ECE-GY 6263 Game Theory	Fall 2019
Lecture 6 — 10/11, 2019	
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1 Overview

In the last lecture we talked about extensive form games,the definition of mixed strategies, behaviors strategies, stakelberg game and kuhn's theorem.

In this lecture we will discuss a little of the Kuhn's Theorem and focus on the multistage game with mixed strategy and behavior strategy.

2 Kuhn's Theorem(perfect recall)

In every game in extensive form, the players have perfect recall, for every mixed strategy of player1 there exists an equivalent behavior strategy.

2.1 Mixed stragegy

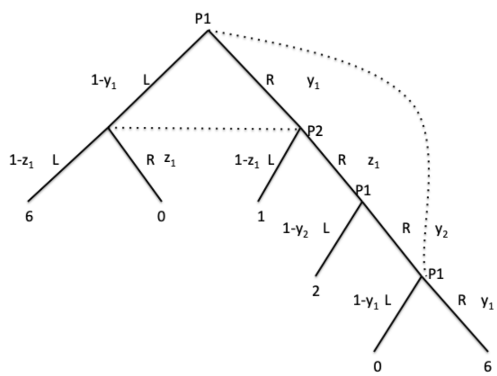

Figure 1: mixed strategy ⇒ behavior strategy

Here is an equation:

$$J(y_1, y_2, z_1) = 6(1-y_1)(1-z_1) + 0(1-y_1)z_1,$$

$$1y_1(1-z_1),$$

$$2y_1z_1(1-y_2) - 0(1-y_1),$$

$$6y_1z_1y_2y_1,$$

$$\max_{z} \min_{y_1y_2} J$$

2.2 Thought experiment

Absent-minded driver game
For the driver who didn't remember his drive path, he can choose to turn right in first block and arrived destination O_3, or he can choose to turn left in first block and keep driving to second block. In second block the driver can choose to turn right and arrive destination O_2 or turn left and arrive destination O_1.

The probability for the driver to choose left and right in each block is q and $1 - q$.

we can get the graph below.

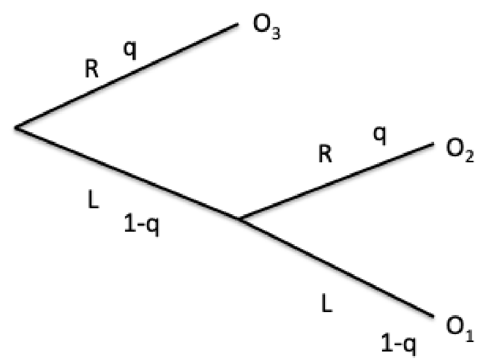

Figure 2: Absent-minded driver game

3 Multistage Game

In the multistage game, players are allowed to act more than once and for each player, he or she may has different information at each stage of the game. We assume the players keep playing game for infinite times, and we can get an infinite tree structure. The tree sturcture has different information for each player at each stage of the game.

For example in a multistrage game, there are two players P1 P2. P1 is the leader to pick a choice between left and right and then P2 pick the choice between left and right. For P2 he doesn't know where he is. Two players keep playing the game infinite turns and we can get the graph below.

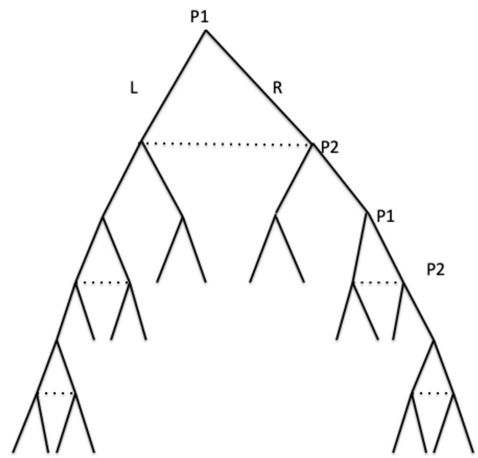

Figure 3: multistage game

3.1 Notations:

a multistage game, 1)A set of players N = {1,2,. . . ,N}
2)t = 0,1,. . . ,T stages, horizontal of game
3)h_r is the history of the pay up to time t.

$$\{(a_1(0), \ldots, a_n(0)), (a_1(1), \ldots, a_n(1)), \ldots, (a_1(t), \ldots, a_n(t))\}$$

$$h_t = h_{t-1} \cup \{a(t-1), \ldots, a_n(t-1)\}$$

$$h_t^i = \{a_i(0), a_i(1), \ldots, a_i(t-1)\}$$

4)Action

$$A_i(t, h_t) \Rightarrow \delta(A_i(t, h_t)) \Rightarrow$$

behavior strategy

$$a_i(t) \in A_i(t, h_t)$$

5)strategy
5.1 for feedback strategy

$$a_i(t) = \mu_i(t, h_t)$$

5.2 for open loop strategy

$$a_i(t) = \mu_i(t)$$

5.3 for open strategy delay

$$a_i(t) = \mu_i(t, h_{t-1})$$

5.4 for distributed funtion

$$a_i(t) = \mu_i(t, h_t^i)$$

6) payoff function

$$J_i(\mu_c, \mu_{-i}) = \sum_{t=0}^{T} u_i(t, \mu_2(t, h_0), \mu_i(t, h_0))$$

$\mu \in P_i : P_i$ All admissible feedback policies

3.2 Definition

A strategy profile $\{\mu_i^*\}_{i=1}^N$ is a subgame perfect Nash equilibrium(SPNE) if given any k, and it associated history $h_k \in H_k$, the notation of $\{\mu_i^*\}_{i=1}^N$ to the horizon $t \in \{k, k+1, \dots T\}$ is also a Nash equilibrium.

ie.

$$J_i^{(k)}(\mu_i^*, \mu_{-i}^*|h_k) \geqslant J_i^{(k)}(\mu_i, \mu_{-i}^*|h_k)$$

for every feasible $\mu_i \in \Gamma_i$ and $h_k \in H_K$.

3.3 Thought experiment

For a game with k stage. $k = T$ $J_i^{(T)}(\mu_i, \mu_{-i}|h_{T-1}) = u_i(T, \mu_i(T, h_T), \mu_{-i}(T, h_T))$

$$\mu_i(T, h_T) = a_i(T) \in A_i(T, h(T))$$

We can set the game like a LR game.

$$\mu_1(T, h_T) = L$$

$$\mu_2(T, h_T) = R$$

So we can list the pure strategy

$$u_1(T, h_T) = \begin{matrix} L & \text{if } h_T \text{ is } \dots \\ R & \text{if } h_T \text{ is } \dots \\ \vdots & \end{matrix} \tag{1}$$

$$u2(T, h_T) = \begin{matrix} \vdots \\ \vdots \end{matrix} \tag{2}$$

$$\{J_1^{(T)}, J_2^{(T)}, \dots . J_N^{T)}\} \to \text{NE} \to \{J_1^{*(T)}, J_2^{*(T)}, \dots, J_N^{*(T)}\}$$

We can get $\{J_1^{*(T)}, J_2^{*(T)}, \dots, J_N^{*(T)}\}$ from $\{\mu_1^*(T, h_T), \mu_2^*(T, h_T), \dots, \mu_N^*(T, h_T)\}$

$$\Rightarrow J_i^* = NE_i\{J_1^{(T)}, \dots, J_N^{(T)}\}$$

For the stage $k = T - 1$

$$J_i^{(T-1)}(\mu_i, \mu_{-i}|h_{T-1}) = u_i(T-1, \mu_i(T-1, h_{T-1}), \mu_{-i}(T-1, h_{T-1})|h_{T-1}) + u_i(T, \mu_i(T, h_T), \mu_{-i}(T, h_{T-1}|h_T))$$

$$h_T = h_{T-1} \cup \{a_1(T-1), \ldots, a_N(T-1)\} + J_i^{(T)^*}|_{h_T}$$
$$J_i^{(T-1)*}|_{h_(T-1)} = NE_i\{J_i^{(T-1)}, J_2^{(T-1)}, \ldots, J_N^{(T-1)}\} \Leftarrow$$
$$J_i^{(T)*}|_{h_(T)} = NE_i\{J_i^{(T)}, J_2^{(T)}, \ldots, J_N^{(T)}\} \Leftarrow$$

References

[1] T. Basar and G.J. Olsder, Dynamic Noncooperative Game Theory, 2nd edition, Classics in Applied Mathematics, SIAM, Philadelphia, 1999

[2] D. Fudenberg and J. Tirole, Game Theory, The MIT Press, 1995