On-the-Job Wage Dynamics

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Abstract

This paper assesses wage setting and wage dynamics in a search and matching framework where (i) workers and firms on occasion meet multilaterally; (ii) workers can recall previous encounters with firms; and (iii) firms cannot commit to future wages and workers cannot commit to not searching on the job. The resulting progression of wages (from firms paying just enough to keep their workers) yields a compensation structure consistent with well established but difficult to reconcile observations on pay dynamics within jobs at firms. Along with wage tenure effects, serial correlation in wage changes and wage growth are negatively correlated with initial wages.

Keywords: Wage dynamics; stock-flow matching; on-the-job search; no commitment

JEL classifications: J31, J63, J64

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1 Introduction

How do firms and employees agree wages? Evidence from internal labour markets and from worker-firm matched data reveals that the job alone does not determine compensation. Instead, a rich and dynamic picture of pay emerges. In particular,

- similar workers in the same position are not paid the same wage;[1]
- job tenure generally has a positive impact on wages although nominal wage cuts occur with regularity;[2]
- serial correlation occurs in wage changes so that there are predictable winners and losers;[3]
- initial labour market conditions matter such that cohorts who earn more on entry maintain their advantage through time — after controlling for composition differences, the progression of a cohort's wage depends in part on the average starting wage.[4]

These observations are challenging to collectively obtain in competitive labour models, in models of internal labour markets, and in standard job search models.

To account for these findings, this paper uses on-the-job search frictions in a market without worker and firm commitments.[5] Search as specified in this

[1]See Mortensen (2005) for an overview on wage dispersion. Baker et al. (1994a) find a strong individual component to pay determination. Job levels or positions are important to compensation, but there is also substantial individual variation in pay within levels as well as in the growth rate of pay. There are likewise large overlaps in pay across levels. Wage jumps at promotions are much smaller than differences in mean pay across levels.

[2]Elsby and Solon (2019) review the evidence from worker-firm administrative data across multiple countries and find that between 10% and 25% of job stayers experience a year-on-year wage cut. See also Baker et al. (1994a), Baker et al. (1994b); McLaughlin (1994); and Card and Hyslop (1997).

[3]See Baker et al. (1994b); Lillard and Weiss (1979); and Hause (1980).

[4]Baker et al. (1994a), Baker et al. (1994b) find that after controlling for composition differences, the progression of a cohort's wage depends in part on the average starting wage. See also Kahn (2010); Oyer (2006); and Oreopoulos et al. (2004), Martins et al. (2012). Although their findings are somewhat different, Beaudry and DiNardo (1991) also report that cohorts matter.

[5]Waldman (2012) reviews the literature on internal labour markets and considers a variety of explanations for wage dynamics based on imperfect information linked to human

paper differs, however, from conventional 'black-box' random matching as well as from directed search frictions. Although the labour search literature claims numerous insights and successes (such as generating equilibrium wage dispersion among similar workers), it does not readily yield a sufficiently rich pattern of compensation over time for a particular worker-job match. This incomplete picture may stem from the underlying specification of search frictions rather than from the general search and matching approach. This paper therefore adopts an alternative matching specification, the stock-flow specification, which offers a plausible, empirically valid microfoundation for search frictions and matching dynamics.[6]

The stock-flow matching framework posits two natural as well as relatively novel features of on-the-job search. In particular, workers and firms

i. on occasion (but not every time) encounter each other multilaterally during job search

ii. can remember past encounters.

In a labour market with stock-flow matching, when a seller, i.e. a worker, goes on the market in search of a partner, he or she immediately becomes fully informed about the number of suitable buyers in the stock, i.e. the stock of job vacancies. If lucky, the worker finds several viable options and matches quickly. If the worker is unlucky, the market turns up few (more precisely only one) or possibly no viable opportunities. In the event that no acceptable vacancies exist in the marketplace, the worker must wait (possibly alongside other workers seeking similar work) to match with the flow of new jobs.

Recall of past encounters readily follows from the full revelation of all opportunities and competition in the marketplace. Matched and unmatched traders remember their past marketplace experiences. As time proceeds, the gradual flows in and out govern the number of traders active in the marketplace so that unmatched workers who constantly visit the market

capital acquisition, job assignment, learning and tournaments. These explanations offer insights but abstract from some broad market considerations of competition.

[6]The matching framework used here is most closely related to the matching models of Taylor (1995), Coles (1999) and Lagos (2000). Emerging empirical evidence indicates this framework has more validity than random matching. See Coles and Smith (1998), Petrongolo and Pissarides (2001), Andrews et al. (2013), Gregg and Petrongolo (2005), Coles and Petrongolo (2008), and Kuo and Smith (2009).

anticipate and find only negligible changes, if any, between one day and the next. Employed workers can likewise recall their last visit to the job market and who was there at the time. On the other hand, they are unaware of the intervening turnover since that last visit occurred. Job opportunities and competition turn over, but the worker and the employer do not directly observe this turnover unless the worker actively engages in on-the-job search. If they have not visited the market since matching, the market may have changed, at random, more profoundly over a substantial period of continuous employment.

Consider wage determination in this set-up with on-the-job search when firms cannot commit to future wages and workers cannot commit to not search while employed.[7] After job search reveals the number of currently available jobs, all suitable firms bid for the worker's services. If the worker finds that only one job option is currently available, the firm offers a monopsony payoff that claims all of the gains to trade for the firm. On the other hand, with more than one firm involved, the firms engage in competitive Bertrand bidding. This time, the worker extracts the gains to trade. At the outset of the employment relationship, wage dispersion obtains and depends on the number of competitive bidders found at that time.

Now suppose that at any time after a firm and worker pair up, the firm can update its offer. In other words, as the firm cannot commit to future wages, a new wage is offered in each instant. The worker can either accept the latest offer or go again to the market to elicit bids. Although the pair perfectly remember their last visit, they do not know what has happened in the market since that visit. The firm updates its wage offer knowing that as time proceeds, firms and workers come and go and the number of prospective bidders in the market evolves randomly. The worker must physically visit the market to learn the actual number of bidders currently in the market.

This process provides a new source of wage progression with tenure at a firm. Employers will want to avoid bidding with the (anticipated) firms in the market and keep the worker away from the market with a sufficiently high wage offer. Such an offer outbids the evolving threat of on-the-job search, not the actual firms. The resulting progression of wages from firms paying just enough (in a specific time period) to keep their workers from on-the-job search yields a compensation structure consistent with the above findings.

[7] Taylor (1995) and Coles and Muthoo (1998) examine wages in this set-up without on-the-job search.

No-search wages face two countervailing forces from turnover in the market. Previous bidders gradually leave the market and new options enter the market. Outside options therefore can rise or fall depending on this birth and death process. Wages not only differ at the outset, they also evolve in different patterns. For monopsony wages, the unfortunate history (from the worker's perspective) fades and the outside option improves. Low initial wages rise over time. For competitively bid wages, the more favorable history that led to high initial wages fades and eventually a less attractive expectation of the number of new firms matters more. Although wages start at different points and evolve in different patterns, they ultimately converge with sufficiently long tenures.

Job availability and turnover jointly determine wage dynamics and wage dispersion. Initial wages and their subsequent progression at a job within a firm combine to create a distribution of wages at a point in time. Although it is difficult to formulate and evaluate an explicit expression for the distribution, numerical methods reveal sensible shapes for a range of parameters. In a homogeneous environment, the cross section of wages is dispersed around an interior mode with prominent tails on both sides. The model can also generate reasonable mean-min ratios and thus overcome the lack of frictional wage dispersion found in standard search models by Hornstein et al. (2011).

The key determinant of compensation is the expected payoff from search. As an employment spell progresses, the search option evolves thereby driving the results. Potential competition drives wages but frictions limit its full scope. As in Yamaguchi (2010) and Bagger et al. (2014) the outside option evolves as potential partners come and go but unlike those papers, firms react to the threat of search rather than the trigger of an actual job offer for renegotiation. Because stock-flow matching in effect builds in duration dependence, the evolving threat of on-the-job search and not its realization determines wages. As a result, turnover is less pronounced.

The next section describes the general framework and the process governing vacancy turnover. Section 3 describes the worker's and firm's decisions in this economy. Section 4 derives the payoffs in the job center as workers and firms search. Section 5 derives payoffs for existing worker-firm pairings, and Section 6 derives wage dynamics. Section 7 describes a numerical example of the wage progression, wage dispersion, and the impact of job tenure. The last section concludes.

2 Economic Environment

Homogeneous workers and homogeneous firms populate an economy with a small, highly specialized labor market. Both agents are risk neutral, discount the future at rate $r > 0$ and maximize expected lifetime payoffs.

The economy operates over an infinite sequence of discrete time periods of length $dt > 0$. Each time period consists of two sub-periods or stages - an internal labour market phase and a job search phase. At the start of time ($t = 0$) the economy is empty with entry occurring randomly over time.

At any point in time, a worker is said to be "attached" to a particular firm's job if the worker produced output for that firm in the previous period. If the worker did not produce output for a firm in the previous period, the worker is unattached or equivalently unemployed and actively looking for a job. A firm without an attached worker is a vacancy that is also actively looking to recruit a worker. Unemployed workers receive flow payoff $b\,dt$ per period. Vacant jobs incur the flow cost $c\,dt$. When a worker agrees to produce for a firm, the worker generates output $x\,dt > b\,dt$. To keep the exposition and notation uncluttered, workers and jobs live forever.[8]

2.1 Sub-period activity

In the first sub-period (the internal labor market stage), an attached firm offers its worker a wage $w\,dt$ in the current period. The worker then either accepts or rejects this wage. A worker who accepts the offer receives the wage payment, generates the per period output and remains attached to the firm as they both move on to the next time period.

New firms and new workers then enter the economy at the start of the second sub-period. As time progresses, new workers individually enter the economy (between the first and second sub-period) at the constant, exogenous Poisson rate $\alpha > 0$. For dt small, $\alpha\,dt$ is the approximate probability that a new worker enters in period t. Likewise, new firms each with a single,

[8]Job destruction shocks can be incorporated (death and discounting are related) but some caution would be needed. A familiar approach specifies that workers become unemployed whereas firms leave the market following a job destruction shock. Given equal and exogenous arrival rates, this specification would lead the number of workers growing unboundedly higher than the number of firms. Endogenous firm entry would, of course, remedy this difficulty. This paper abstracts from worker-firm separations and from an endogenous number of vacancy/firms as in Pissarides 2000.

indivisible job or vacancy enter in the same manner and at the same rate but independently of workers. Over time the population in the economy is therefore balanced with equal expected numbers but at any given point in time there may be either more workers or more firms.

These new entrants enter the job search stage and visit the labour market looking for partners. For simplicity, unemployed workers pay no search costs to visit the market. A worker who rejected the internal wage offer likewise enters the labour market in this second stage of the time period. The rejected firm goes to the market as well.

During this stage, an attached worker pays a search cost $\xi > 0$ to participate and solicit alternative wage offers in the labor market. By rejecting an offer, the attached worker in effect chooses to enter the job market and check the posted list of vacancies, if any. The worker is said to be searching on-the-job (as detailed below) as the attached firm remains a feasible employment option. Moreover, because an attached worker who accepts a first stage offer does not visit the labor market, the worker is unable to search on-the-job without the firm becoming aware of this activity. In effect, if a worker in any period would like to try their luck in the labour market, the paired firm becomes aware of this activity before per period production takes place. The firm fully observes or perfectly monitors the search activity of an attached worker and can modify its first stage pay offer in the second stage.

2.2 Stock-flow matching

Following the stock-flow matching approach (see Smith 2020 for an overview) information about the availability of firms and workers in the job search stage is centralized. Unemployed/unattached workers including any entrants as well as offer-rejecting workers all register their availability at a job centre, on a website, or on some other established platform as soon as they enter the market looking for partners. Vacant and new entrant jobs along with any rejected firms similarly post the availability of their employment opportunities. The firm maintains their listing until the job attaches or hires a worker.

Agents in this centralized marketplace are perfectly informed about all available trading opportunities that have registered there. When any worker enters the marketplace, he or she immediately observes the number of vacancies in the market as well as any other workers in the job centre. After the worker checks the list of posted vacancies, there are no frictions or delays in processing the information. All information regarding the viability of a position is immediately made clear and common knowledge at the job centre.

Given the number workers and jobs in the marketplace, a complete information, competitive auction occurs. Each worker observes a proposed wage offer from each firm, including all of the just rejected firms who can update their wage offers. Firms post their wage offer knowing the number of workers and competing firms in the market in the second stage of the current period. Although all decision making is based on expectations of future behaviour, wage offers are for only the current period. Firms cannot commit to future wage payments in their offers. Workers likewise cannot commit to withholding future search for other employers. For entrant workers, the acceptance of a wage offer thus corresponds to their initial wages.

In this auction, the process of pairing workers and jobs occurs within the second stage of the period. One by one, an arbitrarily chosen worker either selects and accepts one of the wage offers or passes on all offers. All of the other agents in the market observe if a worker and firm pair together. This matched pair then both leave to immediately produce output and transfer the agreed payment. The matching process continues until all workers have had a chance to pair up. Unsatisfied (i.e. unmatched) workers and firms remain behind as unemployed workers and vacant firms who both wait for further trading opportunities. As such, there are no impediments to trade such as coordination frictions in the second stage after entry from either new born entrants, unattached agents, or attached agents.

2.3 Beliefs

When an attached firm makes its internal market stage offer and when the worker subsequently decides to accept or reject the attached firm's offer, they are both unaware of the entry of any workers and of jobs since they last visited the market place. In particular, since the date the two first became attached or since the last time the worker searched on the job, whichever is shortest, the Poisson arrival processes govern their beliefs about what other agents they expect to encounter in the job center. Thus, the attached firm and worker beliefs in the stages before on the job search are based on the workers and firms that were there at the last visit and the duration since its last visit.

Since workers enter at Poisson rate α, workers and firms share the belief that the probability of i new entrants over a duration τ since the last auction is given by

$$\frac{e^{-\alpha\tau}(\alpha\tau)^i}{i!}$$

A symmetric belief applies to the number of new jobs that entered. Visiting the market reveals all past entry information completely and the worker as well as the firm update their beliefs accordingly.

3 Decisions within periods

Focusing on symmetric, pure Markov strategies, this section describes subperiod behaviour and outcomes at a point in time.

3.1 Second stage job search

Suppose there are $B > 0$ firms bidding for $S > 0$ workers in the open labour market auction. Since agents are homogeneous, the outcome is well understood. The optimal strategy for a worker on the short side contemplating a bid involves a reservation wage strategy - the worker will accept the highest offer (or will randomly select among the highest offers) provided that the offer yields a discounted expected payoff greater than or equal to the continuation value of waiting as an unemployed, unattached worker in the job market. Similarly, when there are more workers than firms in the auction, a firm will likewise have a threshold payoff to having a worker accept its offer. This payoff must be less than or equal to the value of being a vacancy. These continuation payoffs depend on the expected number of traders in future periods.

Given these strategies, the short side of the market determines the outcome of the auction in a second stage of activity. If $B > S$, firms bid up to their threshold bids thereby making them indifferent between hiring and having an open position at the start of next period. Provided there are gains to trade, these bids exceed the worker's reservation wage and workers are willing to accept these bids. All workers are hired leaving $B - S$ vacancies indifferent between waiting and hiring. On the other hand, if $S > B$, firms offer the worker's reservation wage. The worker accepts leaving $S - B$ unemployed.

Immediate trade emerges so that unattached workers and unattached workers do not coexist entering the next period. Moreover, the outcome is the familiar Bertrand result leaving the long side indifferent between being attached or unattached. Basic induction demonstrates that this indifference generates payoff equivalence to adding a buyer-seller pair. The following proposition summarizes these observations.

Proposition 1 *Given $B > 0$ jobs and $S > 0$ workers in the second stage auction,*

i. *immediate trade occurs –* $\min\{B, S\}$ *worker-firm pairs form leaving* $\max\{B-S, 0\}$ *unsatisfied buyers and* $\max\{S-B, 0\}$ *unsatisfied sellers.*

ii. *payoff equivalence occurs – a $B > 2$ and $S > 2$ auction yields the same payoffs to an auction with $B - 1$ buyers and $S - 1$ sellers. By induction these payoffs are equivalent to an auction with either one buyer ($S \geq B = 1$) or one seller ($S = 1 < B$) that generates the long-side payoff to waiting in the job center with* $\max\{S - B, 0\}$ *sellers and* $\max\{B - S, 0\}$ *buyers respectively.*

iii. *if $S \geq B$, firms offer the workers' their reservation wage which workers accept (with indifference)*

iv. *if $S < B$, firms competitively bid up the wage offer (which workers accept) until they are indifferent between hiring and continued waiting with an open vacancy in the next period second stage*

Proof. Results follow from the exposition in the text. Proofs of subsequent propositions are in the Appendix. ■

It is important to note, however, that the payoffs to going forward are contingent on net agent entry, in this case the difference between B and S, not the actual levels of buyer and seller entry.[9]

3.2 First stage internal labour market

In the first stage, an attached worker again adopts a reservation wage strategy for accepting the associated firm's internal offer and forgoing on-the-job search. Given this reservation wage in the first stage, the attached firm's

[9] Coles and Muthoo (1998) demonstrate that in the stock-flow framework without on-the-job search there is a unique Markov equilibrium in which exchange occurs immediately.

choice is effectively whether to offer this first stage reservation wage. Anything above this threshold lowers profit and anything below results in moving to a second stage auction. As the worker rejects any offer below its no-search threshold wage, any offer below this reservation wage triggers a second stage visit to the market.

4 Payoffs in the Job Center

Proposition 1 describes (without completely characterizing) the outcome of the second sub-period labour market auction. This section completes the characterization of the job center outcomes by deriving the lifetime discounted expected payoffs for workers and firms as they enter (or if they already exist unattached in) the market.

From Proposition 1, immediate trade occurs in the second sub-period so that unattached jobs and unemployed workers do not coexist over time. From point [i.] only unattached workers or only unattached firms carry over into the next period. Moreover, point [ii.] implies that the on-the-job search decisions of other attached worker-firm pairs do not affect the payoffs in the second sub-period. Adding another worker along with another firm is payoff equivalent.

Given payoff equivalence, the payoff relevant active agents in the second sub-period are those who entered in previous periods but are without an attached partner along with any new entrants this period. Moreover, payoffs can be derived from auctions with only one lone trader matching with one or more potential partners on the other side of the market. Therefore, let

$$N_t = Stock\ of\ workers\ who\ entered - Stock\ of\ vacancies\ that\ entered$$

denote the history of net agent entry up through date t. N_t represents <u>either</u>

- the number of traders in competition on the same side of the market

<u>or</u>

- the number of potential partners on the other side.

Bids and accepted offers in both sub-periods will depend on this history of market entry. If $N_t \in \mathbb{N}^+ = \{1, 2, 3, ...\}$, there is at least one unemployed worker on the long side of the market waiting for a firm to enter the market with a vacancy. Moreover, immediate trade in the second period implies there are exactly $\max\{0, N_t\}$ workers and $-\min\{0, N_t\}$ vacant jobs waiting in the marketplace carried over across time periods.

When a new firm enters the market at date t, history changes so that $N_t = N_{t-dt} - 1$. If at date $t - dt$ there are at least two available workers waiting, then at date t when the second stage auction bidding takes place, the entering vacancy has at least $N_{t-dt} = N_t + 1$ potential workers. Thus, for this case, accounting for the change in history of firm entry, we have $N_t \in \mathbb{N}^+ = \{1, 2, 3, ...\}$.

Since immediate trade occurs in the second sub-period, two relevant cases arise for a new worker entering in the job center:

- Long-side Case: No viable jobs are waiting in which case the worker becomes unemployed after entry.
- Short-side Case: The market has one or more excess firms available, in which case the firm or firms compete against each other (if more than one firm) for the new worker resulting in immediate employment.

Similar cases apply when a firm with a new open vacancy enters the job center.

4.1 The Long-side of the Market

To derive the reservation payoffs in the second sub-period job search stage, let $V(N_t)$ denote the expected payoff to a worker waiting on the long side of the market who has $N_t - 1$ other workers competing for employment. Since there are no jobs currently available as this worker waits for jobs to appear, standard dynamic techniques imply that

$$V(N_t) = b\,dt + \frac{1}{1 + rdt}[\alpha dt\, V(N_t + 1) + \alpha dt\, V(N_t - 1) + (1 - 2\alpha)\, V(N_t)] + O(dt^2) \qquad N_t \in N^+ \quad (1)$$

The worker receives net flow payments b while waiting. During a short interval of duration dt, a competing worker arrives with probability $\alpha\, dt$ and increases the number of available workers by one. With the same probability a firm arrives during this interval and the workers all pursue this job. Bertrand-like competition makes the worker indifferent between employment and waiting with one less competitor. If there are no other workers when a firm arrives, the worker receives the boundary payoff $V(0)$ as one firm bids for one worker.

To find the respective employer shares with two or more firms bidding, now consider the case in which firms with a vacancy are waiting on the long side of the market for a worker to enter, that is $N_t \in \mathbb{N}^- = \{-1, -2, -3, ...\}$. Applying the same logic, the payoff for these firms while they wait is given by

$$\begin{aligned}\Pi(N_t) = -c\,dt + \frac{1}{1+rdt}[\alpha dt\,\Pi(N_t+1) + \alpha dt\,\Pi(N_t-1) \\ + (1-2\alpha)\,\Pi(N_t)] + O(dt^2) \quad N_t \in N^- \quad (2)\end{aligned}$$

where $\Pi(0)$ is the associated boundary condition with one firm bidding for one worker.

Proposition 2 *The discounted expected payoffs for unattached workers and unattached firms are given by*

$$V(N_t) = \lambda^{N_t}(\bar{V} - b/r) + \frac{b}{r} \qquad N_t \in \mathbb{N}^+$$

and

$$\Pi(N_t) = \lambda^{-N_t}(\bar{\Pi} + c/r) - \frac{c}{r} \qquad N_t \in \mathbb{N}^-$$

where

$$\lambda = \frac{r + 2\alpha - (r^2 + 4r\alpha)^{1/2}}{2\alpha}$$

The boundary conditions are

$$V(0) = \frac{\alpha(1-\lambda)(x+b+c) + rb}{r(r + 2\alpha(1-\lambda))}$$

and

$$\Pi(0) = x/r - V(0)$$

Proof. See Appendix. ■

4.2 The Short-side of the Market

If an attached worker-firm pair visit the market (that is, if on-the-job search occurs at some point in the relationship), the pair can re-attach with a renegotiated wage reflecting the newly updated circumstances revealed at the job center. Re-attachment is not only feasible, it is also as good as any other available attachment. Re-attachment can thus constantly recur making the match indefinite. The joint match payoff for an infinitely-lived pairing equals total discounted production x/r. There is no job creation margin so the wage simply divides match rents.[10]

Proposition 3 *The firm's share in auctions with more than one worker is given by*

$$\Pi(N_t) = \frac{x}{r} - V(N_t) \qquad N_t \in \mathbb{N}^+$$

Workers payoff from multiple competing firms in an auction is given by

$$V(N_t) = \frac{x}{r} - \Pi(N_t) \qquad N_t \in \mathbb{N}^-$$

Proof. Follows from the splitting of the match payoff x/r and the long side payoffs in Proposition 2. ■

5 Attached Workers and Firms

Once a worker and firm join together in employment, the joint payoff to a match is x/r. The firm's wage payments over time (offered and accepted during the first sub-period) determine the allocation of this joint payoff as they share the value of the match. A wide variety of compensation schemes - that is promised, committed payments over time paid in the first sub-period coupled with a commitment not to search - can deliver these shares so wages are indeterminate when commitment is feasible. This section establishes that the constant threat of on-the-job search during the match along with the inability to commit to future wage payments pins down wages in the internal labour market of the first sub-period.

[10]The critical point is that the match payoff is constant over time. It is readily seen that Proposition 5 holds for any constant match payoff. An alternative approach for establishing the results is to impose no on-the-job search and then demonstrate that this choice is optimal.

In the first stage of each period, an attached firm simply chooses between offering their attached worker's (first sub-period) reservation wage at the time or inducing on-the-job search with a lower offer. If on-the-job search occurs, the outcome is common knowledge - both the worker and the firm become informed about the number of available employment opportunities for the worker in the job centre.[11] Given the number of viable opportunities found in the job centre, a new auction results and the new or re-negotiated wage depends on the number of searching workers and vacancies in the job centre at that moment of on-the-job search.

5.1 On-the-Job Search

An attached pair does not observe entry or turnover in the job center until they visit the job center. Expectations of finding potential partners and the payoff to on-the-job search therefore depend on whom they last saw there (the known participants from the auction that last attached the worker to the firm), and the duration (determining the expected flows in and out of participants) since that auction.

More specifically, at date t, the expected payoff to on-the-job search for an attached worker who

- last visited the job centre a duration $\tau > 0$ ago
- was hired at the job center when the history of net entry at that time was $N_{t-\tau}$

is given by

$$W(t; N_{t-\tau}, \tau) = -\xi + \sum_{k=-\infty}^{\infty} V(k) f(k; N_{t-\tau}, \tau)$$

where $f(k; N_{t-\tau}, \tau)$ is the probability of observing history k given a duration τ since initial history $N_{t-\tau}$. Let $F(k; N_{t-\tau}, \tau)$ denote the cumulative distribution for f.

The Poisson arrival processes govern turnover (unobserved during attachment duration τ) in the job center. The (Poisson) number of unobserved

[11] Common knowledge rules out the possibility that a worker visits the job centre and calls for an auction only if conditions are favorable. As demonstrated below the firm can infer worker behavior from its wage offer.

workers less the (Poisson) number of unobserved vacancies follows a Skellam distribution. (See Irwin 1937, Skellam 1946.)[12] For $N_{t-\tau} = 0$, the probability of observing a history of new net entry of k after duration τ at date t is given by

$$\Pr(N_{t+\tau} = k \mid N_t = 0) = f(k; 0, \tau) = e^{-2\alpha t} \sum_{j=\max\{0,-k\}}^{\infty} \frac{(\alpha t)^{2j+k}}{j!(j+k)!}$$

The law of motion for f

$$\begin{aligned} f(N_t; N_{t-\tau}, \tau + d\tau) &= (1 - 2\alpha d\tau) f(N_t; N_{t-\tau}, \tau) \\ &+ \alpha d\tau \, f(N_t + 1; N_{t-\tau}, \tau) - \alpha d\tau \, f(N_t - 1; N_{t-\tau}, \tau) \end{aligned}$$

implies

$$\dot{f}(N_t; N_{t-\tau}, \tau + d\tau) = \alpha f(N_t + 1; N_{t-\tau}, \tau) - 2\alpha f(N_t; N_{t-\tau}, \tau) + \alpha f(N_t - 1; N_{t-\tau}, \tau)$$

Proposition 4 *The worker payoff to search while employed is*

$$\begin{aligned} W(t; N_{t-\tau}, \tau) &= -(\Pi(0) + c/r) \sum_{i=1}^{\infty} \lambda^i f(-i; N_{t-\tau}, \tau) \\ &+ (V(0) - b/r) \sum_{j=0}^{\infty} \lambda^j f(j; N_{t-\tau}, \tau) \\ &+ F(-1; N_{t-\tau}, \tau)(x - b + c)/r + b/r - \xi \end{aligned}$$

which (after dropping the history and duration notation in f) evolves according to

$$\dot{W}(t; N_{t-\tau}, \tau) = \frac{\alpha(1-\lambda)^2}{\lambda} \left[-(\Pi(0) + c/r) \sum_{i=1}^{\infty} \lambda^i f(-i) + (V(0) - b/r) \sum_{j=0}^{\infty} \lambda^j f(j) \right]$$

Proof. See Appendix. ■

[12] This probability can also be expressed for any k using a modified Bessel function of the first kind

$$\Pr(N_{t+\tau} = k \mid N_t = 0) = f(k; 0, \tau) = e^{-2\alpha\tau} I_k(2\alpha\tau)$$

where $I_k(z) = I_{|k|}(z)$. Alternatively,

$$\Pr(N_{t+\tau} = k, k > 0 \mid N_t = 0) = f(k; 0, \tau) = e^{-2\alpha t} \sum_{j=k}^{\infty} \frac{(\alpha t)^{2j-k}}{j!(j-k)!}$$

with negative values for k found by the symmetry of Skellam distribution.

5.2 Accepting an Internal Offer

The worker's payoff from not going to the market at any point in time t depends on the first sub-period internal wage offer at the time. Suppose a firm offers the instantaneous wage $\hat{w}(t; N_{t-\tau}, \tau)\, dt$ (for the current interval of duration dt) to its worker from the last period. This worker has a duration $\tau > 0$ of continuous employment without an intervening visit to the job market since attachment began at date $t - \tau$, at which time it had a history of net entry $N_{t-\tau}$.

Let $E(t; N_{t-\tau}, \tau)$ denote the expected payoff at date t to employment in the first sub-period given the relevant history and duration. If the worker accepts the current internal wage offer and decides not to search at this point in time, it follows that the worker's expected payoff is this period's wage offer plus the discounted payoff of either search next period or continued attachment next period, whichever is larger:

$$\begin{aligned} E(t; N_{t-\tau}, \tau) &= \hat{w}(N_{t-\tau}, \tau)dt \\ &+ \frac{1}{1 + rdt} \max\{E(t; N_{t-\tau}, \tau + dt), W(t; N_{t-\tau}, \tau + dt)\} + O(dt^2). \end{aligned}$$

Manipulating and letting dt become small gives

$$rE(t; N_{t-\tau}, \tau) = \hat{w}(t; N_{t-\tau}, \tau) + \max\{\dot{E}(t; N_{t-\tau}, \tau), \dot{W}(t; N_{t-\tau}, \tau). \qquad (3)$$

A firm can clearly offer a sufficiently high wage such that $E(t; N_{t-\tau}, \tau) \geq W(t; N_{t-\tau}, \tau)$. In this case, because search activity is observable, the firm effectively bribes the worker to not to visit the job centre after duration τ. Moreover, if the firm chooses to offer such a no-search wage, the firm would optimally offer the lowest possible wage so that the no search condition holds with equality

$$E(t; N_{t-\tau}, \tau) = W(t; N_{t-\tau}, \tau).$$

Proposition 5 *In the first sub-period, an attached firm offers its worker the worker's reservation wage $w(t; N_{t-\tau}, \tau)$ for all t which the worker always accepts.*

Proof. See Appendix. ■

Given there is a positive cost of visiting the job centre, it is efficient for the worker and the firm to save the search cost and split the match payoff within the current employment match at any given τ. In this economy, on-the-job search is a wasteful, rent seeking activity. Once a match is formed, search does not generate any further gains to trade (such as finding a better match) or match specific rents. The relationship does not fundamentally change when the worker visits the job centre. There are no new opportunities generated by a visit - existing opportunities are merely realized. Search does not change the expected gains to trade at any given point in time, it just reallocates the division of these benefits. Since workers and firms share the same risk neutral, intratemporal preferences, and since all firms are identical, there is no potential role for meaningful on-the-job search.

6 Per Period Wages

Let $w(t; N_{t-\tau}, \tau)$ denote the lowest wage that makes an attached worker willing to forgo visiting the job centre. As time progresses, Proposition 5 reveals that subsequent first sub-period offers will continuously make the worker indifferent between accepting the internal first sub-period wage offer and on-the-job search. The worker accepts the internal offer and remains attached to the firm so that

$$rE(t; N_{t-\tau}, \tau) = w(t; N_{t-\tau}, \tau) + \dot{E}(t, N_{t-\tau}, \tau)$$

This process leads to the following characterization of wages that firms pay their workers after duration $\tau > 0$ given that market history $N_{t-\tau}$ at the date of hiring.

Proposition 6 *For $\tau > 0$, equilibrium wages are given by*

$$w(t; N_{t-\tau}, \tau) \quad = \quad b - r\xi + F(-1; N_{t-\tau}, \tau)(x - b + c)$$

Proof. See Appendix. ∎

Note that at the time of matching, the distribution $f(k; N_t, \tau = 0)$ is degenerate at the realized value of N_t so that $F(-1; N_t, \tau = 0)$ is either zero or one, depending on whether or not more than one firm is bidding. In addition, when there is a lone firm bidding (that is, for $N_t \geq 0$), the switch (at implicit duration $\tau = 0$) from costless job search as an unattached worker to costly potential on-the-job search requires an additional bonus compensation in the pay offer at the job center. As noted, when such bidding in the second sub-period job market takes place, workers will accept offers where they are indifferent between working and waiting, implying that the payoff at the start of employment equals the payoff to waiting. To satisfy indifference and equate visiting the market in the next instant for an attached worker and for an unattached worker requires a compensating signing on bonus equal to the search cost ξ.

In contrast, when unattached workers have more than one suitor ($N_t < 0$), the firms, not workers, become indifferent between hiring and continued search. The worker strictly prefers employment from Bertrand bidding over continued search which eliminates the need to compensate for the potential on-the-job search cost. Competing offers bid up wages so that such workers are initially paid their marginal product (plus the firm's search costs less the flow value of workers search costs) when there is more than one bidder.[13]

Proposition 7 *Initial wages depend on the short side of the market:*

$$w(t; N_t < 0, \tau = 0) = \lim_{\tau \to 0} w(t; N_t < 0, \tau) = x + c - r\xi$$

$$w(t; N_t \geq 0, \tau = 0) = \xi + \lim_{\tau \to 0} w(t; N_t \geq 0, \tau) = b + (1 - r)\xi$$

Proof. See Appendix. ■

From Proposition 6, wages at date t, are positively related to the probability $F(-1; N_{t-\tau}, \tau)$ that the current employer will find competition for the worker's services. The worker is prepared to accept a lower wage and avoid re-negotiating the terms of employment when there is a higher probability that the employer can become a monopsonist. This probability of competition evolves over time with the duration of employment, rising or falling depending on the state when attachment first occurred.

After the first payment, wages can increase or decrease depending on the initial market conditions. The symmetry of the Skellam distribution (given equal arrival rates), implies that wages for $N_{t-\tau} < 0$ are decreasing over time ($\dot{w} < 0$) and are increasing for $N_{t-\tau} \geq 0$. If $N_{t-\tau} \geq 0$, the firm was initially in a monopsonistic position on the short side of the market. As turnover

[13]The firms search costs are an artifact of not dropping out of the market. If the firm received flow benefits rather than costs, the firm would not pay above marginal costs by the same logic that workers receive b.

occurs in the job center, the probability that the current employer remains a monopsonist decreases over time. The outside option of the worker therefore improves and the firm increases its wage offer to avoid the worker visiting the job centre. On the other hand, starting with multiple initial bidders, i.e for $N_{t-\tau} < 0$, the same turnover increases the likelihood over time that the current employer could become a monopsonist. The worker's outside option and hence $w(t; N_{t-\tau}, \tau)$ decreases with τ, as the threat of any potential loss brought about by an induced visit to the job centre increases over time.

The following proposition formalizes this reasoning.

Proposition 8 *Wage progression satisfies*

$$\dot{w}(t; N_{t-\tau}, \tau) \quad = \quad \alpha[f(0; N_{t-\tau}, \tau) - f(-1; N_{t-\tau}, \tau)](x - b + c)$$

Proof. See Appendix. ■

Although wages start and evolve very differently from different states N_t, all wages limit to the same value as the employment spell becomes long:

$$\lim_{t \to \infty} w(t; N_{t-\tau}, \tau) = (x + b + c)/2 - r\xi \tag{4}$$

The Skellam distribution flattens out over time with variance $2\alpha\tau$. In the limit as $\tau \to \infty$, all wages converge as the history of initial conditions recedes. The Skellam process governing vacancy turnover implies that the distribution of N_t converges to a distribution with a mean zero and variance equal to ∞ as $t \to \infty$. The history of the initial state fades (including $N_t = 0$) over time so that eventually all workers face the same prospects in the job centre. Since the effect of N_t is only transitory, wages converge to a unique wage.

7 Numerical example

Initial wages take on two possible values tied to worker search costs ξ and either to (i) unemployment benefits b or (ii) productivity x less firm search costs c. The evolving likelihood of being on the short side of the market then governs wage progression as described by $w(t; N_{t-\tau}, \tau)$ for duration τ and initial hiring history $N_{t-\tau}$. The principle parameter controlling the rate of churning and hence the likelihood of being on the short side in the market is α.

This Poisson arrival rate of agents on both sides of the market determines the evolution of the Skellam distribution at the core of the wage determination process described in Proposition 6. The baseline specification used below is that the probability of an arrival of at least one worker in a given month equals one half so that $\alpha = 0.6931$.

7.1 Wages and turnover

To gauge the impact of the arrival rate parameter α, Figure 1 plots wage progressions for two values of α and various initial conditions for $N_{t-\tau}$. The high α used for the top panel in Figure 1 is three times the baseline case; the low value used in the bottom panel of Figure 1 is one third the baseline value. The remaining five parameters (x, b, c, ξ, r) given in Table 7.1 are standard values.

Table 1. Parameter Values

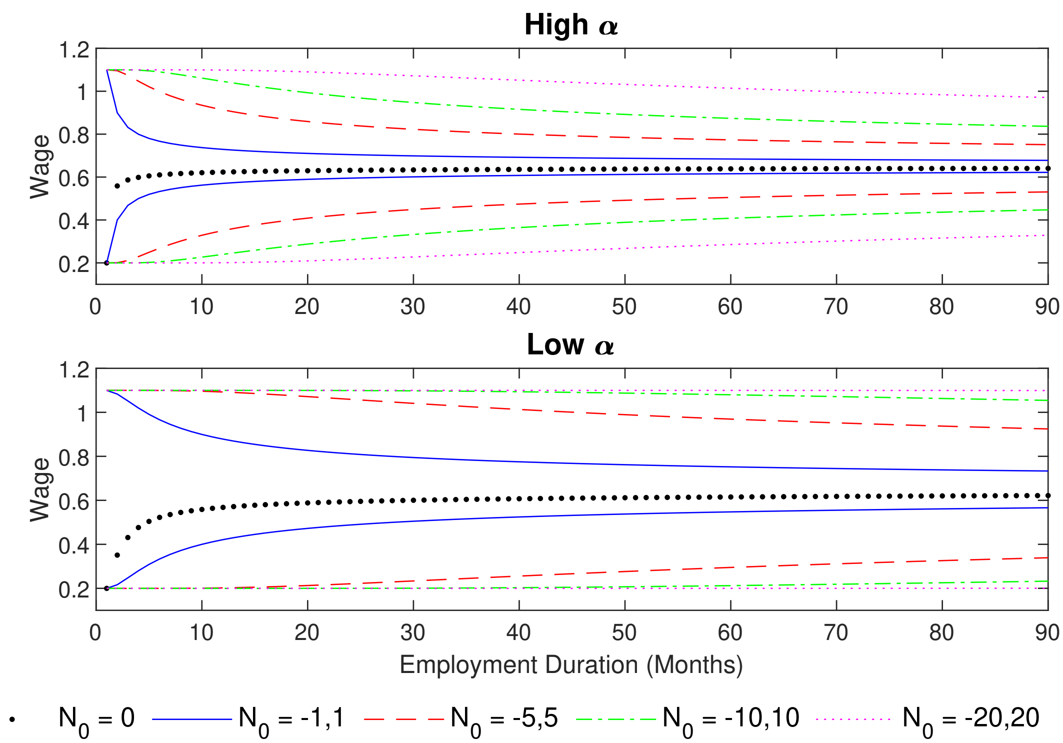
Parameter	Value
x	1
b	.20
c	.10
ξ	0.05
r	0.0042

A comparison of the two panels in Figure 1 reveals that wage persistence increases as turnover in the marketplace falls, that is, as the economy moves further away from a competitive setting. The high α turnover plot in the top panel of Figure 1 converges faster than the low α plot in Figure 1. However, persistence lasts even for the high value α. For $N_{t-\tau} = -1$, after ten years the wage remains more than 11% above the long run wage whereas the wage for an $N_t = 0$ is 4% below the long run wage. The impact of the initial conditions fades over time but the convergence is most pronounced early on in the employment spell. Nonetheless, wage changes are predictable, both positive and negative, serially correlated and persistent, which all conform with the evidence noted above.

7.2 Wage dispersion

In the model, workers and firms inhabit a small, particular market to highlight the essential mechanics of wage determination with on-the-job search.

Figure 1: Wages by Duration



An observed statistical market will typically contain numerous such entities. As observed statistics are broader than the model, the approach adopted here replicates and aggregates the model across a number of small markets to mimic familiar statistics. The objective is not to generate accurate quantitative predictions nor to precisely measure the model. The model abstracts from prominent features that are likely to affect entry as well as pay and thereby alter the alignment of the model with data. This evaluation should be viewed as a demonstration that under some fairly generic parameterizations, the model can qualitatively deliver compensation patterns within specific jobs that are broadly consistent with established empirical regularities.

Simulated entry of workers and firms occurs over 120 periods or ten years. Repeating the exercise over 100 markets all with the same initial $N_0 = 0$ yields a panel of wages for employed workers as well as information on unemployment and vacancies. Cross section wages from the last period are computed using all of the employer-employee pairs that formed during the ten year long period. These wages are conditional on the initial N_t at hiring and the subsequent duration of employment.

Figure 2 presents the simulated distribution of wages at the end of the ten year period. The ratio of the mean wage to the minimum wage is 2.99.[14] Wage dispersion exhibits a rough symmetry with three local peaks aligning with the two potential initial wages and the long run limiting value of wages. The three local modes emerge primarily as an artifact of the Poisson entry rates. In the model, there is no ergodic steady state. The variance of the Skellam distribution linearly increases with duration τ. As time progresses the likelihood of long queues of either workers or firms increases which anchors some wages at more extreme values of the long side of the market as measured by N_t.

It is plausible that at some point workers and firms will find alternatives to markets with extreme values for N_t.[15] Although the associated value matching and smooth pasting tools associated with potential bounds are understood, the impact on the evolution of the Skellam distribution of agents F is less so. To offer a simple picture of the potential impact, Figure 3 presents the same wage distribution but for workers who matched with $-10 <$

[14]The associated U-V ratio equals 1.30 and the unemployment rate is just below eight percent at 7.99%.

[15]With negative flow search costs c, entering firms have negative expected payoffs and prefer to not enter.

Figure 2: Wage Dispersion after 10 Years

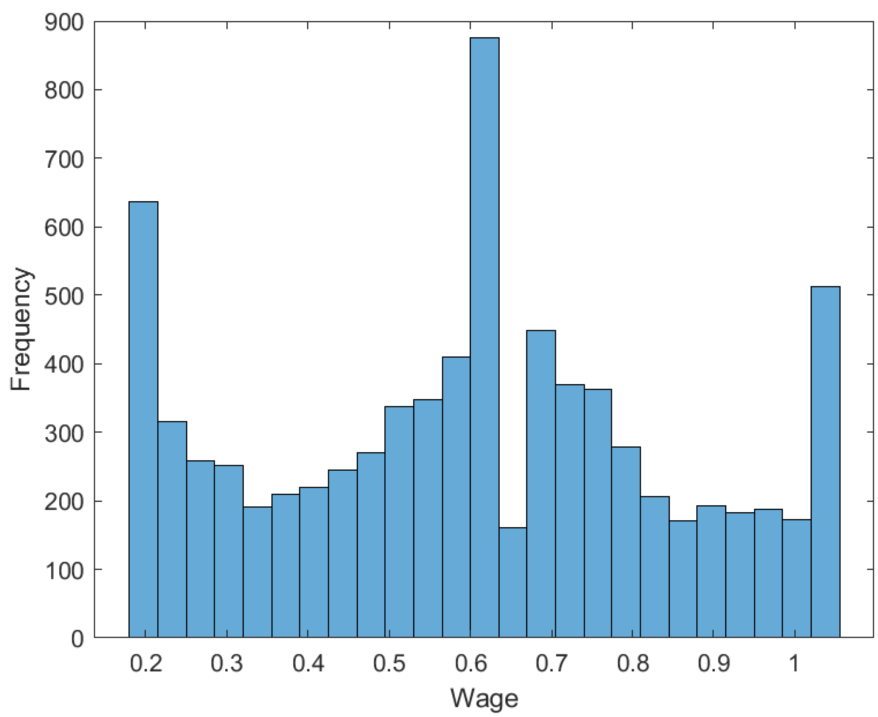
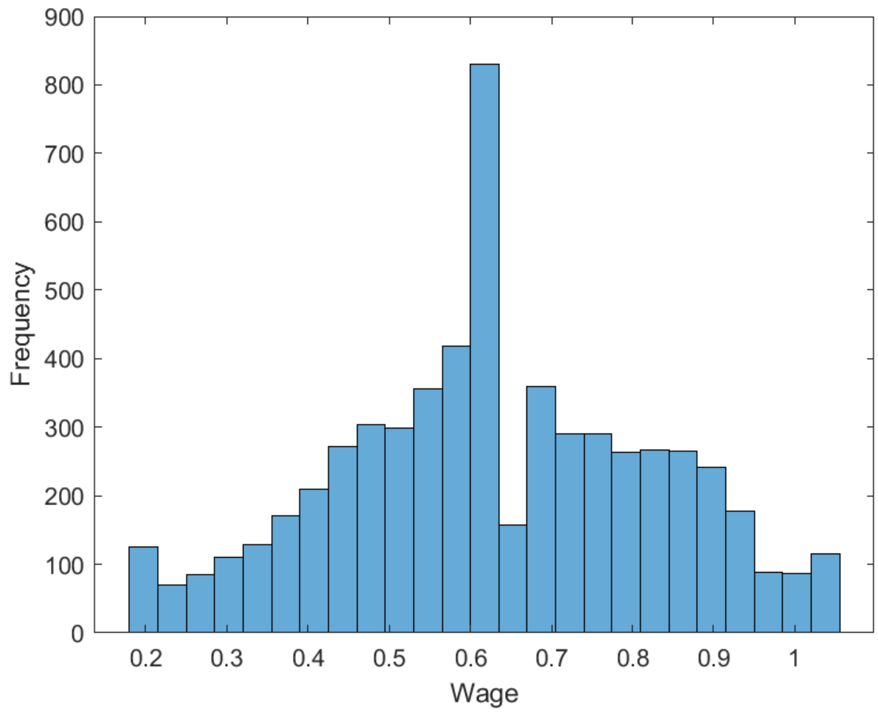


Figure 3: Wage Dispersion after 10 Years with Bounded N_t



$N_t < 10$. The abridged distribution remains roughly symmetric but becomes approximately unimodal around the limited, long run wage.

7.3 Job Tenure

Regressing the cross sectional wage in the last period on tenure and initial hiring conditionals N_t yield the tenure effects. In particular, regressing logged wages on logged tenure and (unlogged) N_t at the time of hiring yields

$$\ln(w) = -0.906\,[0.046] + 0.070\,[0.002] * ln(Tenure) - 0.055\,[0.001] * N \qquad R^2 = 0.870$$

with standard errors in square brackets. Tenure on average raises wages.

Note that in this regression the measure for initial condition N_t at hiring is very exact and precise for the individual. Although the empirical literature documents that initial conditions matter and are persistent, the measures of job competition used are far more general than in the above regression. Local unemployment rates for example are broad measures whereas N_t is very particular to the individuals circumstances. Finding the appropriate benchmark in the model is unclear but rerunning the regression without any such measure does not substantially alter the tenure coefficient estimate or its significance.

8 Conclusion

Search models provide an elegant and powerful framework for understanding why and how wages increase with job tenure. Firms that face a moral hazard problem of workers who are unable to commit to not pursuing attractive outside job offers have an incentive to backload wages in order to retain their workforce. Burdett and Coles (2003) show that with full firm commitment (and with risk-averse workers), backloading generates smoothly increasing wages. Postel-Vinay and Robin (2002) assume that firms do not commit to future wages but instead reset the piece rate they pay a worker each time the worker receives an attractive outside offer, implying that the worker's piece rate also increases stochastically and in discrete steps with tenure in response to competitors' attempts to poach the worker.

This paper likewise assesses wage setting when firms cannot commit to future wages and when workers cannot commit to not search while employed. The stock-flow job search frictions in which (i) workers and firms on occasion meet multilaterally and (ii) workers can recall previous encounters with firms generates a progression of wages with firms paying just enough to keep their workers, as in the familiar competitive labour market. The emerging compensation structure is consistent with well established but difficult to reconcile observations on pay dynamics within jobs at firms. In particular, this framework delivers

- wage dispersion and wage growth dispersion including wage cuts
- persistence - serial correlation in wages that creates predictable winners and losers
- initial conditions matter - which can be broadly viewed as cohort effects

These results here contribute an alternative perspective to the debate on the impact of job tenure on wage growth that allows both wage increases or decreases over time. Some papers find large and significant tenure effects, while others estimate small or insignificant effects (see Abraham and Farber 1987; Altonji and Shakotko (1987); Topel 1991; Dustmann and Meghir 2005; Beffy et al. 2006; Buchinsky et al. 2010). The mixed empirical evidence may tie in with the initial conditions.

In the formulation of this paper, homogeneous workers search for identical jobs. On-the-job search does not lead to better matches. Because such search is costly, it is inefficient but workers are tempted to use it to increase their wages. In equilibrium, firms and workers do not engage in this wasteful rent-seeking behaviour and agree to the efficient outcome. Nonetheless, the ever evolving threat of on-the-job search coupled with the absence of multi-period commitments not only overcomes the well known Diamond paradox but also empirically relevant wage profiles over time.

The avoidance of the option of on-the-job search aligns with the observation that only a small fraction (less than 5 percent) of employed workers are actively searching. Fallick and Fleischman (2004), Nagypál (2005), Nagypál (2008). Jolivet et al. (2006) find that "relative to involuntary mobility (reallocation shocks and lay-offs), voluntary mobility is a rather rare event" in many European countries and in the US. Workers may not search when they are not at risk of job loss but the threat to do so disciplines wages in a way that is consistent with a variety of challenging wage setting observations.

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APPENDIX

Proof of Proposition 2

Manipulating terms in (1) and letting $dt \to 0$ gives

$$\alpha V(N_t + 1) - (r + 2\alpha)V(N_t) + \alpha V(N_t - 1) = -b$$

Following the same approach for (2) likewise produces

$$\alpha \Pi(N_t - 1) - (r + 2\alpha)\Pi(N_t) + \alpha \Pi(N_t + 1) = c$$

The characteristic equations for the homogeneous version of the two second order, linear difference equations have the same two distinct roots. $\lambda < 1$ is the stable root for both. Adding the particular solutions gives general solutions.

To find the boundary conditions, note that

$$\Pi(0) + V(0) = x/r$$

At the boundary $N_t = 0$, the firm again makes an offer to the lone worker that makes the worker indifferent between accepting and rejecting. Now, however, the waiting payoff accounts for the possibility that a worker or a firm may arrive next period. If a firm arrives, the worker would get $x/r - \Pi(1)$.

$$V(0) = \frac{1}{1 + rdt} b\,dt + \alpha dt\,V(1) + \alpha dt\,x/r - \Pi(1) + (1 - 2\alpha)\,V(0)$$

Plugging in

$$V(1) = \lambda(V(0) - b/r) + b/r$$

as well as

$$\Pi(1) = \lambda(\Pi(0) + c/r) - c/r = \lambda(x/r - V(0) + c/r) - c/r$$

and solving give the boundary condition for $V(0)$ The boundary for $\Pi(0)$ follows from the joint payoff. ■

Proof of Proposition 4

Using $V(i) = x/r - \Pi(i)$ for $i < 0$ yields

$$W(t; N_{t-\tau}, \tau) \quad = \quad -\xi + \sum_{i=1}^{\infty}[x/r - \Pi(-i)]f(-i; N_{t-\tau}, \tau) + \sum_{j=0}^{\infty} V(j)f(j; N_{t-\tau}, \tau)$$

Plugging in from Proposition 2 produces

$$\begin{aligned} W(t; N_{t-\tau}, \tau) &= \sum_{i=1}^{\infty} [x/r - \lambda^i(\Pi(0) + c/r) + c/r] f(-i; N_{t-\tau}, \tau) \\ &+ \sum_{j=0}^{\infty} [\lambda^j (V(0) + b/r] f(j; N_{t-\tau}, \tau) - \xi \end{aligned}$$

Note that the law of motion for f yields

$$\begin{aligned} \frac{d}{dt} \sum_{i=1}^{\infty} \lambda^i f(-i) &= \sum_{i=1}^{\infty} \lambda^i [\alpha f(-i-1) - 2\alpha f(-i) + \alpha f(-i+1] \\ &= \frac{\alpha}{\lambda} \sum_{i=1}^{\infty} \lambda^i f(-i) - \frac{\alpha}{\lambda} \lambda f(-1) - 2\alpha \sum_{i=1}^{\infty} \lambda^i f(-i) \\ &+ \alpha\lambda f(0) + \alpha\lambda \sum_{i=1}^{\infty} \lambda^i f(-i) \\ &= \frac{\alpha(1-\lambda)^2}{\lambda} \sum_{i=1}^{\infty} \lambda^i f(-i) - \alpha f(-i) + \alpha\lambda f(0) \end{aligned}$$

Similar derivations yield

$$\begin{aligned} \frac{d}{dt} \sum_{j=0}^{\infty} \lambda^j f(-j) &= \sum_{j=0}^{\infty} \lambda^j [\alpha f(j+1) - 2\alpha f(j) + \alpha f(j-1] \\ &= \frac{\alpha(1-\lambda)^2}{\lambda} \sum_{j=0}^{\infty} \lambda^j f(j) - \frac{\alpha}{\lambda} f(0) + \alpha f(-1) \end{aligned}$$

and

$$\begin{aligned} \frac{d}{dt} F(-1) &= \frac{d}{dt} \sum_{i=1}^{\infty} f(-i) = \sum_{i=1}^{\infty} [\alpha f(-i-1) - 2\alpha f(-i) + \alpha f(-i+1] \\ &= \alpha[f(0) - f(-1)] \end{aligned}$$

Differentiation and substituting in the above relationships gives

$$\begin{aligned}
\dot{W}(t; N_{t-\tau}, \tau) &= -(\Pi(0) + c/r)\left[\frac{\alpha(1-\lambda)^2}{\lambda}\sum_{i=1}^{\infty}\lambda^i f(-i) - \alpha f(-1) + \alpha\lambda f(0)\right] \\
&+ (V(0) - b/r)\left[\frac{\alpha(1-\lambda)^2}{\lambda}\sum_{j=1}^{\infty}\lambda^j f(j) - \frac{\alpha}{\lambda}f(0) + \alpha f(-1)\right] \\
&+ \frac{x-b+c}{r}\alpha\left[f(0) - f(1)\right] \\
&= -(\Pi(0) + c/r)\frac{\alpha(1-\lambda)^2}{\lambda}\sum_{i=1}^{\infty}\lambda^i f(-i) \\
&+ (V(0) - b/r)\frac{\alpha(1-\lambda)^2}{\lambda}\sum_{j=1}^{\infty}\lambda^j f(j) \\
&+ \alpha f(0)\left[-\lambda(x(0) + c/r) - (V(0) - b/r)/\lambda + \frac{x-b+c}{r}\right] \\
&= -(\Pi(0) + c/r)\frac{\alpha(1-\lambda)^2}{\lambda}\sum_{i=1}^{\infty}\lambda^i f(-i) \\
&+ (V(0) - b/r)\frac{\alpha(1-\lambda)^2}{\lambda}\sum_{j=1}^{\infty}\lambda^j f(j) \\
&+ \alpha f(0)(1-\lambda)\left[(x(0) + c/r) - (V(0) - b/r)/\lambda\right]
\end{aligned}$$

Note that

$$V(0) - b/r = \frac{\alpha(1-\lambda)(x-b+c)}{r(r+2\alpha(1-\lambda))}$$

and

$$\Pi(0) + c/r = \frac{(r+\alpha(1-\lambda)(x-b+c)}{r(r+2\alpha(1-\lambda))}$$

so that

$$x(0) + c/r - (V(0) - b/r)/\lambda = \frac{(r\lambda - \alpha(1-\lambda)^2)(x-b+c)}{r\lambda(r+2\alpha(1-\lambda))}$$

It is straight forward to establish that $r\lambda = \alpha(1-\lambda)^2$. Hence

$$\begin{aligned}\dot{W}(t; N_{t-\tau}, \tau) &= -(\Pi(0) + c/r)\frac{\alpha(1-\lambda)^2}{\lambda}\sum_{i=1}^{\infty}\lambda^i f(-i) \\ &+ (V(0) - b/r)\frac{\alpha(1-\lambda)^2}{\lambda}\sum_{j=1}^{\infty}\lambda^j f(j) \qquad \blacksquare\end{aligned}$$

Proof of Proposition 5

The payoff to the firm of paying the lowest no-search wage is

$$\frac{x}{r} - E(t; N_{t-\tau}, \tau) = \frac{x}{r} - W(t; N_{t-\tau}, \tau).$$

In contrast, any wage offer below the no-search threshold $w(N_{t-\tau}, \tau)$ triggers a visit to the job center where all information is revealed. The payoff to a firm inducing the worker to search is determined after the worker pays the search cost. Hence this search payoff to the firm equals

$$\frac{x}{r} - W(t; N_{t-\tau}, \tau) - \xi$$

It is more profitable for the firm to avoid the outcome of worker on-the-job search. The argument applies for any τ hence the jointly optimal outcome is a relationship that avoids incurring search costs. $\blacksquare$

Proof of Proposition 6

As noted, accepting the no-search reservation wages implies

$$E(t; N_{t-\tau}, \tau) = W(t; N_{t-\tau}, \tau).$$

From equation (3), the accepted wage satisfies

$$w(t; N_{t-\tau}, \tau) = rW(t; N_{t-\tau}, \tau) - \dot{W}(t; N_{t-\tau}, \tau)$$

Plugging in for W and for $\dot{W}$ from Proposition 4 and collecting terms gives

$$\begin{aligned}w(t; N_{t-\tau}, \tau) &= -[\Pi(0) + c/r][r - \alpha(1-\lambda)^2/\lambda]\sum_{i=1}^{\infty}\lambda^i f(-i) \\ &+ [V(0) - b/r][r - \alpha(1-\lambda)^2/\lambda]\sum_{j=0}^{\infty}\lambda^j f(j) \\ &+ F(-1)(x - b + c) + b - r\xi\end{aligned}$$

Again, it is straight forward to establish that $r\lambda = \alpha(1-\lambda)^2$. As a result the first two terms in the above equation drop giving the stated wage equation. ■

Proof of Proposition 7

(i) Workers who initially had offers from competing vacancies have $F(-1; N_t < 0, 0) = 1$. Plugging in to the wage equation in Proposition 6 produces

$$\lim_{\tau \to 0} w(t; N_t, \tau)_{|N_t \in \mathbb{N}^-} = x + c - r\xi.$$

(ii) If $N_t \geq 0$, then $F(-1; 0, 0) = 0$. Plugging in gives

$$\lim_{\tau \to 0} w(t; N_t, \tau) = b - r\xi$$

To accept an offer in the job center implies indifference between employment and unattached search. It follows that the payoff to employment conditional on the history of net entry (bidders at the time of attachment N_t and the duration $\tau = 0$ since attachment) is given by

$$\begin{aligned} V(N_t = 0) &= B + w(t, 0, 0)dt + \frac{1}{1 + rdt} E(t; N_{t+dt}, dt) \\ &= B + w(t, 0, 0)dt + \frac{1}{1 + rdt} \left[-\xi + \sum_{k=-\infty}^{\infty} V(k) f(k; N_{t+dt}, dt) \right] \end{aligned}$$

where B is an initial bonus payment paid at the instant of employment. The limit

$$\lim_{\tau \to 0} \sum_{k=-\infty}^{\infty} V(k) f(k; N_{t+dt}, dt) = V(N_t)$$

implies $B = \xi$.

Proof of Proposition 8

Differentiation establishes

$$\dot{w}(t; N_{t-\tau}, \tau) \quad = \quad \dot{F}(-1)(x - b + c)$$

Again the law of motion over time for f gives

$$\dot{F}(-1) = \frac{d}{dt} \sum_{-\infty}^{i=-1} f(i) = \sum_{-\infty}^{i=-1} [\alpha f(i-1) - 2\alpha f(i) + \alpha f(i+1)] = \alpha[f(0) - f(1)]$$

Plugging in gives the desired result. ■