

Rethinking

THE WAY FORWARD

Expectations

FOR MACROECONOMICS

Edited by

Roman Frydman

Edmund S. Phelps

The Imperfect Knowledge Imperative in Modern Macroeconomics and Finance Theory

Roman Frydman and Michael D. Goldberg

Quite apart from the fact that we do not know the future, the future is objectively not fixed. The future is open: objectively open.

Karl R. Popper, *A World of Propensities* (1990)

I confess that I prefer true but imperfect knowledge . . . to a pretense of exact knowledge that is likely to be false.

Friedrich A. Hayek, *Nobel Lecture* (1974)

4.1 Introduction

Modern macroeconomics constructs models of aggregate outcomes on the basis of mathematical representations of individual decisionmaking, with market participants' forecasting behavior lying at the heart of the interaction between the two levels of analysis. Individuals' forecasts play a key role in how they make decisions, and markets aggregate those decisions into prices. The causal processes underlying both individual decisions and aggregate outcomes, therefore, depend on market participants' understanding of the economy and on how they use this knowledge to forecast the future.

Over the past four decades, economists have come to a nearly universal consensus that the Rational Expectations Hypothesis (REH) is the way to represent how rational, profit-seeking market participants forecast the future. Even behavioral economists, who have uncovered much evidence of REH's empirical failure, generally subscribe to this belief and have interpreted their findings as evidence that individuals fall short of "full rationality."

In this chapter, we argue that REH has no connection to how even minimally reasonable profit-seeking individuals forecast the future in real-world markets. We trace the root of REH's insurmountable epistemological difficulties and widespread empirical problems to a single, overarching premise that underpins contemporary macroeconomics and finance theory:

The authors are grateful to the Institute for New Economic Thinking for supporting the research on which this chapter is based.

nonroutine change—change that does not follow mechanical rules and procedures—is unimportant for understanding outcomes.

We also point out that contemporary behavioral finance models rest on the same core premise as their REH-based counterparts. Behavioral finance theorists claim that their portrayals of individual behavior are more "realistic." However, the assumption that nonroutine change is unimportant for understanding individual decisionmaking implies that their models, too, lack plausible microfoundations.

We also sketch an alternative approach to modeling individual behavior and aggregate outcomes: Imperfect Knowledge Economics (IKE). IKE opens macroeconomics and finance models to nonroutine change and the imperfect knowledge that it engenders, which is necessary to render their microfoundations both plausible and compatible with individual rationality.

4.2 The Pretense of Exact Knowledge

On the occasion of his 1974 Nobel lecture, Friedrich Hayek appealed to fellow economists to resist the "pretense of exact knowledge" in economic analysis. Drawing on his prescient analysis of the inevitable failure of central planning, Hayek warned against the lure of predetermination: no economist's model would ever render fully intelligible the causes of market outcomes or the consequences of government policies.

Ignoring Hayek's warning, contemporary macroeconomists and finance theorists have been much less circumspect about the ability of economic analysis to uncover the causal mechanism that underpins market outcomes. In fact, nearly all economists have come to believe that, to be worthy of scientific status, economic models should generate "sharp" predictions that account for the full range of possible market outcomes and their likelihoods.¹ But to construct such models, which we refer to as "fully predetermined," contemporary economists must fully specify in advance whether and how market participants alter their decisionmaking, and whether and how the social context—including economic policies, social and political factors, and institutions—unfolds over time. Contemporary models, therefore, rule out *by design* nonroutine change.

4.3 Assuming Away Nonroutine Change

In modeling the microfoundations of their models, economists relate an individual's preferences, forecasting strategy, and the constraints that she

1. For a comprehensive treatment of the concept of sharp prediction in the context of fully predetermined probabilistic models, see Sargent (1987) and Frydman and Goldberg (2007: chapters 3, 4, and 6).

faces to some set of causal variables. Assuming that an individual chooses the option that, according to her forecasting strategy, will maximize her well-being, an economist represents her decisionmaking in terms of the causal variables and parameters appearing in each of the components—preferences, forecasting strategy, and constraints. The functional form of such a representation of optimal decisions, its parameters, and the properties of the causal variables constitute the causal structure of the microfoundations of macroeconomic models.

An economist formalizes his assumptions about how an individual makes decisions with restrictions that constrain the structure of his model and how it might change over time. Alternative sets of restrictions permit economists to formalize alternative causal accounts of outcomes. Although contemporary macroeconomic and finance models differ in their specifications on both the individual and aggregate levels, they *all* share one core feature: restrictions that exactly relate the properties of the model's causal structure at all points in time, past and future, to the properties of the structure at some "initial" point in time.

4.3.1 The Causal Structure

At each point in time, the structure of an economist's representation is characterized by the following properties:

1. The composition of the set of causal variables.
2. The properties of the joint probability distribution of the causal variables.²
3. A functional form that relates outcomes to the causal variables, which typically includes the signs of partial derivatives. In cases such as the example we examine here, in which the functional form is explicit, economists often restrict the signs of some parameters.

Contemporary macroeconomists and finance theorists assume away non-routine change by fully prespecifying how the structure of their models changes over time, which is illustrated by our simple algebraic example. Later, we use this example to show how assuming away nonroutine change led economists to the nearly universal, yet fundamentally misguided, belief that REH is the only scientific way to represent rational forecasting. We also make use of this simple model to show how IKE provides macroeconomic models with plausible individual foundations.

2. If the model includes additive error terms, the conditions imposed by an economist also specify the joint probability distribution between these terms and the causal variables.

4.3.2 A Fully Predetermined Model of an Asset Price

Our example is motivated by basic supply-and-demand analysis in financial markets. In modeling an individual's demand for and supply of an asset, economists typically relate these to her forecast of the asset's future price and a set of causal variables. Aggregating over individuals and equating aggregate demand and supply typically yields the following representation, in semi-reduced form, for the equilibrium market price at a point in time t :³

$$P_t = a_t + b_t X_t + c_t \hat{P}_{t|t+1}, \quad (1)$$

where $\hat{P}_{t|t+1}$ is an aggregate of market participants' forecasts formed at t of the market price at $t + 1$, (a_t, b_t, c_t) is a vector of parameters, and X_t is a set of causal variables. These variables typically represent the unfolding of economic policies, including those affecting the money supply, interest rates, or tax rules. Sometimes the causal variables include factors that represent other aspects of the social context in which an individual makes decisions, such as institutional and regulatory changes.

Individual forecasts that comprise the aggregate forecast, $\hat{P}_{t|t+1}$, are formed on the basis of forecasting strategies at t . Economists model these strategies by relating them to a set of causal variables, which represents the information sets used by market participants. An aggregate of such representations can be written as

$$\hat{P}_{t|t+1} = \alpha_t + \beta_t Z_t \quad (2)$$

where Z_t is a vector of variables that characterizes the union of information sets used by market participants, and (α_t, β_t) is a vector of parameters.

4.3.3 Fully Predetermining Restrictions

In general, as time passes, individuals alter the way they make decisions. Institutions, economic policies, and other factors also change over time. These changes influence the way aggregate outcomes move over time. Thus, to model how market outcomes unfold over time, an economist will need different structures—different specifications of forecasting, preferences, constraints, decision and aggregation rules, or the processes driving the causal variables—at different points in time to represent individual behavior.

Remarkably, contemporary macroeconomists typically constrain the structure of their models to remain unchanging over time. As we shall discuss shortly, except for random deviations that average out to zero, these models rule out altogether the importance of change on the individual and

3. In Frydman and Goldberg (Chapter 6, this volume), we derive the aggregate representation for the movement of equity prices of the form in (1) from explicit microfoundations.

aggregate levels for understanding outcomes. In those relatively infrequent cases in which contemporary models do allow for change in their structure, they fully prespecify when it occurs. They also specify in advance the structure of the post-change representation of outcomes on the individual and aggregate levels.

To illustrate how this is done, we focus on revisions in forecasting strategies and constrain the structure of the other components of the model to be time invariant. The following constraints in (1) impose time invariance on the nonexpectational components of the model:

- The composition of the set of causal variables, X_t , and the properties of their joint probability distribution remain unchanged at all times, past and future.
- The parameters (a_t, b_t, c_t) are constants, that is, $(a_t, b_t, c_t) = (a, b, c)$ for all t .⁴

In general, the representation of revisions in forecasting strategies may involve a change in the composition of the set of causal variables, Z_t , or even different functional forms. Because these complications would not affect any of our conclusions, we suppose that an economist represents revisions of forecasting strategies with a parametric shift in his aggregate representation at $t + 1$ and that he assumes that these strategies will remain unchanged thereafter:

$$\hat{P}_{t+\tau|t+\tau+1} = \alpha_{t+\tau} + \beta_{t+\tau} Z_{t+\tau} \quad (3)$$

where $\alpha_t \neq \alpha_{t+1}$, $\beta_t \neq \beta_{t+1}$, $\alpha_{t+\tau} = \alpha_{t+\tau+1}$, and $\beta_{t+\tau} = \beta_{t+\tau+1}$ for all $\tau = 1, 2, 3, \dots$. In this example, revisions, which are set to occur only at $t + 1$, are represented by two constants, $\alpha_{(t,t+1)}$ and $\beta_{(t,t+1)}$:⁵

$$\alpha_{(t,t+1)} = \alpha_{t+1} - \alpha_t \quad \text{and} \quad \beta_{(t,t+1)} = \beta_{t+1} - \beta_t \quad (4)$$

Contemporary economists fully prespecify revisions of forecasting strategies, which can be simply represented as constraining $\alpha_{(t,t+1)}$ and $\beta_{(t,t+1)}$ to

4. For the sake of simplicity, we display time-invariance constraints only on the aggregate level. However, the parameters and causal variables in (1) arise from the nonexpectational components on the individual level. Thus, the time-invariance constraints implicitly apply to these components of the model's microfoundations.

5. Except for purely formal complications, our conclusions in this section apply to nonlinear representations. For example, suppose that the representation of the aggregate forecasting strategy at $t + 1$ is a nonlinear function of the causal variables. In such a case, $\alpha_{(t,t+1)}$ and $\beta_{(t,t+1)}$ would be nonlinear functions of the causal variables.

be equal to particular values, say, $\bar{\alpha}$ and $\bar{\beta}$, respectively:

$$\bar{\alpha} = \alpha_{t+1} - \alpha_t \quad \text{and} \quad \bar{\beta} = \beta_{t+1} - \beta_t \quad (5)$$

We refer to such constraints as “fully predetermining restrictions.”⁶

Sometimes fully predetermining restrictions are probabilistic. For example, an influential class of contemporary models fully prespecifies the timing of all changes with a Markov switching process. At any point in time, t , this rule exactly relates the timing of future change and the switch to the fully prespecified post-change structure of (2) to the structure at t .⁷ Frydman and Goldberg (2007: chapters 4 and 6) show that all our conclusions in this chapter, derived in the context of the simple model presented here, apply to models that use fully prespecified probabilistic rules to represent change.

To complete the full prespecification of change in their models, economists also prespecify how the social context in which individuals forecast the future and make decisions unfolds over time. This is typically done by representing the processes that govern the movements of the causal variables with standard time-series models. These movements are driven by stochastic “shocks,” the probability distributions of which are also fully predetermined.

To simplify our presentation, assume that each set of causal factors in (1) and (2), X_t and Z_t , consists of only one causal variable, x_t and z_t , respectively.⁸ We make use of the following simple representations of these variables:

$$x_t = \mu_x(1 - \rho_x) + \rho_x x_{t-1} + \epsilon_t^x \quad (6)$$

$$z_t = \mu_z(1 - \rho_z) + \rho_z z_{t-1} + \epsilon_t^z \quad (7)$$

where $\mu_x, \mu_z, \rho_x, \rho_z$ are constant parameters, $|\rho_x| < 1$, $|\rho_z| < 1$, and ϵ_t^x and ϵ_t^z are random shocks. As is customary in the literature, we refer to the causal variables appearing in the representation of forecasting strategies as “information” and to the shocks to these variables as “news.”

Once an economist portrays causal factors as random variables, his representations become probabilistic. To render them fully predetermined, economists specify in advance the probability distribution governing the random shocks. We follow the usual practice and constrain these shocks

6. The imposition of time invariance, which is common in contemporary models, thus involves a particularly simple form of fully predetermining restrictions: $\bar{\alpha} = 0$ and $\bar{\beta} = 0$.

7. For the seminal formulation of such models, see Hamilton (1988, 1994).

8. The set Z_t often includes endogenous variables, such as the current asset price P_t . We omit such variables here, as allowing for them would complicate our analysis without affecting our general conclusions.

to be drawn from an unchanging distribution with a mean of zero and constant variances, $\sigma_{\epsilon_x}^2$ and $\sigma_{\epsilon_z}^2$, respectively. For the sake of simplicity, we also constrain these shocks to be uncorrelated over time and uncorrelated with each other at every point in time. Such invariant distributions of shocks are a special case of standard probabilistic representations of uncertainty, which we refer to as “fully predetermined probability distributions.”

4.4 Sharp Predictions of Nothing New

The fully predetermined distribution of shocks and the fully predetermined—and time-invariant—processes governing the movements of the causal variables immediately imply that the joint probability distribution of x_t and z_t is also fully predetermined.

Thus, conditional on realizations of the shock, ϵ_t^z , from time t to $t + \tau$, and the structure of processes governing its movement over time, (7), the overarching probability distribution characterizes $z_{t+\tau}$ for all $\tau = 0, 1, \dots$:

$$z_{t+\tau} = \mu_z [1 - (\rho_z)^\tau] + (\rho_z)^\tau z_t + \epsilon_{(t,t+\tau)}^z \quad (8)$$

where

$$\epsilon_{(t,t+\tau)}^z = \sum_{j=1}^{\tau} (\rho_z)^j \epsilon_{t+\tau-j}^z \quad (9)$$

Similarly, $x_{t+\tau}$ can be written in terms of μ_x , ρ_x , and $\epsilon_{(t,t+\tau)}^x$.

The representation in (8) shows that by specifying information to evolve according to a mechanical rule, an economist in effect presumes that he can fully prespecify changes in the social context. Once this presumption is combined with a fully prespecified representation of revisions in forecasting strategies, (5), an economist can produce a “sharp prediction” of the one-period-ahead forecasts and their probabilities at any time $t + \tau$, conditional on the structure of the model and the realization of the causal variable at t :

$$\hat{P}_{t+\tau|t+\tau+1} - \hat{P}_{t|t+1} = \hat{A} + \hat{B}z_t + (\beta_t + \bar{\beta})\epsilon_{(t,t+\tau)}^z \quad \text{for } \tau = 1, 2, \dots \quad (10)$$

where $\hat{A} = \{\bar{\alpha} + (\beta_t + \bar{\beta})\mu_x [1 - (\rho_x)^\tau]\}$, and $\hat{B} = \bar{\beta} + (\beta_t + \bar{\beta}) [(\rho_x)^\tau - 1]$.

Analogously to (8), the representation in (10) decomposes change in the one-period-ahead forecast formed at t and $t + \tau - (\hat{P}_{t+\tau|t+\tau+1} - \hat{P}_{t|t+1})$ into two fully predetermined components. The first is the expectation of change

$\hat{A} + \hat{B}z_t$, which an economist presumes to know exactly, conditional on the structure of the parameters of the forecasting strategies at t ; their change at $t + 1$, $(\bar{\alpha}, \bar{\beta})$; and current and past information, as summarized in z_t . As time passes, this fully predetermined path of forecast revisions varies with new realizations of information triggered by the second component of change: future news, $(\beta_t + \bar{\beta})\epsilon_{(t,t+\tau)}^z$.

This second component represents the only uncertainty in an economist’s model concerning future information that market participants might consider relevant in forming their forecasts. Contemporary models constrain news to evolve according to an overarching probability distribution that fully predetermines its possible realizations and their probabilities in all time periods.

Thus, contemporary representations of forecast revisions assume away the possibility that participants might revise their forecasting strategies, or that the social context might change, in ways that cannot be fully foreseen—that is, that an economist cannot characterize with a standard probability distribution. An analogous argument shows that contemporary models also assume away nonroutine change in other aspects of behavior on the individual and aggregate levels.

As they do for forecast revisions, these models represent change in aggregate outcomes with two fully predetermined components, each presuming that nothing genuinely new can ever happen:

$$P_{t+\tau} - P_t = [A + Bx_t + B^z z_t] + [b\epsilon_{(t,t+\tau)}^x + c(\beta_t + \bar{\beta})\epsilon_{(t,t+\tau)}^z] \quad (11)$$

where $A = b\mu_x [1 - (\rho_x)^\tau] + c\hat{A}$, $B = b [(\rho_x)^\tau - 1]$, $B^z = c\hat{B}$, and \hat{A} and \hat{B} are defined in (10).

4.5 Fully Predetermined Rationality

We have shown how insistence on sharp predictions requires an economist to embrace contemporary models’ core premise that nonroutine change is unimportant for understanding outcomes. Moreover, this premise has led economists to believe that they have discovered a universal way to represent how rational individuals make decisions.

To select and justify the particular parametric functions that they use to represent “rational” preferences in the microfoundations of their models, economists appeal to a set of a priori assumptions, which postulate that an individual’s choices among the available options follow a consistent pattern. The structure of such preference representations is typically constrained to remain unchanged over time.

Macroeconomists typically assume that the consequences of each option are uncertain. To rank such options, conventional economists have relied on the expected utility hypothesis (von Neumann and Morgenstern 1944). But, to represent expected utility, an economist must represent how a “rational” individual assesses the probabilities that she attaches to the consequences of choosing alternative options, as well as how she revises these assessments over time. Nearly all economists have come to believe that REH provides their models’ fully predetermined microfoundations with the forecasting component needed to generate such assessments.

4.5.1 The Rational Expectations Hypothesis

Prior to REH, economists portrayed market participants’ forecasting strategies with mechanical rules that made no explicit reference to the way the economy works or how the causal process underpinning outcomes might change over time.⁹ In an attempt to incorporate such considerations into representations of forecasting, John Muth proposed the Rational Expectations Hypothesis: market participants’ forecasts “are essentially the same as the predictions of the relevant economic theory” (Muth 1961: 316).

Muth’s idea was that economists, by representing participants’ forecasting strategies with a model that adequately described their understanding of the causal process underpinning outcomes, would be able to “make sensible predictions about the way expectations would change . . . when the structure of the system is changed” (Muth 1961: 315–316).

Muth was well aware that the term “rational expectations” suggests some notion of rationality. Indeed, he explicitly warned that REH should not be viewed as a normative hypothesis about how rational individuals should forecast the future. As he put it, “at the risk of confusing this *purely descriptive hypothesis* with a pronouncement as to what firms ought to do, we call such expectations ‘rational’ ” (Muth 1961: 316, emphasis added).

Even viewed as a purely descriptive hypothesis, it is far from clear how REH should be used to describe market participants’ forecasting strategies. To implement REH, economists had to take a stand on the question of “the relevant economic theory” to which the hypothesis refers.

Muth, in illustrating REH, implemented it in a time-invariant model of a market for an agricultural good that is produced with a production lag. This lag requires that farmers must form forecasts of its future market price to decide how much of the good to produce. Muth represented these forecasts by equating farmers’ expectations regarding the market price at t to be equal to the prediction of that price, implied by his own model, at $t - 1$.

9. The most widely used rule was called “adaptive expectations,” originally formulated by Cagan (1956), Friedman (1956), and Nerlove (1958).

Examining his model, Muth then observed that, were his representation of the aggregate of participants’ forecasting strategies to differ from his model’s sharp prediction—the conditional probability distribution—of the market price, his representation of participants’ forecasting would result time and again in obvious and systematic forecast errors. Although Muth’s model represented an aggregate outcome in a particularly simple market, it is easy to see that this implication holds in the context of any fully predetermined model. For example, the forecast errors, $(P_{t+1} - \hat{P}_{t|t+1}^{FP})$, resulting from a fully predetermined non-REH forecasting strategy, represented in our model with (2) and (5), are obviously and systematically correlated with the information at time t :

$$E_M \left[(P_{t+1} - \hat{P}_{t|t+1}^{FP}) | I_t \right] = C^{(1)} + b(\rho_x)^t x_t + C^{(2)} z_t \quad (12)$$

where E_M is the expectation of the fully predetermined conditional distribution produced by our example, I_t is the information available at t , (x_t, z_t) , and $C^{(1)}$ and $C^{(2)}$ are constants that depend on the parameters of the model and its representation of forecasting strategies at t and $t + 1$.

Nevertheless, because the model is fully predetermined, its prediction error, $(P_{t+1} - E_M[P_{t+1}|I_t])$, is *by design* uncorrelated with I_t :

$$E_M \left[(P_{t+1} - \hat{P}_{t|t+1}^M) | I_t \right] = 0 \quad (13)$$

where $\hat{P}_{t|t+1}^M = E_M [P_{t+1}|I_t]$.

Comparing expressions analogous to (12) and (13) in the context of his model, Muth remarked that, were a fully predetermined model to impute non-REH forecasting strategies to participants, “there would be opportunities . . . to profit from the knowledge—by inventory speculation . . . or by selling a price forecasting service to the firms” (Muth 1961: 318). The “knowledge” that such a service would presumably sell is the superior price predictions produced by an economist’s own fully predetermined model.

It also follows that the imposition of REH in any fully predetermined model eliminates the correlation between forecast errors and current information implied by fully predetermined non-REH representations of the aggregate of forecasting strategies. To illustrate this point and to display the structure of the REH representation, $\hat{P}_{t|t+1}^{RE}$ (which will be useful in our later discussion), we note that because this representation is defined by

$$\hat{P}_{t|t+1}^{RE} = E_M [P_{t+1}|I_t] \quad (14)$$

it immediately follows that¹⁰

$$\hat{p}_{t|t+1}^{\text{RE}} = \alpha^{\text{RE}} + \beta^{\text{RE}} x_t \quad (15)$$

where

$$\alpha^{\text{RE}} = \frac{a}{(1-c)} + \frac{(1-\rho^x)\mu_x}{(1-c)(1-c\rho^x)} \quad \text{and} \quad \beta^{\text{RE}} = \frac{b\rho^x}{(1-c\rho^x)} \quad (16)$$

By design, one-period-ahead forecast errors, $(P_{t+1} - \hat{P}_{t|t+1}^{\text{RE}})$, are represented with a linear combination of future news, $(\epsilon_{t+1}^x, \epsilon_{t+1}^z)$. Consequently, Muth (1961: 318) observed that, *in the context of his model*, “profit opportunities would no longer exist if the aggregate expectation of [market participants] is the same as the prediction of the theory.”

Muth did not explicitly acknowledge that these implications crucially depend on his model’s core premise that its fully predetermined specification adequately represents how the market price unfolds over time. Once an economist presumes that he has found a fully predetermined account of outcomes, it follows as a matter of straightforward logic that not making use of this “knowledge” would imply passing up obvious profit opportunities. Nevertheless, Muth was ambivalent about treating such “profit opportunities” as pointing to opportunities in real-world markets. Remarkably, he seemed to treat his observation that REH could somehow result from profit-seeking behavior as a purely theoretical artifact of his model’s assumptions. Although his model suggested a connection between REH and rational forecasting, he did not alter his interpretation of REH as a “purely descriptive hypothesis,”¹¹ and he did not soften his warning that it should not be confused with “a pronouncement of what [market participants] ought to do.”

4.5.2 The End of Ambivalence

Lacking a normative or other justification for using REH to represent individual forecasting, macroeconomists working in the 1960s largely ig-

10. In deriving the REH representation in (15), we follow standard practice and constrain the parameter c_t in (1) to be equal to or less than unity. We recall that for the sake of simplicity, each of the parameters a_t , b_t , and c_t in that specification is also assumed to be unchanging over time; that is, $a_t = a$, $b_t = b$, and $c_t = c$. Allowing for fully predetermined change would modify the REH representation, but it would not affect any of our conclusions.

11. As we shall argue shortly, REH is neither a descriptively nor a normatively adequate hypothesis concerning forecasting in real-world markets. In fact, the insurmountable flaws in both of these interpretations stem from the core premise of the contemporary approach. For an extensive discussion, see Frydman and Goldberg (2011: chapters 3 and 4).

nored it in modeling forecasting behavior.¹² However, many models developed at that time, particularly large-scale econometric models aimed at explaining time series of aggregate outcomes, were characterized by inconsistency between their representations of individual forecasting and their structure on the aggregate level. Robert Lucas focused on this inconsistency as these models’ fundamental flaw. As he recounted in his Nobel lecture,

The prevailing strategy for macroeconomic modeling in the early 1960s held that the individual or sectoral models . . . could . . . simply be combined in a single [macroeconomic] model. But models of individual decisions over time necessarily involve expected future prices. . . . However, . . . [aggregate] models assembled from such individual components implied behavior of actual prices . . . that bore no relation to, and were in general *grossly inconsistent* with, the price expectations that the theory imputed to individual agents. [Lucas 1995: 254–255, emphasis added]

Echoing Muth, Lucas observed that models involving such “gross inconsistency” imply that an economist imputes to market participants forecasting strategies that generate obvious and systematic forecast errors. But, because Lucas took for granted the premise that a fully predetermined model could provide an adequate account of how “actual prices” evolve over time, he interpreted the obvious forecast errors—implied by non-REH representations in such a model—as a symptom of irrationality on the part of participants in real-world markets. Thus, in contrast to Muth, he presumed that the ostensibly easily detectable, yet unexploited, correlations between these forecast errors and readily available information, which are an artifact of his model’s assumptions, point to obvious, yet unrealized, profit opportunities in real-world markets. As he later emphatically put it, “if your theory reveals profit opportunities, you have the wrong theory” (Lucas 2001: 13) of “actual prices.”

In a leap of faith that would change macroeconomics and finance for generations, Lucas brushed aside Muth’s ambivalence and presumed that the “right theory” is a fully predetermined model in which REH characterizes how rational individuals forecast future market outcomes.

12. Indeed, when Phelps organized a milestone conference in 1969 on the role of expectations in modeling the microfoundations of macroeconomic theories, the papers collected in the conference volume (Phelps et al. 1970) made no use of REH, and it is not even listed in the index.

4.5.3 *The REH Revolution: Model Consistency as a Standard of Rationality*

Lucas's belief of what constitutes the "right theory" gained wide acceptance among macroeconomists and finance theorists. REH was embraced by a vast majority of economists, spanning the Chicago free-market and MIT's New Keynesian schools. REH's imposition of exact consistency between the predictions of market outcomes implied by an economist's own fully predetermined model and individuals' forecasting strategies quickly became the standard way to represent how rational individuals think about the future.

Because it could be applied in every fully predetermined model, the REH standard had much to recommend it to economists who believe that fully predetermined accounts of market outcomes are within reach of economic analysis. Faith in the divine apotheosis of economic theory led economists to hypothesize that every time one of them formulates a fully predetermined model, he has discovered such an account of market outcomes. Once an economist entertained such a hypothesis, it seemed reasonable to suppose that profit seeking would compel market participants to search for such a model, which they should be able to discover, because, after all, an economist already did.

4.5.4 *The Misleading Narrative of REH*

Lucas's (2001) informal account of why he found REH so compelling highlights the narrative that led so many economists to share his belief. Lucas considered a simple, fully predetermined model of a market that attributes to firms in each period the forecast that a market price will remain constant at its current level, whereas the model predicts that the market price nonetheless rises period after period. According to Lucas (2001: 13), "in such a model, you could see profit opportunities that firms were passing up. Why couldn't they see these opportunities, too?"

Such informal arguments have underpinned the widespread belief that REH somehow follows from the assumption of profit maximization—that it simply represents the idea that market participants optimally use the information available to them.

However, even before REH reached its ascendancy in macroeconomics, critics pointed out that this narrative has no foundations. Contrary to Lucas's presumption, the early critics of REH showed that, even under the fanciful assumption that an economist has discovered a fully predetermined account of market outcomes, profit seeking would, in general, neither compel nor lead market participants to forecast according to a particular economist's model.¹³

13. See Frydman (1982, 1983) for a rigorous demonstration of this point in the context of a widely used class of REH models with decentralized information, such as those developed by

Thomas Sargent, one of the most forceful early advocates of REH, acknowledged these critical arguments and recognized that treating REH as a descriptively or normatively plausible hypothesis about how market participants forecast the future is "misleading":

The idea of rational expectations is sometimes explained informally by saying that it reflects a process in which individuals are inspecting and altering their forecasting records. . . . It is also sometimes said that [REH embodies] the idea that economists and the agents they are modeling should be placed on equal footing: the agents in the model should be able to forecast and profit-maximize and utility-maximize as well as . . . the econometrician who constructed the model. [Sargent 1993: 21]

He then pointed out that

these ways of explaining things are suggestive, but *misleading*, because they make [REH] sound less restrictive and more behavioral than it really is. [Sargent 1993: 21, emphasis added]

4.6 The Orwellian World of "Rational" Microfoundations

Despite such seemingly incisive criticisms, nearly all economists, including Sargent himself, have continued to use REH to model "rational" microfoundations. Because REH, by design, imposes exact consistency between the sharp prediction—a single probability distribution of outcomes—implied by an economist's aggregate model and the probability distribution representing participants' forecasting strategies, REH forces an economist to represent forecasting on the individual level with a single overarching probability distribution.¹⁴

Thus, REH determines forecasting on its models' aggregate and individual levels jointly. And, because in "rational expectations models, people's beliefs are the outcomes of our theorizing,"¹⁵ these models lack genuine microfoundations. After all, they rule out the possibility that "people's beliefs" could affect outcomes in a way that an economist cannot fully prespecify.

Lucas (1973) and Stiglitz (2001). For an extensive discussion of REH's epistemological flaws, see Frydman and Phelps (1983) and Phelps (1983).

14. Sometimes economists allow for more than one way for an individual to alter her decisionmaking strategy. As in other contemporary models, however, such "multiple-equilibrium" models fully prespecify the set of decisionmaking strategies to which an individual may switch from an initial strategy. Indeed, these models typically use REH to predetermine fully each of the forecasting strategies to which an individual may switch. For a formal demonstration of how such models disregard nonroutine change, see Frydman and Goldberg (2007: chapter 6).

15. Sargent in an interview with Evans and Honkapohja (2005: 566).

Economists have interpreted REH-based models' microfoundations in one of two ways. The REH forecasting strategy is thought to represent either how market participants forecast the future in the aggregate, or how every one of them does so individually. Both interpretations suffer from insurmountable difficulties, owing to flaws that can be traced to the contemporary approach's core premise.

4.6.1 *The Market Renamed*

In proposing REH, Muth thought of it as a “purely descriptive hypothesis” about “the market's” forecasting strategy. Moreover, he did not claim that REH presupposes that every market participant must forecast according to “the relevant economic theory”: REH “does not assert that the scratch work of entrepreneurs resembles the system of equations [in an economist's model] in any way” (Muth 1961: 317). Moreover, REH did not imply “that predictions of [individuals] are perfect or that their expectations are all the same” (Muth 1961: 317).

Although Muth's formulation of REH as a hypothesis about the market sidestepped the diversity of forecasting strategies, he did suggest that REH is compatible with it. This belief seems to be widely shared. Many economists view REH as an approximation that enables them to capture in a parsimonious way the complexities that the diversity of forecasts, and market participants' revisions of their forecasting strategies, pose for understanding outcomes.

However, the interpretation that REH approximates the market's—rather than an individual's—forecasting strategy somehow had to be reconciled with REH's use in modeling the microfoundations of aggregate models. Because REH, by design, obscures the distinction between individual forecasting and aggregate prediction, the separation between the two levels was accomplished by renaming “the market,” which was henceforth known as a “representative agent.” Soon, the purely definitional aggregate of market participants' strategies began to be viewed as the way a single “representative” individual thinks about the future.¹⁶ Because, after Phelps's

16. The contemporary models' presumption that the aggregate of market participants' forecasting strategies adequately approximates how a single representative agent thinks about the future disregards the key distinction between an individual's and the market's allocation of resources. As Hayek (1945, 1948) argued in his prescient analysis of the inevitable failure of socialist planning, helping society cope with and take advantage of the diversity of knowledge, “which is not given to anyone in its totality” (Hayek 1945: 520), is the key to understanding what markets do. For a rigorous demonstration of how Hayek's critique of socialist planning implies the incoherence of contemporary models of the market, see Frydman (1983). For an extensive discussion of the close affinity of these models to the ideas underlying central planning, see Frydman and Rapaczynski (1993, 1994) and Frydman and Goldberg (2011: chapter 2).

milestone 1969 conference, macroeconomists began to take their models' microfoundations seriously, this obfuscation became an important component of REH's misleading narrative, opening the way to its widespread use in modeling “rational” foundations of contemporary macroeconomic and finance models.¹⁷

4.6.2 *The Pseudodiversity of Rational Expectations*

Calling the market's forecasting strategy that of a “representative agent” was supposed to sidestep the problem caused by constraining representations of individual forecasting to a single probability distribution; namely, that REH could not explicitly model the diversity of market participants' forecasting strategies.¹⁸ The belief, shared by many economists, that the rational representative agent's forecasting strategy adequately captures this diversity stood reality—forecasting in real-world markets—on its head: the rational representative agent neither represents how participants' diversity unfolds over time, nor is it compatible with individual rationality.

The claim that the representative agent's forecasting strategy approximates micro-level diversity overlooks REH's requirement that the proportions of participants holding particular views of the future, together with their forecasting strategies, must unfold over time in a fully predetermined way. The same mechanical rules are presumed to characterize the pseudodiversity of participants' forecasting strategies, so that any change in the proportion of market participants holding different views, or any revisions in their views, must be mechanically tied to each other to ensure that REH holds in the aggregate at all points in time—past, present, and future.

To illustrate this point, we allow for a minimal degree of diversity in our simple example of the REH model in (1), (14), and (15). In accordance

17. The belief of Lucas and most other economists that the representative agent's forecasting strategy adequately captures such diversity has underpinned macroeconomic and policy analysis around the world. For example, a representative agent's forecasting strategy plays a key role in Lucas's (1976) famous critique of Keynesian macroeconomic models. Moreover, even the most prominent critics of orthodox theory use REH as a “matter of convenience” in modeling microfoundations. See, for example, Stiglitz (2010). For an extensive discussion of this point, see Frydman and Goldberg (2008, 2011).

18. Although REH models by design rule out explicit representation of the diversity of forecasting strategies, they have been used to represent heterogeneity of market participants' forecasts. Such representations suppose that every participant forecasts according to REH, and that differences in their forecasts arise solely from participants' access to, or their choosing to rely on, different information. Such formulations include Lucas's (1973) model with decentralized information and Stiglitz's (2001) models with asymmetric information. As Frydman (1982, 1983) showed rigorously and Frydman and Goldberg (2011: chapter 1) discuss extensively, our critique of the standard REH models that ignore decentralized information also applies to models that allow for heterogeneity of information used by market participants in forming their forecasts.

with the contemporary approach, we represent forecasting strategies of two groups of participants—say, bulls and bears in an equity market—with two *distinct* fully predetermined analogues to (2):

$$\hat{P}_{t|t+1}^{(i)} = \alpha^{(i)} + \beta^{(i)} Z_t^{(i)}, \quad i = 1, 2 \quad (17)$$

where $(\alpha^{(i)}, \beta^{(i)})$ are parameters and $Z_t^{(i)}$ denotes the two causal variables appearing in the representation of the forecasting strategies, $i = 1, 2$. Consequently, the market's—the representative agent's (RA's)—forecasting strategy can be written as:

$$\hat{P}_{t|t+1}^{\text{RA}} = w \hat{P}_{t|t+1}^{(1)} + (1-w) \hat{P}_{t|t+1}^{(2)} \quad (18)$$

where w and $(1-w)$ are the aggregation weights that sum up to unity.¹⁹

We suppose that the causal factors in (17) follow the same kind of autoregressive process as in (7):

$$z_t^{(i)} = \mu_{z^{(i)}}(1 - \rho_{z^{(i)}}) + \rho_{z^{(i)}} z_{t-1}^{(i)} + \epsilon_t^{z^{(i)}}, \quad i = 1, 2 \quad (19)$$

To simplify our presentation, we specify explicitly the relationships between $z_t^{(i)}$ and the causal factor appearing in the economists' aggregate model, x_t in (1). To this end, we suppose that the vector of shocks $(\epsilon_t^x, \epsilon_t^{z^{(1)}}, \epsilon_t^{z^{(2)}})$ is normally distributed and uncorrelated over time. Using standard formulas enables us to express the causal factors in terms of x_t as follows:

$$z_t^{(i)} = \gamma_0^{(i)} + \gamma_1^{(i)} x_t + \eta_t^{(i)}, \quad i = 1, 2 \quad (20)$$

where

$$\gamma_0^{(i)} = \mu_{z^{(i)}}(1 - \rho_{z^{(i)}}), \quad \gamma_1^{(i)} = \frac{\text{Cov}(\epsilon_t^x, \epsilon_t^{z^{(i)}})(1 - \rho_x)}{\sigma_{\epsilon^x}^2(1 - \rho_x \rho_{z^{(i)}})}$$

$\text{Cov}(\cdot)$ denotes the covariance operator, and, by construction, $E[\eta_t^{(i)} | x_t] = 0$, $i = 1, 2$.

We are now ready to illustrate the pseudodiversity that underpins the belief that REH approximates the diversity of market participants' forecasting strategies. Substituting (20) into (18) yields the following expression for the market's—representative agent's—forecast:

$$\hat{P}_{t|t+1}^{\text{RA}} = \alpha^{\text{RA}} + \beta^{\text{RA}} x_t + \eta_t \quad (21)$$

19. These weights are typically wealth shares of each group as a percentage of the total wealth of all market participants.

where α^{RA} and β^{RA} are functions of w , $\alpha^{(i)}$, $\beta^{(i)}$, and $\gamma_1^{(i)}$ (which we explicitly write out in (22) and (23)); and $\eta_t = [w\beta^{(1)}\eta_t^{(1)} + (1-w)\beta^{(2)}\eta_t^{(2)}]$. Because $E[\eta_t | x_t] = 0$, a comparison of (15) with (21) shows that for $\hat{P}_{t|t+1}^{\text{RE}}$ to approximate $\hat{P}_{t|t+1}^{\text{RA}}$ up to a random shock, η_t , which is uncorrelated with the causal variable in an economist's model, x_t , the parameters of the bulls' and bears' forecasting strategies must satisfy the following constraints:

$$\alpha^{\text{RE}} = \alpha^{\text{RA}} = w(\alpha^{(1)} + \beta^{(1)}\gamma_0^{(1)}) + (1-w)(\alpha^{(2)} + \beta^{(2)}\gamma_0^{(2)}) \quad (22)$$

and

$$\beta^{\text{RE}} = \beta^{\text{RA}} = w\beta^{(1)}\gamma_1^{(1)} + (1-w)\beta^{(2)}\gamma_1^{(2)} \quad (23)$$

Thus, the claim that REH “does not assert that expectations are all the same” (Muth 1961: 317) requires that participants' forecasting strategies are tied to each other and to the economist's REH model according to fully predetermined rules, such as those in (22) and (23) in all time periods. Consequently, when any group of participants alters their forecasting strategies, the strategies of the others must change to ensure that REH holds in the aggregate.

By focusing on the market and renaming it a representative agent, REH does abstract from the differences among participants' forecasting strategies. But, in presuming that an economist's fully predetermined model adequately approximates the predictions of the aggregate forecast, REH does not approximate the diversity underpinning outcomes in real-world markets. Rather, it abstracts from its models' already constructed pseudodiversity, which evolves according to rigid, prespecified mechanical rules and has no connection with how differences of views in real-world markets unfold over time.²⁰

4.6.3 The Incoherence of the “Rational” Representative Agent

Beyond its inherent incompatibility with how participants revise their forecasting strategies in real-world markets, fully predetermined pseudodiversity renders incoherent the very notion of rational microfoundations

20. Some contemporary economists interpret Muth's claim that REH is compatible with diversity as hypothesizing that market participants' forecasting strategies differ from some common aggregate—the market's strategy—by a random error term that averages to zero. However, this definition of diversity is just another, slightly weaker, version of the assumption of unanimity: on average, each market participant's forecasting strategy conforms to the same mechanical rule. Under this interpretation, the way in which the diversity unfolds over time is represented with a random shock around this common rule (as illustrated by η in (21)). Because contemporary models fully prespecify the probability distribution of such shocks, this specification is just another representation of how REH's pseudodiversity unfolds over time.

based on REH's representative agent. If the representative agent indeed stood for the views of market participants who make use of different forecasting strategies, every one of them would obviously be irrational, in the sense that they ignore systematic forecast errors and thereby forgo obvious profit opportunities endlessly. This conclusion follows immediately from the observation that in the context of a fully predetermined model, $(\hat{P}_{t|t+1}^{(i)} - \hat{P}_{t|t+1}^{RA})$ is systematically correlated with $z_t^{(i)}$. Because, under REH, $\hat{P}_{t|t+1}^{RA} = E_M [P_{t+1}|I_t]$, the forecast errors $(\hat{P}_{t|t+1}^{(i)} - \hat{P}_{t|t+1}^{RA})$ implied by each of the diverse forecasting strategies are systematically correlated with the information that an economist supposes underpins each of these strategies. Thus, microfoundations of contemporary macroeconomic and finance models that are based on a "rational" representative agent construct could hardly be called "rational," whatever this term might mean.

4.6.4 *The Distorted Language of Economic Discourse*

Distorting or inverting the meaning of notions like "rationality" or "the representative agent" in contemporary macroeconomics and finance has had a profound impact on public debate. When economists invoke rationality to justify their public-policy recommendations, noneconomists interpret such statements to mean that the recommendations are based on scientific representations of how reasonable people behave in the real world. In addition, because economists claim that their conclusions follow as a matter of straightforward logic,²¹ those who doubt their claims have often been portrayed as being akin to creationists or flat-earthers.

To understand the assumptions that underpin the language of these constructions is to comprehend that the standard of rational forecasting purportedly provided by REH turns the very notion of rationality on its head. What economists imagine to be rational forecasting would be considered obviously irrational by anyone in the real world who is minimally rational. After all, a rational profit-seeking individual understands that the world around her will change in nonroutine ways. She simply cannot afford to believe that, contrary to her experience, she has found a true overarching forecasting strategy, let alone that everyone else has found it as well.

The distorted language of economic discourse has also had a profound impact on the development of economics itself. Behavioral economics provides a case in point. After uncovering massive evidence that contemporary economics' standard of rationality fails to capture adequately how individuals actually make decisions, the only sensible conclusion to draw was that

this standard was utterly wrong. Instead, behavioral economists concluded that individuals are less than fully rational or are irrational.

To justify such a conclusion, behavioral economists and nonacademic commentators argued that the REH-based standard of rationality works—but only for truly intelligent investors.²² Most individuals lack the abilities needed to understand the future and compute correctly the consequences of their decisions.²³

In fact, REH requires no assumptions about the intelligence of market participants whatsoever.²⁴ Rather than imputing to individuals superhuman cognitive and computational abilities, REH presumes just the opposite: market participants forgo using whatever cognitive abilities they do have. REH supposes that individuals do not engage actively and creatively in revising the way they think about the future. Instead, they are presumed to adhere steadfastly to a single mechanical forecasting strategy at all times and under all circumstances. Thus, contrary to widespread belief, in the context of real-world markets, REH presumes that participants are obviously irrational. When new relationships begin driving asset prices, they supposedly look the other way, and thus either abjure profit-seeking behavior altogether or forgo profit opportunities that are in plain sight.

4.7 The Predictable Empirical Difficulties of Fully Predetermined Rationality

In real-world markets, participants must rely on their own imperfect understanding of which variables are important for forecasting and how those variables are related to future outcomes. No participant, let alone an economist, knows in advance how she will revise her forecasting strategies or how the social context will change as the future unfolds.

Thus, even if a fully predetermined model might adequately represent the past relationship between causal variables and aggregate outcomes in a selected historical period, its structure would cease to be adequate

22. Having embraced the fully predetermined notion of rationality, behavioral economists proceeded to search for reasons, mostly in psychological research and brain studies, to explain why individual behavior is so grossly inconsistent with it.

23. For example, an important class of models in the behavioral finance literature, originated by DeLong et al. (1990a,b), contrasts the behavior of "fully rational" participants, whom they refer to as "smart" investors, with those who are "less-than-fully rational." Even Simon (1971), a forceful early critic of economists' notion of rationality, regarded it as an appropriate standard of decisionmaking, though he believed that, for various cognitive and other reasons, it was unattainable for most people. To underscore this view, he coined the term "bounded rationality" to refer to departures from the supposedly normative benchmark.

24. For an extensive discussion, see Frydman and Goldberg (2011: chapters 2, 3, and 4).

21. For a recent example, see Cochrane (2009). For further discussion, see Frydman and Goldberg (2011: chapter 1).

at moments that no one can fully prespecify.²⁵ Such contingent change implies that the statistical estimates generated by fully predetermined models of asset prices vary in significant ways as the time period examined is changed. Correlations between price changes and informational variables that might be found in the data over some stretch of time eventually change or disappear, and are replaced by new relationships.

Because participants' forecasting is the key factor underpinning the causal process in asset markets, models of these markets are particularly prone to such irregular temporal instability. For example, Fama and MacBeth (1973) and others report favorable estimates of the Capital Asset Pricing Model (CAPM), which is widely used in academia and industry, over a sample that runs until 1965. However, when the sample was updated to include the 1970s and 1980s, and additional variables were added to the analysis, the results implied that the CAPM was "atrocious as an empirical model" (Berg 1992: D1). Commenting in an interview with *Institutional Investor* on the temporal instability of correlations in asset-price data, Nobel laureate William Sharpe quipped that "it's almost true that if you don't like an empirical result, if you can wait until somebody uses a different [time] period . . . you'll get a different answer" (Wallace 1980: 24).

It is not surprising that models that disregard the importance of non-routine change in driving outcomes have repeatedly failed to predict outcomes in real-world markets, let alone predict them "sharply." In examining the widely reported empirical difficulties of REH-based models for price and risk movements in currency markets, Frydman and Goldberg (2007: chapters 7 and 8) trace their failures to the premise that fully predetermined accounts of price and risk movements are within reach of economic analysis.

Although both REH and behavioral economists largely missed the connection between the failure of REH models and the core premise on which they rest, their research has helped to uncover these models' dismal empirical performance. After considering many econometric studies of REH models, Maurice Obstfeld and Kenneth Rogoff concluded in their magisterial book on international macroeconomics that

the undeniable difficulties that international economists encounter in empirically explaining nominal exchange rate movements are an embarrassment, but one shared with virtually any other field that attempts to explain asset price data. [Obstfeld and Rogoff 1996: 625]

25. Even when it comes to past relationships, there are many possible models that might adequately describe the causal processes underpinning outcomes in any selected historical period. For an argument that subjective judgments play a key role in understanding the past, see Frydman and Goldberg (2011: chapter 11).

Drawing on extensive laboratory and psychological studies, behavioral economists also reached the conclusion that microfoundations based on an economist's a priori notion of rationality were inconsistent with empirical evidence, and replaced them with formalizations of their empirical findings on how individuals "actually" behave. But, despite their focus on the "psychological realism" of their representations, behavioral macroeconomists and finance theorists embraced the core premise of the contemporary approach. Consequently, they formalized their empirical findings with mechanical rules, thereby basing their accounts of aggregate outcomes on fully predetermined microfoundations.

4.8 The Irrelevant "Inconsistency" of Behavioral Finance Models

Representing market participants as robots who act according to rules fully prespecified by an economist is odd for an approach that claims the mantle of psychological realism.²⁶ As we have argued, fully predetermined models are anything but realistic. Indeed, whether they appeal to a priori assumptions about how a rational market participant should behave or to empirical findings about how they actually behave, fully predetermined models disregard by design crucial features of real-world markets.²⁷

Although behavioral models have gained a significant following among economists and nonacademic commentators in recent years, a large segment of macroeconomists continue to view behavioral explanations with considerable skepticism. This position seems to be related to Lucas's arguments for REH, which many found so convincing. Because non-REH behavioral models' microfoundations are internally inconsistent with their representations on the aggregate level, Lucas argued that such models are "the wrong theory."

But, as we have argued, fully predetermined models are the wrong theory on both the individual and aggregate levels. Thus, consistency between these levels has no connection to rationality in real-world markets, and inconsistency within these models is not, as Lucas and his followers believe, a symptom of departures from full rationality in those markets. The consistency of participants' fully prespecified forecasting strategies with an economist's representation of aggregate outcomes is, to put it bluntly, beside

26. Camerer et al. (2004) argue that greater psychological realism is the main advantage of behavioral models over their "fully rational" counterparts.

27. Another oddity of the behavioral approach is that some behavioral economists continue to rely on REH. For an influential example, see Barberis et al. (2001). Because our critique of REH-based fully predetermined rationality also applies to these models' microfoundations, we focus here on non-REH behavioral models.

the point. Imputing such strategies to market participants merely presumes that every one of them disregards nonroutine change, and that their—and the economist's own—understanding of the causal process underpinning market outcomes is inherently imperfect.

4.9 The Fatal Flaw

We have argued that there is an inherent conflict between the objective of modeling market outcomes on the basis of mathematical, yet plausible, microfoundations and contemporary economists' insistence that their models produce sharp probabilistic predictions of change. Regardless of whether they are “fully rational” or “less-than-fully rational,” fully predetermined microfoundations are incompatible—and, indeed, have no connection—with profit seeking in real-world markets. Thus, to open macroeconomic models' foundations to minimally reasonable decision-making, let alone individual rationality, economists must jettison their core premise that nonroutine change is unimportant for understanding market outcomes.

We should emphasize that our critique of contemporary models is not that they are abstract or mathematical. Useful scientific models are those that abstract from features of reality that are irrelevant for an adequate account of the phenomenon that the model seeks to explain. The hope is that the omitted considerations really are relatively unimportant to understanding the phenomenon.

The need to exclude many potentially relevant considerations is particularly acute if one aims to account for outcomes with mathematical models, which ipso facto make use of a few assumptions to explain complex phenomena. So the bolder an abstraction that one seeks, the more important it is to scrutinize the assumptions that are “termed crucial . . . on the grounds [of their] intuitive plausibility or capacity to suggest, if only by implication, some of the considerations that are relevant in judging or applying the model” (Friedman 1953: 26).²⁸

The fatal flaw of contemporary macroeconomic and finance models is that they rule out by design the crucial factors—participants' revisions of forecasting strategies and how the diversity of these strategies and the social context unfold over time—that underpin the market outcomes they are

28. Contemporary economists brush off criticism that their assumptions are unrealistic by invoking the dictum put forth by Milton Friedman (1953: 23) in his well-known essay on economic methodology: “theory cannot be tested by the ‘realism’ of its assumptions.” In fact, at no point did Friedman suggest that economists should not be concerned about the inadequacy of their models' assumptions. For an argument that Friedman's influential essay has been misinterpreted as legitimizing contemporary models' core assumptions, see Frydman and Goldberg (2011: chapter 1).

attempting to explain. No one can fully specify these factors in advance. Only when we abandon contemporary economists' mechanistic conception of science can we hope to develop models that might account for how market outcomes unfold over time and that are compatible with profit-seeking behavior and individual rationality in the real world. Indeed, we show in Frydman and Goldberg (Chapter 6, this volume) that, by stopping short of fully prespecifying change, IKE can account for movements in asset prices and risk that extant approaches have found so difficult to explain.

4.10 Opening Macroeconomics and Finance Theory to Imperfect Knowledge and Diversity

We make use of our simple example in (1) and (2) to illustrate how, by not fully prespecifying change, economic analysis can escape contemporary models' insurmountable epistemological and empirical difficulties. As before, for the sake of simplicity, we continue to impose the invariance restriction on the parameters and causal variables in (1) and focus on the representations of market participants' forecasting strategies in (2).

We begin by jettisoning fully predetermining restrictions on how participants revise their forecasting strategies in the aggregate. Thus, we rewrite the representation of this aggregate in (10) at time $t + 1$ in terms of the structure of its representation and the realization of the causal variable at t :

$$\hat{P}_{t+1|t+2} - \hat{P}_{t|t+1} = \hat{A}_{(t,t+1)} + \hat{B}_{(t,t+1)}z_t + (\beta_t + \beta_{(t,t+1)})\varepsilon_{t+1}^z \quad (24)$$

where $\hat{A}_{(t,t+1)} = \alpha_{(t,t+1)} + (\beta_t + \beta_{(t,t+1)})\mu_z(1 - \rho_z)$, $\hat{B}_{(t,t+1)} = \beta_{(t,t+1)} + (\beta_t + \beta_{(t,t+1)})[\rho_z - 1]$, and $(\alpha_{(t,t+1)}, \beta_{(t,t+1)})$ represent revisions of forecasting strategies, which, for convenience, we repeat here from (4):

$$\alpha_{(t,t+1)} = \alpha_{t+1} - \alpha_t \quad \text{and} \quad \beta_{(t,t+1)} = \beta_{t+1} - \beta_t \quad (25)$$

Analogously to (11), we can also write the change in the market price as

$$P_{t+1} - P_t = [A_{(t,t+1)} + Bx_t + C_{(t,t+1)}z_t] + \eta_{t+1} \quad (26)$$

where $A_{(t,t+1)} = b[\mu_x(1 - \rho_x) + (\rho_x - 1)] + c\hat{A}_{(t,t+1)}$, $B = b(\rho_x - 1)$, $C_{(t,t+1)} = c\hat{B}_{(t,t+1)}$, $\hat{A}_{(t,t+1)}$ and $\hat{B}_{(t,t+1)}$ are defined in (24) and (25), and

$$\eta_{t+1} = b\varepsilon_{t+1}^x + c(\beta_t + \beta_{(t,t+1)})\varepsilon_{t+1}^z \quad (27)$$

Because the unrestricted model in (24) and (26) does not constrain in any way the probability distribution for outcomes at $t + 1$ or beyond, it is trivially compatible with nonroutine change on both the individual and aggregate levels, as well as with the diversity of forecasting strategies and

their revisions. Consequently, unless further restrictions are imposed on revisions of forecasting strategies, the unrestricted representation has no empirical content: it is compatible with any time path of outcomes and with any causal process that underpins them.

4.10.1 Methodological Extremes: Animal Spirits versus Fully Predetermined Accounts of Outcomes

The unrestricted model illustrates radical uncertainty, a situation in which individual decisions cannot be adequately represented with a standard probability distribution. Keynes (1921, 1936) and Knight (1921) forcefully argued that most business decisions are fraught with such uncertainty. An extreme version of radical uncertainty is often thought to force participants to act according to their “animal spirits,” psychological impulses that are largely disconnected from any fundamental considerations that might drive outcomes. However, unless participants’ forecasting strategies could be connected, at least during some periods, to the causal factors observable by economists, no formal economic theory with empirical content would be possible. As Phelps (2008: A19) has recently put it, “animal spirits can’t be modeled.”

Contemporary models, with their core premise that fully predetermined causal accounts of individual decisionmaking and market outcomes are within reach of economic analysis, occupy the opposite methodological extreme. Searching for such accounts, economists constrain their models in a severe way: they fully prespecify change. Their models represent outcomes at each point in time—and thus how they unfold over time—with a single overarching conditional probability distribution. The relationships between the moments of this distribution and the set of causal variables constitute the empirical content that can be confronted with the time-series evidence on market outcomes.

Like the opposite (animal spirits) extreme of the methodological spectrum, the contemporary approach to macroeconomics and finance theory is inherently in conflict—though for very different reasons—with the objective of developing empirically relevant economic theory. As we have argued, contemporary models’ core premise leads to intractable epistemological problems, which inevitably translate into gross inconsistencies between their supposedly sharp predictions of market outcomes and the empirical record.

4.10.2 IKE’s Nonstandard Probabilistic Formalism

IKE stakes out an intermediate position between radical uncertainty (which, in its extreme, animal spirits, version, denies that economists can formulate testable mathematical models of any features of the causal process driving

change) and the contemporary presumption that a standard conditional probability distribution can fully and adequately represent this process.

Although it stops short of imposing fully predetermining restrictions on change, IKE aims to explain outcomes with mathematical models that can be confronted with empirical evidence. To this end, IKE relies on nonstandard probabilistic formalism to formulate its mathematical representations of forecasting strategies and their revisions.

To illustrate how IKE opens economic models to nonroutine revisions of forecasting strategies and their diversity, and to compare its representations with its fully predetermined counterparts, we formulate an IKE version of the representation of diversity in (17) and (18). This example represents diversity on the micro level with the forecasting strategies of two groups of participants—say, bulls and bears—in an equity market:

$$\hat{P}_{t|t+1}^{(i)} = \alpha_t^{(i)} + \beta_t^{(i)} Z_t^{(i)}, \quad i = 1, 2 \quad (28)$$

where $(\alpha_t^{(i)}, \beta_t^{(i)})$ are parameters and $Z_t^{(i)}$ denotes the two causal variables appearing in a representation for each of the forecasting strategies for $i = 1, 2$, respectively.

Consequently, the aggregate of forecasting strategies can be written as

$$\hat{P}_{t|t+1} = w \hat{P}_{t|t+1}^{(1)} + (1 - w) \hat{P}_{t|t+1}^{(2)} \quad (29)$$

where, as before, $\hat{P}_{t|t+1}$ denotes the aggregate of participants’ forecasting strategies, and w and $(1 - w)$ are the aggregation weights that sum up to unity.

To model how aggregate outcomes unfold over time, we write the updating of forecasts generated by (28) and (29), which drive these movements, as follows:

$$\begin{aligned} \hat{P}_{t+1|t+2}^{(i)} - \hat{P}_{t|t+1}^{(i)} &= \alpha_{(t,t+1)}^{(i)} + \beta_{(t,t+1)}^{(i)} Z_t^{(i)} \\ &+ \left(\beta_t^{(i)} + \beta_{(t,t+1)}^{(i)} \right) \left(Z_{t+1}^{(i)} - Z_t^{(i)} \right), \quad i = 1, 2 \end{aligned} \quad (30)$$

and

$$\begin{aligned} \hat{P}_{t+1|t+2} - \hat{P}_{t|t+1} &= w \left(\hat{P}_{t+1|t+2}^{(1)} - \hat{P}_{t|t+1}^{(1)} \right) + (1 - w) \left(\hat{P}_{t+1|t+2}^{(2)} - \hat{P}_{t|t+1}^{(2)} \right) \\ &= \alpha_{(t,t+1)} + \beta_{(t,t+1)} Z_t \\ &+ \left(\beta_t + \beta_{(t,t+1)} \right) \left(Z_{t+1} - Z_t \right) \end{aligned} \quad (31)$$

where $\alpha_{(t,t+1)} = w \alpha_{(t,t+1)}^{(1)} + (1 - w) \alpha_{(t,t+1)}^{(2)}$, $\beta_{(t,t+1)} Z_t = w \beta_{(t,t+1)}^{(1)} Z_t^{(1)} + (1 - w) \beta_{(t,t+1)}^{(2)} Z_t^{(2)}$, and $(\beta_t + \beta_{(t,t+1)}) (Z_{t+1} - Z_t)$ is defined analogously.

This representation shows that the updating of participants' forecasts stems from two sources: revisions of forecasting strategies, as represented here by $\alpha_{(t,t+1)}^{(i)}$ and $\beta_{(t,t+1)}^{(i)}$, and new information on the causal variables, $(Z_{t+1}^{(i)} - Z_t^{(i)})$.

For the sake of simplicity, we continue to maintain the time-invariance restrictions in (1). This enables us to relate the representation of change in the market price to revisions in forecasting strategies and new information on the causal variables $Z_{t+1}^{(i)}$ and X_{t+1} :

$$P_{t+1} - P_t = b(X_{t+1} - X_t) + c(\hat{P}_{t+1|t+2} - \hat{P}_{t|t+1}) \quad (32)$$

In contrast to contemporary models, IKE recognizes that our knowledge of the causal process underpinning outcomes is inherently imperfect. Consequently, IKE does not fully prespecify which causal variables may be relevant, or when and how these variables may enter an economist's representation of forecasting behavior. In this way, IKE models remain open to nonroutine changes in the ways that individuals in real-world markets forecast the future.

However, like any scientific theory, IKE must presume that purposeful behavior exhibits regularities, even if these regularities are only qualitative, context-dependent, and relevant at a time that no one can fully specify in advance. IKE explores the possibility that revisions in forecasting strategies and changes in the social context can be characterized with qualitative mathematical conditions. In portraying an individual's forecasting strategy at a given point in time, we use a conditional probability distribution, but we do not fully prespecify how it unfolds over time. Instead, we impose qualitative restrictions on this change.

4.10.3 Partially Predetermined Probability Distributions

In the representation (30), revisions of forecasting strategies and changes in the social context are reflected in $\alpha_{(t,t+1)}^{(i)}$ and $\beta_{(t,t+1)}^{(i)}$, and $(Z_{t+1}^{(i)} - Z_t^{(i)})$, $i = 1, 2$, respectively. Thus, for the model to generate empirically relevant implications, we need to specify qualitative conditions for both these components of change.

For example, in Chapter 6 of this volume, we follow this approach in specifying the microfoundations of a model of swings in asset prices and risk. We build on an idea that goes back to Keynes (1936): Faced with imperfect knowledge concerning the causal process driving market outcomes, a participant tends to revise in guardedly moderate ways her thinking about how fundamentals influence the prospects of her investments. There are stretches of time during which she either maintains her forecasting strategy or revises it gradually.

How one would formalize "guardedly moderate revisions" depends on the context. In general, doing so requires a specification of both the formation of forecasting strategies and the baseline against which revisions of these strategies are judged. In our recent work, we have developed a formalization of such revisions in the context of modeling asset-price swings.²⁹

To complete the microfoundations of our simple IKE model, we also need to specify qualitative conditions for changes in the fundamental variables. The empirical record suggests that many of these fundamentals tend to trend in particular directions for long stretches of time. Although these trends generally vary in magnitude, for the sake of simplicity, we constrain them to be time invariant and assume that these fundamentals are characterized by processes with fixed drifts in (6) and (8).

Because IKE imposes qualitative restrictions on the structure of its models, it can characterize how an individual revises her forecasting strategy, regardless of whether she is a bull or a bear. Although bulls and bears forecast prices to move in opposite directions, if both revise their strategies in guardedly moderate ways, and fundamentals trend in unchanging directions between two successive periods—say, $t - 1$ and t , and t and $t + 1$ —it can be shown that their price forecasts would move in the same direction in each of these periods.

IKE uses nonstandard probabilistic formalism to represent the qualitative conditions making up its models' microfoundations and their predictions for aggregate outcomes. In contrast to fully predetermined models, IKE represents forecast revisions with myriad distributions implied by (30) and constrains these distributions to share common qualitative features. We refer to such representations as "partially predetermined probability distributions."

For example, it can be shown that if fundamentals were to trend in unchanging directions and a participant were to revise her forecasting strategies in guardedly moderate ways over two successive periods, the mean of any of the myriad conditional distributions of change implied by IKE representations of her forecast revisions in (30) would be the same in each period. Consequently, during time periods in which these conditions characterize forecast revisions of "most" market participants—as measured by the relative weight of their positions in the market—the aggregate forecast in (31) will tend to move in one direction over time. In connecting how the aggregate price forecast unfolds over time to individual behavior, we make use of assumptions about diversity in the qualitative way that trends in fundamentals impact bulls' and bears' forecasts (see Frydman and Goldberg, Chapter 6, this volume).

29. In Frydman and Goldberg (Chapter 6, this volume), we present a revised version of the model developed in our earlier work, as well as a more complete formal demonstration of the main assertions sketched in this section.

The qualitative conditions on the micro level, together with our assumptions about diversity, generate empirically relevant implications on the aggregate level. Specifically, price swings in asset markets will occur during stretches of time in which trends in market fundamentals are persistent, and participants, on the whole, interpret the impact of these trends on their price forecasts in a qualitatively similar manner, as well as revise their strategies in guardedly moderate ways.

4.11 The Contingency of IKE's Representations

One would not expect participants to revise their forecasting strategies in guardedly moderate ways or fundamentals to continue to trend in the same direction forever. One would also not expect the way these decisions translate into aggregate outcomes to remain unchanged, even if by qualitative conditions. Indeed, the conditions that make up an IKE model are both qualitative and contingent. Probabilistic representations generated by an IKE model are not only compatible with myriad post-change probability distributions, but their structure also undergoes change at moments that no one can fully prespecify. Thus, IKE models are open to nonroutine change and recognize the importance of the imperfect knowledge and diversity of forecasting strategies that such change engenders.

The contingent and qualitative nature of the conditions that characterize IKE's approach to macroeconomics and finance theory plays a crucial role in its ability to deliver empirically relevant accounts of swings in asset prices and risk. Qualitative regularities, like market participants' tendency to revise their forecasts in guardedly moderate ways for stretches of time, play a key role in an IKE model of protracted upswings and downswings. But because these conditions are also contingent, they are consistent with the irregular structural change in macroeconomic and finance models that, as we discussed earlier, has been pointed out in many studies. The contingency of these conditions also plays a key role in IKE's ability to account for the reversals that occur when markets eventually self-correct.

Asset-price swings end when market participants' tendency toward guardedly moderate revisions of their forecasting strategies—or the qualitative similarity of how trends in fundamentals impact their price forecasts—no longer holds. This contingency is likely to be related to that of the tendency of fundamentals to trend in the same direction. For example, news about how trends in fundamentals change may lead participants to revise their forecasting strategies so substantially as to spell the end of a price swing in one direction and the start of a new one in the opposite direction.

Thus, by specifying qualitative and contingent models, IKE accounts for the irregular duration and magnitude of asset-price swings that we observe in real-world markets. The stretches of time in which fundamentals trend in

unchanging directions and revisions are guardedly moderate are irregular. As a result, one of our IKE model's important predictions is irregular swings in asset prices and risk.

4.12 Opening Microfoundations to Genuine Diversity and Individual Rationality

Beyond accounting for the pattern of asset-price swings in real-world markets, the contingency of IKE models renders them compatible with the coexistence of bulls and bears in asset markets and the rationality of both positions, despite their contradictory predictions of price movements.

An IKE model explains the presence of both bulls and bears in the market at every point in time by the fact that no participant can predict with certainty when trends in fundamentals may switch direction; when other participants may cease to revise their forecasting strategies in guardedly moderate ways; or when the qualitative impacts of how trends in fundamentals influence their price forecast will diverge substantially. Because a price swing may continue or end at any time, betting one way or the other does not involve any obvious irrationality on the part of participants who hold either bullish or bearish views.

4.13 Fundamentals and Psychology in IKE's Microfoundations

Behavioral economists have uncovered important insights into the role of psychological factors in individual decisionmaking. However, the contemporary approach's core premise has constrained behavioral theorists either to disregard these insights or to formalize them with mechanical rules that are fully prespecified by an economist. This has led to a widespread belief that the salience of psychological factors should be viewed as a symptom of less than "full rationality."³⁰

But once we acknowledge the inherent imperfection of market participants' and economists' knowledge, we should also acknowledge that both fundamental and psychological considerations play a role in rational decisionmaking:

We are merely reminding ourselves that human decisions affecting the future, whether personal or political or economic, cannot depend on strict

30. Many other influential claims in contemporary economics are an artifact of the belief that the litmus test of rationality is the absence of psychological factors in individual decisionmaking. For a critical discussion of this belief in the context of modeling individual decisions in asset markets, swings in asset prices, and thinking about the rationale for and scope of financial regulation, see Frydman and Goldberg (2011).

mathematical expectation, since the basis for making such calculations does not exist; and . . . that our *rational* selves [are] choosing between alternatives as best as we are able, calculating where we can, but often falling back for our motive on whim or sentiment or chance. [Keynes 1936: 162, emphasis added]

For Keynes, unlike for contemporary behavioral economists, reliance on psychological factors in decisionmaking is not a symptom of irrationality. He emphasizes that even though rational individuals in the real world use knowledge of facts, such as information about fundamentals, our understanding of the processes underpinning outcomes is inherently imperfect; thus, calculation alone is insufficient in decisionmaking.

Precisely because IKE models comprise conditions that are *both* qualitative and contingent, they can incorporate both sets of factors in economic analysis. In our IKE model of swings in asset prices and risk, the qualitative and contingent conditions that constrain revisions of forecasting strategies play an important role in accounting for upswings, reversals, and downswings. These conditions are motivated by Keynes's insights into markets, as well as by subsequent psychological research (see Edwards 1968; Shleifer 2000; and references therein). Moreover, their contingent formalization in our IKE model reflects the widely held view that psychologically based conditions, especially in asset markets, are subject to change that no one can fully foresee.

To be sure, to account for the asset-price swings that we actually observe in markets, we must look beyond psychological and other nonfundamental considerations.³¹ Indeed, the purely psychological and nonfundamental accounts of asset markets overlook the possibility that in forecasting price movements, participants look to fundamental factors that they think will move the market over whatever time horizon interests them for the purpose of forecasting returns. Any confidence and optimism that might exist in the market would quickly evaporate if, say, earnings and overall economic activity consistently moved in the “wrong” direction.

4.14 Sharp versus Contingent Predictions

The evidence that psychology alone cannot drive asset-price movements is good news for the possibility of empirically relevant economic theory: after all, fundamental considerations are, for the most part, the only tangible

31. For example, we make use of Keynes's (1936) insight that price departures from estimates of benchmark values play a key role in how participants assess the riskiness of their speculative positions.

factors that economists can use to develop models that might enable them to distinguish among alternative explanations of outcomes.

However, although predictions in both IKE and REH models are driven by fundamental considerations, IKE's qualitative and contingent predictions are inherently different from contemporary models' requirement of sharp predictions.

The aim of contemporary economists to find a model that can predict the complete set of future market outcomes and probabilities is not the first such endeavor in the social sciences. In his seminal refutation of the claim that “historicism” might one day enable social science to “predict the future course of history,” Karl Popper pointed out that any such approach is futile “to the extent to which [historical developments] may be influenced by the growth of our knowledge” (Popper 1957: xi–xii).

Because market outcomes—especially outcomes in financial markets—crucially depend on changing understandings of the process and psychology that underpin those outcomes on both the individual and aggregate levels, our critique of contemporary macroeconomics and finance theory can be viewed as further refutation of the historicist's vain ambition.

Although Popper was strongly critical of attempts to develop fully predetermined accounts of history, he was quick to point out that his

argument does not, of course, refute the possibility of every kind of social prediction; on the contrary, it is perfectly compatible with the possibility of testing social theories—for example economic theories—by way of predicting that certain developments will take place *under certain conditions*. It only refutes the possibility of predicting historical developments to the extent to which they may be influenced by the growth of our knowledge. [Popper 1957: xii, emphasis added]

The contingent predictions generated by our IKE model of asset-price swings exemplify what Popper would regard as a feasible goal of economic theory. Although our model predicts that, under certain conditions, an asset price will undergo a sustained movement in one direction, it does not predict when such upswings or downswings will begin or end.

Beyond building on Popper's insights concerning the possibility, scope, and character of predictions in the social sciences, our IKE model of asset-price swings exemplifies Hayek's claim that “our capacity to predict will be confined to . . . general characteristics of the events to be expected and not include the capacity for predicting particular individual events” (Hayek 1978: 33). Although an IKE model, by design, stops short of predicting “particular individual events,” such as when the swing will begin and end, it does generate predictions concerning their “general characteristics”—for example, that they tend to be quite persistent. Thus, by examining the

persistence and related features of swings in asset prices and risk implied by alternative models, an economist may compare explanations of economic phenomena. Johansen et al. (2010) and Frydman et al. (2012a,b) develop such an approach to econometric testing and conclude that an IKE model provides a significantly better account than standard and REH-based bubble models of swings in currency markets.³²

These studies show that, despite placing imperfect knowledge and non-routine change at the center of economic analysis and limiting our ambition solely to generating qualitative predictions, IKE models may still yield “predictions which can be falsified and which therefore are of empirical significance” (Hayek 1978: 33).

4.15 Probing the Frontier of Formal Macroeconomic and Finance Theory

In Frydman and Goldberg (2007) and our recent technical studies, we show how IKE models shed new light on salient features of the empirical record on exchange rates that have confounded international macroeconomists for decades. In Frydman and Goldberg (2011), we focus on how recognizing the centrality of nonroutine change and imperfect knowledge enables us to understand better the process by which financial markets, particularly equity markets, help society allocate its capital, and why asset-price swings are an integral part of this process.

IKE also provides a new way to explain why asset-price swings sometimes become excessive and shows how the hitherto neglected relationship between financial risk and price swings can help us to understand how excessive swings come to an end. This analysis provides a conceptual framework for prudential policy aimed at dampening excessive price swings—and thus at reducing the social costs inflicted when they reverse direction.

Although the application of IKE to financial markets appears promising, it is too early to claim broader usefulness for it in macroeconomic and policy modeling. If qualitative regularities can be established in contexts other than asset markets, IKE’s nonstandard probabilistic formalism can show how to incorporate them into mathematical models. However, when revisions of forecasting strategies (or more broadly, change on the individual and aggregate levels) cannot be adequately characterized with qualitative and contingent conditions, empirically relevant mathematical models of how market outcomes unfold over time may be beyond

32. Our approach to testing the implications of IKE versus REH models of swings makes use of cointegrated vector autoregressive methodology and inference. For book-length treatments, see Johansen (1996) and Juselius (2007).

the reach of economic analysis. In this sense, IKE explores the frontier of what formal macroeconomic and finance theory can deliver. How far, and in which contexts, this boundary can be extended is the crucial open question.

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