Noisy Memory and Asset Price Fluctuations

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Introduction

- Why are asset prices so volatile and sensitive to news?
  - Shiller (1981), De Bondet and Thaler (1987), Cutler, Poterba, and Summers (1990), and more
- How to model belief formation of asset prices?
  - One extreme: Rational Expectations + Full Info
  - The other extreme: Expectations separate from the model
    - e.g. Extrapolative belief based on statistical projection
- A tractable model that lies in between?

What I do

- de Silveira and Woodford (2019)
  - With a limit on the complexity of memory,
    more reccent news will be given disproportionate weight
  - A simple explanation for why consumption over-reacts to news
- Research question:

Can the theory of noisy memory predict
a realistic asset prices dynamics?

I show it can

- Compared to perfect memory rational expectations,
  - asset prices are more sensitive to news
  - asset prices are more volatile and serially correlated
  - trade volume is larger

despite assuming a very low dimensional underlying shock

A model of noisy memory

- DM observes a realization of

$$d_t \sim \mathcal{N}(\mu, \sigma_d^2) \quad i.i.d$$

  σ_d^2 is known, but μ is not known

- DM has a prior distribution for μ

$$\mu \sim \mathcal{N}(0, \omega)$$

- There's a limit on the precision of memory
  - DM has imperfect memory of the past d_{t-j}
    - Past observations are accessible only through memory
  - DM optimally chooses a memory state $\{m_t\}$

Storing memory is costly

- Cost of storing a memory
  - $I_t = \theta \times$ mutual info between (m_t, d_t) and m_{t+1}

- m_{t+1} is a noisy function of (m_t, d_t)
  - If m_{t+1} is not informative about (m_t, d_t), $I_t = 0$
  - If m_{t+1} is completely informative about (m_t, d_t), $I_t = \infty$

Consumption and portfolio choice

- DM maximizes

$$\mathbb{E}\sum_{j=1}^{\infty}\beta^{j}[-\exp(-\alpha c_t)-\theta I_t]$$

subject to

$$c_t + p_t\theta_{t+1} + \theta^{rf}_{t+1} = p_t\theta_t + R\theta^{rf}_t, \quad \forall t$$

- p_t: price of risky assets
- θ_{t+1}: holdings of risky assets
- θ^{rf}_{t+1}: holdings of risk-free assets

Optimal portfolio choice

- More risky assets if high expected returns and low variance

$$\theta^i_{t+1} = \frac{E^i_t p_{t+1} + d_t - Rp_t}{\kappa V^i_t p_{t+1}}$$

where $\kappa \equiv \alpha \frac{R-1}{R}$

- DM forms subjective beliefs based on cognitive states (m^i_t, d_t)

- Memory is used to improve an estimate of μ

$$\mu | m^i_t \sim \mathcal{N}(m^i_t, \sigma^2_\pi(t))$$

Optimal memory choice

- Optimal estimate of μ

$$\mathbb{E}[\mu | m_t^i, d_t] = (1 - \gamma(t))\, m_t^i + \gamma(t)\, d_t$$

with gain coefficient $\gamma(t) \equiv \frac{\sigma_\pi^2(t)}{\sigma_\pi^2(t) + \sigma_d^2}$

- Optimal memory stores a noisy record of $\mathbb{E}[\mu | m_t^i, d_t]$

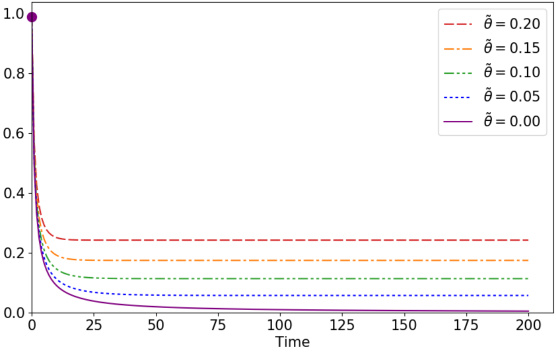
$$m_{t+1}^i = \lambda(t)\, \mathbb{E}[\mu | m_t^i, d_t] + \omega_{t+1}$$

where $\omega_{t+1} \sim \mathcal{N}(0, \sigma_\omega^2(t+1))$

- We numerically solve for $\{\gamma(t)\}$, which completely determines $\{\lambda(t)\}$ and $\{\sigma_\omega^2(t+1)\}$

Gain coefficient in the long run

- When $\theta = 0$, $\gamma(t) \to 0$
  - Precision $(\frac{1}{\sigma_\pi^2})$ linearly increases over time
- When $\theta > 0$, $\gamma(t) \to \gamma > 0$



Consequences of $\gamma > 0$ on asset prices

- Price of risky assets in equilibrium (assuming fixed supply)

$$p_t = p_0(\theta) + p_m(\theta)\bar{m}_t + p_d(\theta)d_t$$

  - $\bar{m}_t \equiv \int m_t^i di$
  - $p_d(\theta)$ increases in θ

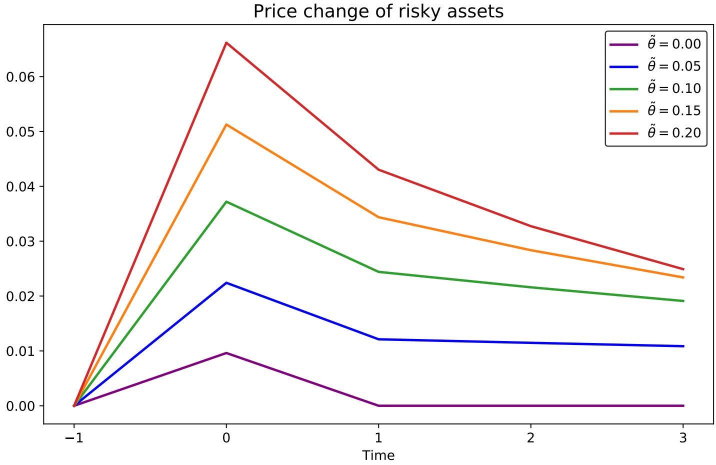
- By backward substitution of $\bar{m}_{t+1} = \lambda[(1-\gamma)\bar{m}_t + \gamma d_t]$,

$$p_t = p_0(\theta) + p_m(\theta)\lambda\gamma\sum_{j=0}^{t}\rho^j d_{t-1-j} + p_d(\theta)d_t$$

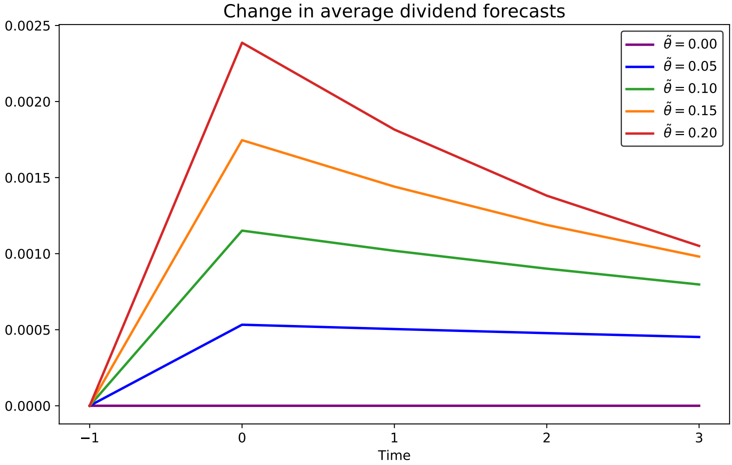
  where $\rho \equiv \lambda(1-\gamma)$

- When $\theta = 0$, small fluctuations + i.i.d process
- When $\theta > 0$, big fluctuations + long lag!

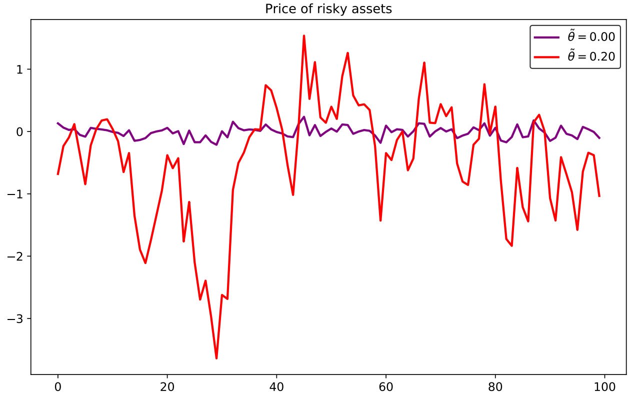
Asset prices over-react to news of dividend



Dividend forecasts extrapolate news of dividend

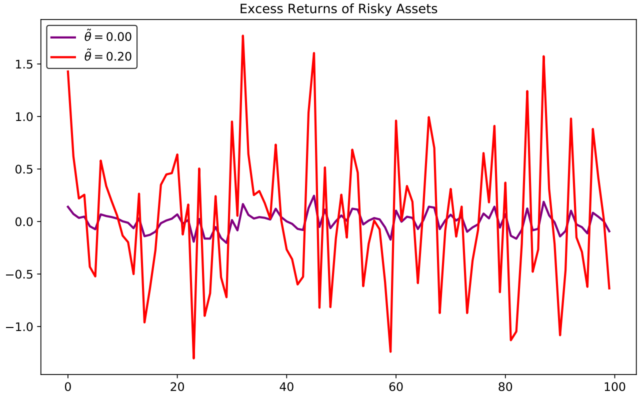


Asset prices are more volatile and serially correlated



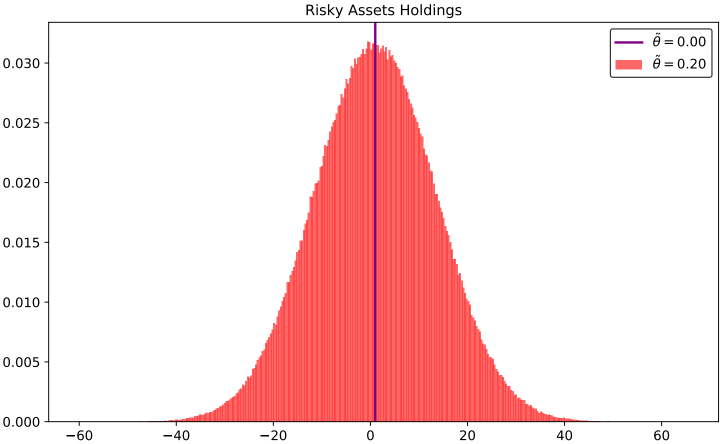
◂ Standard deviation ◂ Auto-correlation

Excess returns are higher and more volatile



◂ Average Excess Returns ◂ Std Excess Returns

Asset holdings distribution is non-degenerate

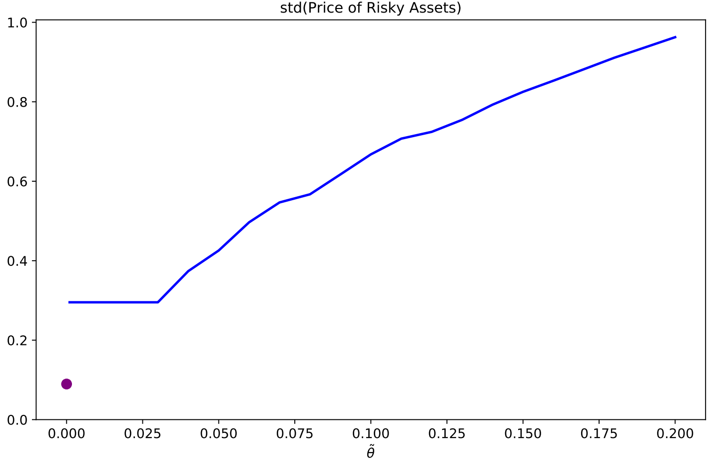


◂ Trade volume ◂ Trade volume comparison

Conclusion

- Theory of noisy memory can predict a realistic feature of asset price fluctuations
- Optimal Bayesian Inference is yet to be rejected as a useful modeling tool
- Future work includes
  - Empirically more relevant dividend process
  - Incorporation into a quantitative model

Asset prices are more volatile



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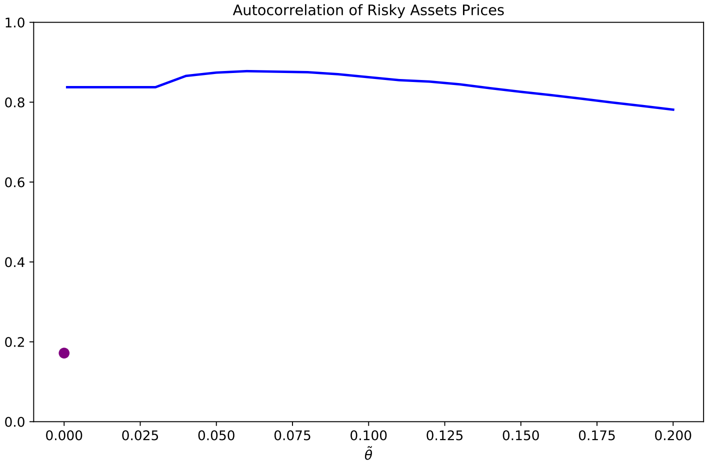
Asset prices are more serially correlated



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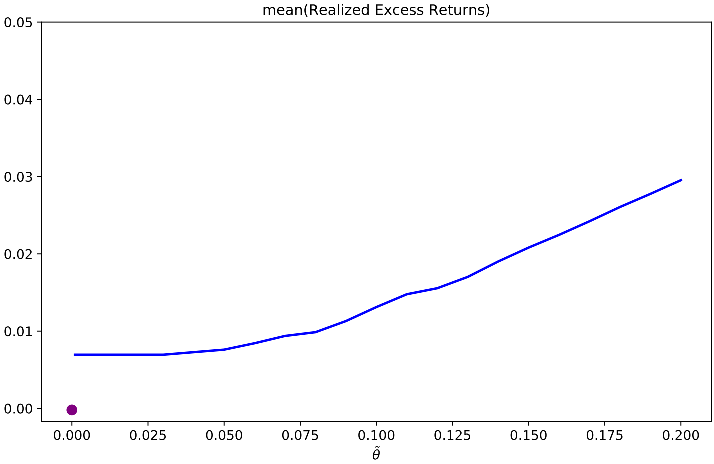
Excess returns are higher



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Excess returns are more volatile

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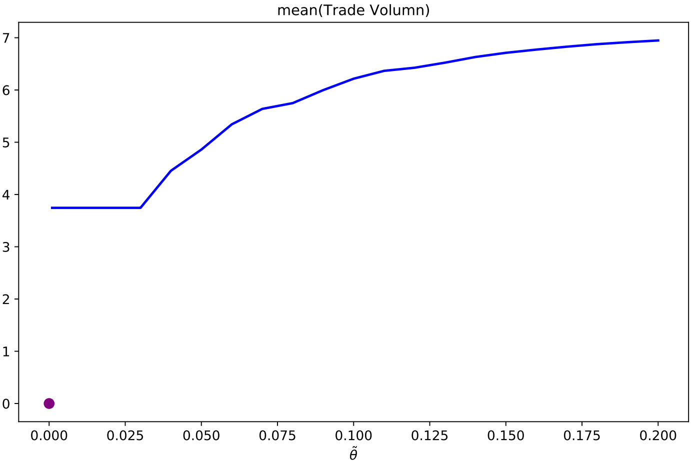
Trade volume is larger



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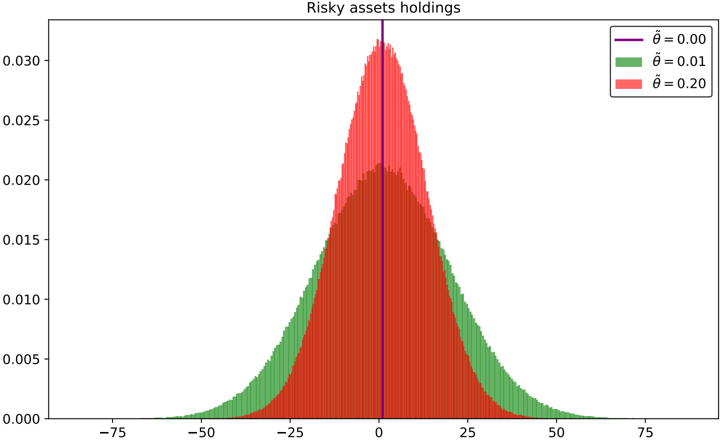
Asset holdings distribution is more centered to zero



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