Excess Volatility from Increasing Overreaction

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Motivations

Excess volatility and Beliefs

Conclusions

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  - *beyond discount rates*

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- Long maturity yields $\approx$ Expected future short maturity yields

Giglio and Kelly (2018): US Treasury Bonds

Factors=3, R^2=100.0%
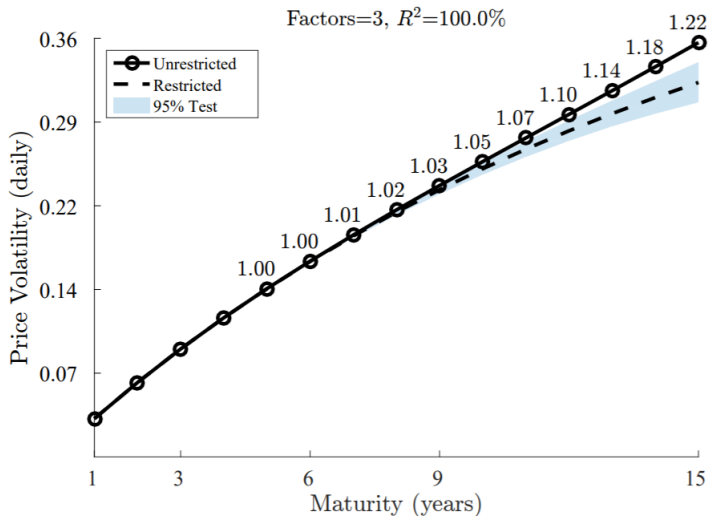
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   - confirm increasing over-reaction
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3. Explain increasing over-reaction *and* excess volatility with a model of diagnostic expectations
   - key ingredient: agents over-react more in more volatile environments

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- Forecast revision $FR_t^m := \hat{y}_{t+1|t,m} - \hat{y}_{t+1|t-1,m}$

Testing Under/Over-reaction to Information

Coibion and Gorodnichenko (2015) regression:
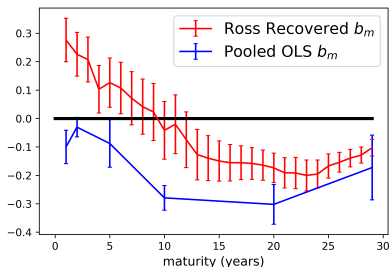$$FE_{t+1}^m = a_m + b_m FR_t^m + \epsilon_{t+1}^m$$

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- both survey and recovered forecast exhibit increasing over-reaction

- recovered beliefs display under-reaction at short end, over-reaction at long end

Survey Data and Ross Recovered Beliefs

- survey data and recovered beliefs are strongly correlated
- departures at short maturities: heterogeneity?
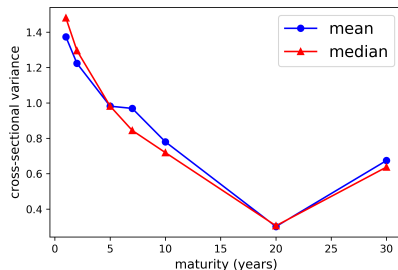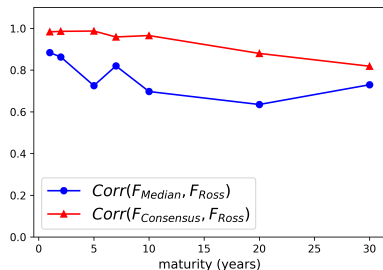
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$$\underbrace{\mathbb{Q}^{\theta}(X_{t+1}|X_t)}_{\text{Diagnostic Probability}} \quad \propto \quad \underbrace{\mathbb{Q}(X_{t+1}|X_t)}_{\text{Objective Probability}} \quad \overbrace{\left(\frac{\mathbb{Q}(X_{t+1}|X_t)}{\mathbb{Q}(X_{t+1})}\right)^{\theta}}^{\text{Representativeness}}$$

Affine models under Rational and Diagnostic Expectations

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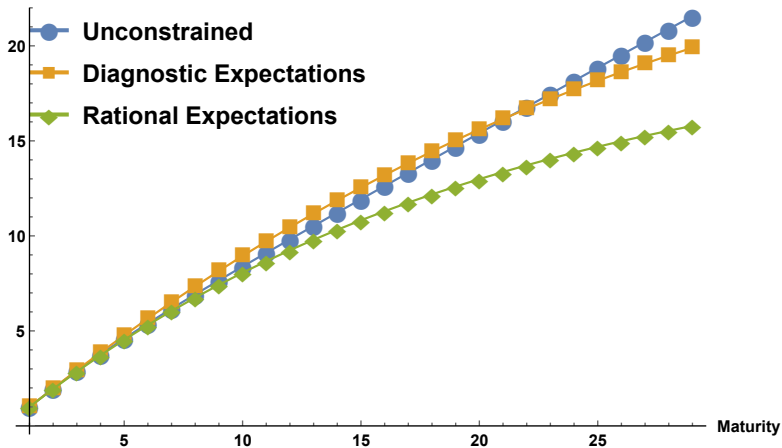
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Excess Volatility: Affine Three Factor Model

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- Rationalize both increasing over-reaction and excess volatility within diagnostic expectations. Key ingredient: agents over-react more in more volatile environments