Multidominance in light of conflicting formalizations of minimalism and syntactic economy

Daniil M. Ozernyi
Department of Linguistics,
Northwestern University

Workshop on Multidominance in Syntax
June 25, 2023
UMass-Amherst

The roadmap

Northwestern

- First, I will go through some of the Collins and Stabler (2016) and point out several details of the proposal, which, to my view, constitute inconsistencies or imprecisions. Moving closer to multidominance, I discuss some of the points in Collins and Groat (2018) and Chomsky (2020).
- This discussion will lead me to claim that **there is no meaningful distinction between merge-based sets and graphs or trees (= directed graphs), particularly in the context of labeled merge-based sets.**
- Then, I will argue that Chomsky's putative, ill-defined, and ambiguity-ridden "third-factor" economy gives rise to many interpretations – and on some of them **(even conservative!)**, **multidominant derivations are more economic than conservative merge-based ones**.

Why

Northwestern

Both parts are equally important. While I want to draw attention to economy and multidominance in the second part, I think it very important to draw attention to handwavy formalizations which prevent us from meaningful discussions of crucial notions, including economy.

What did C&S aim to do?

Northwestern

What was the objective of Collins and Stabler (2016)?

> *The goal of this paper is to give a precise, formal account of certain fundamental notions in minimalist syntax, including Merge, Select, Transfer, occurrences, workspace, labels, and convergence. (C&S, 43)*

Let's scrutinize whether the authors succeed in this venture.

The instruments

Northwestern

> *We use basic set theory to represent syntactic objects, with standard notation: $\in$ (is an element of), $\cup$ (set union), $\subseteq$ (is a subset of), and $\subset$ (is a proper subset of). The empty set is written $\{\}$ or $\emptyset$. Given sets S and T, the difference $S - T = \{x|x \in S, x \notin T\}$. Angle brackets are used for sequences $\langle S_1, S_2, ..., S_n\rangle$, and when $n = 0$, the empty sequence is written $\langle\rangle$ or ϵ. (C&S, 43)*

There are many, many questions about these tools. What is "basic set theory"? It should be **always** understood that there's no one "basic set theory" just like there is no one "theory of grammar".

Basic set theory?

Northwestern

Choice of formal tools has very far-reaching consequences for formalization. For example, C&S might have assumed:

- naive set theory
  but it is incoherent (cf. Russel's paradox); situating anything in this theory seems rather inadvisable;
- "standard" set theory for any set-theorist or mathematician, by considerable consensus, is ZFC...
- "basic" set theory is probably Z^- or something weak of the sort, most mathematics can be done in Z^-;[1]
- we stop to consider axioms of ZFC now:[2]

[1] Small models: $V_{\omega+\omega}$ in ZFC is a model of Z^-.

[2] I will use a slightly unconventional notation: instead of $x \in A$, I will write $\in(x, A)$. This is in order to emphasize that $\in$ is a predicate just as $>, <$, &c. – except set theory only has one predicate.

ZFC

Northwestern

Extensionality. $\forall A \forall B(\forall Ax(\in(x, A) \leftrightarrow \in(x, B)) \rightarrow A = B)$
Union. $\forall A \exists U \forall x(\in(x, U) \leftrightarrow (\exists b \in(b, A)) \in(x, b))$
Pairs. $\forall a \forall b \exists P \forall x(\in(x, P) \leftrightarrow (x = a \vee x = b)$
Powersets. $\forall A \exists P \forall x(\in(x, P) \leftrightarrow (\forall z \in(z, x)) \in(z, A))$
Infinity. $\exists I(\in(\emptyset, I) \wedge \forall x(\in(x, I))(\in(x \cup \{x\}), I))$
Separation. $\forall A \exists S \forall x(\in(x, S) \leftrightarrow (\phi(x) \wedge \in(x, A)))$
Foundation. $\forall A(\forall x \neg \in(x, A) \vee (\exists b \in(b, A))(\forall x \in(x, A)) \neg \in(x, b))$
Replacement. $\{y : \exists x \in(x, A) \phi(x, y)\}$ exists for any A if ϕ is functional.
WO. There's a choice function.

Have C&S used any of these in their paper? It does not seem so. A more pressing problem – were ordered pairs defined anywhere? It does not seem so either. However, what are ordered pairs?

Wiener. $\langle x, y \rangle : \{\{\{a\}, \emptyset\}, \{\{b\}\}\}$
Hausdorff. $\langle x, y \rangle : \{\{a, 1\}, \{b, 2\}\}$
Kuratowski. $\langle x, y \rangle : \{\{a\}, \{a, b\}\}$
&c.

Then, what is
$UG : \langle PHON{-}F, SYN{-}F, SEM{-}F, Select, Merge, Transfer \rangle$?
Recall, C&S define this as "sequences" – but there is no such thing as sequences in the axioms we have just seen. It is not clear what these are and what their structure is. Yet, the structure of the UG-object is very important for a theory of grammar.

Two perspectives

Northwestern

On the one hand, we can say "we have a true theory, e.g., Z^-", and since we can represent countable sequences in this theory (via whatever means), we are fine using the finite sequences. This is a "mathematical" view.

On the other hand, however, we need to know why the structure is what it is linguistically. I.e., why C&S's UG is what it was (what was it underlyingly?, e.g., on Kuratowski's conception); why not $\langle\langle phon-f, syn-f, sem-f\rangle\langle select, merge, transfer\rangle\rangle$. Are all of these innate?

We also ought not forget that in ZFC (and naive set theory – despite its incoherency), **there are no objects other than sets and the empty set**.[3] How to reconcile this with C&S' proposal?

Clearly, the system in C&S does not capture lexical items as a well-order.[4] So a set theory with a specific axiom is needed, one such that accommodates *urelements* – ie, items that are not sets and might not be abstract.[5] However, even if we assume that ZFCU, i.e. ZFC with urelements is taken (or KPU[6] or NGBU[7] or something else[8]), problems persist:

[3]Here we remember von Neumann and his ordinals. (Or Zermelo and his ordinals).

[4]Though it would be a considerable achievement to find a reason why the set of features or lexical items $LI \cong X \subset \mathbb{N}, |X| < \aleph_0$; importantly, $\cong$.

[5]Incidentally, there have been arguments to this effect in Behme (2015). I do not endorse Behme's views in the least, but I do believe in fundamentally biological nature of whatever units we take to be elemetary, features or otherwise (in a zeitgeisty-naturalistic way of Button and Walsh (2018)). This underlies much of the biolinguistic program.

[6]Kripke-Platek s.t., Barwise (1973); Barwise (1975).

[7]Von Neumann-Godel-Bernays s.t.

[8]Tons of them, really. Can make up your own. Avron and Grabmayr (2023) did.

A tangent

Northwestern

Metaphyically speaking, it is important that we decide whether the features we say are innate and are real chemical things (as I am agnostic they are) – or whether we want to model them entirely abstractly a-la von Neumann ordinals.

Mathematically speaking, however, it is not as important – any sentence in our theory of grammar (so only involving countable SOs) we can make in either a theory with urelements or in a theory extending one of the weak theories like Z^{-}, McLane, $\text{GST}^{(?)}$.

Axioms

Northwestern

Abstracting from the fact that set theory was developed for reasons which had little to nothing to do with linguistics – hence there is no reason to think set theory is a good tool to formalize in – let's consider axioms as a phenomenon. They serve a purpose in ZFC or other set theories. They say which constructions are possible and which construction are not possible in a theory. For example, theory Z^- alone (or ZF) cannot describe, e.g. large cardinal property.[9]

As such, axioms of a theory of grammar could or should serve the same purpose: tell is which constructions are acceptable. However, these axioms, which seem to be crucial, are not given in C&S. Some definitions even seem redundant or repeating the axioms:

[9] E.g., $ZFC \nvdash \exists \kappa, \kappa > \alpha, \alpha = \omega_\alpha$.

Pairs?

Northwestern

Definition 14. Given any two distinct syntactic objects A, B, $Merge(A, B) = \{A, B\}$. *(C&S, 47)*

But, axiom of pairs (recall: $\forall a \forall b \exists P \forall x (\in(x, P) \leftrightarrow (x = a \vee x = b))$ of any basic set theory (even naive one) makes this very redundant. (Pairs says: for any a, b, $\{a, b\}$ exists).

What is this "merge" definition doing, then? It is very unclear.

Larger picture

Looking at the larger picture, depending on the set theory C&S might have chosen, the number of constructions they admit is different. Surely, in stating a formalism of a theory of grammar, a number of questions must be answered before it is a viable formalism.[10] Specifically:

[10] Definition of what a formalism is and what requirements we impose on it – as well as why adopt it in the first place is a separate question. At this point, let's just assume following Chomsky (1990) that formalism is a way to avoid ambiguity in stating our theory.

Many questions

Northwestern

We adopt certain language (first-order? second-order? why one vs the other) in which we state our theory $\mathcal{T}$ as a set of axioms (what axioms? e.g., axioms of PA^2). We then would benefit from building some model $\mathcal{M} = (D^{\mathcal{M}}, c^{\mathcal{M}}, R^{\mathcal{M}}, f^{\mathcal{M}})$, specifying what exactly is the domain of the model $D^{\mathcal{M}}$ (perhaps the finite set of features, Marantz (1997)), what exactly are constant(s) $c_1^{\mathcal{M}}, ..., c_n^{\mathcal{M}}$, the relation(s) $R_1^{\mathcal{M}}, ..., R_n^{\mathcal{M}}$ and the functor(s) $f_1^{\mathcal{M}}, ..., f_n^{\mathcal{M}}$.[11,12]

Without answers to these questions, is our theory really made precise?

[11] Then we would be able to say, e.g., that there are many theories of grammars that satisfy some core Minimalist Grammar $\mathscr{MG}$ ($\mathcal{M}_1, ...\mathcal{M}_n \vDash \mathscr{MG}$).

[12] For example, $\mathcal{Z} = (\mathbb{Z}, 0^{\mathcal{Z}}, 1^{\mathcal{Z}}, +^{\mathcal{Z}}, \times^{\mathcal{Z}})$. Or Q; see Boolos (1998) and such for Robinson arithmetic or PAs.

Why?

Northwestern

As such, I am not sure what it is that formalization in C&S achieves. The formal basis remains undefined, some axioms are made redundant by whatever set theory is assumed.

However, the linguistic questions also persist (though they arguably fall outside the immediate scope of C&S): if this is the formalization of a fragment of a theory of grammar, then can the answers to questions of availability, but more importantly, learnability, be given? I.e., how (formally) learner comes by such a UG? The units are also not specified – are features valued TF? TFX? something else? Ostensitve definitions of sets of features does not suffice.

Broadly similar objections apply to Collins and Groat (2018) and similar work. For example, it is incorrect to assume that graph-theoretic objects are inherently not set-theoretic or cannot be expressed as such. After all, set theory can serve as foundations of math, graph theory included (particularly a version where only directed graphs are permissible). On a similar note, recall that $\{a, b\} = \in (a, X) \wedge \in (b, X)$. [13]

[13] It is never made clear why this is a conjunction or relation and not a simple truth-functional conjunction. Set X is never discussed in Chomsky either. It is simply incorrect to claim over and over that this (axiom of pairs) is the simplest operation. I think $a \wedge b$ might do just fine in a pinch.

More on graphs

Northwestern

Okay, let multidominant object be a graph-theoretic object. However, what if we argue that graphs are primary and sets are secondary? I.e.,

> *"Given some structured object a in the world, we may (in theory, at least) represent its hereditary constituency relation by means of a graph and thereby obtain a 'set-theoretic' model of a by moving from the graph to the set it depicts-namely, the set that corresponds to the top node of the graph." (Devlin 1994, p. 150)*

Indeed, we can have a reasonable axiom which says that every directed graph corresponds to exactly one set, i.e. is uniquely realized by $\in$ (anti-foundation axiom of Peter Aczel, sec. Keith Devlin, 1994). A nice consequence – every set can be pictured by a tree, every tree can be captured by a set.

There is no reason to assume that somehow sets are fundamental. We assume so because, presumably, Chomsky said so. This is not a very good reason. Unless we find a good reason, given this bijection from sets to graphs, we need to abandon this uninformed assumption.

Chomsky's unidominant claims Northwestern

> *The tree notations are kind of convenient, but they're very misleading and you should really pay no attention to them. For one thing, a tree notation kind of leads you to suggest that there has to be something at the root of the tree. But that's conflating compositionality with projection. And in fact you often don't have anything at the root of the tree — for example, every exocentric construction. (Chomsky (2020), page 37f)*

As mentioned above, few things are as misleading omitting to state that $Merge(a,b) = \{a,b\} = \in(a,X) \wedge \in(b,X)$ has to involve some never-specified set X. **Note** that this **not** labelling: $\in(a,X) \wedge \in(b,X) \neq \in(a,A) \wedge \in(b,A) \wedge \in(A,C) \wedge \in(a,C)$ such that $\{a,\{a,b\}\}$.

"The simplest"

Northwestern

Chomsky, again, and again, and again, states:

> *The simplest object constructed from* α *and* β *[two arbitrary SOs] is the set* $\{\alpha, \beta\}$*, Chomsky (1994)*
>
> *The operation Merge forms K [an SO] from* α, β*. Minimally, K should consist only of* α *and* β*, so* $K = \{\alpha, \beta\}$ *Chomsky (1998)*
>
> *NS [narrow syntax, bare computational component of language faculty] has one operation that comes "free," in that it is required in some form for any recursive system: the operation Merge, which takes two elements* α, β *already constructed and creates a new one consisting of the two; in the simplest case,* $\{\alpha, \beta\}$*. The operation yields the relation* $\in$ *of membership, and assuming iterability, the relation dominate (contain) and term-of. Chomsky (2001)*

Moving on

Northwestern

At this time, let's move on to multidominance proper, and the last claim: Chomsky's putative, ill-defined, and ambiguity-ridden "third-factor" economy gives rise to many interpretations – and on some of them **(even conservative!)**, **multidominant derivations are more economic than conservative merge-based ones**.

Economy

Northwestern

Various "economy conditions" have been posed by Chomsky and colleagues, e.g.:

- *Inclusiveness*: No new features are introduced by $\mathcal{C}_{\mathrm{HL}}$.[14]
- *PIC.* In phase $\mathscr{P}$ with head H, the domain of H is not accessible to operations outside $\mathscr{P}$, only H and its edge are accessible to such operations.
- *NTC.* SOs remain unchanged by Merge;
- &c.

In this conservative view, for example, SO a from $\{c, \{a, b\}\}$ should not be available for $M(a, \{c, \{a, b\}\}) = \{a, \{c, \{a, b\}\}\}$.

[14] Computational procedure for human language.

UCLA Northwestern

So there's got to be something about the definition of MERGE that's telling you can't create Parallel Merge structures. And if you think about it for a second, you can see what it is. Too many accessible elements have been produced. The right definition of MERGE, when we get to it, should allow only one new accessible object—namely the one you're constructing. When you put P and Q together, you're constructing a new object, the set $\{P, Q\}$. That set is accessible to computation, but nothing else should be. That should be the right definition of MERGE. (It might seem as though this excludes internal MERGE, but as we will see directly, it does not). Parallel Merge is adding two new accessible objects. It's adding the new set $\{b, c\}$ but also the new occurrence of b in $\{b, c\}$. And that's too many. (Chomsky (2020), page 39f)

Thinking a second

Indeed, let's think about it for a second. The first meaningful question we can pose after thinking a second is *What restricts accessibility and why?*, i.e. *What is economy?*

> *There are other factors, sometimes called third factor properties, just general principles of growth and development, which are independent of language – maybe natural laws. For a computational system like language, the natural one to look at is computational efficiency, just assuming, for the reasons I mentioned last time, that nature seeks the simplest answer: so computational efficiency plus the simplest computational operation plus whatever contribution data makes, that should yield the I-language attained. That's the goal. This program has been called the Minimalist Program. (Chomsky (2020), page 21)*

So, we do not know what economy is (= restrict resources = computational efficiency). This is "a conjecture", "an idea" lacking precise definition, linguistic or otherwise. Let's try to make sense of this and watch out for surprising consequences.

Economy

Northwestern

There are different processes involved in the derivation, hence there must be different types of economy or computational efficiency – mirroring those processes. Let us distinguish at least four types of resources/costs:

(1) a. memory costs: storing items in workspace;
    b. processing costs: putting/taking items in/from workspace and using them in derivation requires;
    c. search costs: costs for pulling objects from The Lexicon (the Select operation, seemingly);
    d. accessibility costs: being able to access items in previous steps of the derivation.

Select

So, the question is being posed as follows: how do multidominant vs merge-conservative derivations fare in terms of the costs? We investigate exactly that.

Importantly, we do not consider search costs – as we'll see any asymmetry is mirrored by processing costs. Further, we can just consider search costs a subtype of processing costs.[15]

[15] More specifically, however, it is important that we understand that in absence of a specific definition of lexicon and its structure, we cannot talk about searching and selecting meaningfully. Some investigations are suggestive, like Emonds (2003)'s Syntacticon or Pustejovsky (1998) but they are far from minimalistic proposals which could provide a viable theory of lexicon for minimalists. I will come back to this point.

Taking thins for granted

Northwestern

Of course, we here are taking a whole lot of things for granted. For example, that there is such a thing as a workspace, &c.

Derivations

Now, let's compare the derivations. For conservative:
(li – number of lexical items, num = number of occurrences of movement; see full code at the end of the slides)

```
def conservative(li, num):
  memory = 0, processing = 0, accessibility = 0, n = 0
  sentence = [], workspace = []
  # forming worspace and storing items there
  for i in li:
    workspace.append(i)
    memory += 1, accessibility += 1, processing += 1, n += 1
  # add a few repeated items to the workspace as occurrences
  for i in range(num):
    workspace.append(random.choice(li))
    memory += 1, accessibility += 1, processing += 1, n += 1
  # derivation: merge costs for processing = 3; two to pull the objects and one to merge
  for i in range(len(workspace)-1):
    n += 1, processing += 3, accessibility -= 1, memory += 0
return null
```

Listing 1: Conservative

Derivations (cont’d)

Northwestern

For multidominant:

```python
def multidominant(li, num):
 memory = 0, processing = 0, accessibility = 0, n = 0
 sentence = [], workspace = []
 # forming worspace and storing items there
 for i in li:
  workspace.append(i)
  memory += 1, accessibility += 1, processing += 1, n += 1
 # derivation; merge costs for processing = 3; two to pull the objects and one to merge
 for i in range(len(workspace)-1):
  n += 1, processing += 3, accessibility += 0 # newly object isn’t accessible
  memory += 1
# account for movement
 for i in range(num):
  n += 1, processing += 3, memory += 1
  return null
```

Listing 2: Multidominant

Visualizing

Let's say there's 10 lexical items and 5 instances of movement:

(Items accessible at any given point are in blue, memory expense is in red, and processing is in green.)

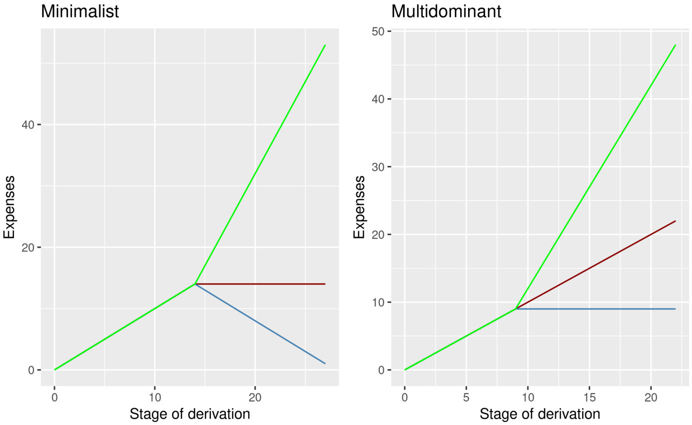

Figure: 10 li, 5 mv

Visualizing (cont'd)

Let's say there's 100 lexical items and 200 instances of movement (everything moves twice):

(Items accessible at any given point are in blue, memory expense is in red, and processing is in green.)

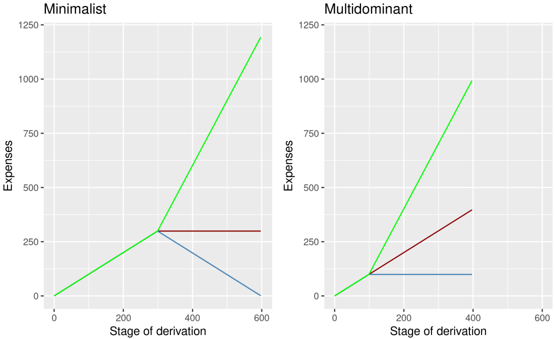

Figure: 100 li, 200 mv

Not a surprise

Northwestern

It is not at all a big surprise, then, when we see that indeed, multidominant structure puts fewer objects in workspace and is more eonomic once there's more instances of movement than unique LIs. Occurrences, in the conservative merge, just clutter the workspace. Now, are there conceptions on which conservative merge wins? Yes, the ones on which costs for the three operations are different and not the same as we stipulated, or when the costs of search is not equivalent to processing and occurrences are not searched for but just copied locally in the WS. All of these have to be justified and not just stipulated in a profoundly definitionless way as in Chomsky (2020) and much of the preceding work.

Whoa

The are few things we know about language. It's fundamentally not linear, for example, and prone towards long-distance dependencies of arbitrary length. Another one: everything moves. As such, it is not outlandish to assume that there's more instances of movement than LI put in the WS. The code above shows a viable model of calculating resources across three axes, and total resources for configurations where $|LI| < |Mv|$ is lower for multidominant derivations. As such, the biggest pillar of Chomskian objection to multidominance, viz. putative third-factor "economy" actually advocates **for** multidominance, not against it.

~~Conclusions~~ Suggestions Northwestern

- **Before** formal-talk and debates in the area, it is suggested to read Partee, Meulen, and Wall (1993) and then some Kracht. Hopefully something on logic, set theory, and model theory along the way.
- Abandon the definitionless way of doing things and either find a solid ground and definitions for economy, or abandon it.
- Propose a clean and precise formalization of a theory of grammar and hopefully acquisition. (Perhaps agree on the purposes of fornalization and what constitutes one.)
- Think about epistemic compactness (Church (1926)) and empirical viability.
- It seems multidominance along the lines of parallel merge can accommodate reanalysis in parsing, no? Isn't it economical to have merge as a parsing module too. We seem to forget to talk about parsing module, generally. It's also UG; very much so.

Two takeaways

Northwestern

No. 1

We are in dire need of precision in our discussions of theories of syntax. Such precision could be given by a formal system which leaves no space for ambiguity. However, it seems recent formal development in minimalist syntax only aggravate the existing issues. Just taking Chomsky's arbitrary conjectures on board does not help either.

No. 2

Given no definition whatsoever of Chomskian third-factor economy, a second's thought can discern at least three kinds of syntactic economy. These, defined and modelled, point to multidominance being more economic than Chomsky's conservative merge.

References I

Northwestern

Avron, Arnon and Balthasar Grabmayr (Feb. 2023). "Breaking the Tie: Benacerraf's Identification Argument Revisited". In: *Philosophia Mathematica* 31(1), pp. 81–103. ISSN: 1744-6406. DOI: 10.1093/philmat/nkac022. URL: https://doi.org/10.1093/philmat/nkac022 (visited on 04/15/2023).

Barwise, Jon (1973). "Admissible sets over models of set theory". In: *Generalized Recursion Theory*. Ed. by J. E. Fenstad and P. Hinman. Proceedings of the 1972 Oslo symposium. Elsevier: North-Holland, Amsterdam.

Barwise, Jon (1975). *Admissible Sets and Structures: An Approach to Definability Theory*. Perspectives in Logic 7. Association for Symbolic Logic. ISBN: 3-540-07451-1.

References II

Northwestern

Behme, Christina (2015). "Is the ontology of biolinguistics coherent?" en. In: *Language Sciences* 47, pp. 32–42. ISSN: 0388-0001. DOI: 10.1016/j.langsci.2014.07.012. URL: https://www.sciencedirect.com/science/article/pii/S0388000114000758 (visited on 05/17/2022).

Boolos, George (1998). *Logic, logic, and logic*. Harvard University Press: Cambridge, MA. ISBN: 0-674-53766-1.

Chomsky, Noam (1990). "Topic...comment: On formalization and formal linguistics". en. In: *Natural Language & Linguistic Theory* 8(1), pp. 143–147. ISSN: 1573-0859. DOI: 10.1007/BF00205534. URL: https://doi.org/10.1007/BF00205534 (visited on 12/23/2021).

Chomsky, Noam (1994). *Bare phrase structure*. MIT Occasional Papers in Linguistics 5. MIT Working Papers in Linguistics: Cambridge, MA.

References III

Northwestern

Chomsky, Noam (1998). *Minimalist inquiries: The framework.* MIT Occasional Papers in Linguistics 15. MIT Working Papers in Linguistics: Cambridge, MA.

Chomsky, Noam (2001). *Beyond explanatory adequacy.* MIT Occasional Papers in Linguistics 21.

Chomsky, Noam (2020). *The UCLA Lectures.* URL: https://ling.auf.net/lingbuzz/005485 (visited on 04/23/2022).

Church, Alonzo (1926). "On irredundant sets of postulates". en. In: *Bulletin of the American Mathematical Society* 32(6), pp. 626–628. ISSN: 0002-9904, 1936-881X. DOI: 10.1090/S0002-9904-1926-04271-3. URL: https://www.ams.org/bull/1926-32-06/S0002-9904-1926-04271-3/ (visited on 03/22/2022).

References IV

Northwestern

- Collins, Chris and Erich Groat (2018). *Copies and Repetitions*. URL: https://ling.auf.net/lingbuzz/003809.

- Collins, Chris and Edward Stabler (2016). "A Formalization of Minimalist Syntax". en. In: *Syntax* 19(1), pp. 43–78. ISSN: 1467-9612. DOI: 10.1111/synt.12117. URL: https://onlinelibrary.wiley.com/doi/abs/10.1111/synt.12117.

- Marantz, Alec (Jan. 1997). "No Escape from Syntax: Don't Try Morphological Analysis in the Privacy of Your Own Lexicon". In: *University of Pennsylvania Working Papers in Linguistics* 4(2). URL: https://repository.upenn.edu/pwpl/vol4/iss2/14.

- Partee, Barbara H., Alice Meulen, and Robert E. Wall (1993). *Mathematical Methods in Linguistics*. en. Studies in Linguistics and Philosophy 30. Springer Dodrecht. ISBN: 978-90-277-2244-7. URL: https://link.springer.com/book/10.1007/978-94-009-2213-6.

Thanks!

Acknowledgements: I am very grateful to Masaya Yoshida for many, many discussions – some relating to this topic and for teaching me about syntax and being flexible with all of my ideas. I also thank Sean Ebels-Duggan for making sure (on a very short notice) I do not say anything nonsensical about set theory. DISCLAIMER: ***Given the potential of some these ideas to be divisive – trivially – views that were expressed in this presentation are my own and not necessarily shared by Masaya or Sean.***

Daniil M. Ozernyi

Benjamin W. Slivka Hall
2332 Campus Dr.
Northwestern University
Evanston, IL 60208

doz@u.northwestern.edu

Slides available at **dozernyi.com/work**

Derivations – bonus

Northwestern

```
from prettytable import PrettyTable
import random

lexicon = [str(i) for i in range(1,100)]

def minimalist(li, num):
  t = PrettyTable(['#', 'Memory', 'Items accessible', "Processing (sum)"])
  memory = 0
  processing = 0
  accessibility = 0
  n = 0
  t.add_row([f'{n}', f"{memory}", f"{accessibility}", f"{processing}"])
  sentence = []
  workspace = []
  # forming worspace and storing items there
  for i in li:
    workspace.append(i)
    memory += 1
    accessibility += 1
    processing += 1
    n += 1
    t.add_row([f'{n}', f"{memory}", f"{accessibility}", f"{processing}"])
  # add a few repeated items to the workspace as occurrences
  for i in range(num):
    workspace.append(random.choice(li))
    memory += 1
    accessibility += 1
    processing += 1
    n += 1
    t.add_row([f'{n}', f"{memory}", f"{accessibility}", f"{processing}"])
```

Derivations – bonus (cont'd)

Northwestern

```
# derivation
# merge costs for processing = 3; two to pull the objects and one to merge
for i in range(len(workspace)-1):
  n += 1
  processing += 3
  accessibility -= 1
  memory += 0
  t.add_row([f'{n}', f"{memory}", f"{accessibility}", f"{processing}"])
  with open('minimalist.csv', 'w', newline='') as f_output:
    f_output.write(t.get_csv_string( ))
return print(t)
```

Derivations – bonus (cont'd)

Northwestern

```
def multidominant(li, num):
  t = PrettyTable(['#', 'Memory', 'Items accessible', "Processing (sum)"])
  memory = 0
  processing = 0
  accessibility = 0
  n = 0
  t.add_row([f'{n}', f"{memory}", f"{accessibility}", f"{processing}"])
  sentence = []
  workspace = []
  # forming worspace and storing items there
  for i in li:
    workspace.append(i)
    memory += 1
    accessibility += 1
    processing += 1
    n += 1
    t.add_row([f'{n}', f"{memory}", f"{accessibility}", f"{processing}"])
  # derivation
  # merge costs for processing = 3; two to pull the objects and one to merge
  for i in range(len(workspace)-1):
    n += 1
    processing += 3
    accessibility += 0 # newly object isn't accessible
    memory += 1
    t.add_row([f'{n}', f"{memory}", f"{accessibility}", f"{processing}"])
  # account for movement
  for i in range(num):
    n += 1
    processing += 3
```

Derivations – bonus (cont'd)

Northwestern

```
    memory += 1
    t.add_row([f'{n}', f"{memory}", f"{accessibility}", f"{processing}"])
    with open('multidominant.csv', 'w', newline='') as f_output:
      f_output.write(t.get_csv_string( ))
  return print(t)
```

Derivations – bonus (cont'd) Northwestern

```

lexicon = [str(i) for i in range(1,100)]

minimalist(lexicon, 200)

multidominant(lexicon, 200)
```