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A Unified Electro-Gravity Theory of a Spinning Electron, and the Fundamental Origins of the Fine Structure Constant and Quantum Concepts

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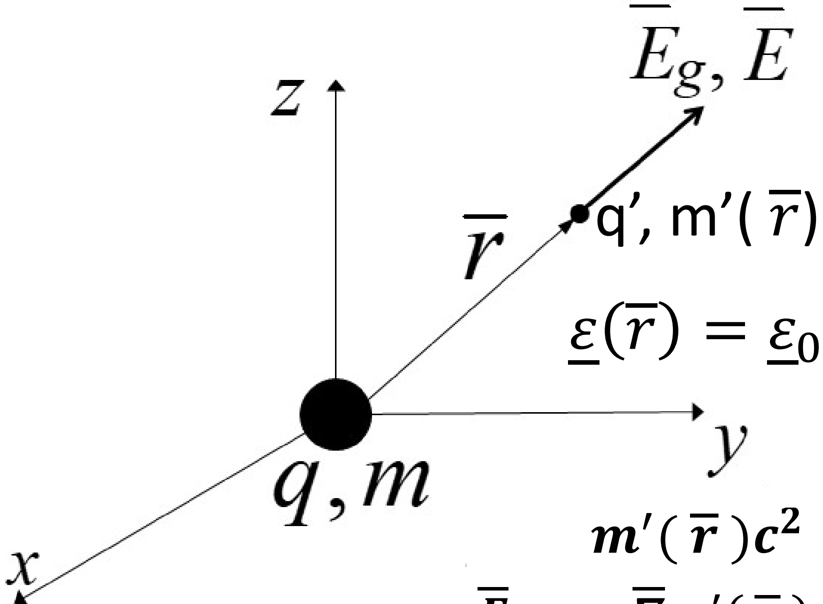
Outline of Presentation

- A Modified/Unified Electro-gravity (UEG) theory for a ``Static'' Electron, without spin: Origin of the Fine-Structure Constant
- Casimir Effect as a UEG effect : Validation of the UEG theory as the origin of the Fine-Structure Constant
- UEG Theory of a Spinning Electron: Re-validation of the origin of the Fine-Structure Constant; Origins of quantum principles
- Conclusion

Electro-Gravitational $\bar{\boldsymbol{E}}_{\boldsymbol{g}}$ and Electric $\bar{\boldsymbol{E}}$ Fields of an Elementary Particle with Charge q and Mass m



Alternate Energy Densities:

$\boldsymbol{W_\tau}$ and $\boldsymbol{W'_\tau = W_\tau + \bar{\nabla} \cdot (\zeta \hat{r} W_\tau)}$

$\boldsymbol{W'_\tau \gg W_\tau \,,\; W'_\tau \approx \bar{\nabla} \cdot (\zeta \hat{r} W_\tau)}$

$$\underline{\varepsilon}(\bar{r}) = \underline{\varepsilon}_0 \underline{\varepsilon}_r(\bar{r}) = 1/\varepsilon(\bar{r})$$

$\boldsymbol{\gamma = UEG\ Constant}$

$$\boldsymbol{m'(\bar{r})c^2 = m'_0 \underline{\varepsilon}_r(\bar{r})c^2}$$

$$\boldsymbol{\bar{F}_g = -\bar{\nabla} m'(\bar{r})c^2 = -c^2 m'_0 \bar{\nabla} \underline{\varepsilon}_r(\bar{r})}$$

$$\bar{E}_g = \frac{\bar{F}_g}{m'_0} = -c^2 \bar{\nabla} \underline{\varepsilon}_r(\bar{r}),$$

$$\bar{\nabla} \cdot \bar{E}_g = -\frac{4\pi G W'_\tau}{c^2} \approx -\frac{4\pi G}{c^2} \bar{\nabla} \cdot (\zeta W_\tau \hat{r})$$

$$\bar{E} = \frac{q}{4\pi r^2} \underline{\varepsilon}(r) \hat{r},$$

$$\bar{E}_g \approx -\frac{4\pi G \zeta}{c^2} W_\tau \hat{r} = -\gamma W_\tau \hat{r}$$

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Complete Unified Electro-Gravity (UEG) Solutions for the Inverse-Permittivity Function and Mass of a Static Electron

$$\bar{E}_g = -c^2\bar{\nabla}\underline{\varepsilon}_r(r) \simeq -\gamma W_\tau \hat{r}, \quad \frac{\partial \underline{\varepsilon}_r(r)}{\partial r} \simeq \frac{\gamma W_\tau}{c^2} = \frac{\gamma D^2 \underline{\varepsilon}_r'}{2c^2\varepsilon_0} = \frac{\gamma}{16\pi^2 r^4 \varepsilon_0 c^2}\int_0^q \underline{\varepsilon}_r(q)\,q\,dq$$

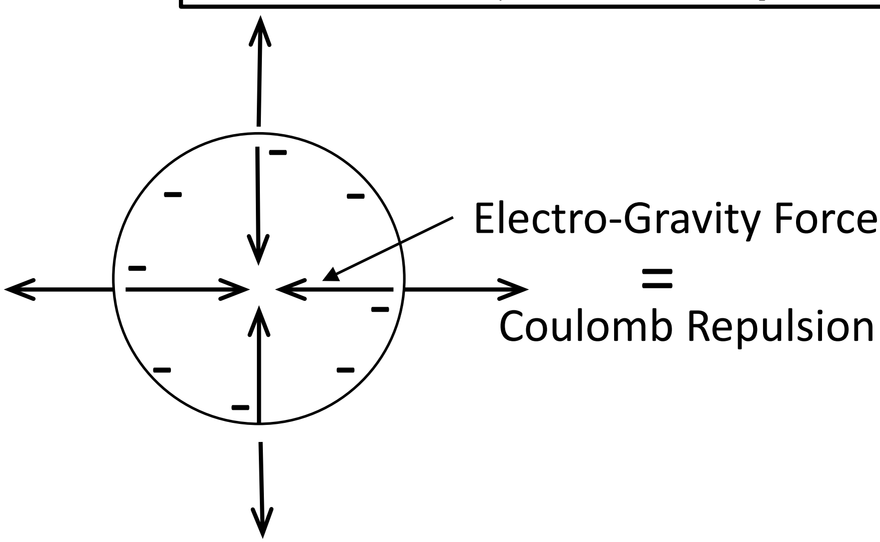
$$\frac{\partial^2 \underline{\varepsilon}_r(r,q)}{\partial(r^{-3})\partial(q^2)} \simeq -\frac{r_\mu^3}{4q^2}\underline{\varepsilon}_r(r,q), \quad r_\mu^3 = \frac{\gamma q^2}{24\pi^2\varepsilon_0 c^2}, \quad \boxed{r_\mu = 5.14\times10^{-16}\gamma^{1/3}}$$

$$\boxed{\underline{\varepsilon}_r(r) = 1 - \frac{t^2}{2^2[1!]^2} + \frac{t^4}{2^4[2!]^2} - \frac{t^6}{2^6[3!]^2} + \ldots = J_0(t), \quad t = \left(\frac{r_\mu}{r}\right)^{1.5}}$$

$$\underline{\varepsilon}_r'(r) = \frac{2}{q^2}\int_0^q \underline{\varepsilon}_r(r,q)\,q\,dq = 1 - \frac{t^2}{2^2[1!]^2\times 2} + \frac{t^4}{2^4[2!]^2\times 3} - \frac{t^6}{2^6[3!]^2\times 4} + \ldots = \left(\frac{2}{t}\right)J_1(t)$$

$$\boxed{W = m(r=r_e')c^2 = \iiint_\tau \frac{q^2\underline{\varepsilon}_r'}{32\pi^2 r^4\varepsilon_0}d\tau = \int_r^\infty \frac{q^2\underline{\varepsilon}_r'(r)}{8\pi r^2\varepsilon_0}dr = m_\mu c^2\sum_{k=0}^{\infty}\frac{(-1)^k t^{(2k+2/3)}}{2^{2k}(k!)^2(k+1)(3k+1)}, \quad m_\mu = \frac{q^2}{8\pi\varepsilon_0 r_\mu c^2}}$$

$$\boxed{m_\mu = 2.49\times10^{-30}\times\gamma^{-1/3}}$$



The new electro-gravitational attraction counters the self-repulsive force of the charge, resulting in a self-consistently stable charge structure.

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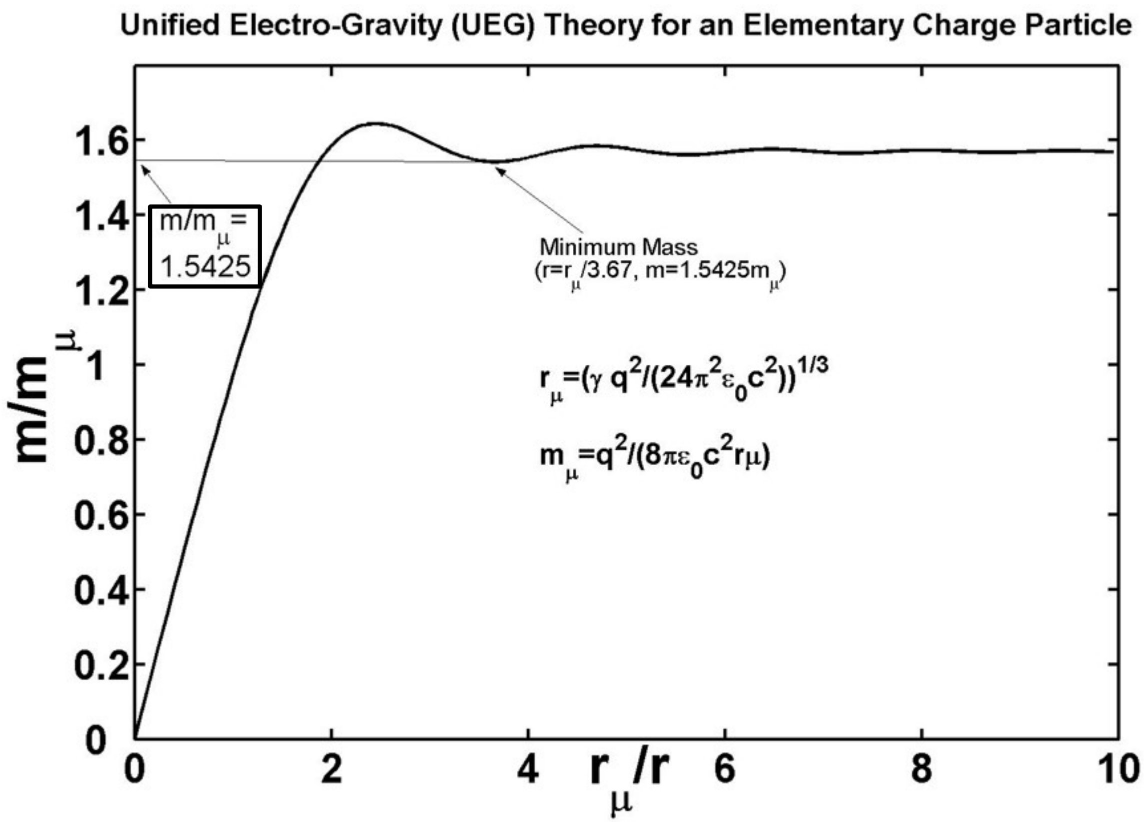
The UEG and Fine-Structure Constants Related by the Normalized Mass Function of an Elementary Charge Particle



$$\left(\frac{m_\mu}{m'_e}\right)^3 = \frac{3q^4}{64\pi c^4 \varepsilon_0^2 \gamma m_e'^3} = \frac{3r_e'^2 \pi}{\gamma m'_e}, \; m'_e = \frac{q^2}{8\pi\varepsilon_0 c^2 r'_e}$$

$$\boxed{\frac{4\gamma m'_e}{r_e'^2} = 12\pi\left(\frac{m'_e}{m_\mu}\right)^3 = 12\pi \times (1.5425)^3 = 138.360 \simeq \frac{1}{\alpha}, \; \alpha = \text{Fine-Structure Constant}}$$

N. Das, American Physical Society Meeting, Washington D.C., April 18-21, 2020

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Dimensionless Relationship Between the UEG Constant, Mass and Classical Radius of an Elementary Charge Particle – Origin of the Fine Structure Constant

$$\frac{m'_e}{m_\mu} = \frac{m_e}{2m_\mu} = 1.5425,\ m_\mu = 2.49\times 10^{-30}\times\gamma^{-1/3} = \frac{m_e}{3.085},\ m_e = 9.1\times 10^{-31}\,\text{kg}$$

$$\gamma^{-1/3} = \frac{m_e}{2.49\times 3.085\times 10^{-30}} = 0.1185,\ \boxed{\text{UEG Constant } \gamma = 5.997\times 10^{2}(\text{m}/\text{s}^2)/(\text{J}/\text{m}^3)}$$

$$m_e = \text{electron mass with spin},\ m'_e = \text{"static" electron mass with no spin} = \frac{m_e}{2}$$

$$\left(\frac{m_\mu}{m'_e}\right)^3 = \frac{3q^4}{64\pi c^4\varepsilon_0^2\gamma m_e'^3} = \frac{3r_e'^2\pi}{\gamma m'_e},\ m'_e = \frac{q^2}{8\pi\varepsilon_0 c^2 r'_e},\ m_e = \frac{q^2}{8\pi\varepsilon_0 c^2 r_e},\ r'_e = 2r_e$$

$$\boxed{\frac{\gamma m'_e}{r_e'^2} = 3\pi\left(\frac{m'_e}{m_\mu}\right)^3 = 3\pi\times(1.5425)^3 = 34.590 \simeq \frac{1}{4\alpha},\ \frac{\gamma m_e}{r_e^2} = 8\times\frac{\gamma m'_e}{r_e'^2} = 276.720 \simeq \frac{2}{\alpha}}$$

α = Fine-Structure Constant

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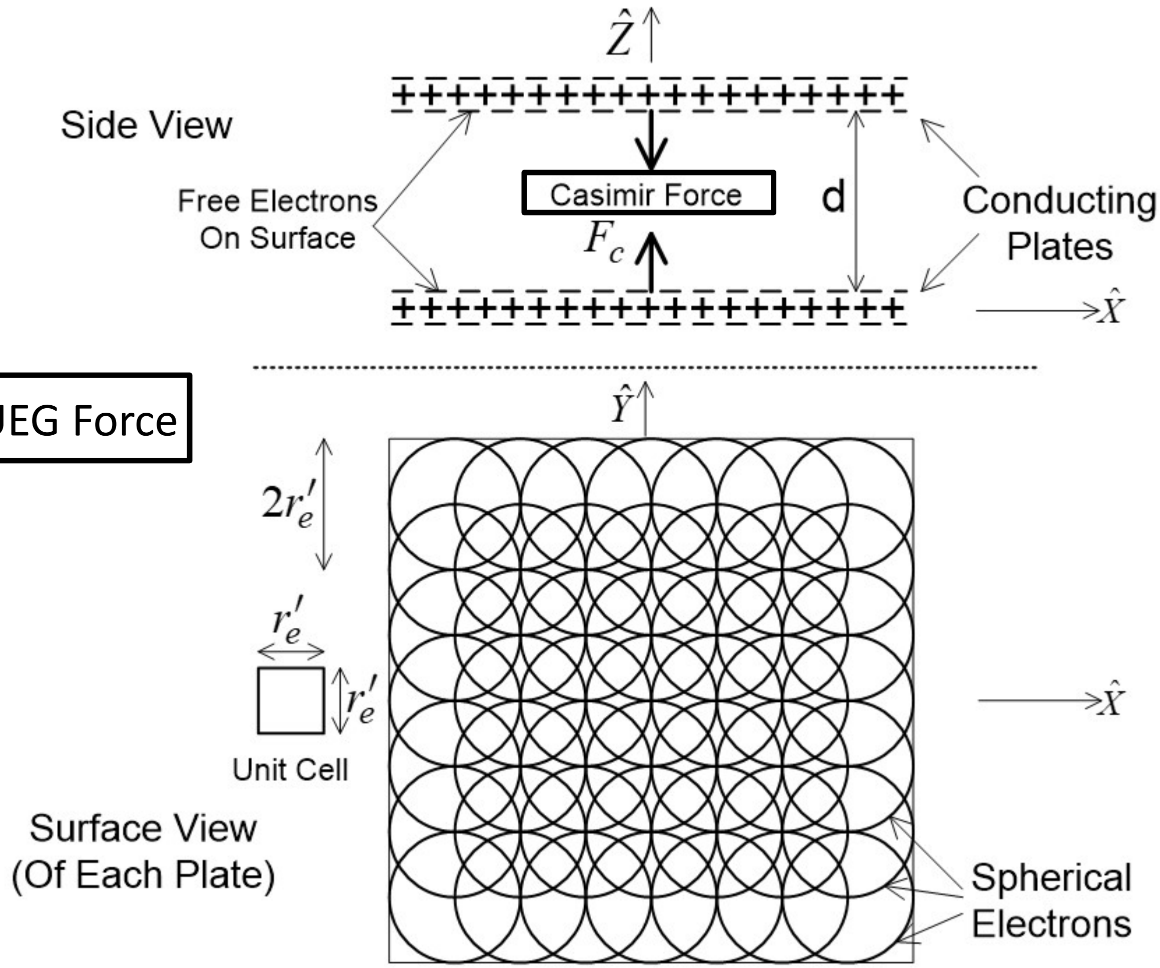
Casimir Effect Modeled as UEG Forces Due to Residual Electric Fields of the Free Electrons in a Conducting Plate

Casimir Force=UEG Force



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Fine-Structure Constant Derived from the Casimir Effect Modeled as an UEG Effect – Validations of the UEG Theory and the Origin of the Fine-Structure Constant

$$\langle w_e \rangle_{spin=\pm\frac{1}{2}} = \frac{\langle Q^2 \rangle}{32\pi^2\varepsilon_0 d^4} = \frac{q^2}{64\pi^2\varepsilon_0 d^4}$$

$$\langle w_e \rangle = \langle w_e \rangle_{spin=+\frac{1}{2}} + \langle w_e \rangle_{spin=-\frac{1}{2}} = 2\langle w_e \rangle_{spin=\pm\frac{1}{2}} = \frac{q^2}{32\pi^2\varepsilon_0 d^4}$$

$$\langle F_0 \rangle_{spin=\pm\frac{1}{2}} = E_{ueg} m_e = \gamma \langle w_e \rangle m_e = \frac{\gamma q^2 m_e}{32\pi^2\varepsilon_0 d^4}$$

$$\langle F_0 \rangle = \langle F_0 \rangle_{spin=+\frac{1}{2}} + \langle F_0 \rangle_{spin=-\frac{1}{2}} = 2\langle F_0 \rangle_{spin=\pm\frac{1}{2}} = \frac{\gamma q^2 m_e}{16\pi^2\varepsilon_0 d^4}$$

$$F_u(\text{UEG Force}) = \langle F_0 \rangle \frac{A}{A_0} = \frac{\gamma q^2 m_e A}{64\pi^2\varepsilon_0 r_e^2 d^4} = F_c(\text{Casimir Force}) = \frac{\hbar c \pi^2 A}{240 d^4}, \quad A_0 = r_e'^2 = 4r_e^2$$

$$\boxed{\frac{\gamma m_e}{r_e^2} = \left(\frac{\pi^3}{15}\right)\left(\frac{4\hbar c\varepsilon_0\pi}{q^2}\right) = \frac{2.067}{\alpha} \simeq \frac{2}{\alpha}, \; \alpha = \frac{q^2}{4\hbar c\varepsilon_0\pi} = \text{Fine-Structure Constant}}$$

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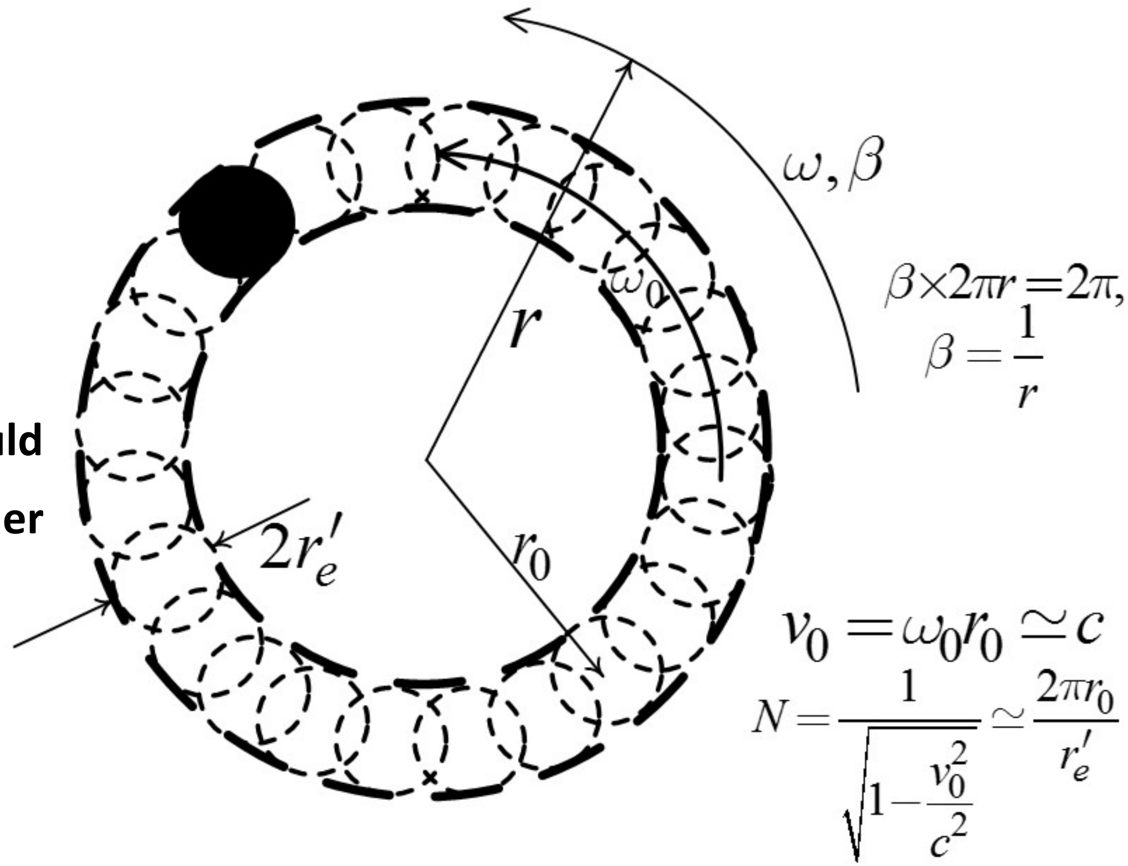
Spinning of a Stable Static Elementary Charge, Supported by the UEG Forces Due to its Own Electromagnetic Fields

Spinning Orbit is an Equivalent <u>Rotational Electro-Gravitational Inertial Frame</u>

The radius r_0 in the local orbital frame would be $\frac{r_0}{N} = \frac{r'_e}{2\pi}$ (much smaller than r_0)



A Spinning Electron Shown on the X-Y Plane

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Dynamical Model of a Spinning Electron, Supported by the UEG Forces Due to its Own Electromagnetic Fields: Re-Confirmation of the Origin of the Fine-Structure Constant

$$\underline{W}_{\tau} = \frac{\varepsilon_0}{2}\underline{E}^2 = \frac{\varepsilon_0}{2}\left(\frac{q}{4\pi\varepsilon_0 r_0^2}\right)^2 (N)^2 \simeq \frac{\varepsilon_0}{2}\left(\frac{q}{4\pi\varepsilon_0 r_0^2}\right)^2 \left(\frac{2\pi r_0}{r_e'}\right)^2$$

$$E_g(\text{Total}) = E_g(\text{UEG, Electric}) = \gamma \underline{W}_{\tau}\left(\frac{4r'^2}{\pi r'^2}\right) \simeq \gamma \frac{\varepsilon_0}{2}\left(\frac{q}{4\pi\varepsilon_0 r_0^2}\right)^2 \left(\frac{2\pi r_0}{r_e'}\right)^2 \left(\frac{4}{\pi}\right) = \frac{v_0^2}{r_0} \simeq \frac{c^2}{r_0}$$

$$\boxed{\frac{\gamma q^2}{2\pi\varepsilon_0 c^2 r_e'^3} = \frac{4\gamma m_e'}{r_e'^2} \simeq \frac{r_0}{r_e'} \simeq \frac{1}{\alpha},\ \frac{\gamma m_e}{r_e^2} \simeq \frac{2r_0}{r_e'} \simeq \frac{2}{\alpha},\ \alpha = \text{Fine-Structure Constant}}$$

$$\left(\frac{4r'^2}{\pi r'^2}\right) = \left(\frac{4}{\pi}\right) = \text{Geometrical transform factor between square/circular grid}$$

$$\text{"Static" electron mass without spin} = m_e' = \frac{q^2}{8\pi\varepsilon_0 c^2 r_e'},\ \text{Electron mass with spin} = m_e = \frac{q^2}{8\pi\varepsilon_0 c^2 r_e},\ m_e \simeq 2m_e',\ r_e \simeq \frac{r_e'}{2}$$

$$\text{Electron spin-angular momentum} = m_e' r_0 v_0 = \frac{q^2 r_0 v_0}{8\pi\varepsilon_0 c^2 r_e'} = \frac{\hbar}{2},\ \frac{r_0}{r_e'} = \frac{4\pi\varepsilon_0 c\hbar}{q^2}\left(\frac{c}{v_0}\right) \simeq \frac{4\pi\varepsilon_0 c\hbar}{q^2} = \frac{1}{\alpha},\ v_0 \simeq c$$

$$\boxed{E_g(\text{UEG, Magneticic}) = E_{gum} = -E_g(\text{UEGravitoMagnetic}) = -E_{gm}\ \text{(Next Slide)}}$$

$$E_g(\text{Total}) = E_g(\text{UEG, Electric}) + E_g(\text{UEG, Magneticic}) + E_g(\text{UEGravito} - \text{Magnetic})$$
$$= E_{gue} + E_{gum} + E_{gm} = E_{gue} = E_g(\text{UEG, Electric})$$

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Dynamical Model of a Spinning Electron, Supported by the UEG Forces Due to its Own Electromagnetic Fields: UEG Forces Due to Magnetic and Gravito-Magnetic Fields

$$\bar{\mu}_S = \hat{z}\frac{\hbar q}{2m_e},\ \bar{H} = \hat{\theta}\frac{\mu_S}{4\pi r^3}\sin\theta + \hat{r}\frac{\mu_S}{2\pi r^3}\cos\theta,\ \bar{E} = \hat{r}\frac{q}{4\pi\varepsilon r^2},\ \bar{v}(EM) = \frac{\bar{E}\times\bar{H}}{(\frac{1}{2}\varepsilon|\bar{E}|^2)} = \hat{\phi}\frac{2\mu_S\sin\theta}{qr} = \hat{\phi}\frac{\hbar\sin\theta}{m_e r}$$

$$\bar{S} = \hat{z}\frac{\hbar}{2} = m_e'\bar{r}\times\bar{v} = \frac{m_e}{2}\bar{r}\times\bar{v} = \hat{z}\frac{m_e}{2}rv_\phi\sin\theta,\ \bar{v}(QM) = \hat{\phi}\frac{\hbar}{m_e r\sin\theta}$$

$$\bar{E}_{gum} = -\hat{r}\gamma < (\frac{1}{2}\mu|\bar{H}|^2) > = -\hat{r}\frac{\gamma\mu\mu_S^2}{32\pi^2 r^6}\frac{\int_0^\pi(\sin^2\theta + 4\cos^2\theta)\sin\theta d\theta}{\int_0^\pi \sin\theta d\theta} = -\hat{r}\frac{\gamma\mu\mu_S^2}{16\pi^2 r^6} = -\hat{r}\frac{\gamma\mu\hbar^2 q^2}{64\pi^2 m_e^2 r^6}$$

$$\bar{E}_{gue} = -\hat{r}\gamma(\frac{1}{2}|\bar{E}|^2) = -\hat{r}\frac{\gamma q^2}{32\pi^2\varepsilon r^4},\ \rho_{vu} = -\varepsilon\bar{\nabla}\cdot\bar{E}_{gue} = -\frac{\gamma q^2}{16\pi^2 r^5},\ \bar{\nabla}\times\bar{H}_{gu} = \bar{J}_{gu} = \rho_{vu}\bar{v}(EM)$$

$$\bar{H}_{gu} = \hat{\theta}H_{gu\theta},\ \frac{1}{r}\frac{\partial(rH_{gu\theta})}{\partial r} = \rho_{vu}v_\phi(EM) = -\frac{\gamma q^2\hbar\sin\theta}{16\pi^2 m_e r^6},\ H_{gu\theta} = \frac{\gamma q^2\hbar\sin\theta}{64\pi^2 m_e r^5}$$

$$\boxed{\bar{E}_{gm} = -\mu < \bar{v}\times\bar{H}_{gu} > = -\mu\bar{v}(QM)\times\bar{H}_{gu} = \hat{r}\mu v_\phi(QM)H_{gu\theta} = \frac{\gamma\mu q^2\hbar^2}{64\pi^2 m_e^2 r^6} = -\bar{E}_{gum}}$$

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Dynamical Model of a Spinning Electron, Supported by the UEG Forces Due to its Own Electromagnetic Fields: Origins of the Energy-Frequency Relationship of the Quantum Theory and the Electron g-Factor

Approximate Model:

$$m_e' v_0 r_0 = \frac{\hbar}{2}, \quad \boxed{\text{Energy} = W = m_e c^2 \simeq 2m_e' c^2 = \frac{\hbar c^2}{v_0 r_0} \simeq \frac{\hbar v_0}{r_0} = \hbar\omega_0 \simeq \hbar\omega,} \quad v_0 \simeq c,\ \omega_0 \simeq \omega,\ m_e \simeq 2m_e'$$

Rigorous Model:

Transformtion between external (unprimed) and its equivalent rotational (prime) inertial frame:

$$e^{j(\omega t - \beta s)} = e^{j(\omega' t' - \beta' s')},\ t = \frac{t' + s' v_0 / c^2}{\sqrt{1 - \frac{v_0^2}{c^2}}},\ s = \frac{s' + t' v_0}{\sqrt{1 - \frac{v_0^2}{c^2}}},\ \beta \times (s = 2\pi r_0) = 2\pi,\ \beta = \frac{1}{r_0}$$

$$\omega = \omega' = \frac{\omega - \beta v_0}{\sqrt{1 - \frac{v_0^2}{c^2}}} = \frac{\omega - v_0 / r_0}{\sqrt{1 - \frac{v_0^2}{c^2}}} = \frac{\omega - \omega_0}{\sqrt{1 - \frac{v_0^2}{c^2}}},\ \omega_0 = \omega(1 - \frac{1}{N}) = \frac{v_0}{r_0} \simeq \omega,\ N = \frac{1}{\sqrt{1 - \frac{v_0^2}{c^2}}} >> 1$$

Energy (W) — UEG wave frequency (ω) — electron g — factor Relationship :

$$J = \frac{\hbar}{2} = m_e' v_0 r_0,\ \mu_J = \frac{q}{2} v_0 r_0 = J \frac{q}{2m_e} g,\ m_e = g m_e',\ W = m_e c^2 = g m_e' c^2 = g c^2 \frac{\hbar}{2 v_0 r_0} = \hbar(\frac{c^2}{v_0^2})(\frac{v_0}{r_0})(\frac{g}{2})$$

$$= \hbar \frac{\omega_0}{(1 - \frac{1}{N^2})} \frac{g}{2} = \hbar\omega \frac{(1 - \frac{1}{N})}{(1 - \frac{1}{N^2})} \frac{g}{2} = \hbar\omega \frac{1}{(1 + \frac{1}{N})} \frac{g}{2},\quad \boxed{\text{Energy} = W = m_e c^2 = \hbar\omega,\ \ \frac{g}{2} - 1 = \frac{1}{N} = \frac{r_e'}{2\pi r_0} \simeq \frac{\alpha}{2\pi}}$$

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UEG Theory of Origins of Other Fundamental Electrodynamic Concepts

- The Fine-Structure Constant α, relates the elementary charge q and Planck's constant $\hbar$, and its value is predetermined as per the UEG theory.

- Therefore, the elementary charge q must also take fixed quantized value, when the angular momentum $\hbar/2$ is known to have a fixed, quantized value

- The UEG fields of one electron, due to its non-linear nature, would mix that of an interacting, colliding electron, or the fields of an interacting electromagnetic/light radiation, resulting in frequency shifts

- This would physically explain frequency shifts in the photo-electric effect, as well as in Raman and Compton Scatterings, under a unified theoretical/physical framework

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Unified Electro-Gravity (UEG) Theory of an Electron, and Origins of the Fine-Structure Constant and Quantum Concepts: Conclusions

- The new UEG theory self-consistently models a stable electron structure.
- A dimensionless constant deduced from the UEG theory is shown to be closely (numerologically) related to the Fine-Structure Constant of electrodynamics.
- The Casimir Effect is shown to be a UEG effect. This re-validates the UEG theory, and its relation to the Fine-Structure Constant.
- The UEG theory also self-consistently models a spinning electron, sustained by the UEG effects of its own electro-magnetic fields.

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Unified Electro-Gravity (UEG) Theory of an Electron, and Origins of the Fine-Structure Constant and Quantum Concepts: Conclusions (Continued)

- The complete UEG model of the spinning electron re-validates the UEG theory, and reveals that the new UEG theory is the physical origins of the Fine-Structure Constant, as well as of many fundamental quantum concepts and phenomena.

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Supplementary Slides:

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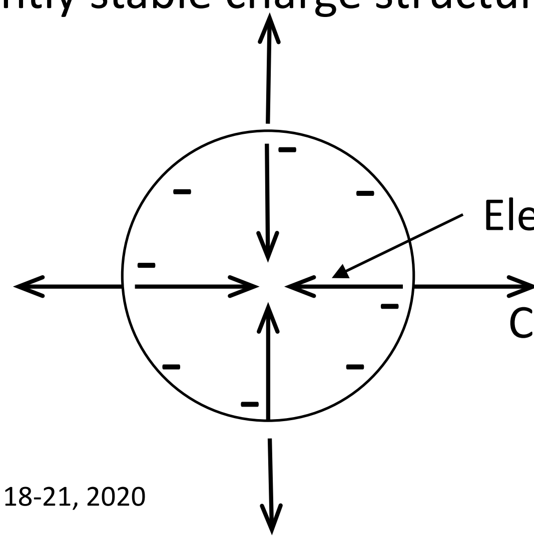
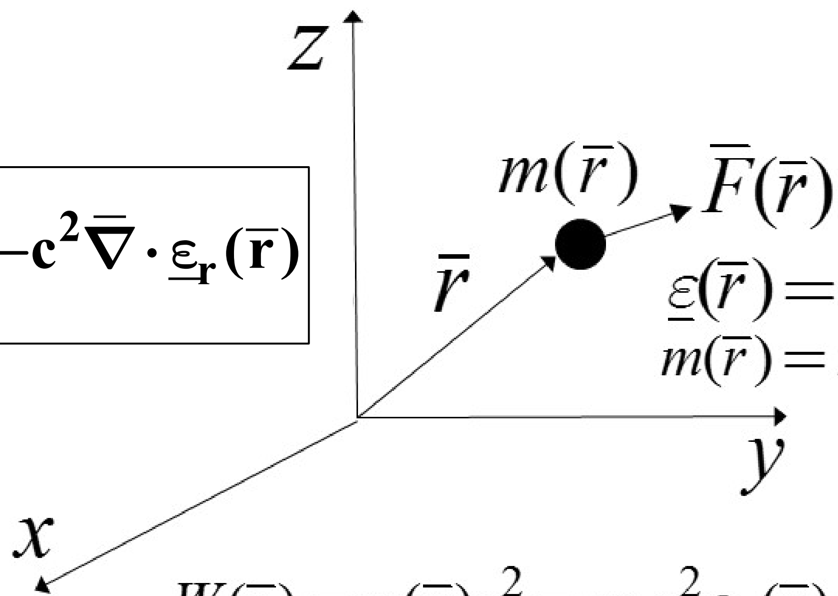
A Modified/Unified Electro-Gravity Theory: The Basic Principles

- Newtonian gravitation is strictly valid only in the external region of a neutral, non-radiating, massive body. The Newtonian gravitation is only a residual effect of strong gravitation in the internal charged structure of the neutral body.
- Gravitation is much stronger in the presence of any electromagnetic field, constituting a Unified Electro-Gravity (UEG) field.
- The UEG field of a charged elementary particle, such as the electron, would counter the self-repulsive force of the charge, resulting in a self-consistently stable charge structure.



Gravitational Field as a Gradient of Inverse-Relative-Permittivity Function

$$\overline{\mathbf{E}}_{\mathbf{g}} = \frac{\overline{\mathbf{F}}}{\mathbf{m_0}} = -\mathbf{c}^2 \overline{\nabla} \cdot \underline{\varepsilon}_{\mathbf{r}}(\overline{\mathbf{r}})$$



$$\underline{\varepsilon}(\overline{r}) = \underline{\varepsilon}_0 \underline{\varepsilon}_r(\overline{r})$$

$$m(\overline{r}) = m_0 \underline{\varepsilon}_r(\overline{r})$$

$$W(\overline{r}) = m(\overline{r})c^2 = m_0 c^2 \underline{\varepsilon}_r(\overline{r})$$

$$\overline{F}(\overline{r}) = -\overline{\nabla} W(\overline{r}) = -m_0 c^2 \overline{\nabla} \underline{\varepsilon}_r(\overline{r})$$

$$\underline{\varepsilon} = 1/\varepsilon,\ \underline{\varepsilon}_0 = 1/\varepsilon_0,\ \underline{\varepsilon}_r = 1/\varepsilon_r,\ \varepsilon = \varepsilon_0 \varepsilon_r$$

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Energy Density W_τ in a Non-Linear Electro-Gravitational Field

- Permittivity distribution in a ``free-space'' medium is dependent on the energy-density distribution of an electro-gravity field in the free-space, or equivalently on the associated source charge

- Electro-gravitational field surrounding a charge is dependent on the permittivity distribution, which in turn is dependent on the electric field itself: The electro-gravitational field is a non-linear field

- Energy density in the non-linear electro-gravitational field must be properly modeled

$$\mathbf{W}_\tau = \int_0^q \bar{\mathbf{E}} \cdot d\bar{\mathbf{D}} = \int_0^q \underline{\varepsilon} \bar{\mathbf{D}} \cdot d\bar{\mathbf{D}} = \int_0^q \frac{1}{2} \underline{\varepsilon} \frac{d(\bar{\mathbf{D}} \cdot \bar{\mathbf{D}})}{dq} dq = \frac{1}{2} |\bar{\mathbf{D}}|^2 \left(\frac{2}{q^2} \int_0^q \underline{\varepsilon}(q) q dq \right)$$

$$\mathbf{W}_\tau = \frac{1}{2} \underline{\varepsilon}' |\bar{\mathbf{D}}|^2 = \frac{1}{2} \underline{\varepsilon}_0 \underline{\varepsilon}_r' |\bar{\mathbf{D}}|^2, \ \underline{\varepsilon}' = \frac{2}{q^2} \int_0^q \underline{\varepsilon}(q) q dq, \ \underline{\varepsilon}_r' = \frac{2}{q^2} \int_0^q \underline{\varepsilon}_r(q) q dq$$

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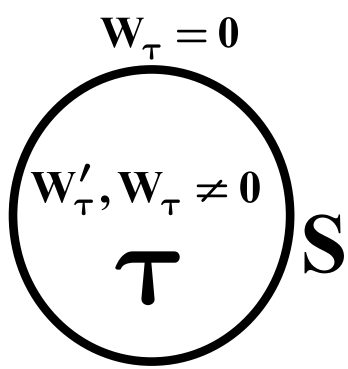
Modified Definition of Gravitation, or Modified Equivalent Energy Density W'_τ, due to an Electromagnetic Field

$$W = \iiint_\tau W'_\tau d\tau = \iiint_\tau (W_\tau + \bar{\nabla}\cdot(\zeta W_\tau \hat{r}))d\tau = \iiint_\tau W_\tau d\tau + \iiint_\tau \bar{\nabla}\cdot(\zeta W_\tau \hat{r})d\tau$$

$$W = \iiint_\tau W'_\tau d\tau = \iiint_\tau W_\tau d\tau + \oint\!\!\oint_S \zeta W_\tau \hat{r}\cdot\overline{dS} = \iiint_\tau W_\tau d\tau,\ W_\tau = 0, \text{on } S$$



$$W'_\tau = W_\tau + \bar{\nabla}\cdot(\zeta W_\tau \hat{r})$$

$$\nabla\cdot\bar{E}_g = -\frac{4\pi G W'_\tau}{c^2} = -\frac{4\pi G W_\tau}{c^2} - \bar{\nabla}\cdot(\gamma W_\tau \hat{r}),\ \gamma = \frac{4\pi G\zeta}{c^2}$$

$$\bar{\nabla}\cdot\bar{E}_g = -c^2\bar{\nabla}\cdot\bar{\nabla}\underline{\varepsilon}_r(r) = -\frac{4\pi G W_\tau}{c^2} - \bar{\nabla}\cdot(\gamma W_\tau \hat{r}) \simeq -\bar{\nabla}\cdot(\gamma W_\tau \hat{r}),\ \frac{4\pi G W_\tau}{c^2} << \bar{\nabla}\cdot(\gamma W_\tau \hat{r})$$

$$\bar{E}_g = -c^2\bar{\nabla}\underline{\varepsilon}_r(r) \simeq -\gamma W_\tau \hat{r},\ \bar{\nabla}\underline{\varepsilon}_r(r) \simeq -\frac{\gamma W_\tau}{c^2}\hat{r},\ \gamma = \text{UEG Constant}$$

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Complete Unified Electro-Gravity (UEG) Solutions for the Inverse-Permittivity Function and Mass of a Static Electron

$$\bar{E}_g = -c^2\bar{\nabla}\underline{\varepsilon}_r(r) \simeq -\gamma W_\tau \hat{r}, \quad \frac{\partial \underline{\varepsilon}_r(r)}{\partial r} \simeq \frac{\gamma W_\tau}{c^2} = \frac{\gamma D^2 \underline{\varepsilon}_r'}{2c^2\varepsilon_0} = \frac{\gamma}{16\pi^2 r^4 \varepsilon_0 c^2}\int_0^q \underline{\varepsilon}_r(q) q\,dq$$

$$\frac{\partial^2 \underline{\varepsilon}_r(r,q)}{\partial(r^{-3})\partial(q^2)} \simeq -\frac{r_\mu^3}{4q^2}\underline{\varepsilon}_r(r,q), \; r_\mu^3 = \frac{\gamma q^2}{24\pi^2\varepsilon_0 c^2}, \; \boxed{r_\mu = 5.14\times 10^{-16}\gamma^{1/3}}$$

$$\boxed{\underline{\varepsilon}_r(r) = 1 - \frac{t^2}{2^2[1!]^2} + \frac{t^4}{2^4[2!]^2} - \frac{t^6}{2^6[3!]^2} + \ldots = J_0(t), \; t = (\frac{r_\mu}{r})^{1.5}}$$

$$\underline{\varepsilon}_r'(r) = \frac{2}{q^2}\int_0^q \underline{\varepsilon}_r(r,q) q\,dq = 1 - \frac{t^2}{2^2[1!]^2\times 2} + \frac{t^4}{2^4[2!]^2\times 3} - \frac{t^6}{2^6[3!]^2\times 4} + \ldots = (\frac{2}{t})J_1(t)$$

$$\boxed{W = m(r = r_e')c^2 = \iiint_\tau \frac{q^2\underline{\varepsilon}_r'}{32\pi^2 r^4\varepsilon_0}d\tau = \int_r^\infty \frac{q^2\underline{\varepsilon}_r'(r)}{8\pi r^2\varepsilon_0}dr = m_\mu c^2 \sum_{k=0}^\infty \frac{(-1)^k t^{(2k+2/3)}}{2^{2k}(k!)^2(k+1)(3k+1)}, \; m_\mu = \frac{q^2}{8\pi\varepsilon_0 r_\mu c^2}}$$

$$\boxed{m_\mu = 2.49\times 10^{-30}\times\gamma^{-1/3}}$$

With a linear medium:

$$\underline{\varepsilon}_r(r) = \underline{\varepsilon}_r'(r) = 1 - \frac{t^2}{2^2 1!} + \frac{t^4}{2^4 2!} - \frac{t^6}{2^6 3!} + \ldots = e^{-t^2/2}$$

$$W = mc^2 = m_\mu c^2 \sum_{k=0}^\infty \frac{(-1)^k t^{(2k+2/3)}}{2^{2k} k!(3k+1)}$$

Coulomb mass/energy:

$$\underline{\varepsilon}_r(r) = \underline{\varepsilon}_r'(r) = 1, \; W = mc^2 = m_\mu c^2 \frac{r_\mu}{r}, \; \frac{m}{m_\mu} = \frac{r_\mu}{r} = t^{2/3}$$

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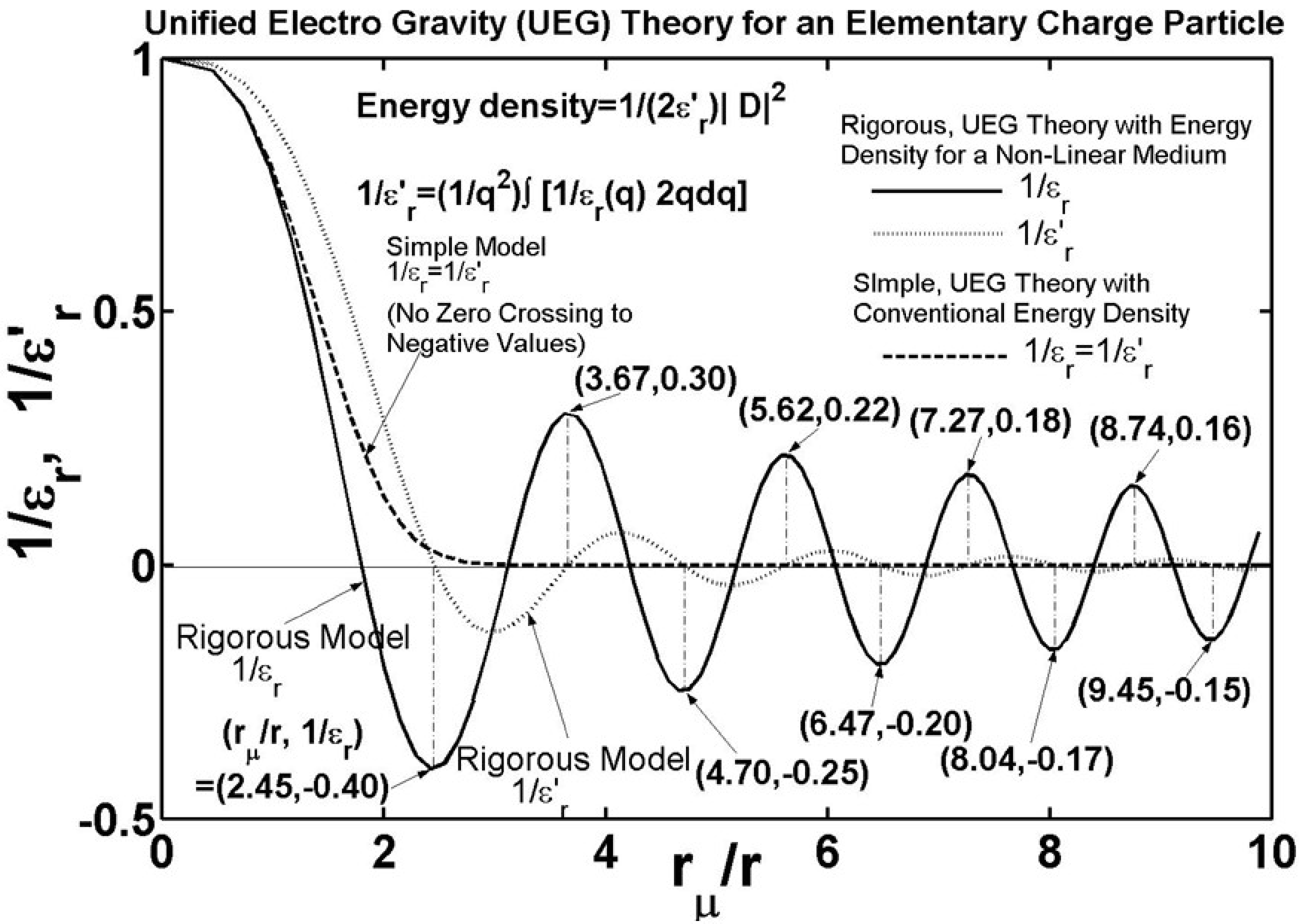
Inverse-Relative-Permittivity Function Around an Elementary Charge Particle



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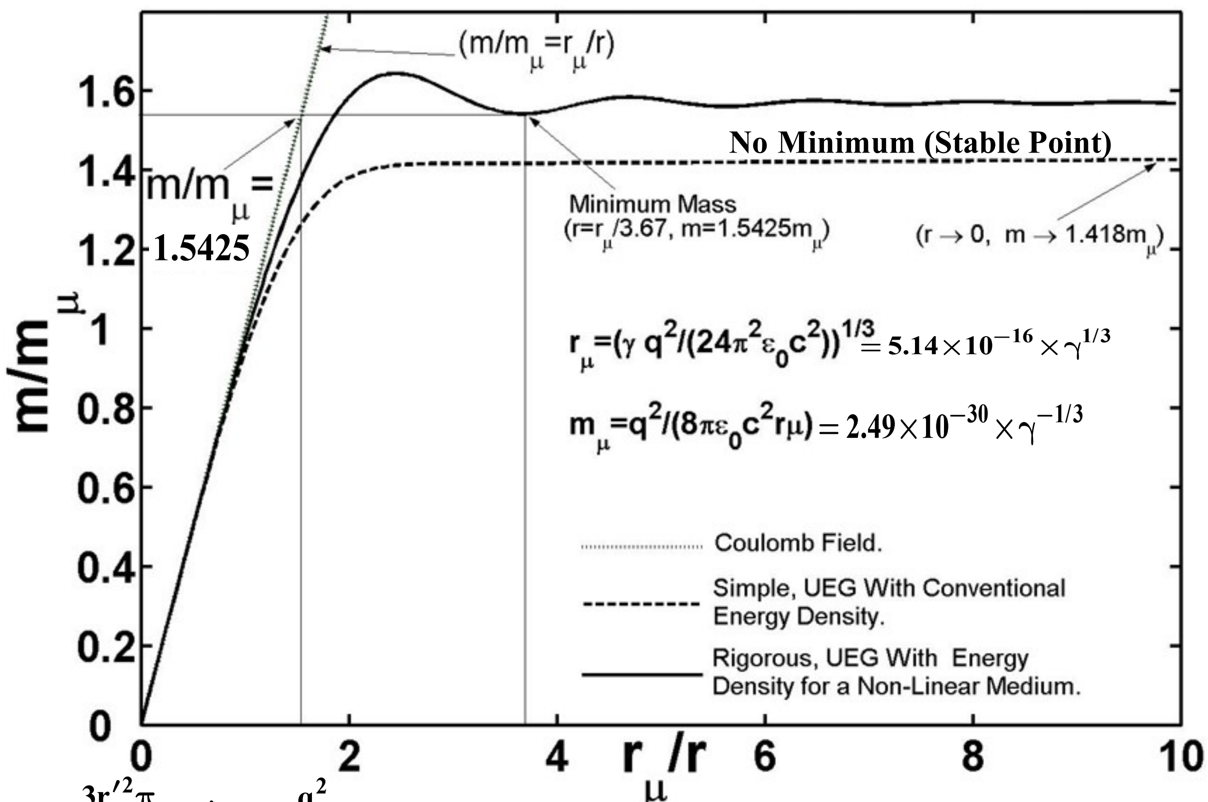
The UEG and Fine-Structure Constants Related by the Normalized Mass Function of an Elementary Charge Particle

Unified Electro-Gravity (UEG) Theory for an Elementary Charge Particle



$$(\frac{m_\mu}{m'_e})^3 = \frac{3q^4}{64\pi c^4\varepsilon_0^2\gamma m_e'^3} = \frac{3r_e'^2\pi}{\gamma m'_e},\ m'_e = \frac{q^2}{8\pi\varepsilon_0 c^2 r'_e}$$

$$\frac{4\gamma m'_e}{r_e'^2} = 12\pi(\frac{m'_e}{m_\mu})^3 = 12\pi\times(1.5425)^3 = 138.360 \simeq \frac{1}{\alpha},\ \alpha = \text{Fine-Structure Constant}$$

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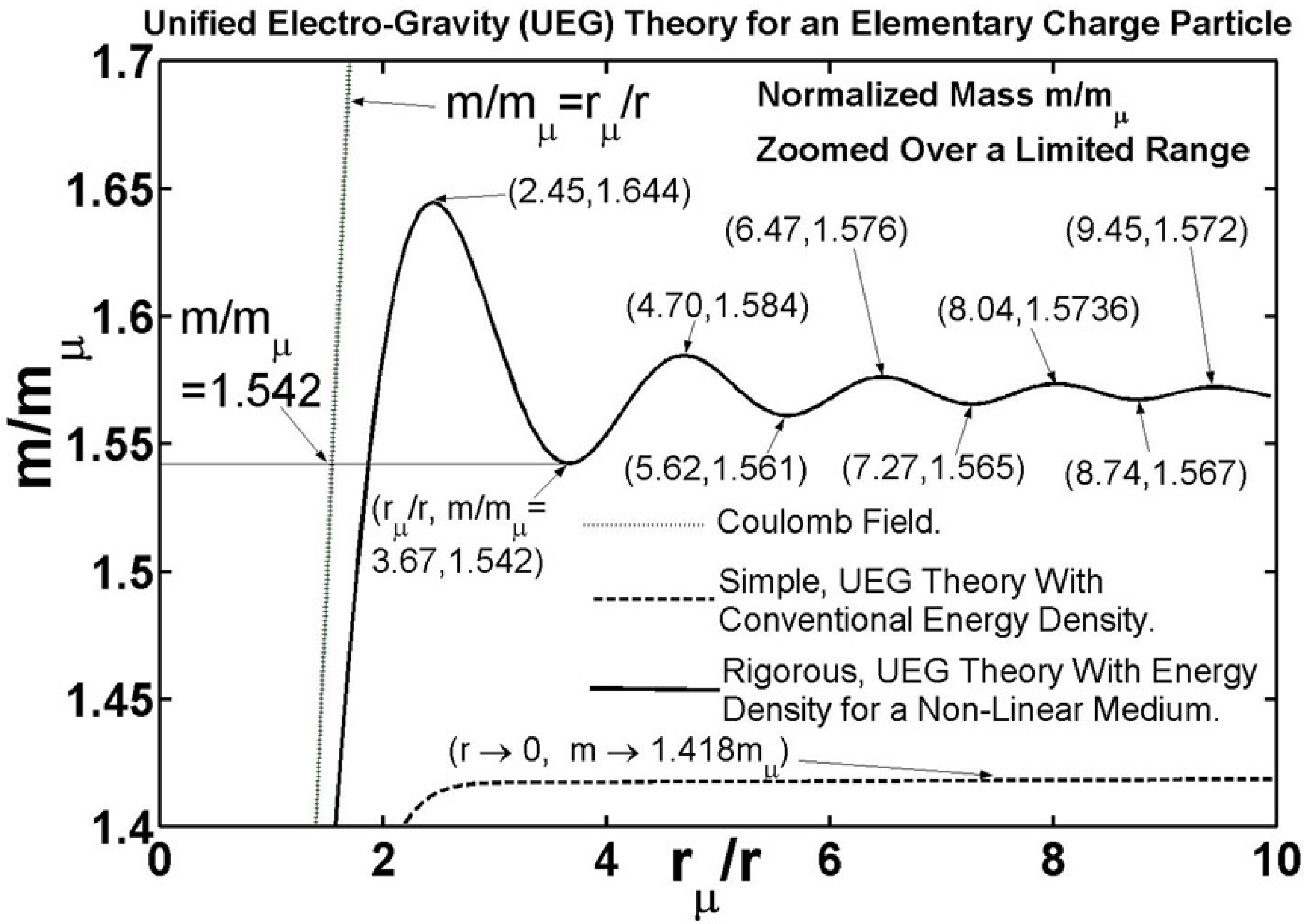
Normalized Mass Function of an Elementary Charge Particle (Expanded Scale)



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