A serial, foveal accumulator underlies approximate numerical estimation

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The approximate number system


Which tree should you harvest from?

The approximate number system

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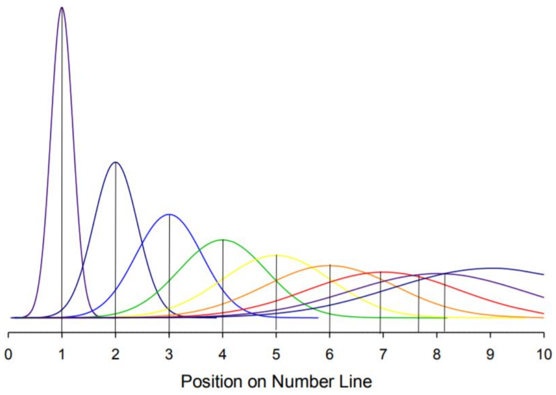
- One commonality: scalar variability
  - Ratio effect: 9:10 is as hard as 90:100.



From Rips, L. J. (2013). How many is a zillion? Sources of number distortion. *Journal of Experimental Psychology: Learning, Memory, and Cognition*

Weber fraction

- The noise in a person's approximate number representations is typically quantified by their **Weber fraction, *w***.
  - *w* is a scalar value, typically assumed to be a stable property of an individual.
- Ubiquitous psychophysical model: for a number of objects *n*, a person's estimate has standard deviation $w \cdot n$.
  - $Estimate \sim Normal(n, w \cdot n)$

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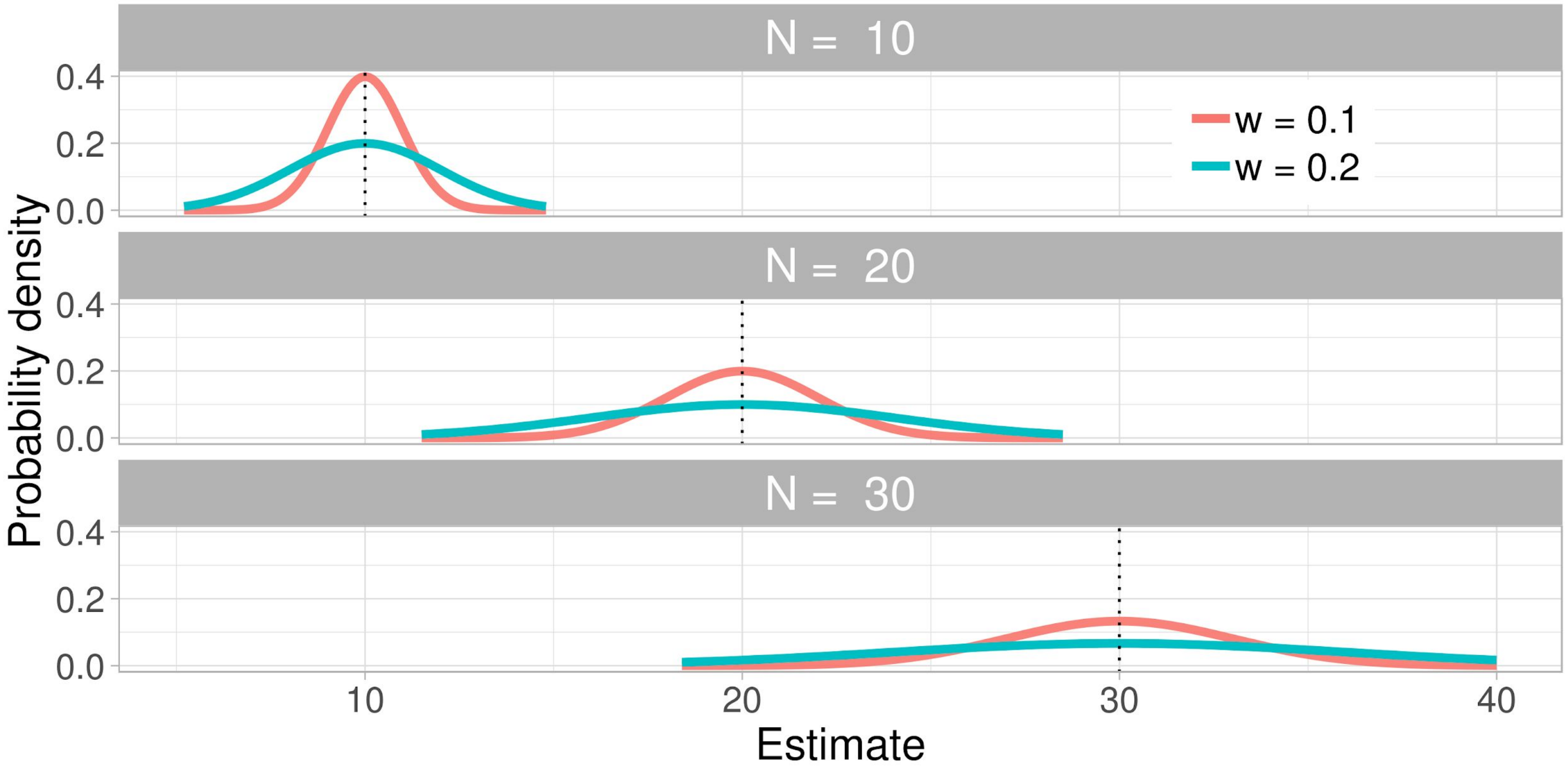

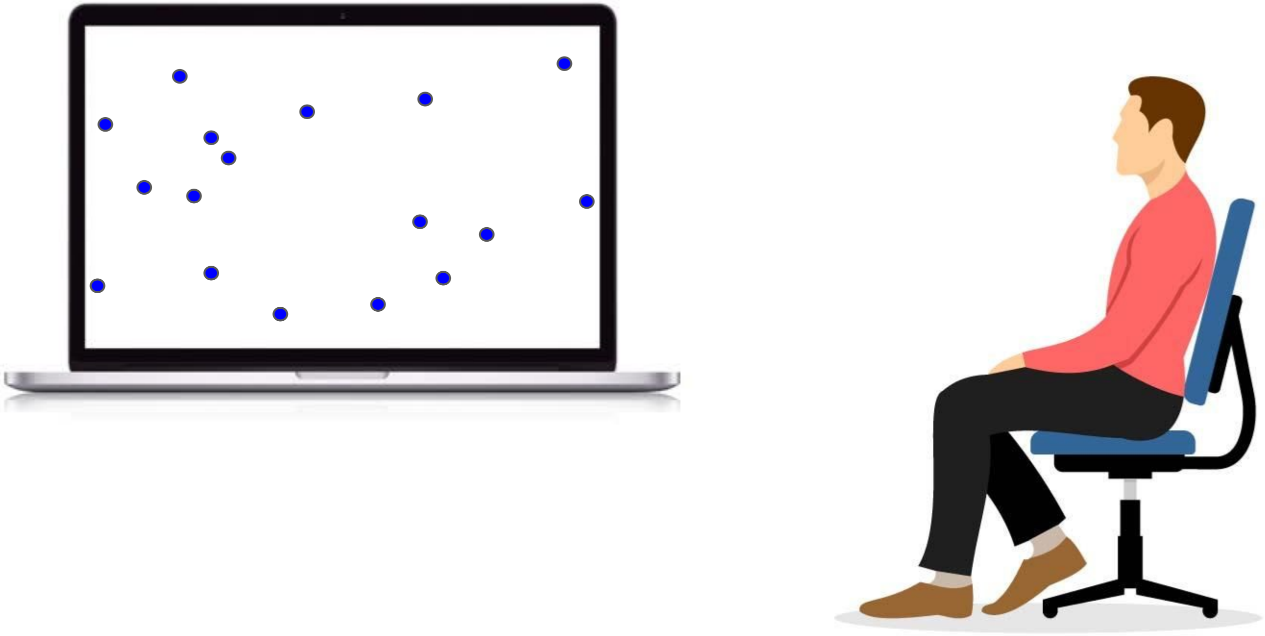

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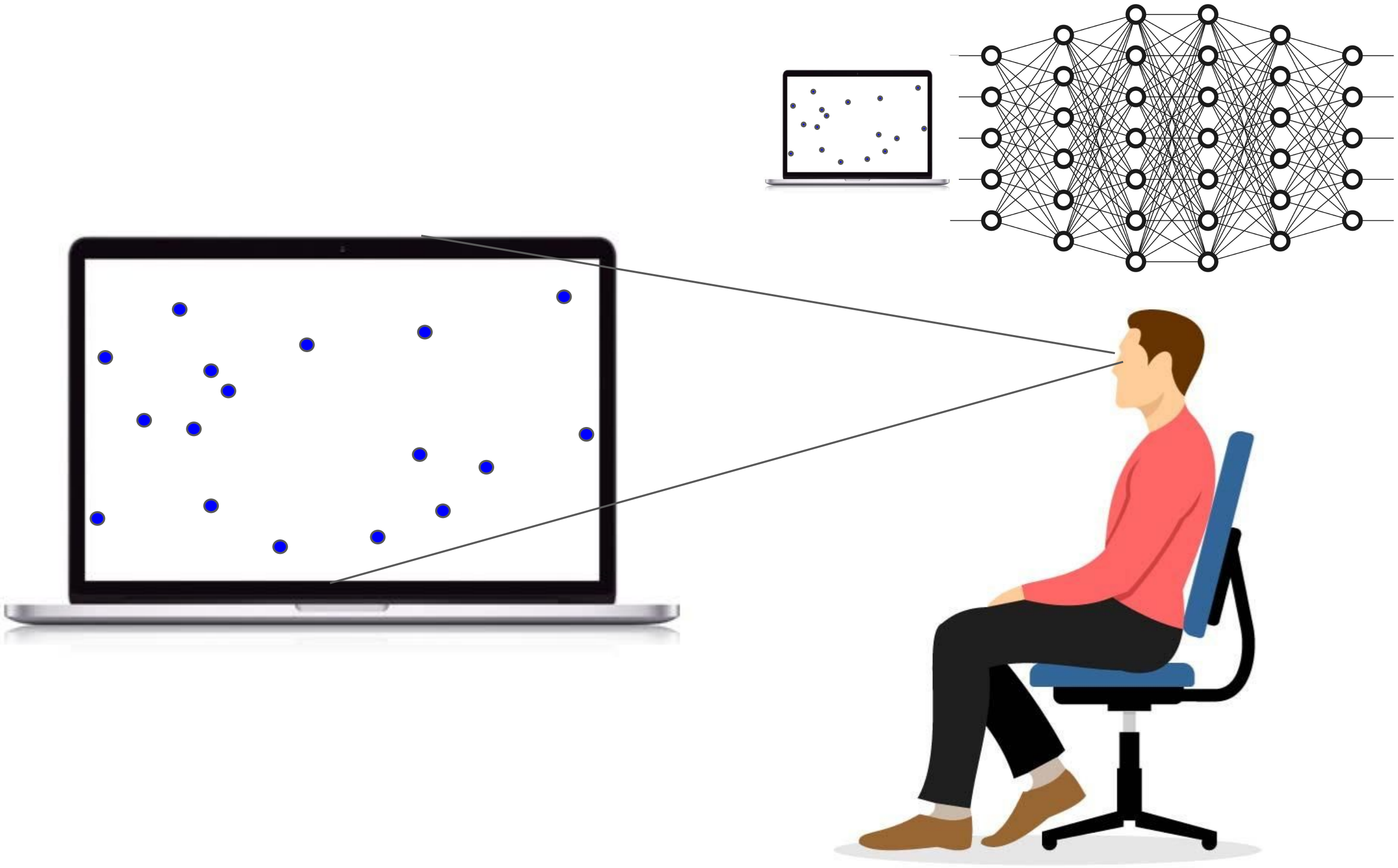

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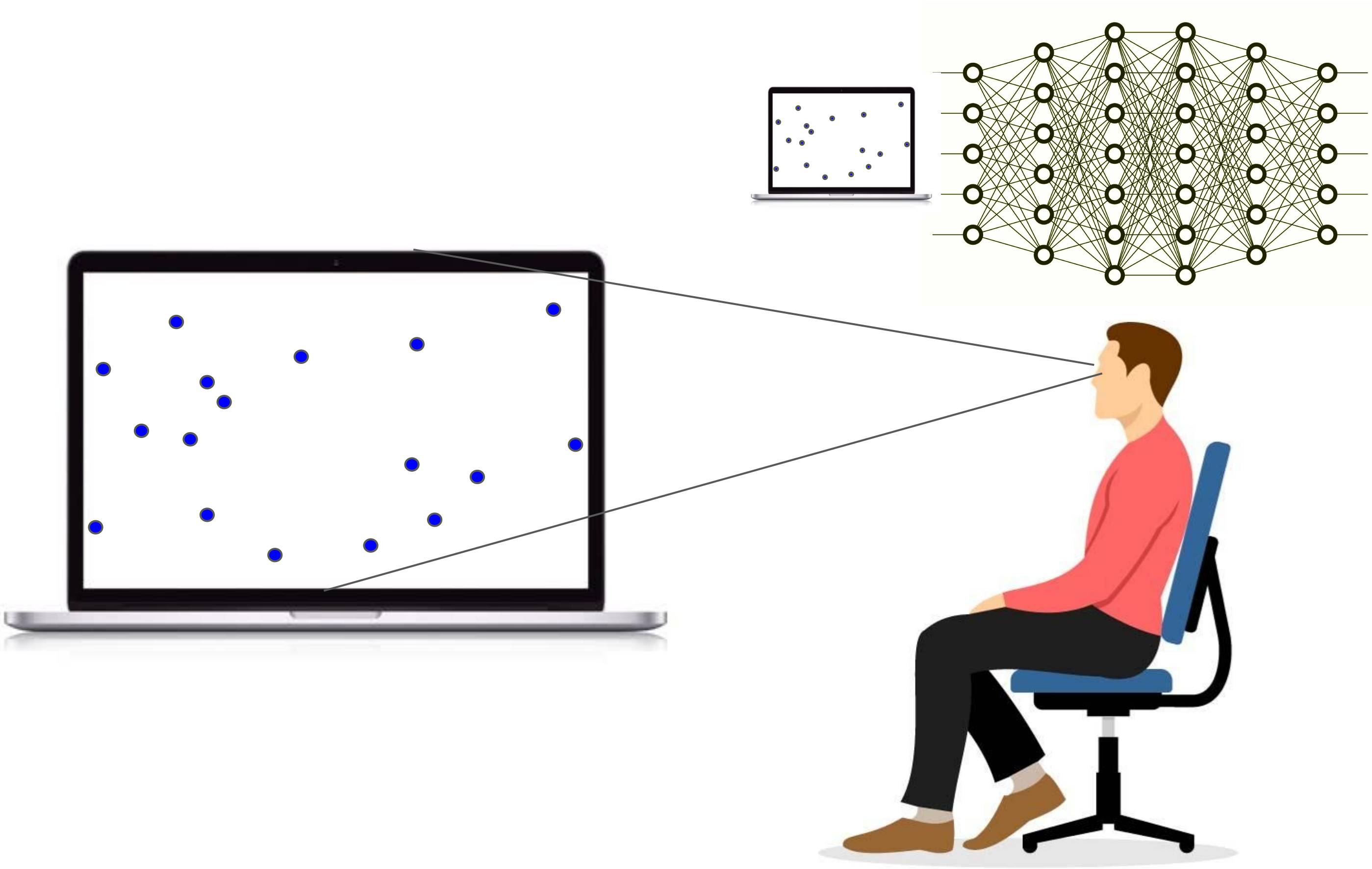

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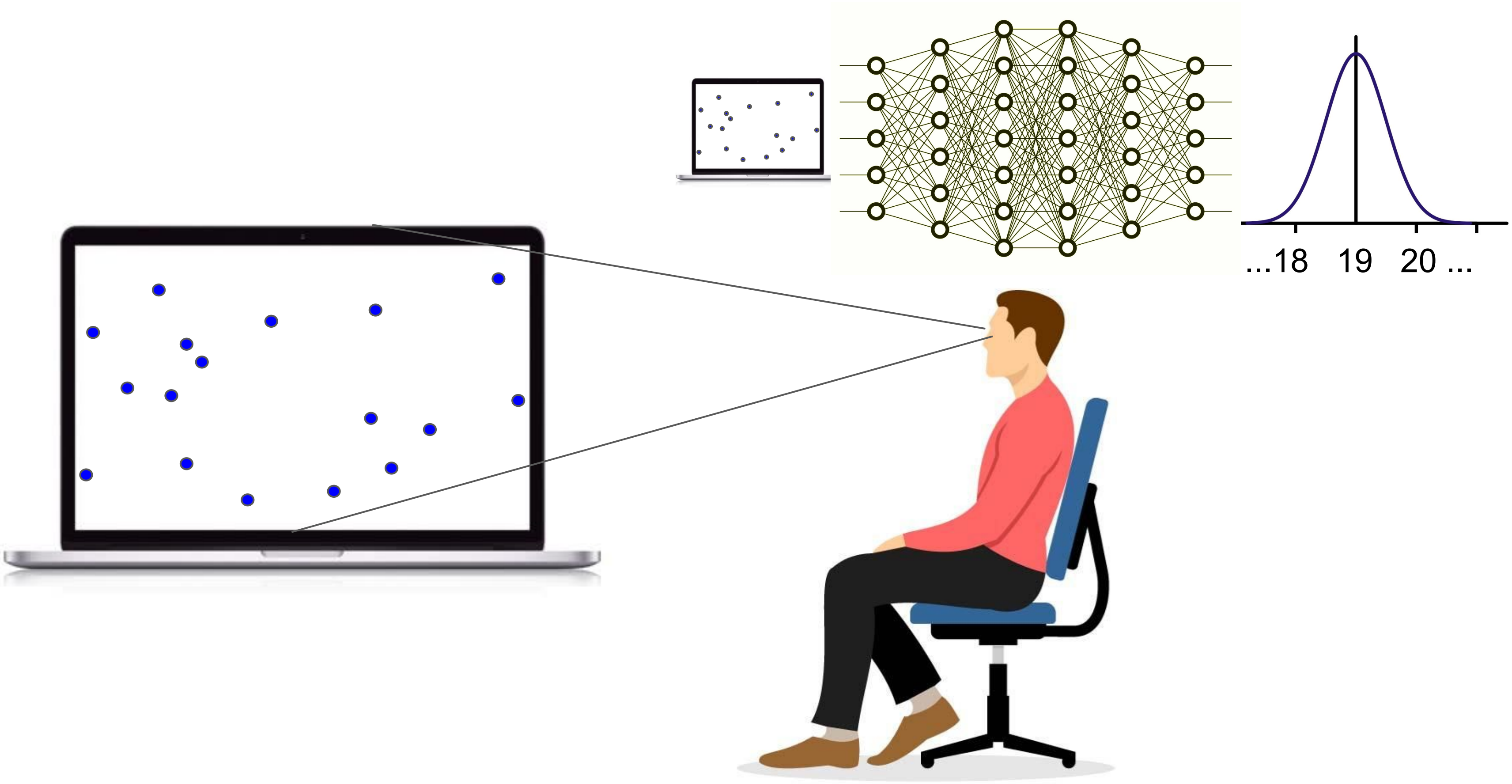

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The picture is a bit muddier

- Weber fractions have poor inter-test reliability.
- Weber fractions have poor test re-test relibaility.
- People consistently underestimate numerosities.
- Continuous increases in exposure duration improve ANS acuity.
    - Not predicted by, e.g., feedforward neural network models.

Price, G. R., Palmer, D., Battista, C., & Ansari, D. (2012). *Acta psychologica*.

Inglis & Gilmore, 2013. Indexing the approximate number system. *Cognition*.

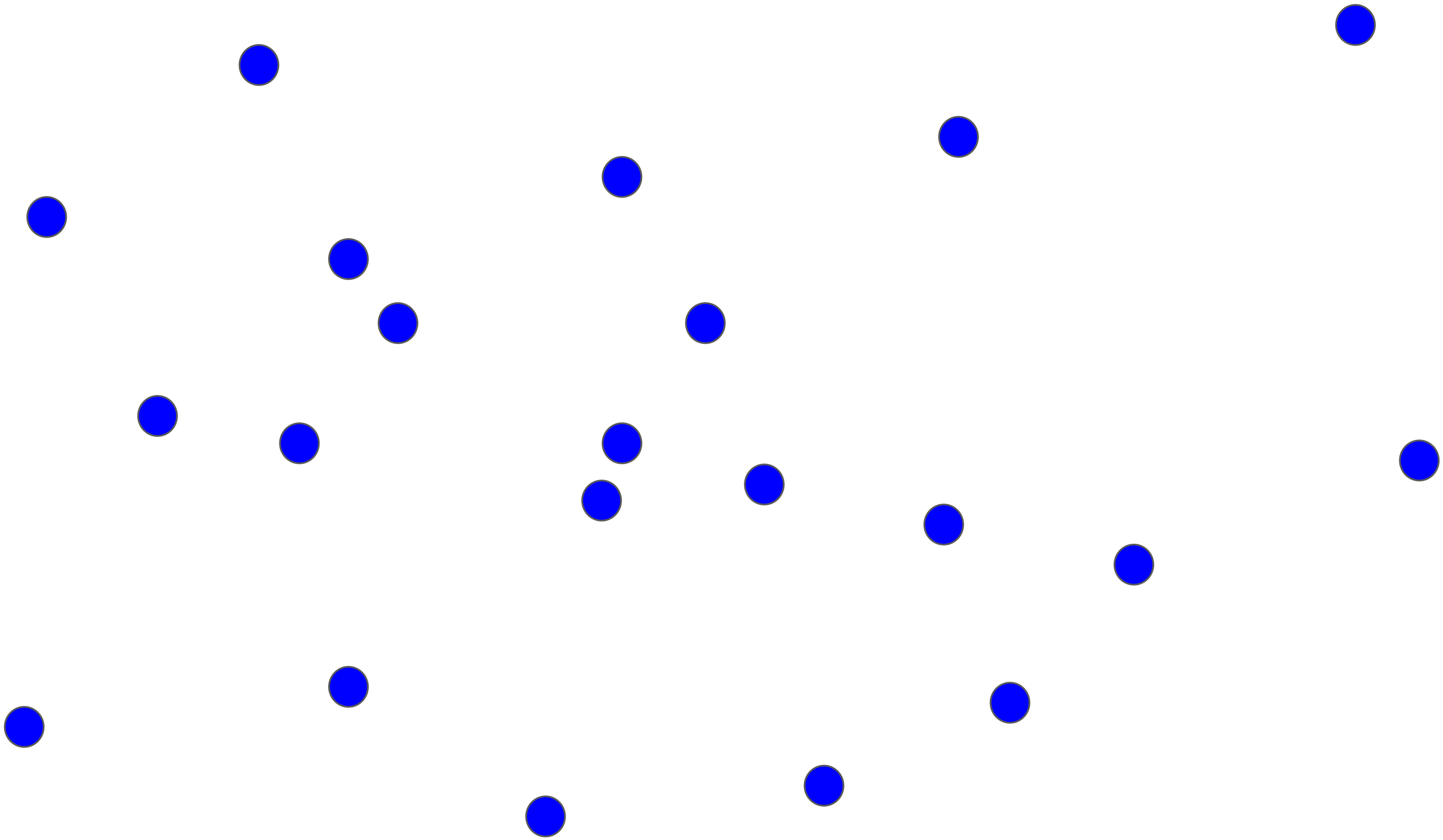
Izard, V., & Dehaene, S. (2008). Calibrating the mental number line. *Cognition*.

Experiments

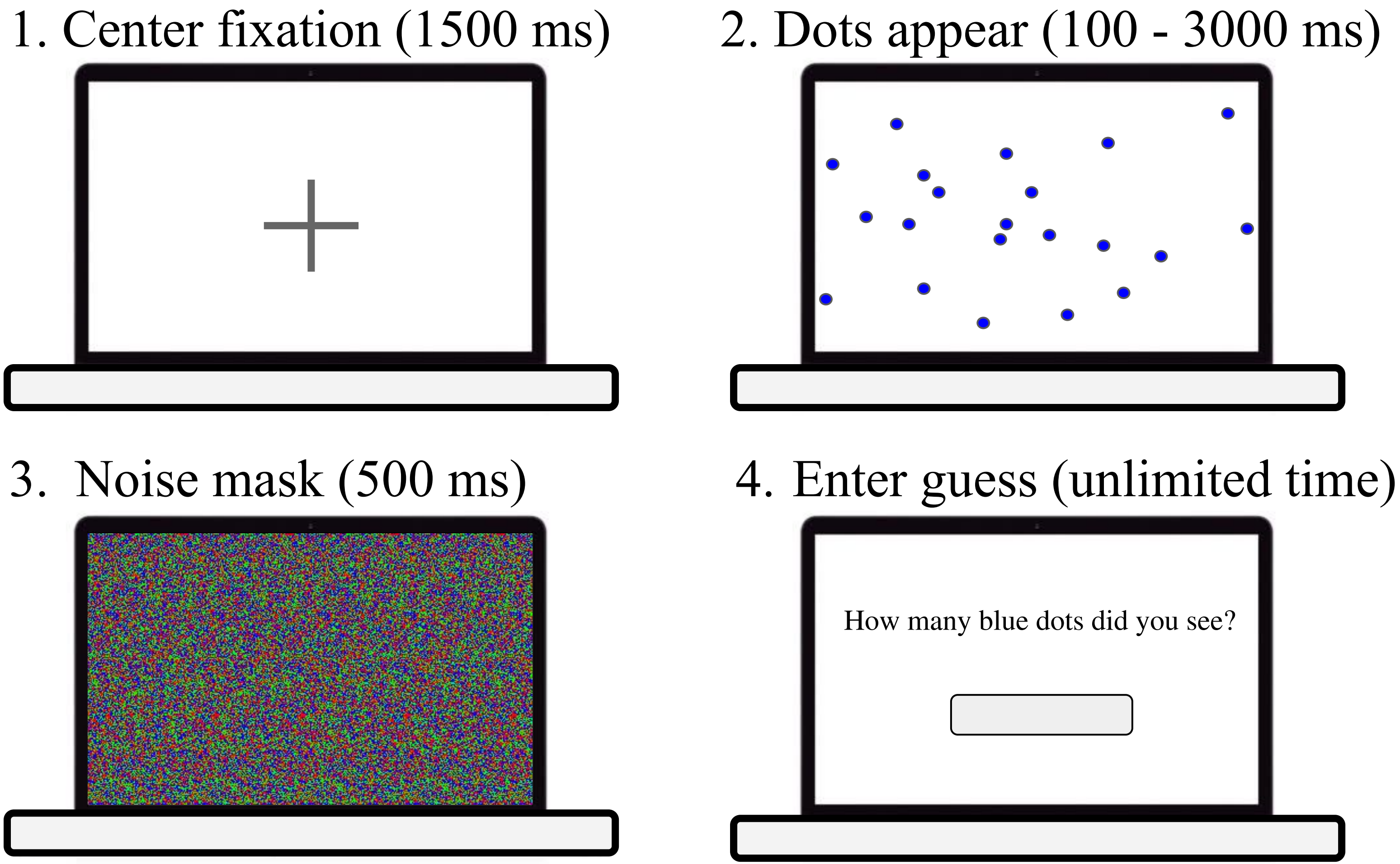
- Want to better understand the mechanisms underlying the ANS.
- Numerical estimation acuity improving with time suggests a serial (non-parallel) processing component.
- We hypothesized that the ANS relies on serial integration across visual fixations.
    - Do visual fixations mediate the link between time and acuity?
- We ran numerical estimation (Exp. 1) and discrimination (Exp. 2) tasks and recorded participants' visual fixations.

Experiment 1

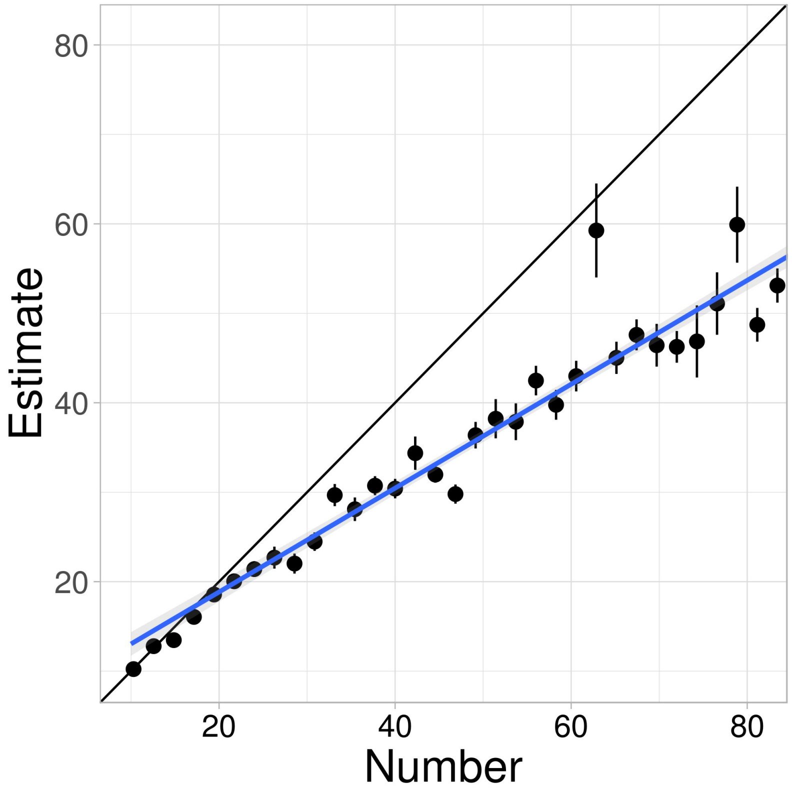
- Ran quantity estimation task (N=27) with 4 time conditions.
    - Dots ranging in number from 10-90 were flashed on the screen for either $\frac{1}{10}$, $\frac{1}{3}$, 1, or 3 seconds.
        - Every participant performed 16 trials of each condition.
- Recorded participants’ gaze using eye-tracker.



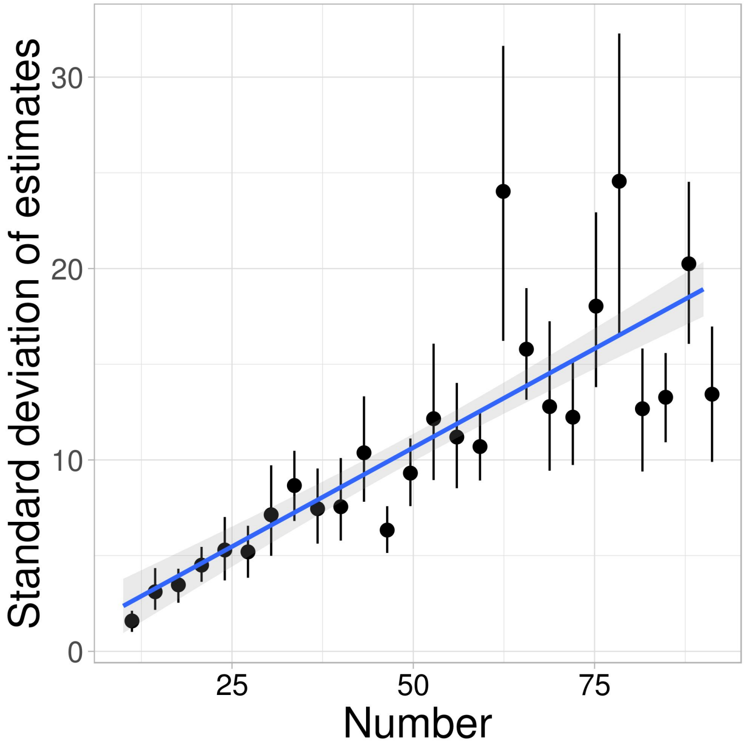
How many dots did you see?


1. Center fixation (1500 ms)
2. Dots appear (100 - 3000 ms)
3. Noise mask (500 ms)
4. Enter guess (unlimited time)
How many blue dots did you see?

Dots shown versus estimates



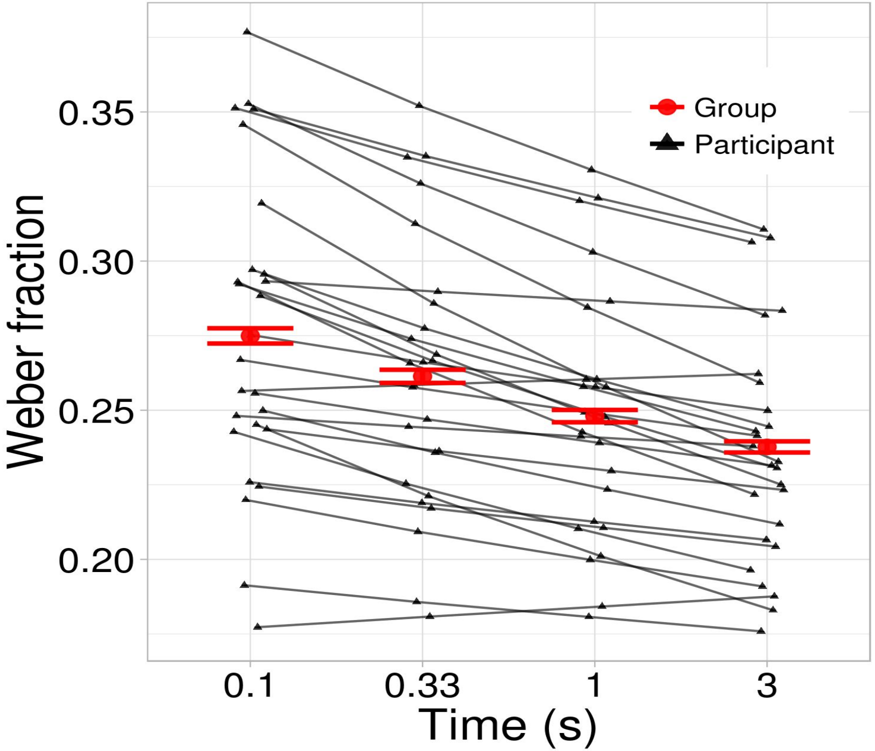
Dots shown versus variability of estimates



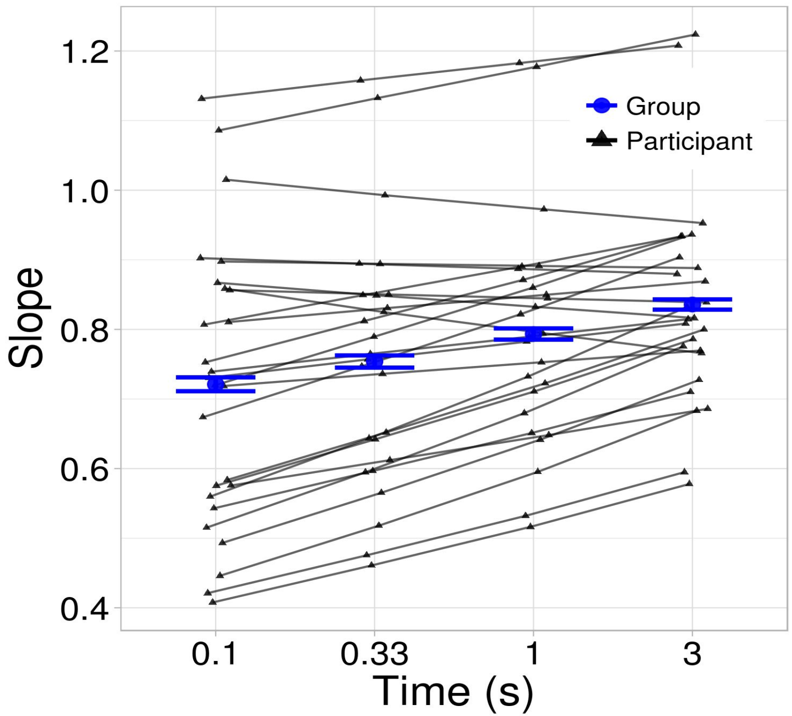
Effect of time on estimation

- Next we can look at the effects of time on mean and standard deviation of estimates.

Acuity increases with time



Mean estimates increase with time



Effect of time on estimation

- People tend to underestimate in each time condition.
  - The degree of underestimation increases with the number of dots shown.
- Accuracy increases with display time.
- People tend to underestimate less with more time.
  - Correspondingly, people tend to guess higher numbers with more time.

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- Used eye-tracker data to compute which dots were fixated.
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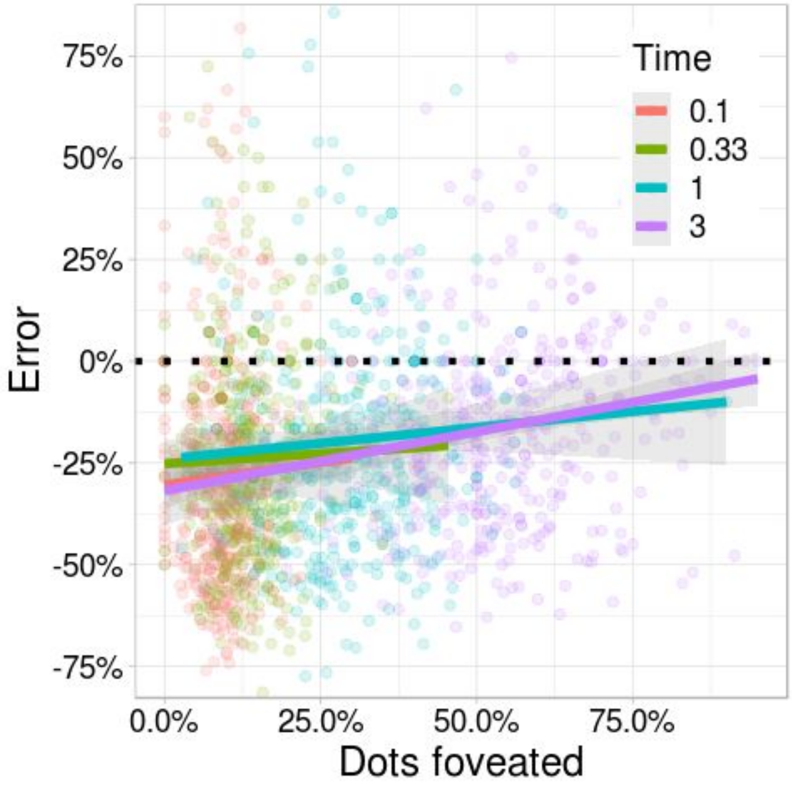
- Does the fraction of dots fixated on in a given trial predict performance?

Bias as a function of visual sample

H1: Fixations *do not* explain effects of time.

H2: Fixations *do* explain effects of time.

Bias as a function of visual sample



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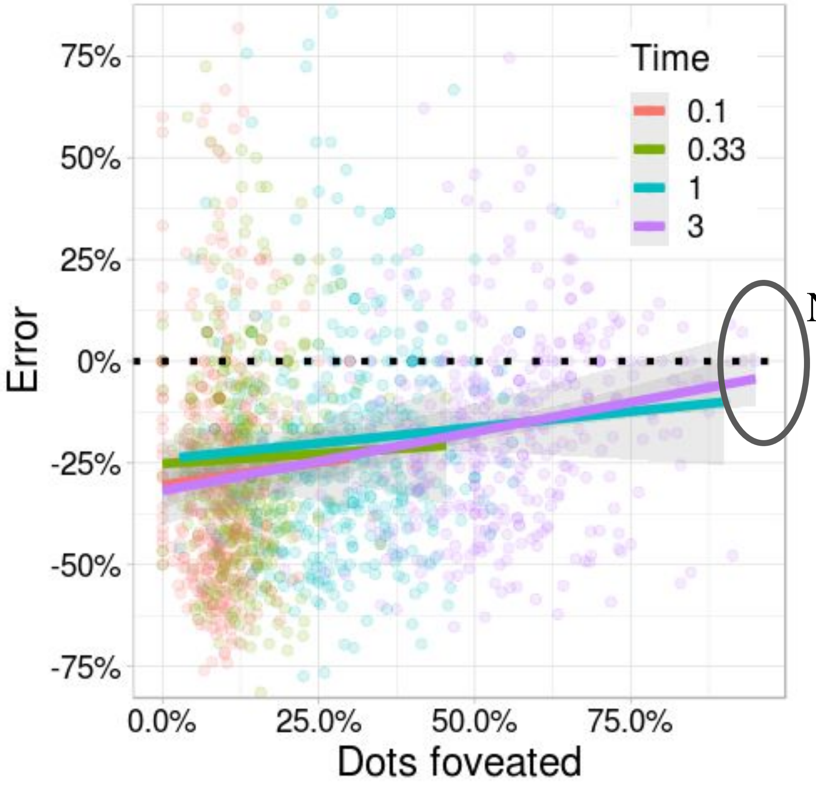
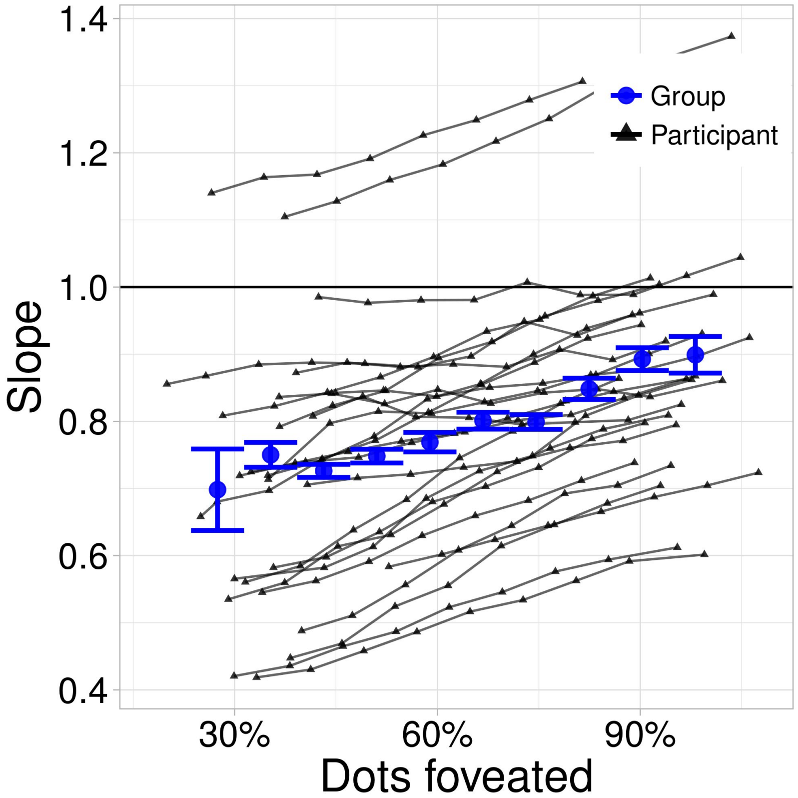
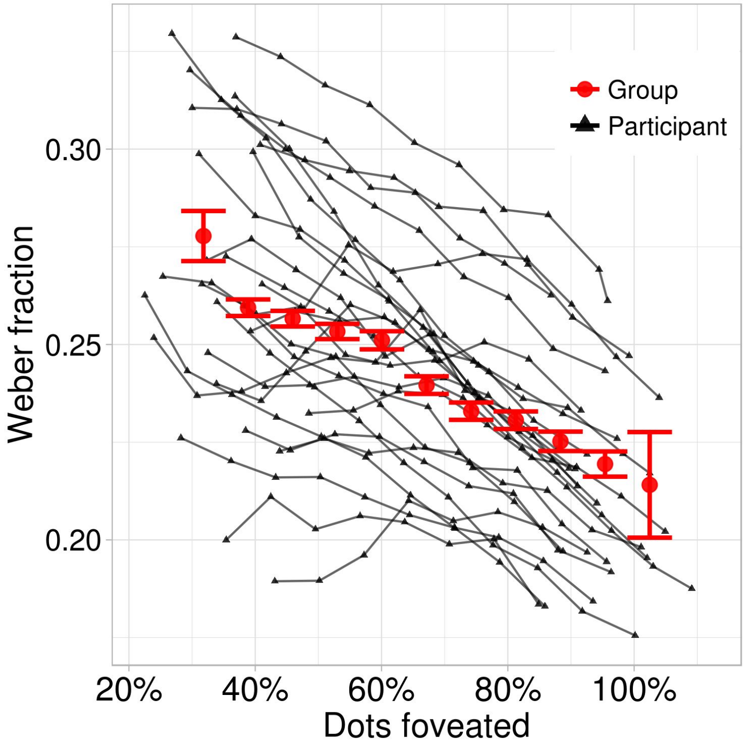


Nearly un-biased

Foveation increases mean estimate



Foveation increases acuity



Summary

- Increased viewing time:
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- Increased viewing time:
  - Increases acuity
  - Decreases underestimation bias (increases mean estimate)
- Both effects are mediated by differences in the visual sample.
- Does this effect hold in a discrimination task?
  - Experiment 2!

Converting visual samples into an estimate

- How do people convert visual samples into numerical estimates?
  - Two possibilities:
    - **Accumulator**: dots within visual gaze are noisily summed:
      - *Estimate = Sum of dots seen*
    - **Density**: accumulated sum is re-normalized by area gazed.
      - *Estimate = (Sum of dots seen)* / *(% area seen)*
  - Other questions:
    - What is the relative contribution of foveal and peripheral dots?
    - How do people deal with re-fixated dots?

How are mean estimates computed?

$$\mu = \overbrace{\beta_{foveal} \cdot (N_{foveal} + \beta_{double} \cdot N_{double})}^{\text{foveal accumulation}} \cdot \underbrace{\left(\frac{1}{A_{foveal}}\right)^{\gamma_{foveal}}}_{\text{re-scaling by foveal area}} + \overbrace{\beta_{peripheral} \cdot N_{peripheral}}^{\text{peripheral accumulation}} \cdot \underbrace{\left(\frac{1}{A_{peripheral}}\right)^{\gamma_{peripheral}}}_{\text{re-scaling by peripheral area}}$$

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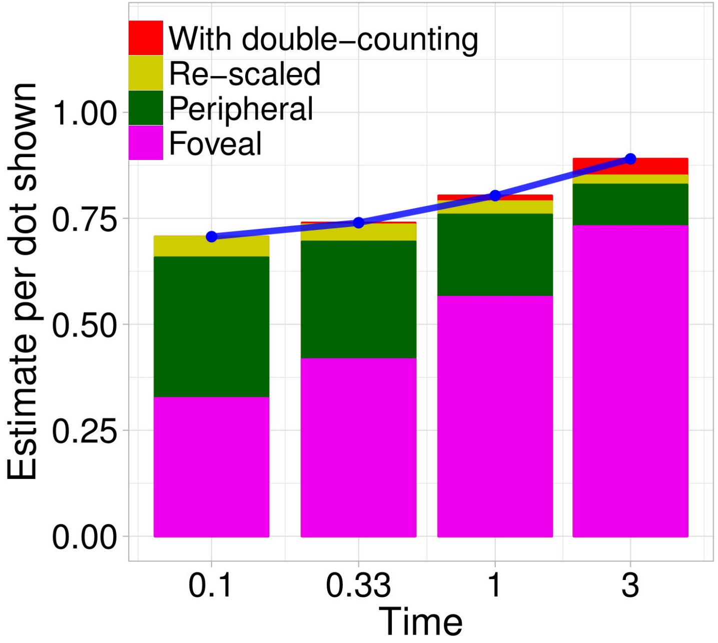

Summary

Analysis reveals that:

1. Foveal dots contribute significantly more to estimates than peripheral dots.
2. People are accumulating a quantity and not adjusting for area.
3. Multiply-fixated dots are not re-counted — suggests people are building a spatial map.

Open questions

- *Why* does foveation increases mean estimates?
- Is foveation just a proxy for attention?
- Is there really an “accumulation” mechanism?
  - Is it just a more general re-sampling mechanism?
- Are people aware of their own internal noise?
  - And are they aware of the degree to which they benefit from increased sampling?
  - Can they use this information in utility calculations?

Thanks:

RA Ashley Bardhan

"Friend of the lab"

Fred Callaway

Colala!

Computation and Language Lab

COLALA

Willa Voorhies

Questions?