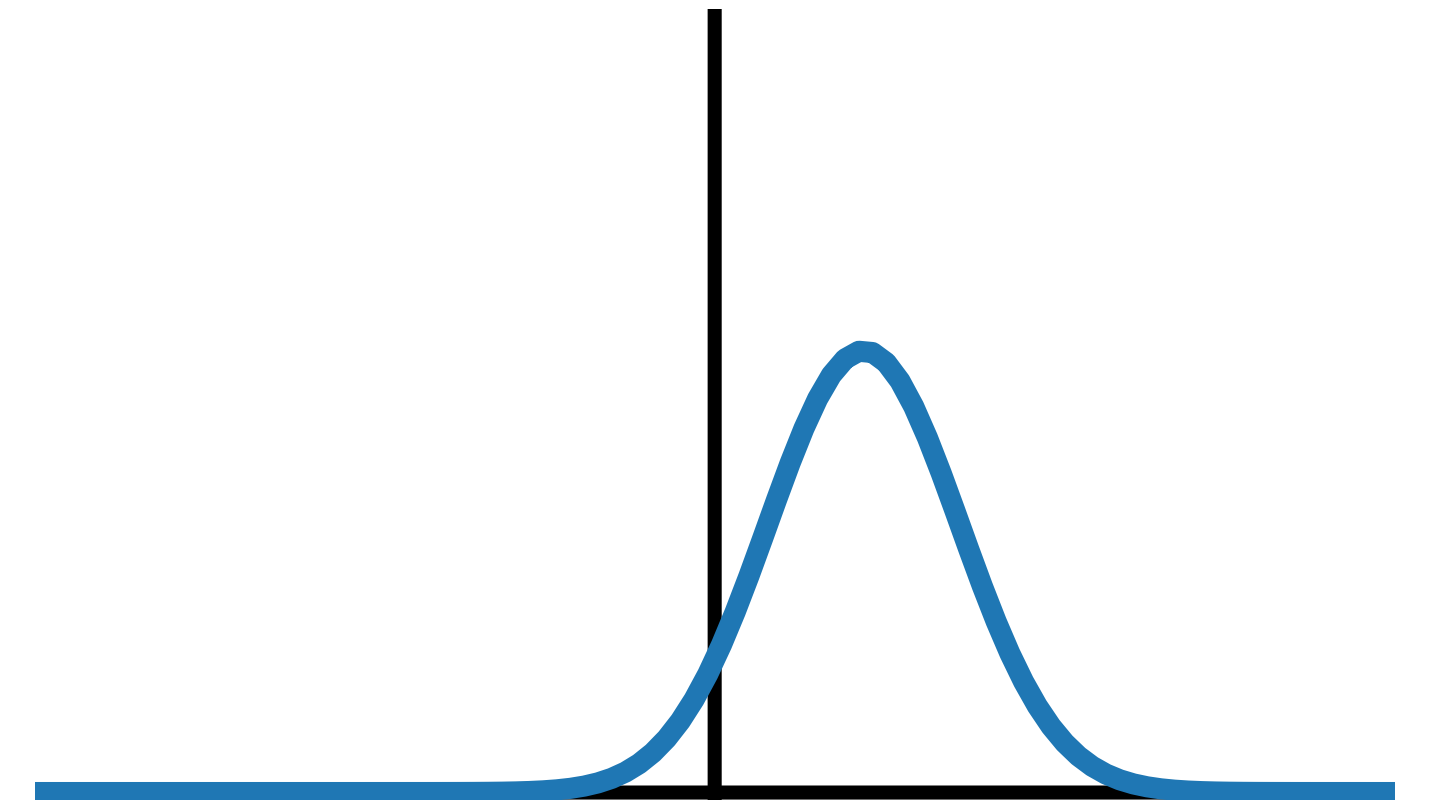
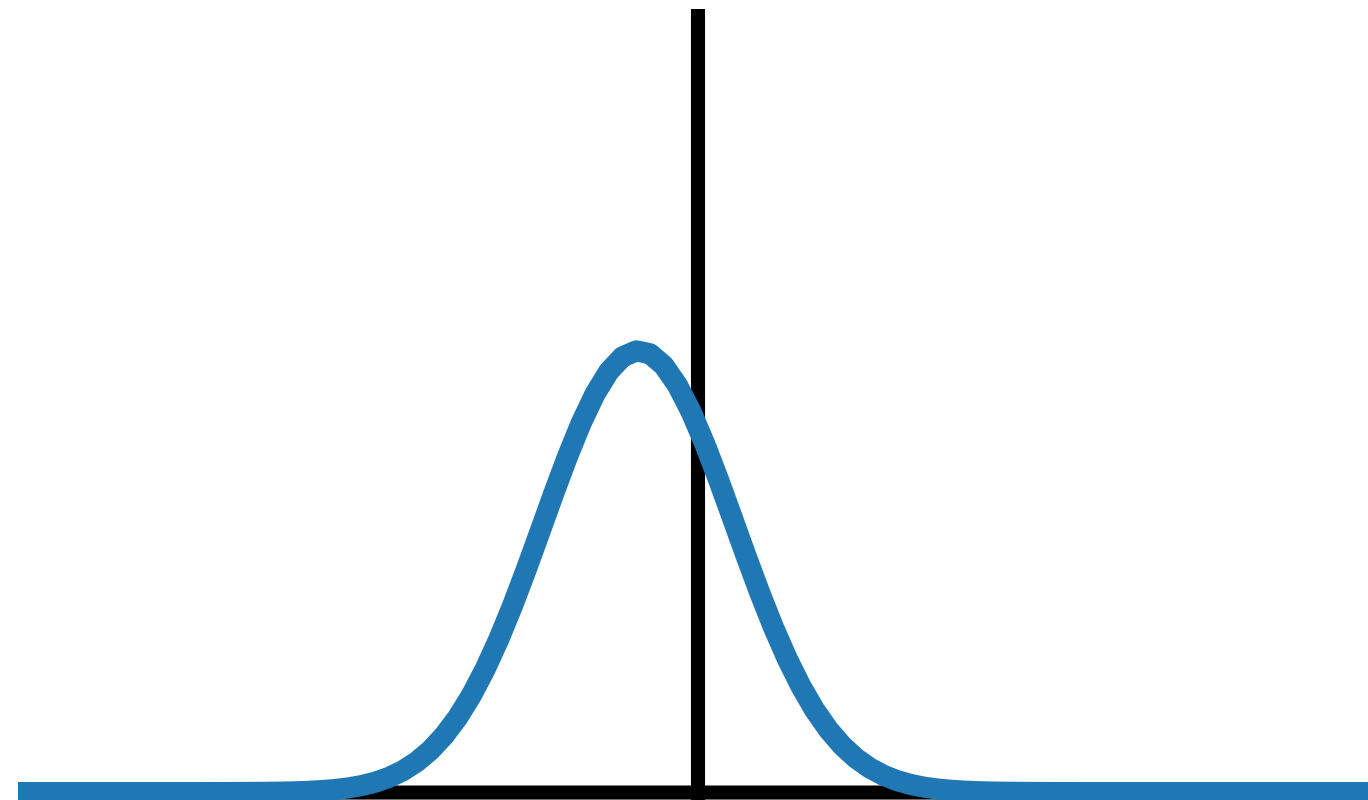
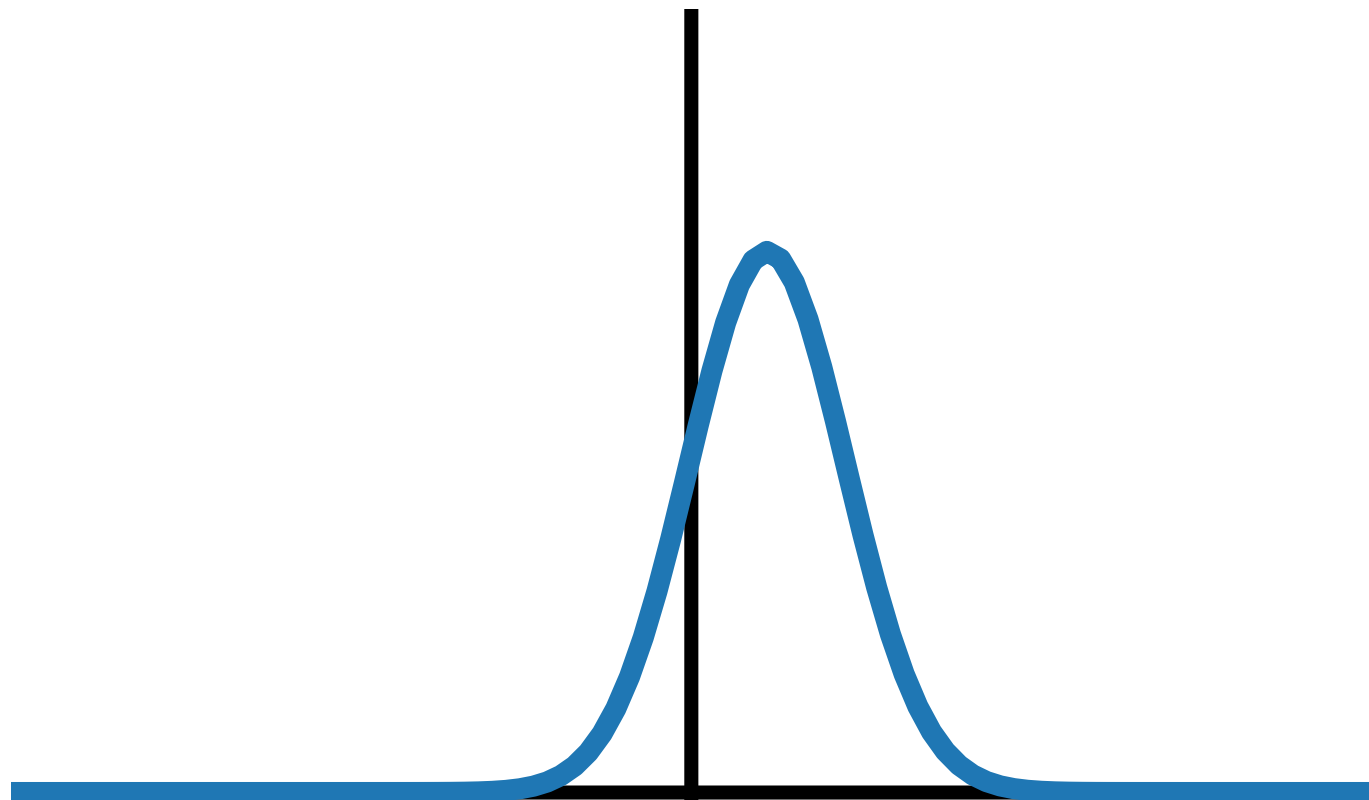
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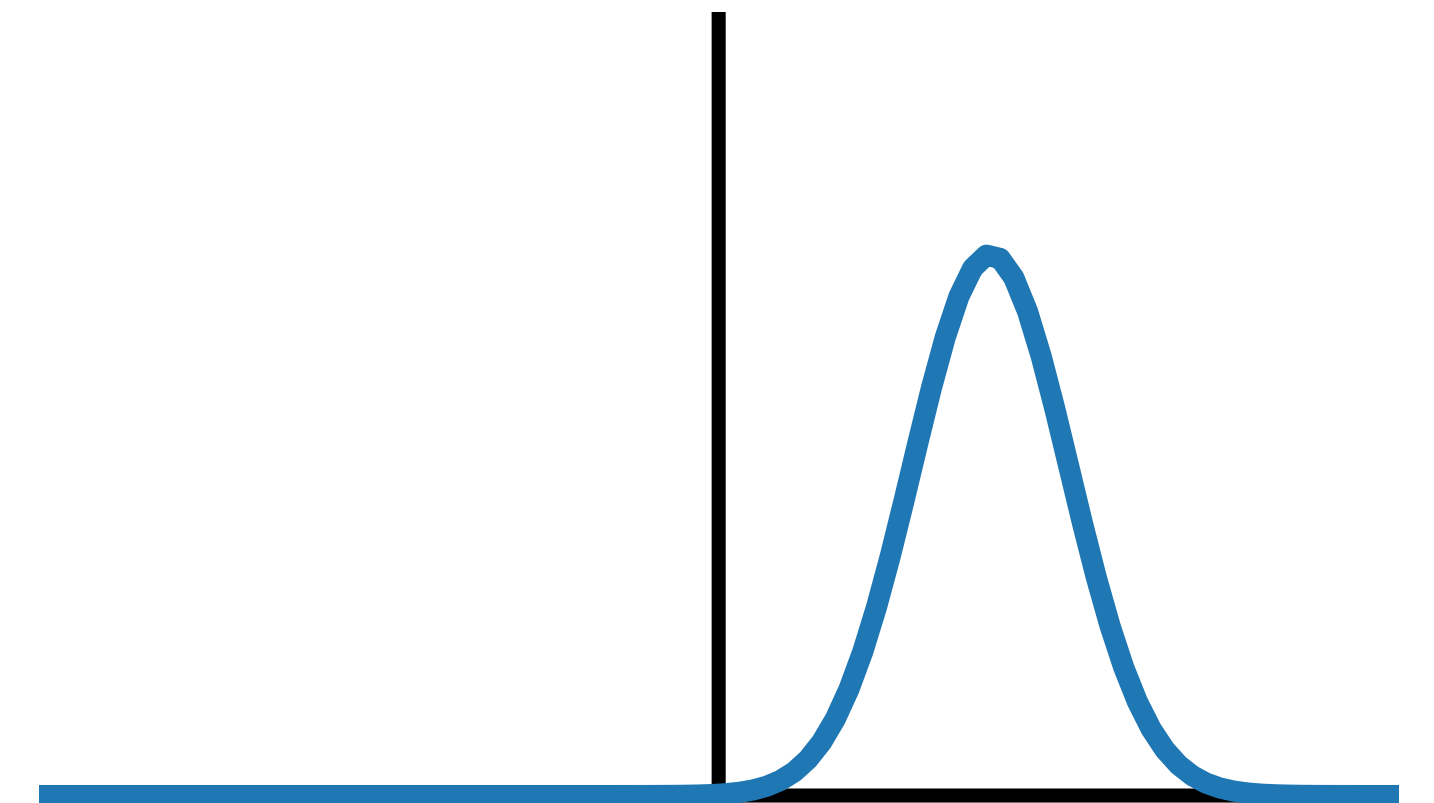
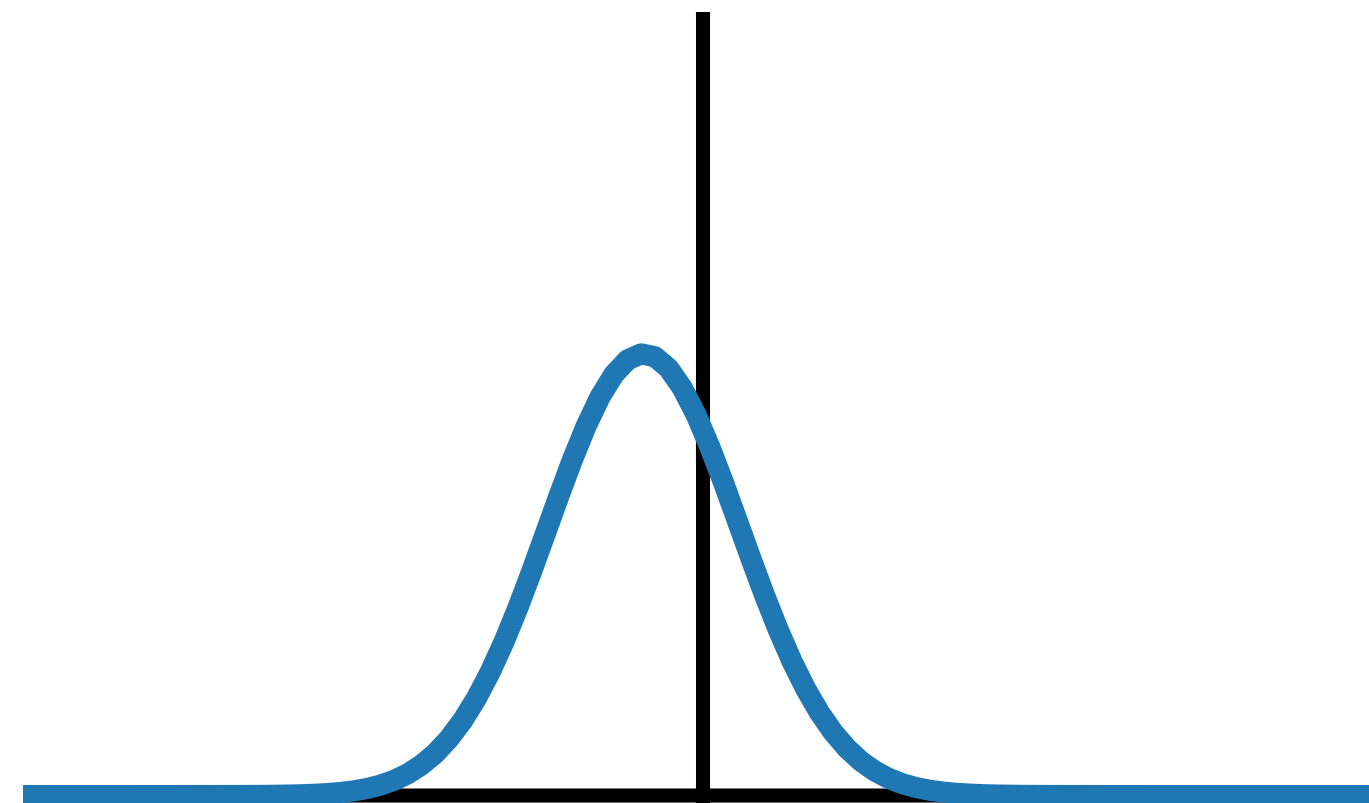
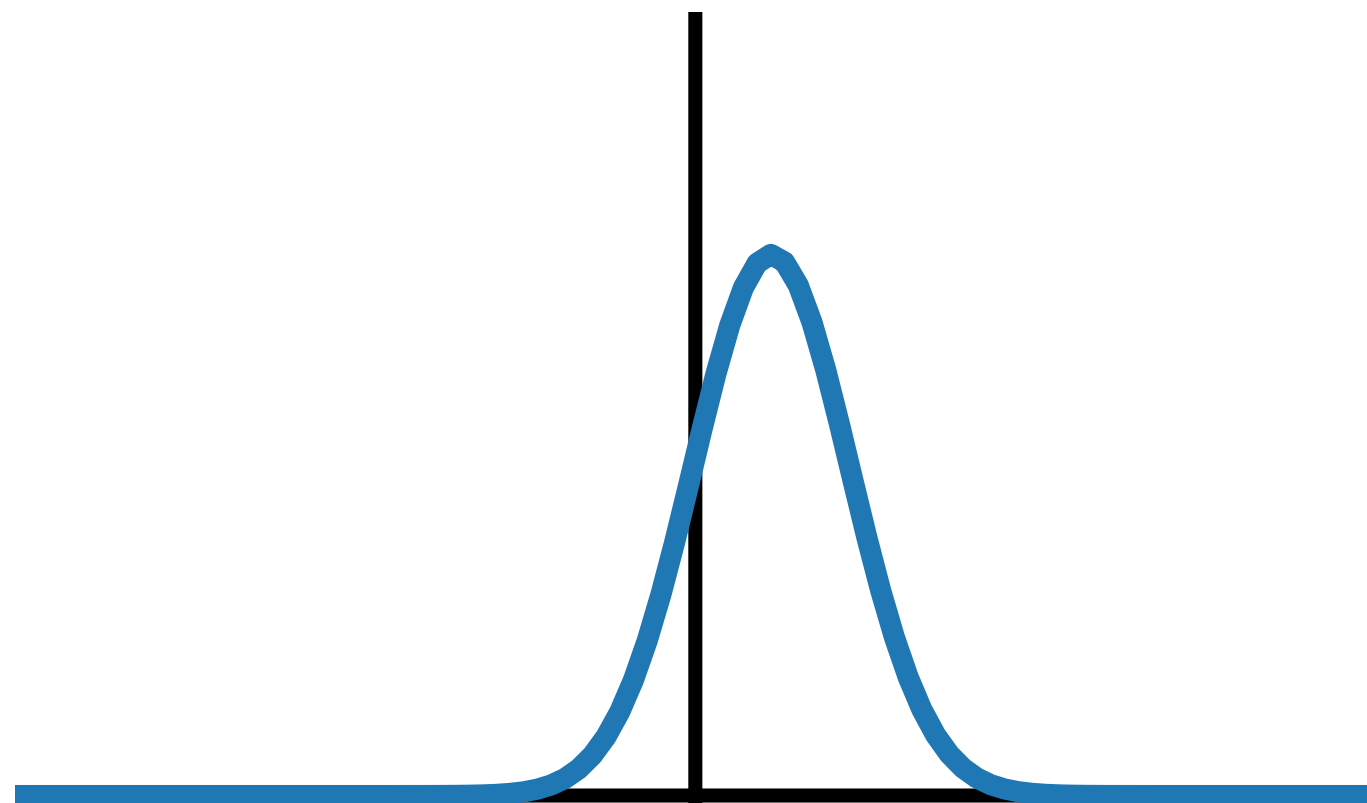


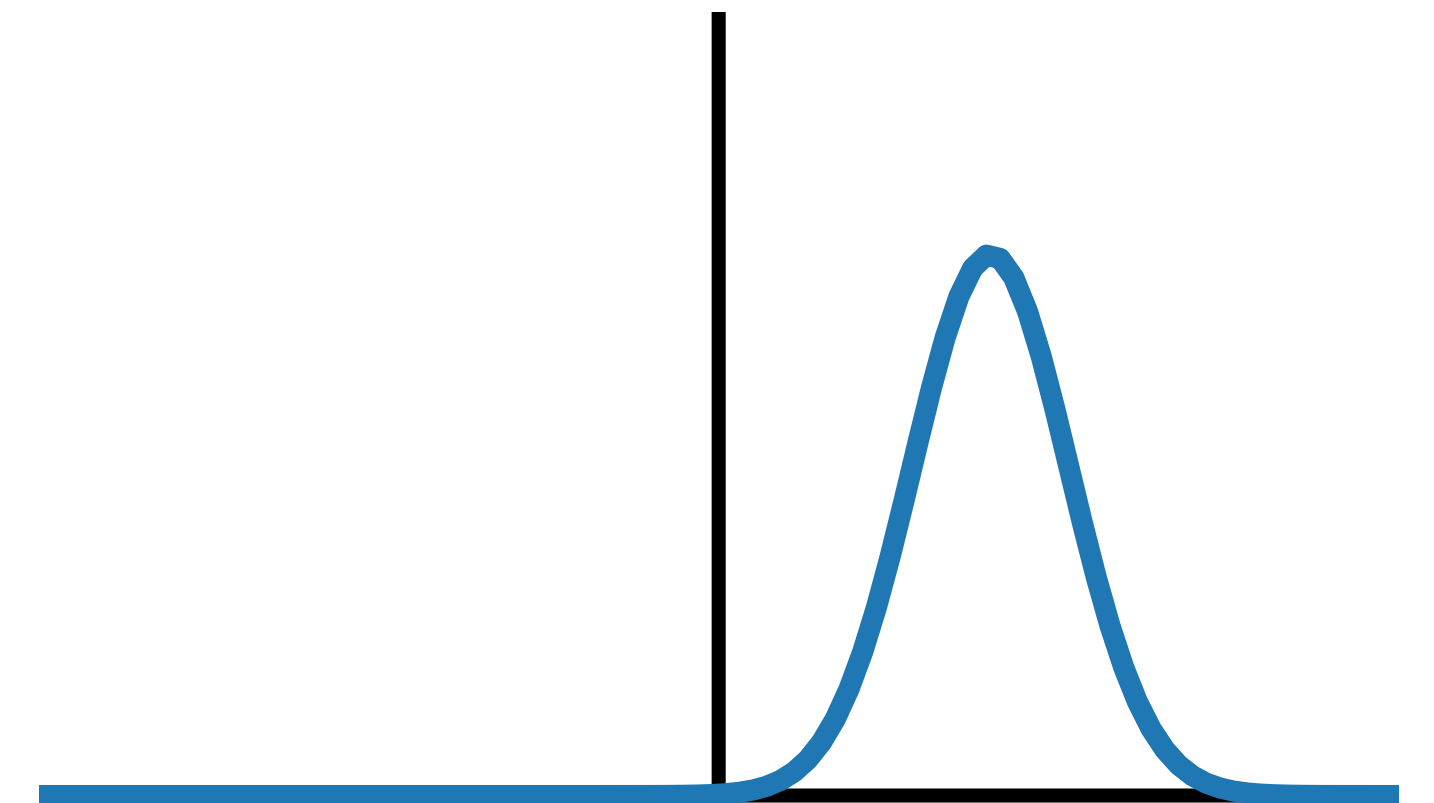
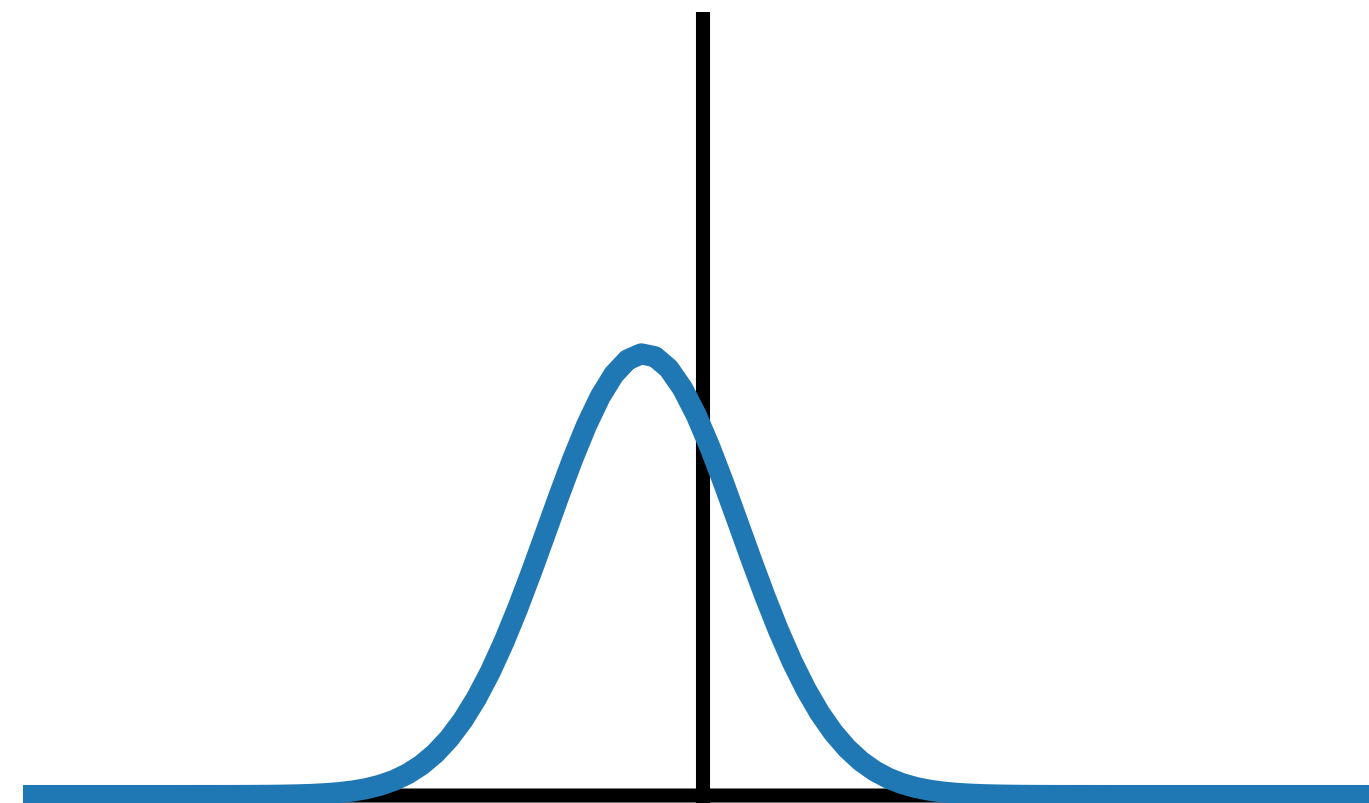
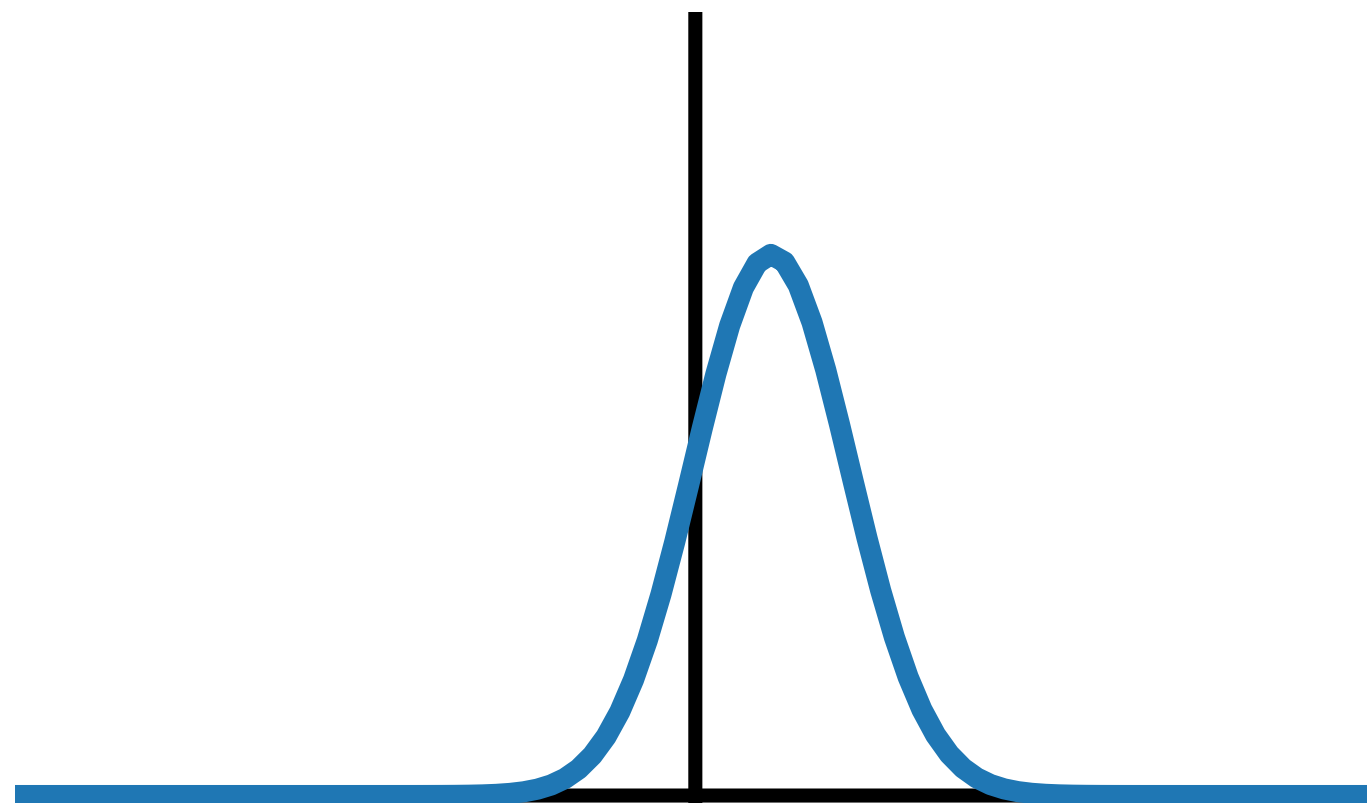
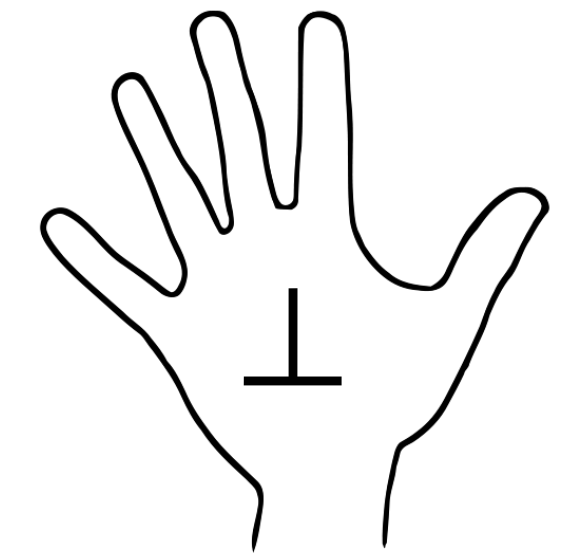


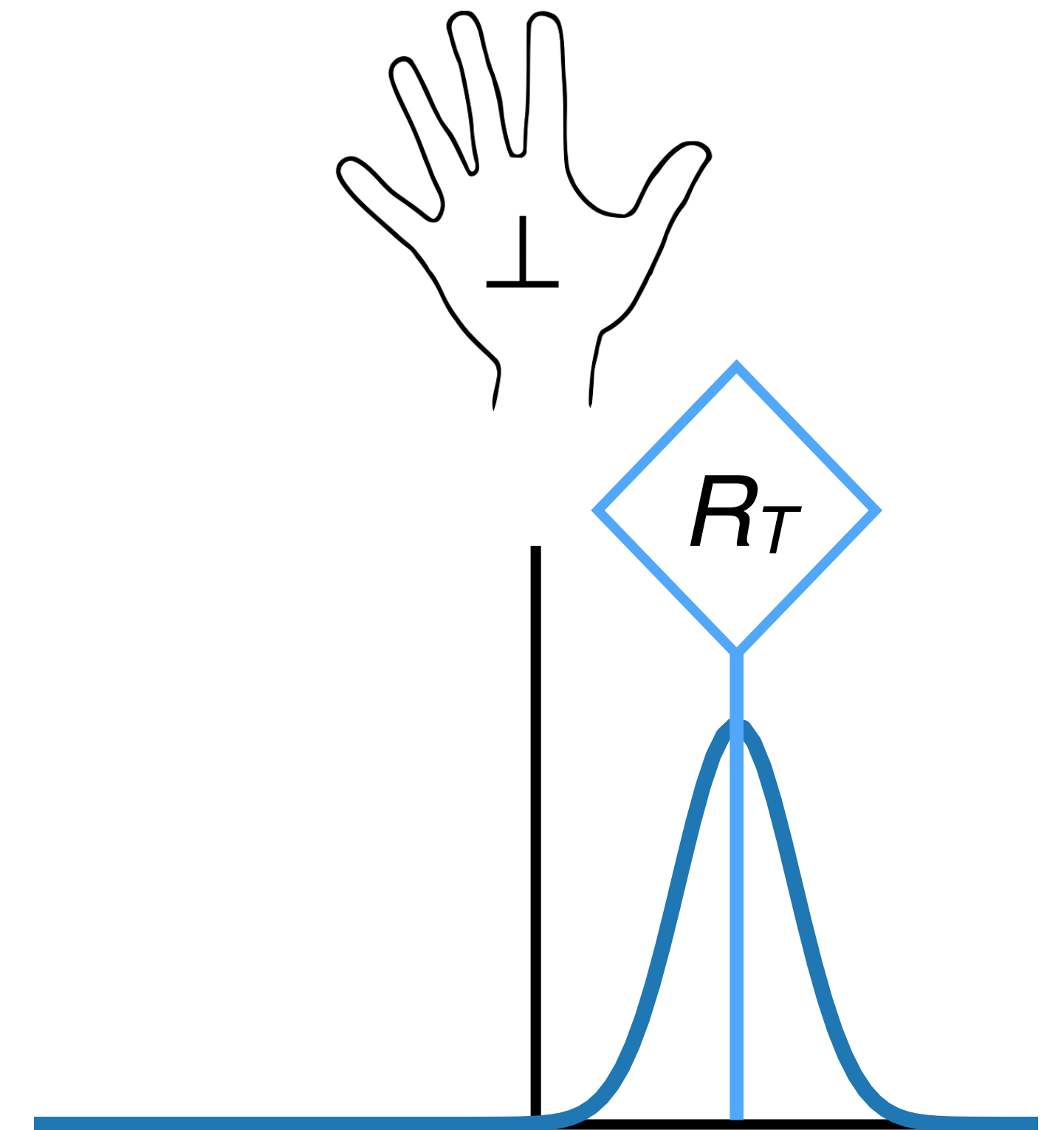
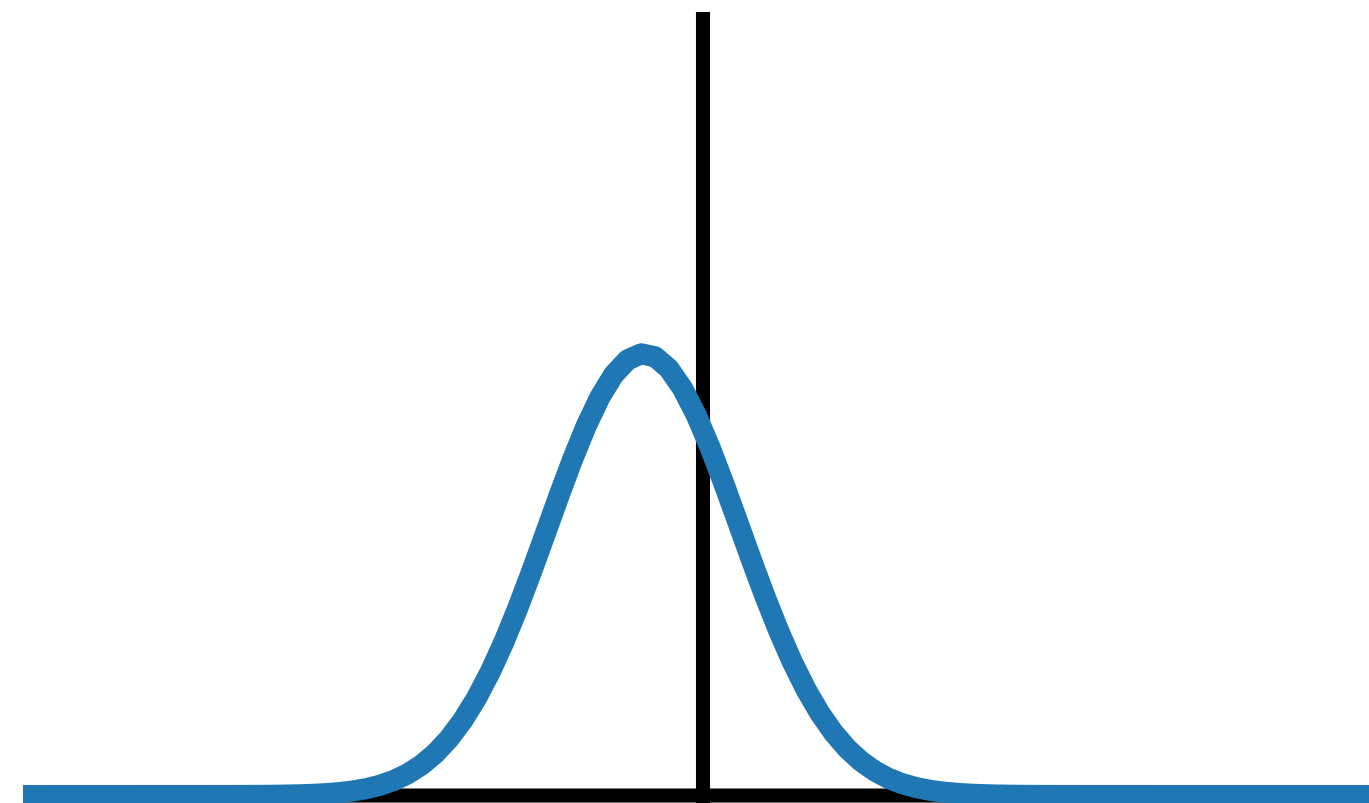
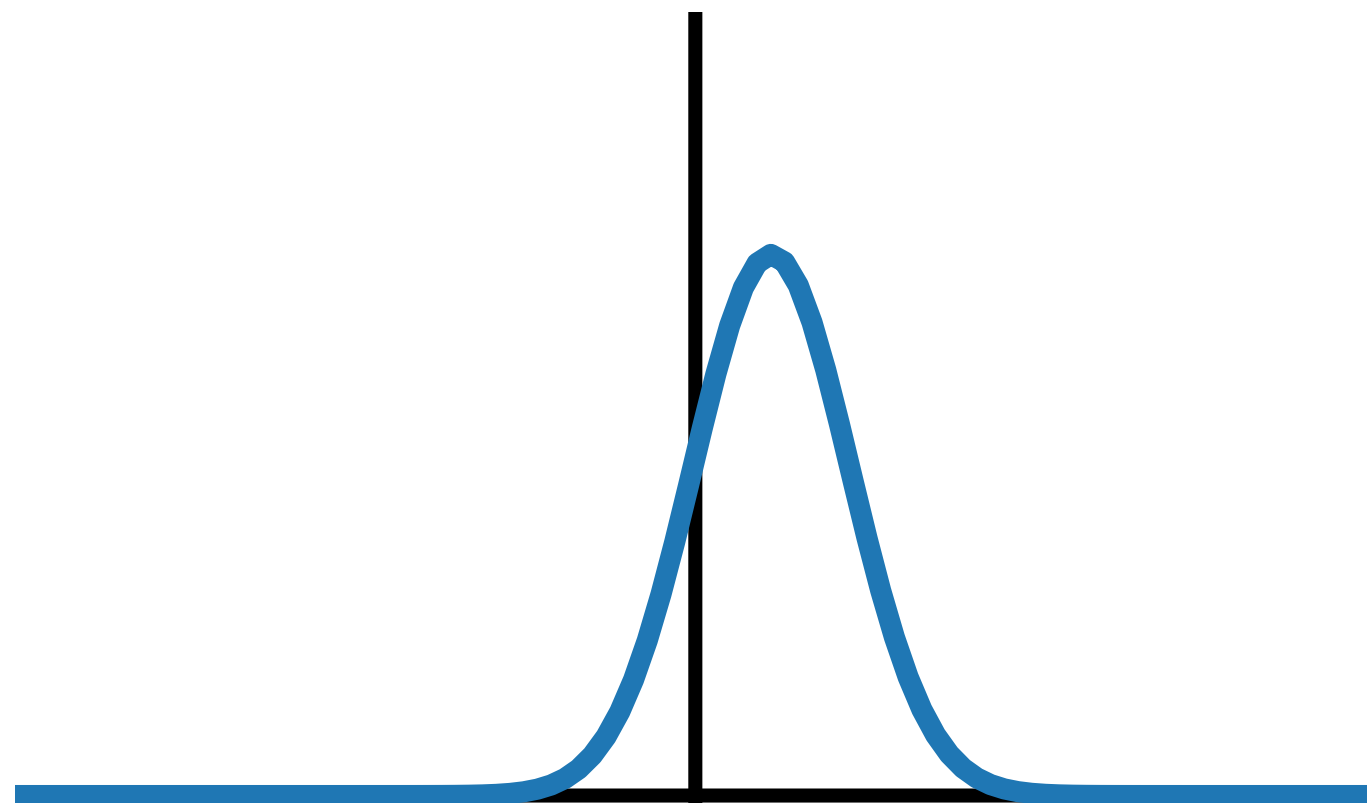










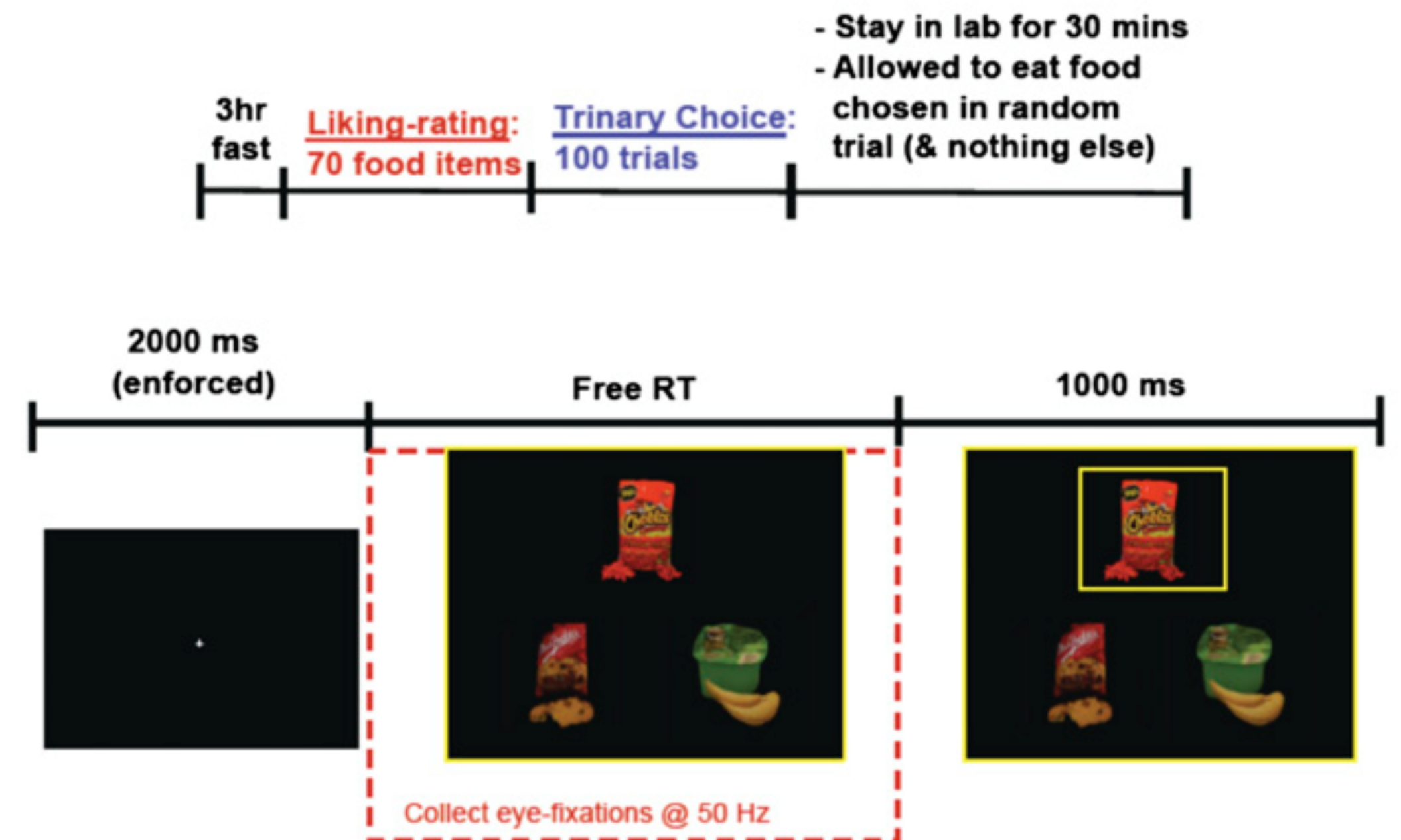


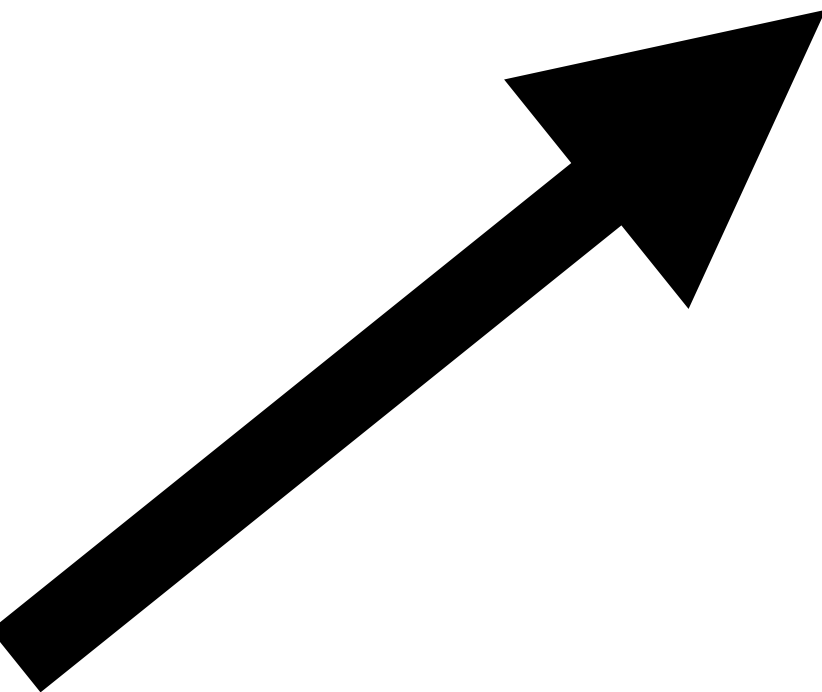
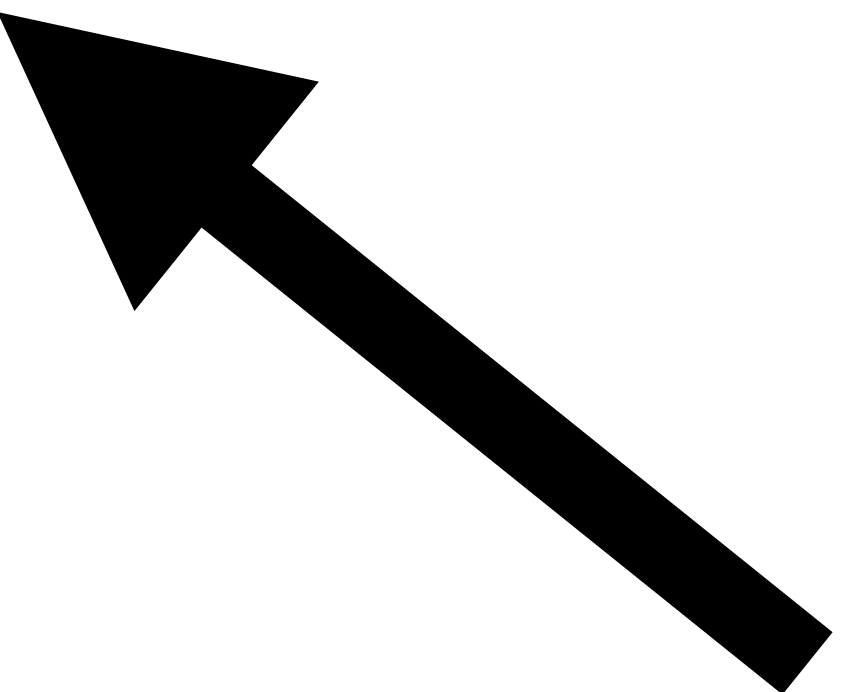
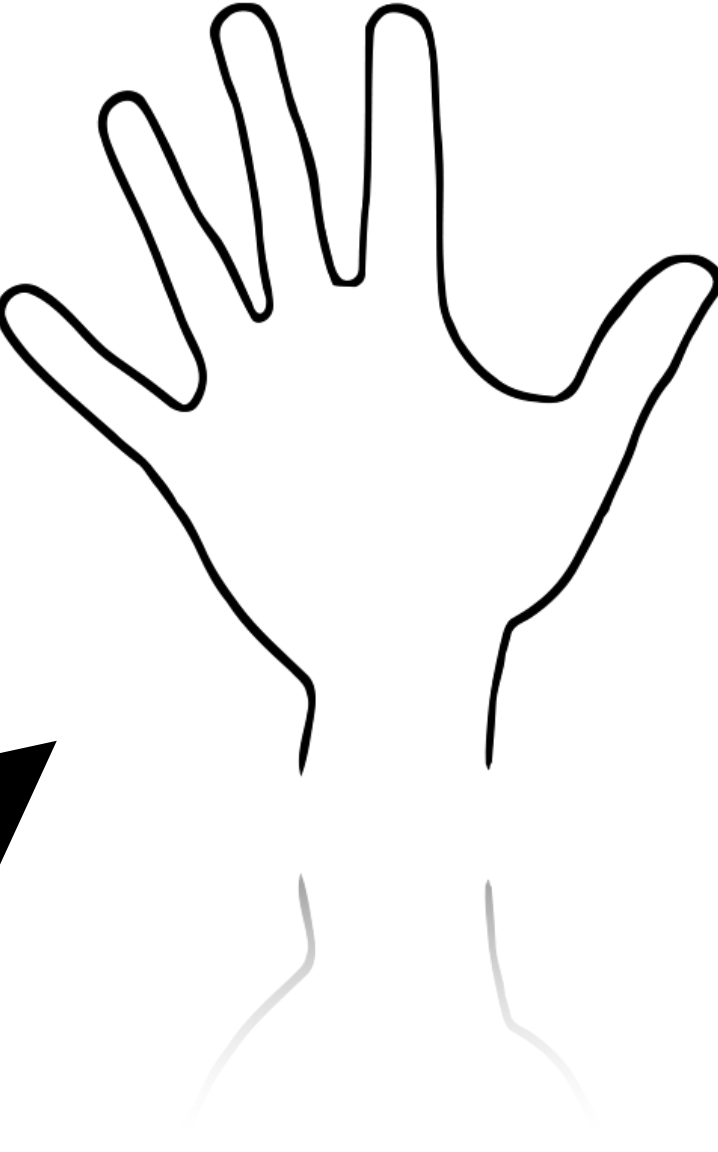
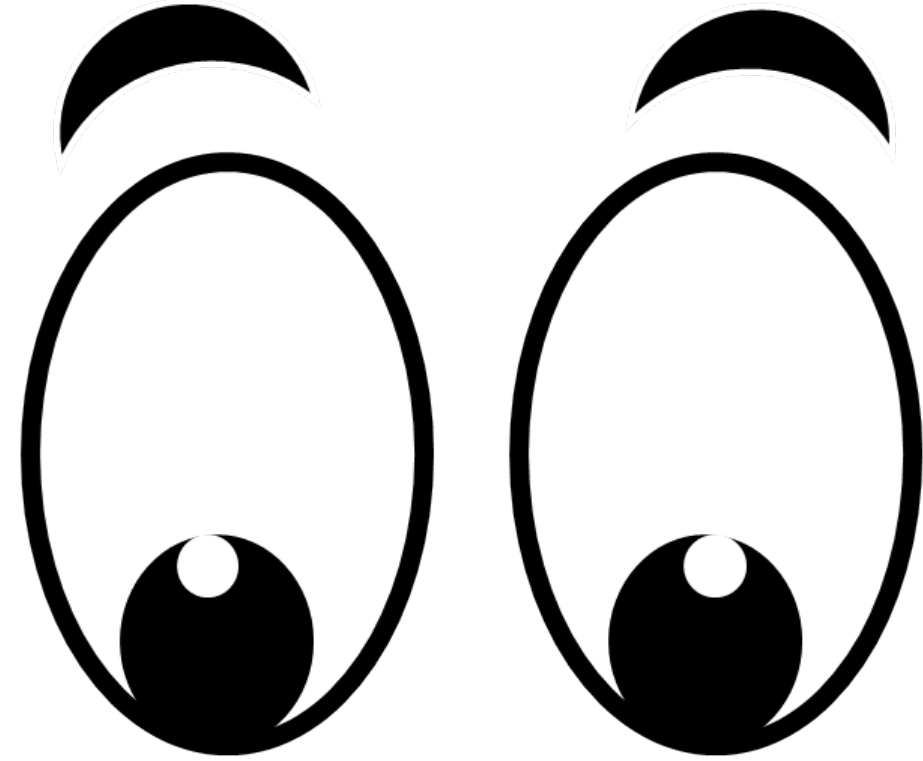
# Results

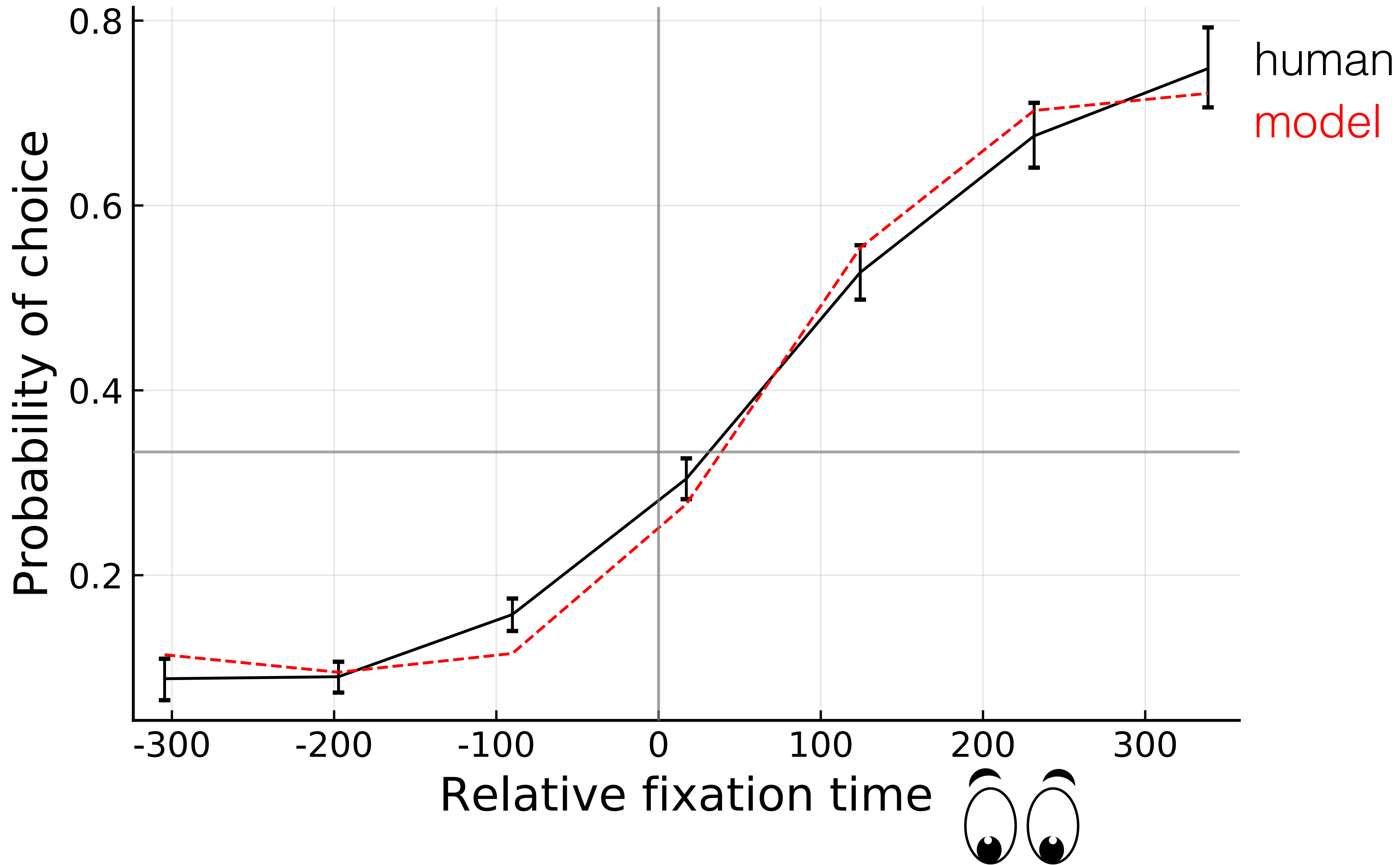
# Krajbich & Rangel (2011)

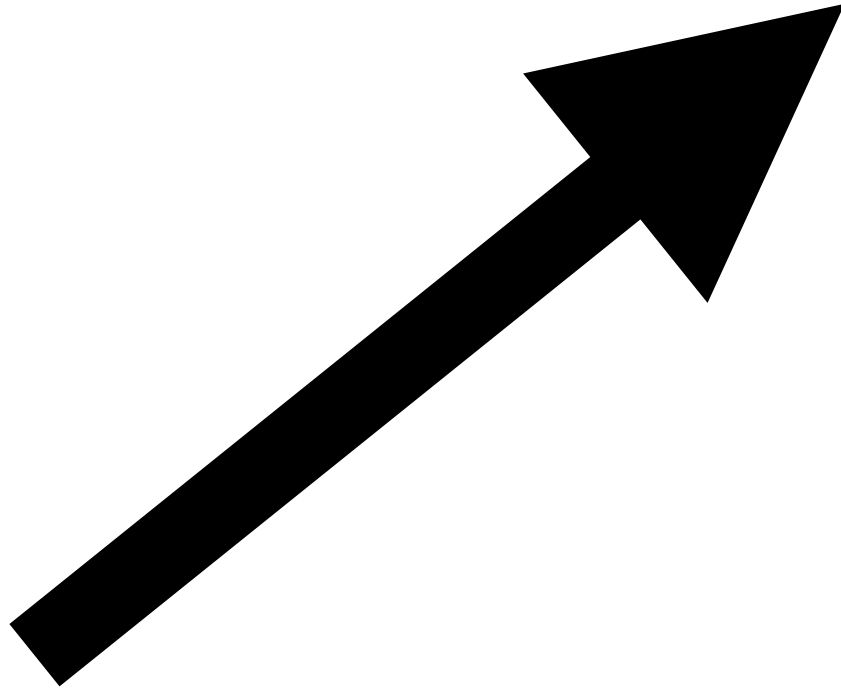
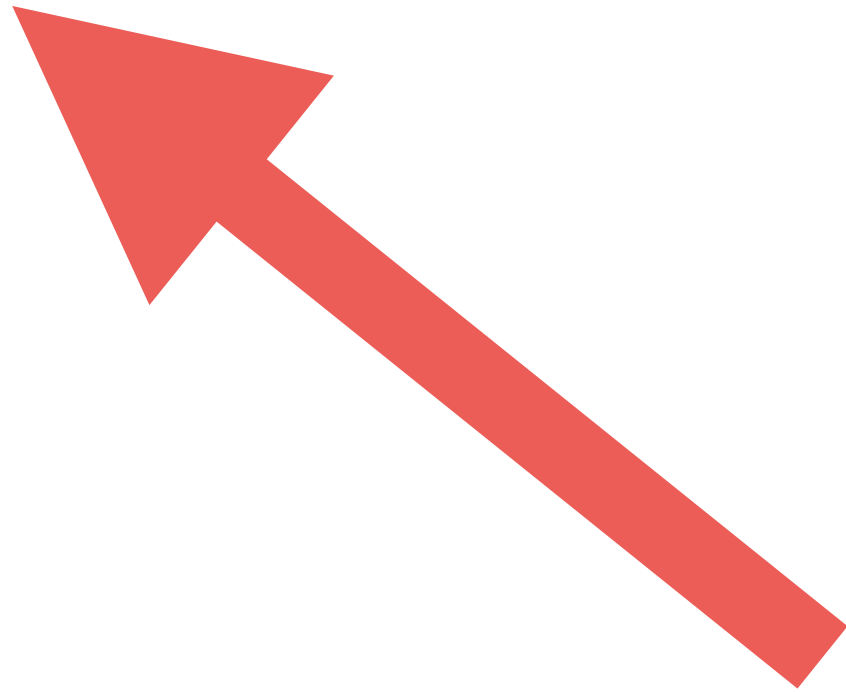
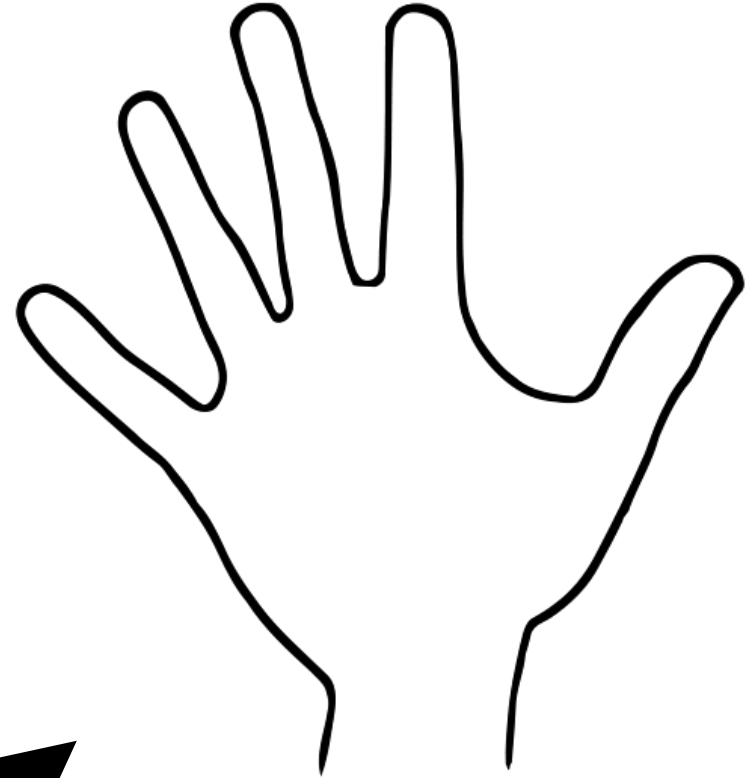
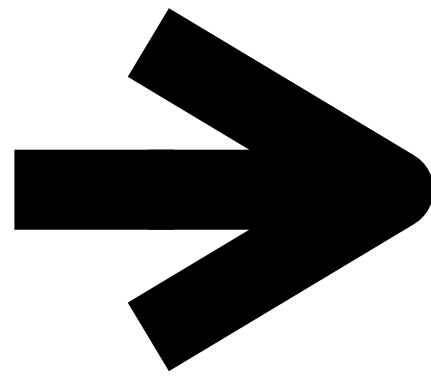
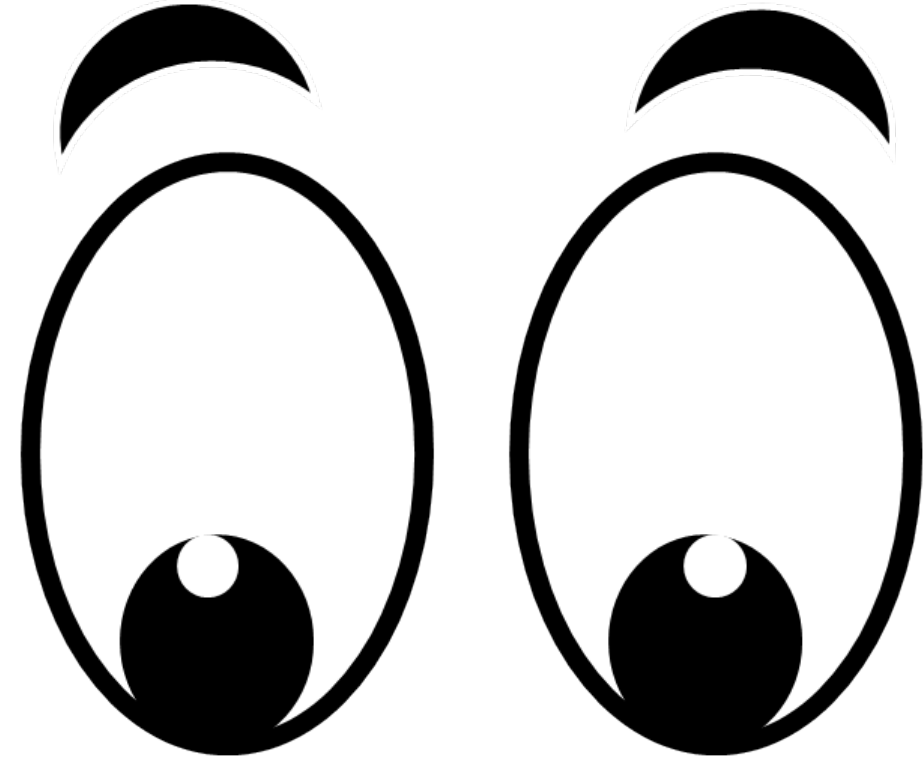
- ▶ Ternary choices between snacks.
- ▶ No time constraint.
- ▶ Eye tracking.
- ▶ Each item rated beforehand.

A

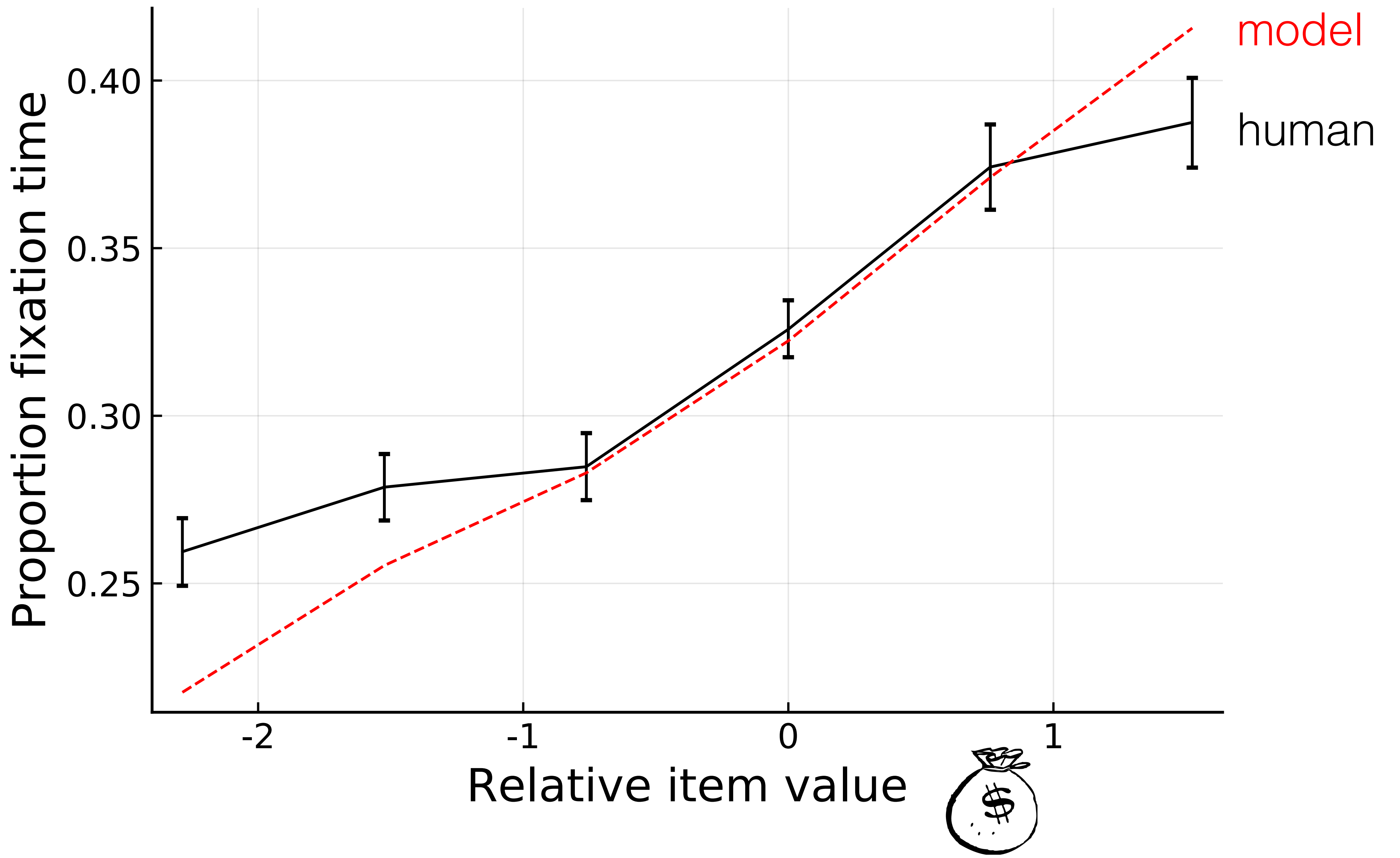
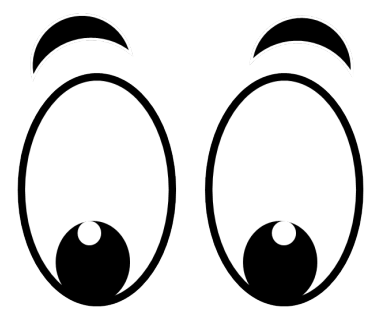






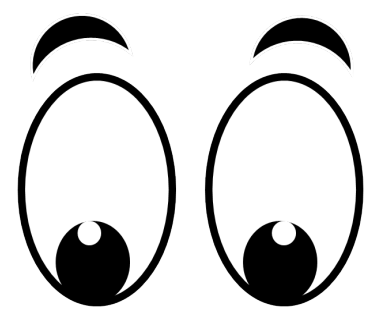




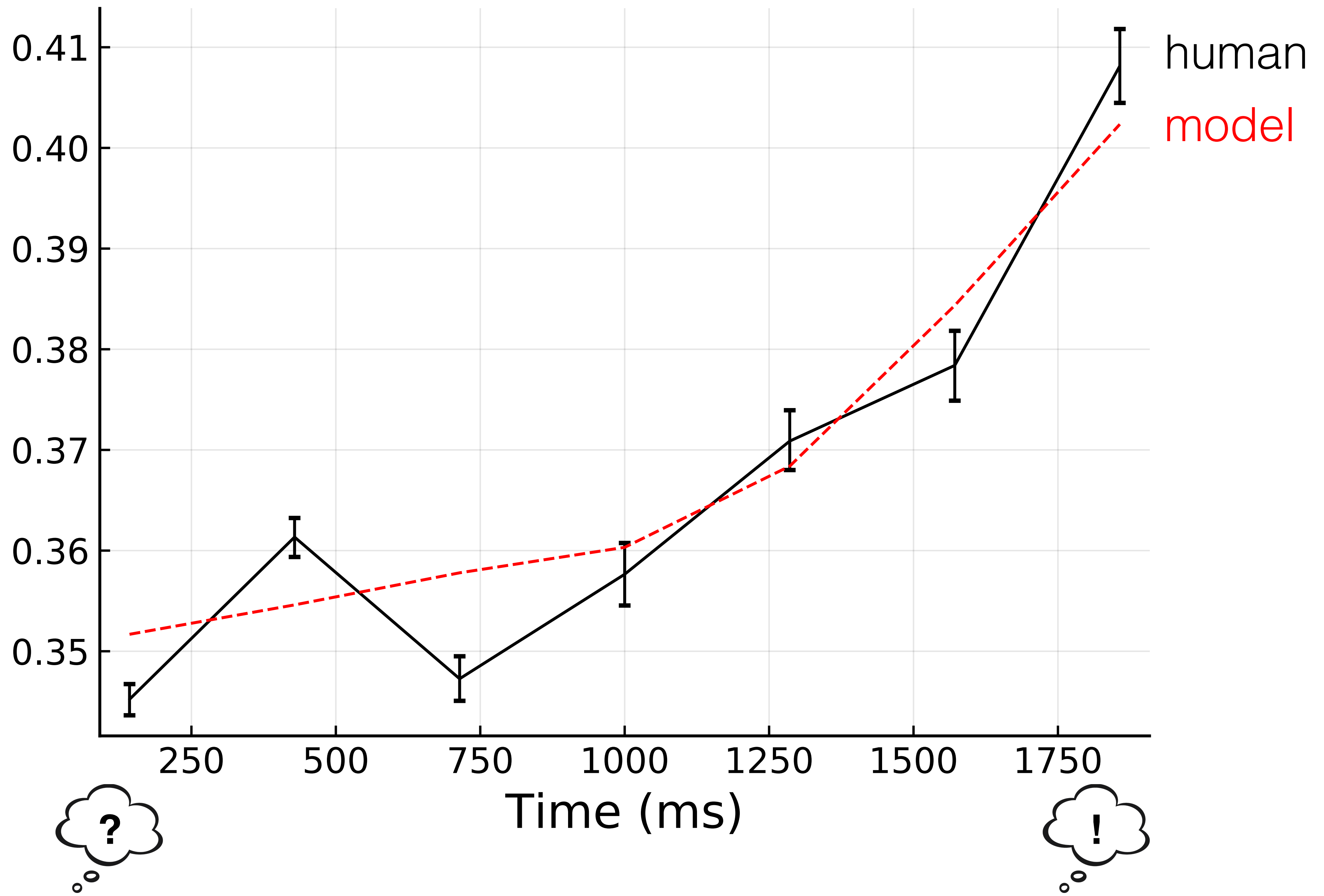


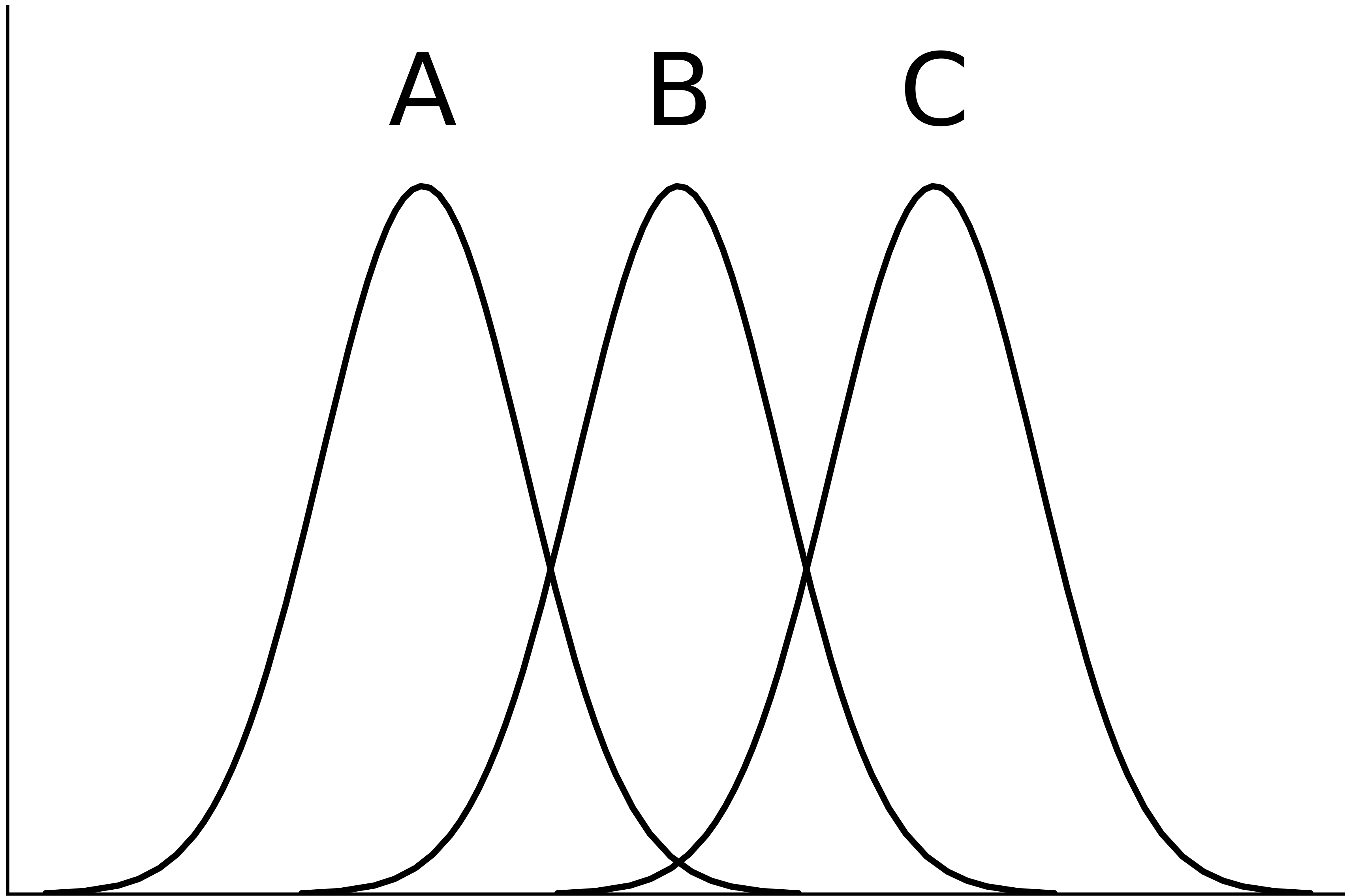
Relative item value

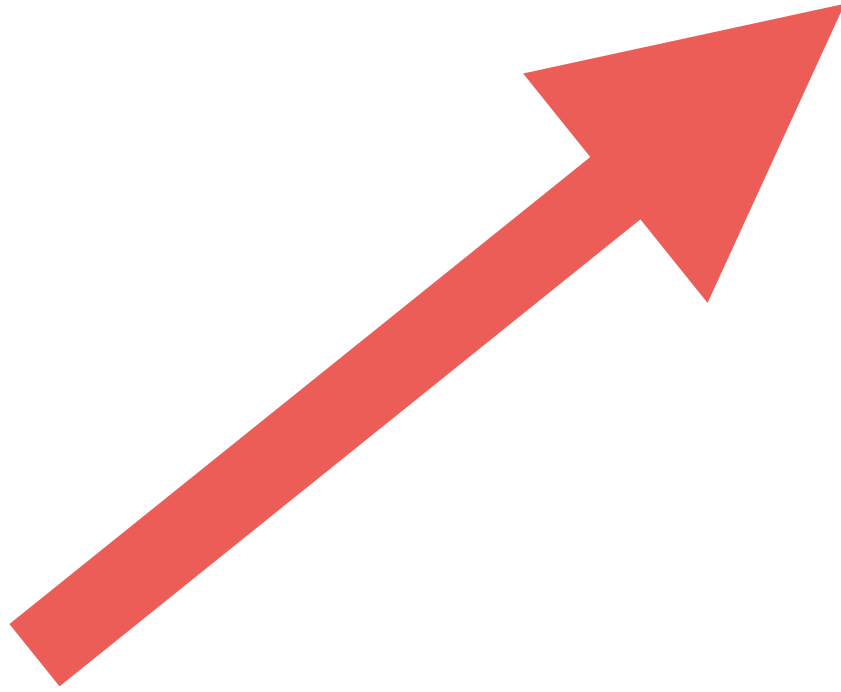
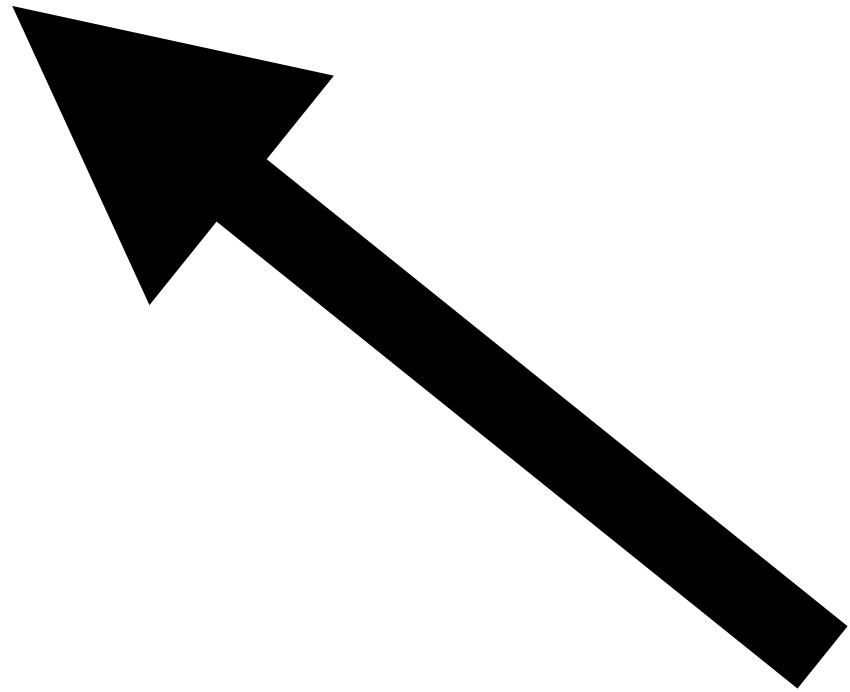
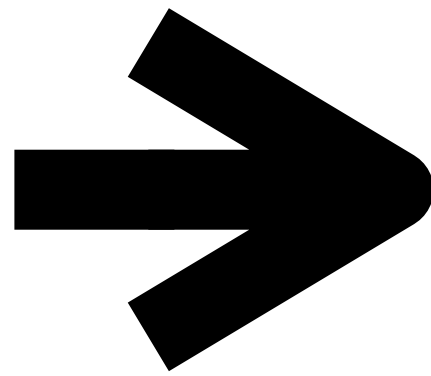
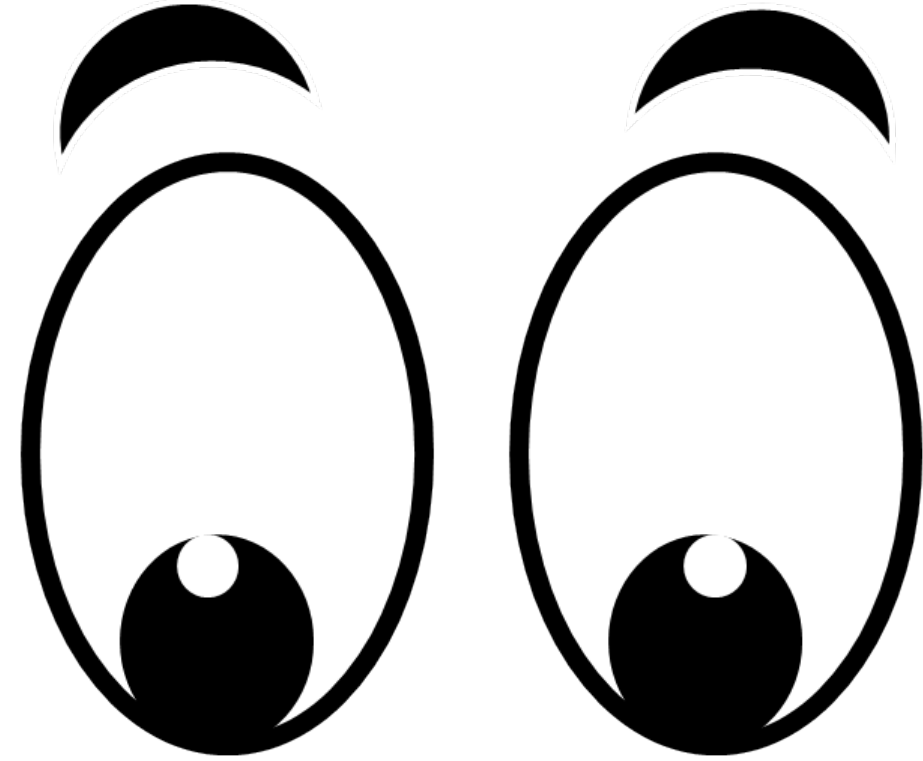


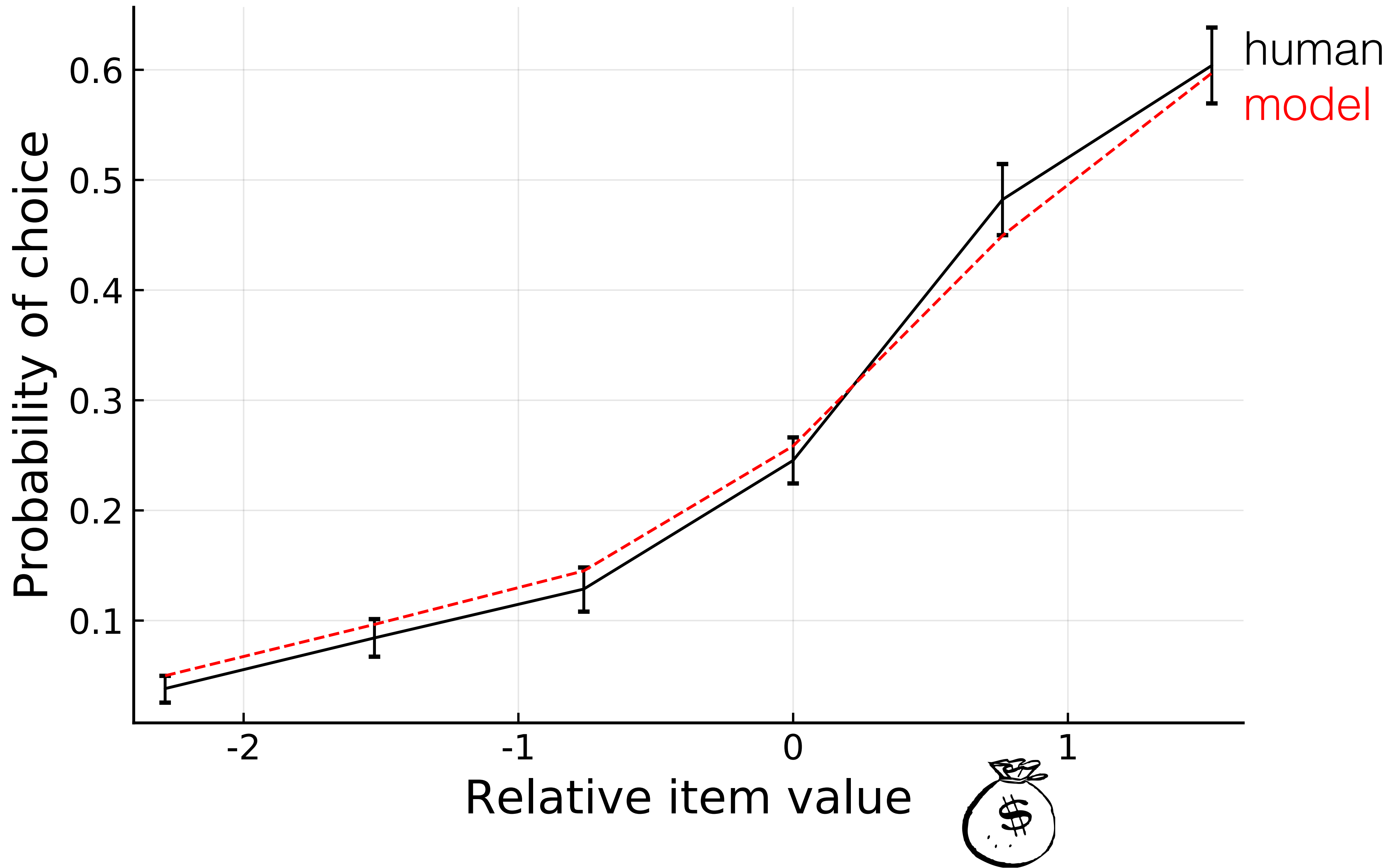


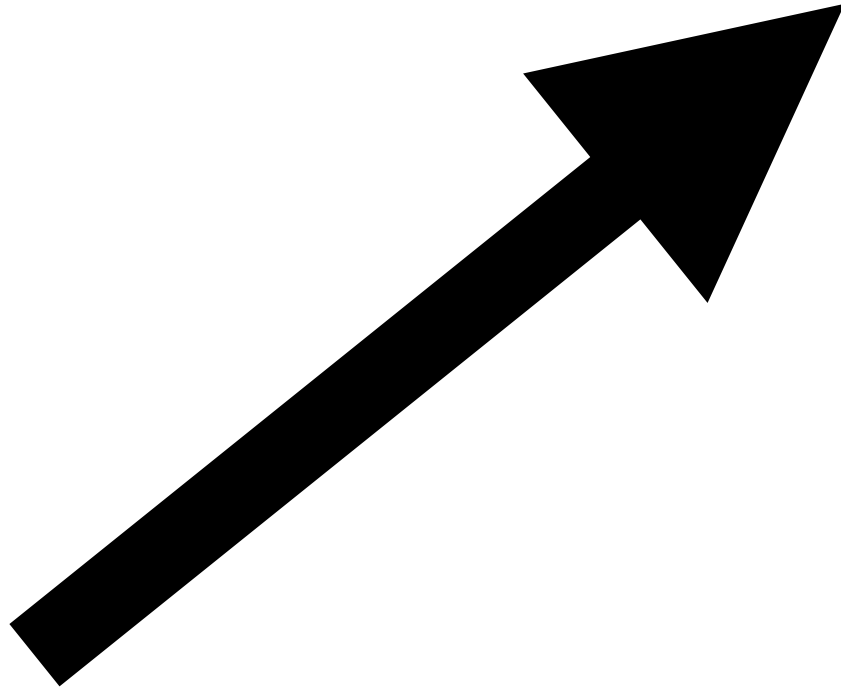
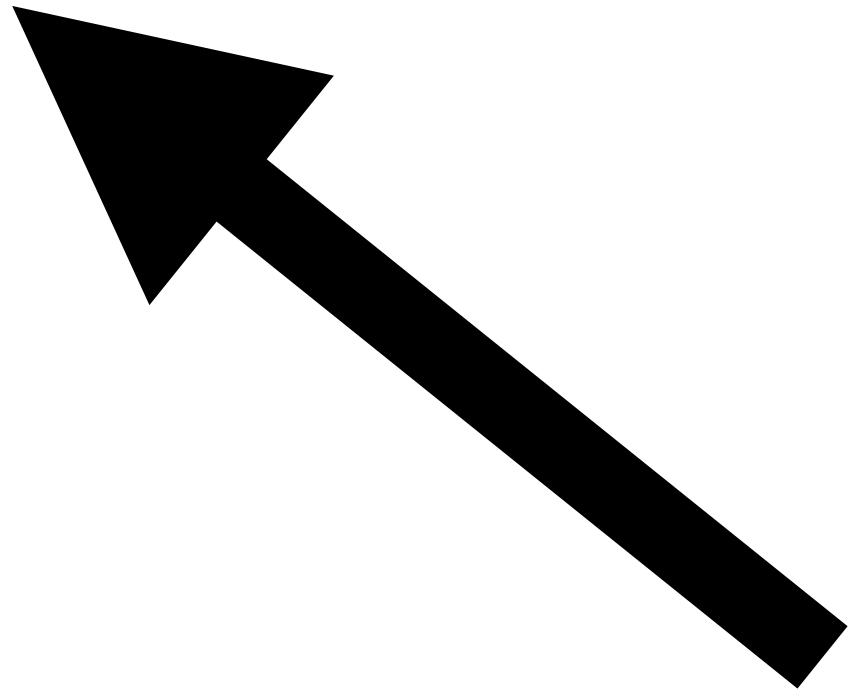
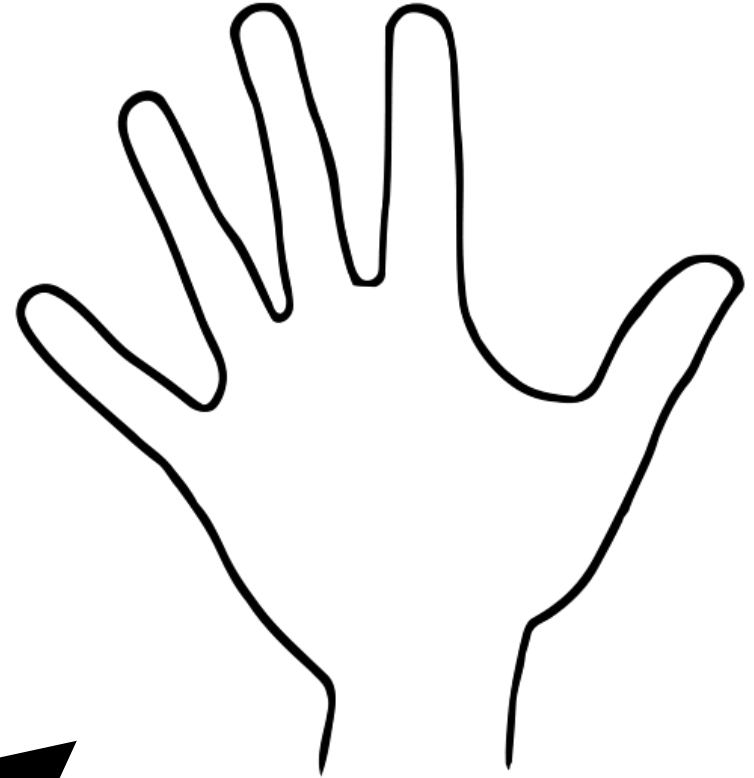
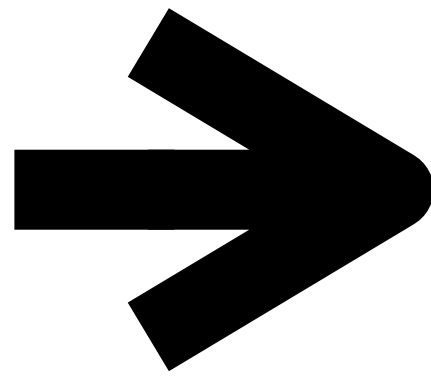
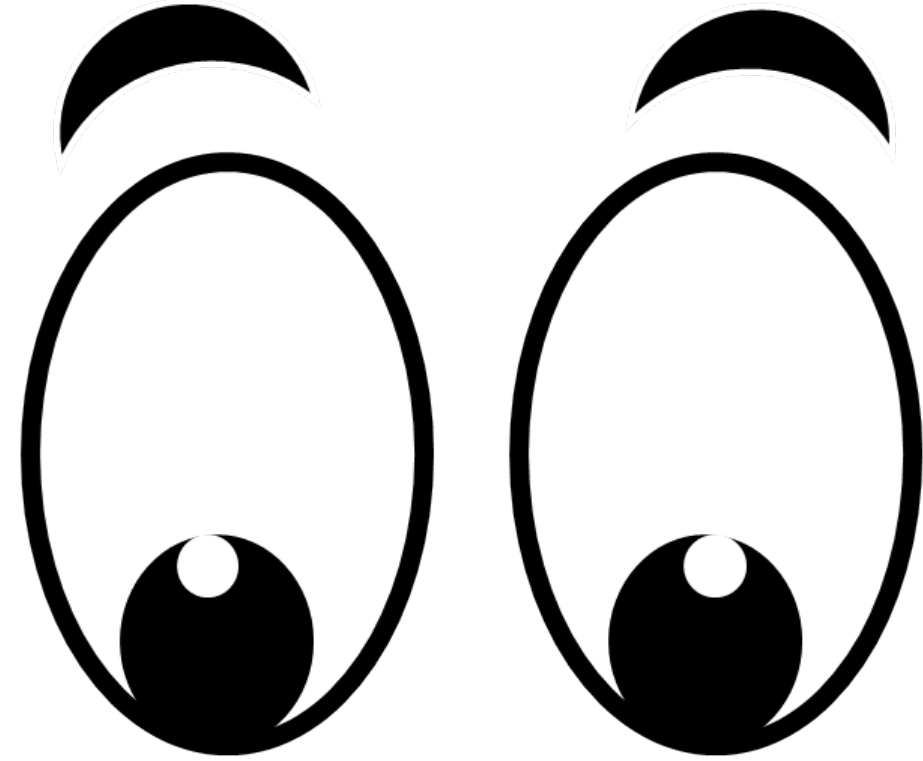
Probability of fixating  
on highest-value item

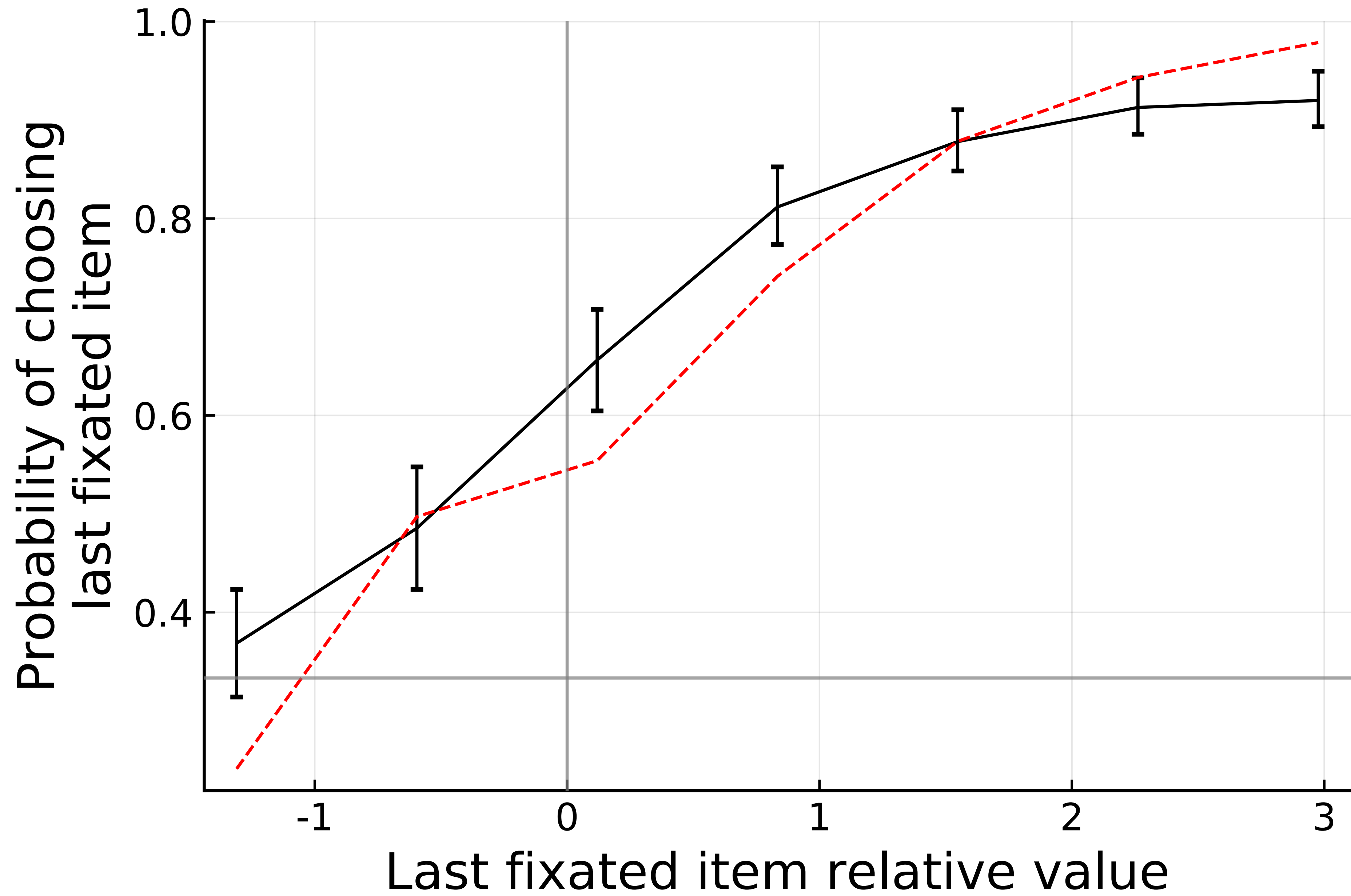












Looking forward



# Looking forward

## **Challenges**

- ▶ Computing a likelihood for parameter estimation.
- ▶ Explaining effects in the  $n = 2$  case.

# Looking forward

## Challenges

- ▶ Computing a likelihood for parameter estimation.
- ▶ Explaining effects in the  $n = 2$  case.

## Takeaways

- ▶ Some apparent “attentional biases” may actually be the result of the rational allocation of attention.
- ▶ Metalevel Markov decision processes provide a principled (and general) way to model cognitive processes in a rational framework.

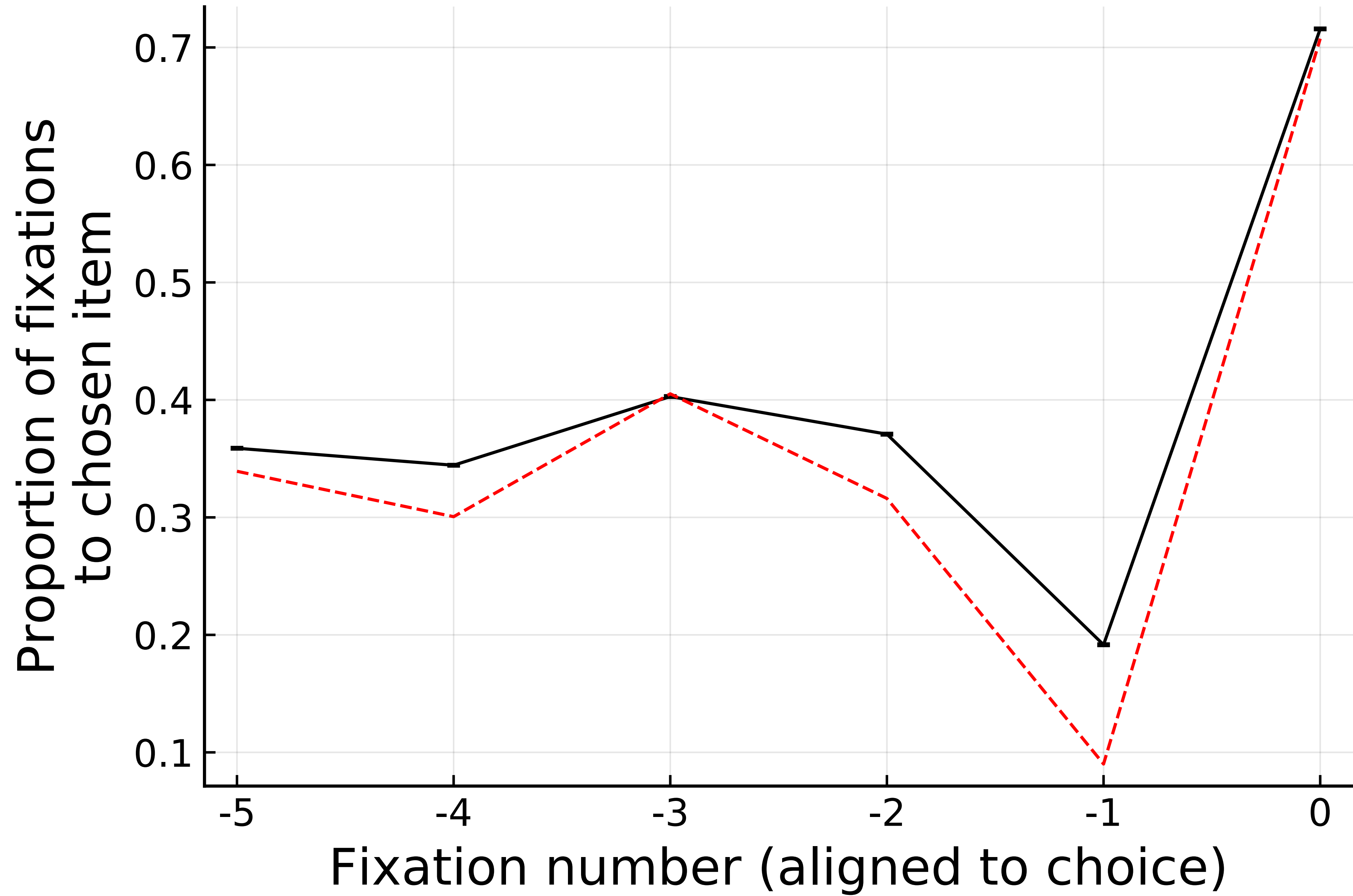
Thanks!

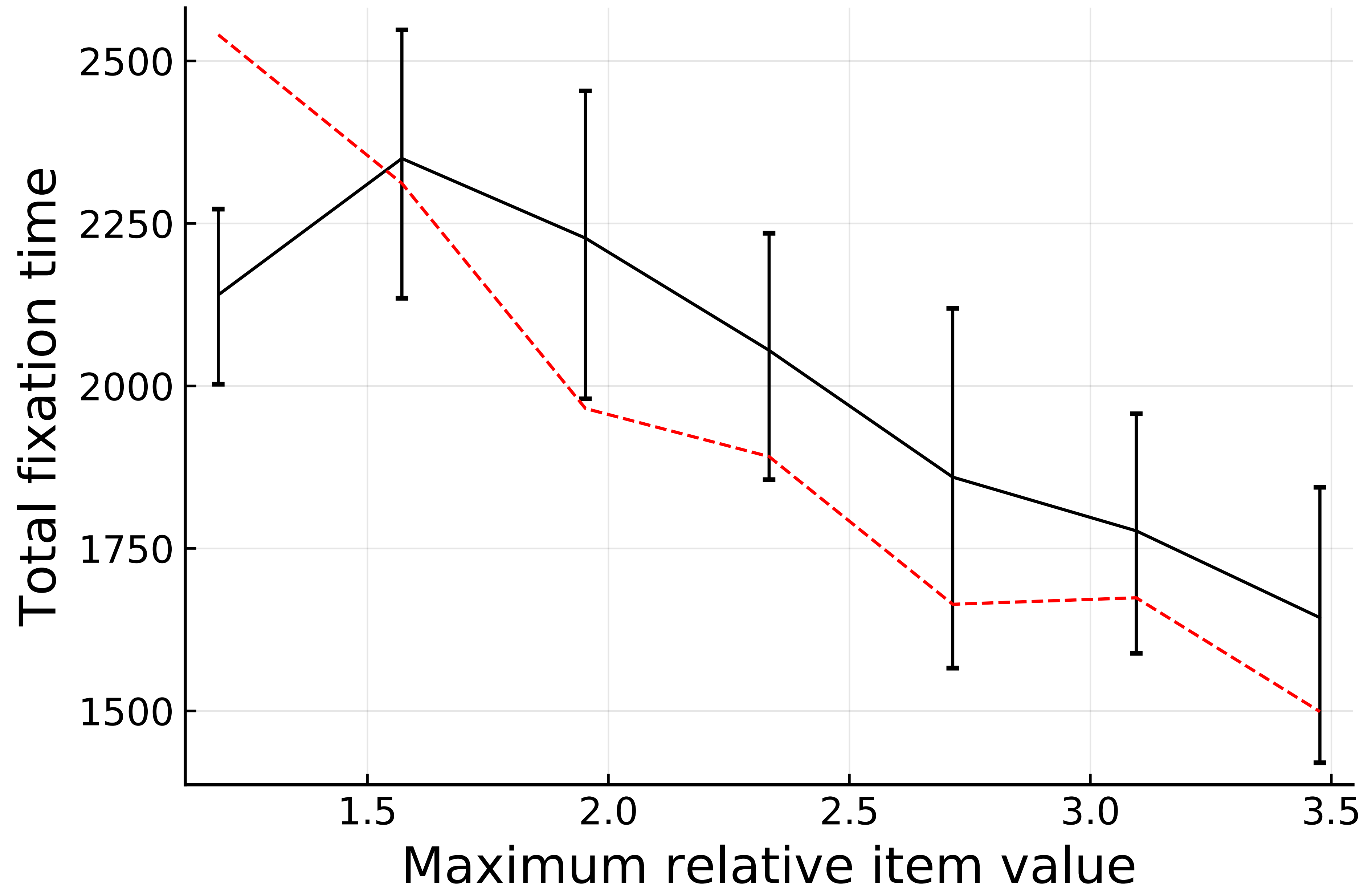


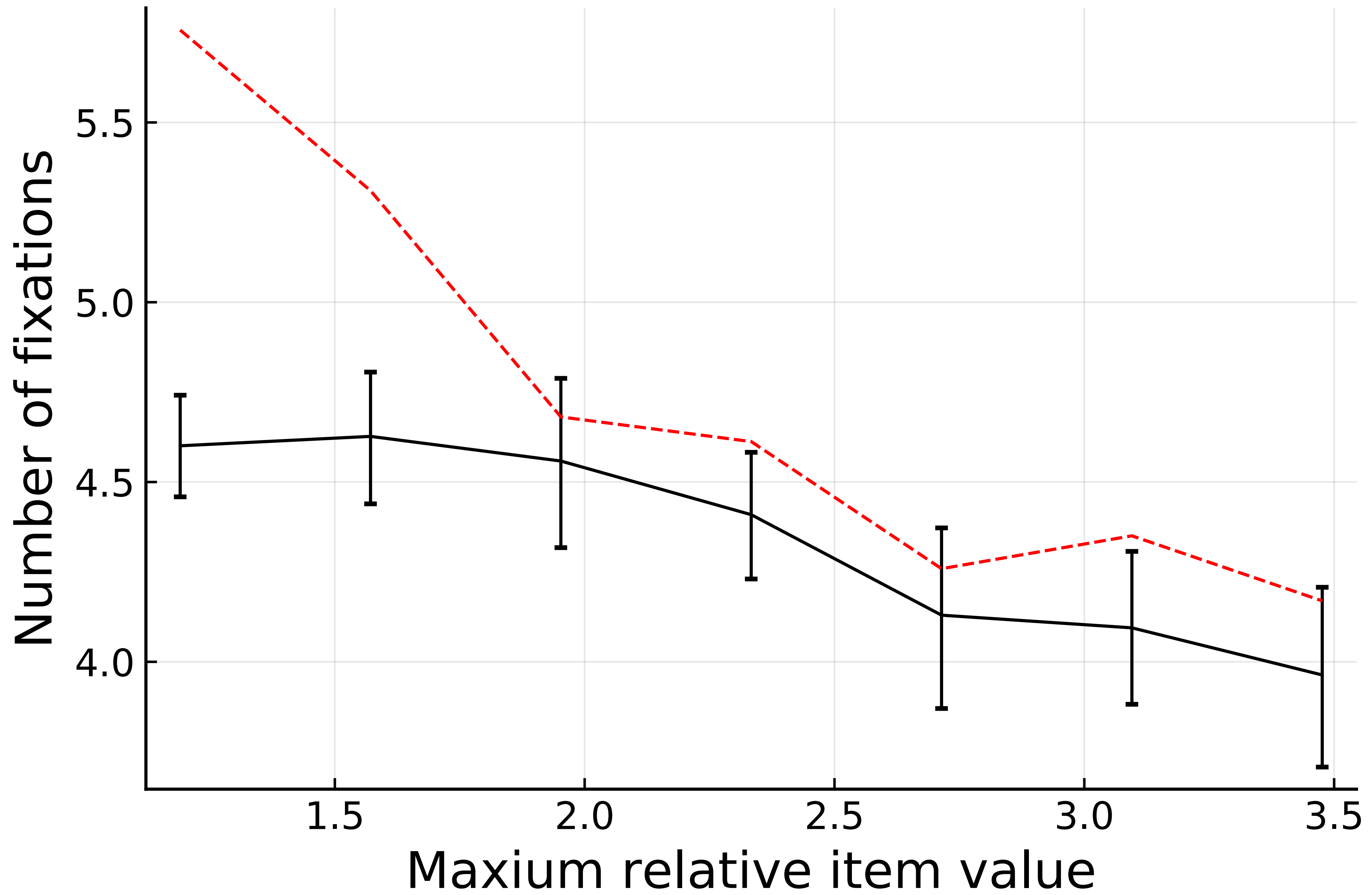
Antonio Rangel



Tom Griffiths



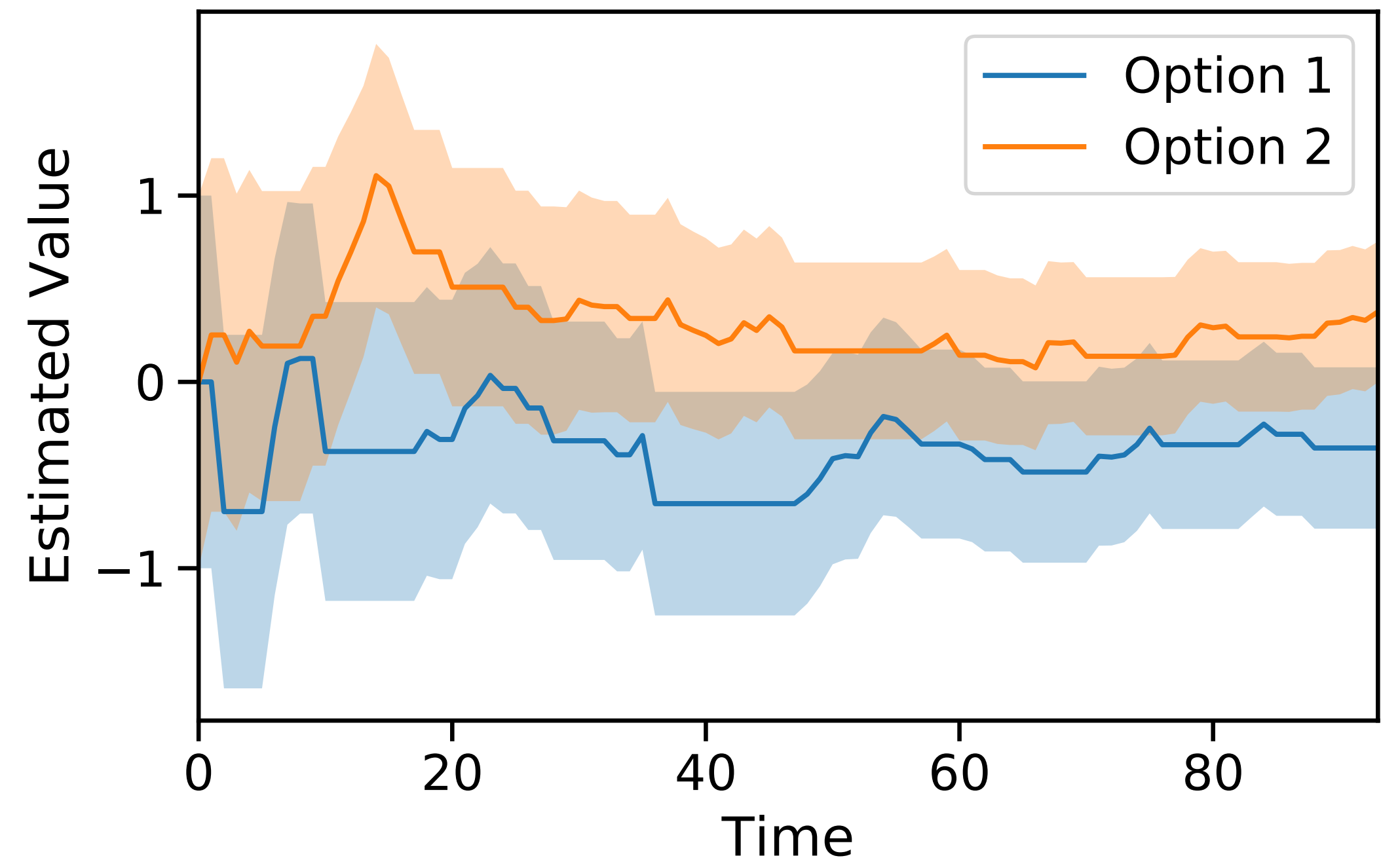




# Relation to drift diffusion models

## Similarities

- ▶ Decision variables represent the accumulation of evidence over time.
- ▶ Accumulators are subject to random walk dynamics.
- ▶ Predicts choices and reaction times.



# Relation to drift diffusion models

## Differences

- ▶ Accumulator is a posterior distribution over value.
- ▶ The variance of the random walk is not constant across time.
- ▶ Decision rule based on expected value of computation.
- ▶ Predicts attention.

