Periodicity detection

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EL9173 Selected Topics in Signal Processing: Audio Content Analysis
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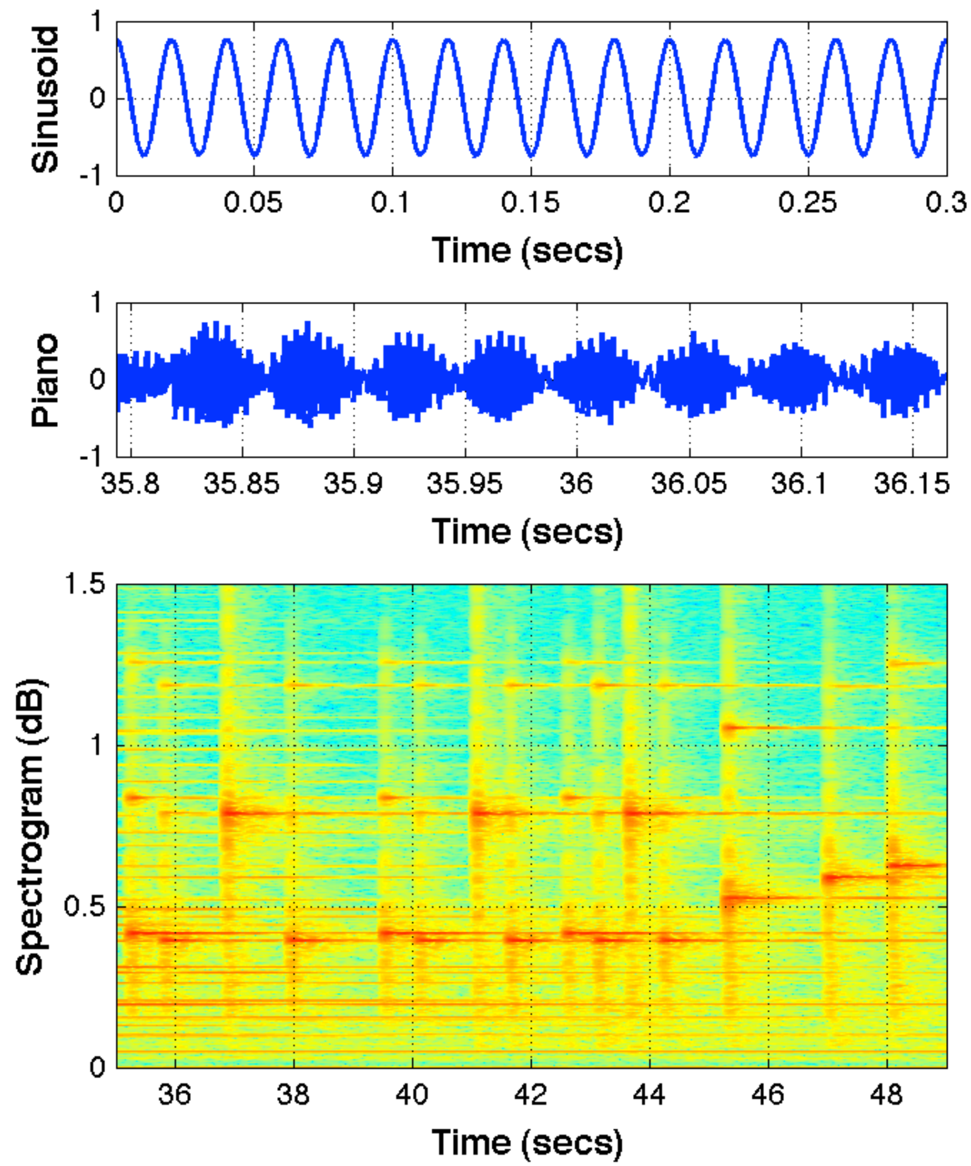
Periodicity detection

- Formally, a periodic signal is defined as:

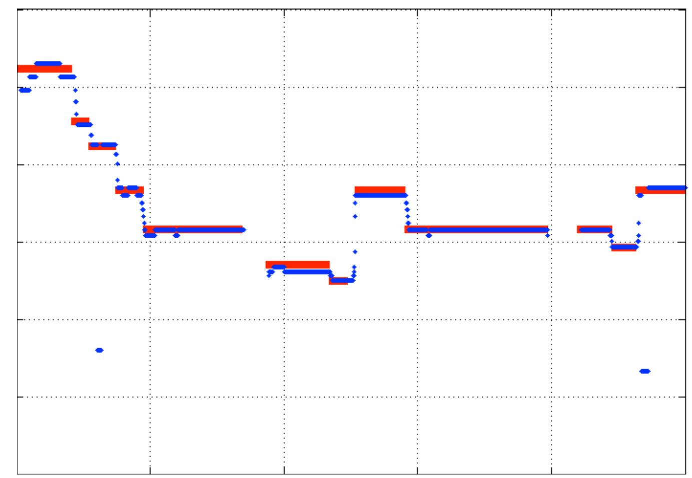
$$x(t) = x(t + T_0), \forall t$$

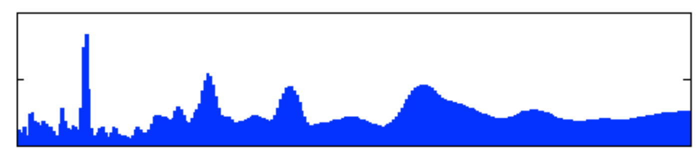
- Detect the fundamental period/frequency (and phase)

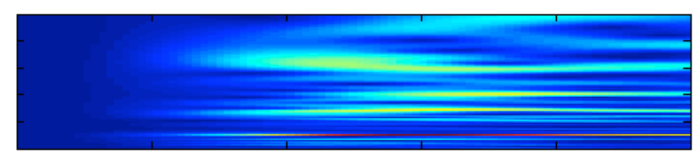


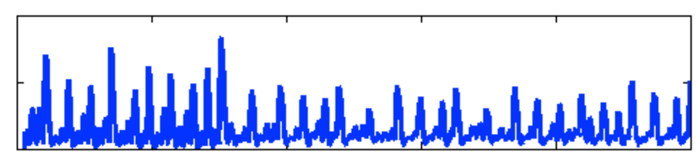
Applications

- At short (pitch) and long (rhythm) time scales:
    - pitch-synchronous analysis
    - voice/sound identification
    - prosodic analysis
    - bioacoustics
    - melodic analysis
    - note transcription
    - beat tracking, segmentation



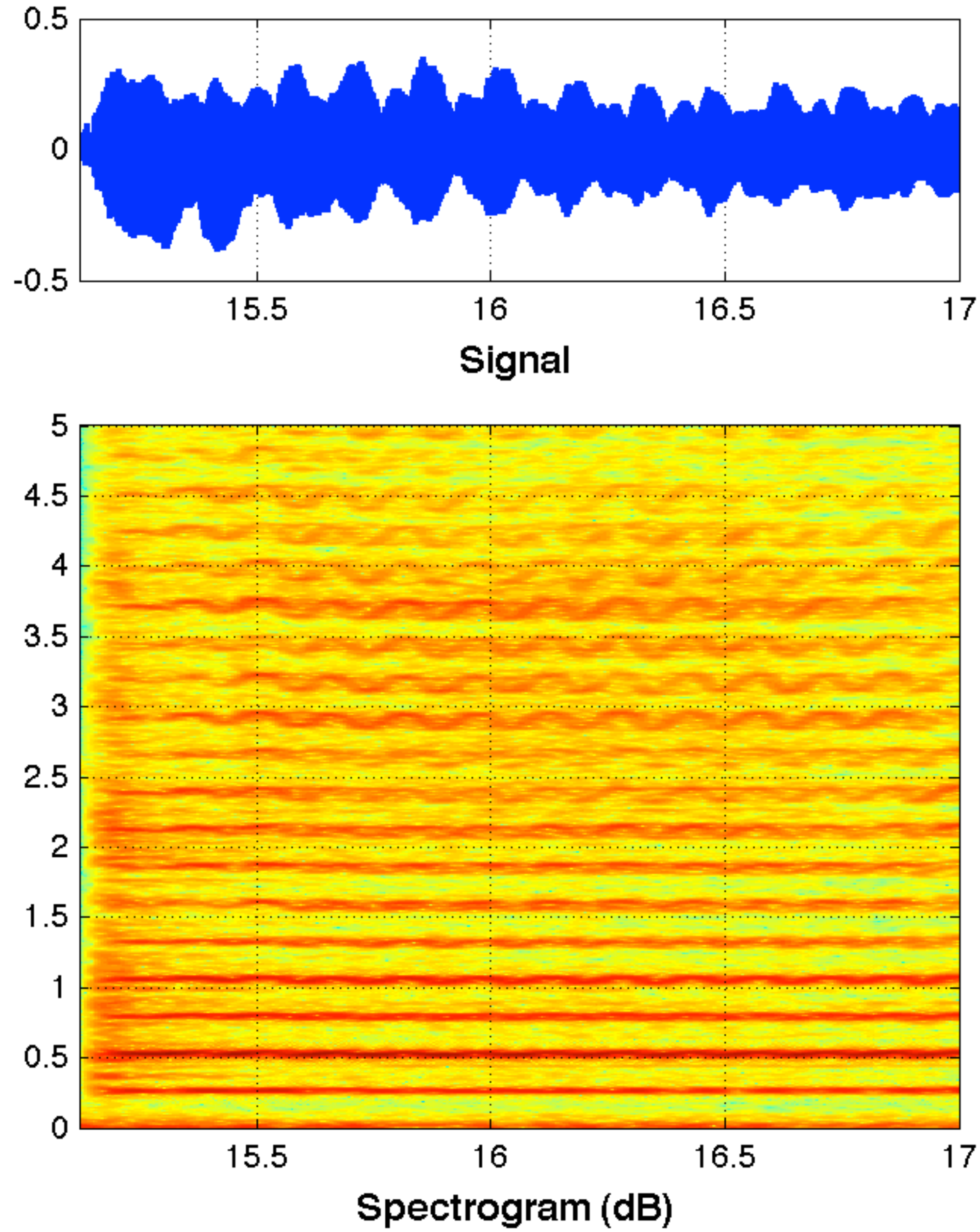






Difficulties

- Quasi-periodicities, temporal variations
- Multiple periodicities associated with f0
- transients and noise
- Polyphonies: information overlap



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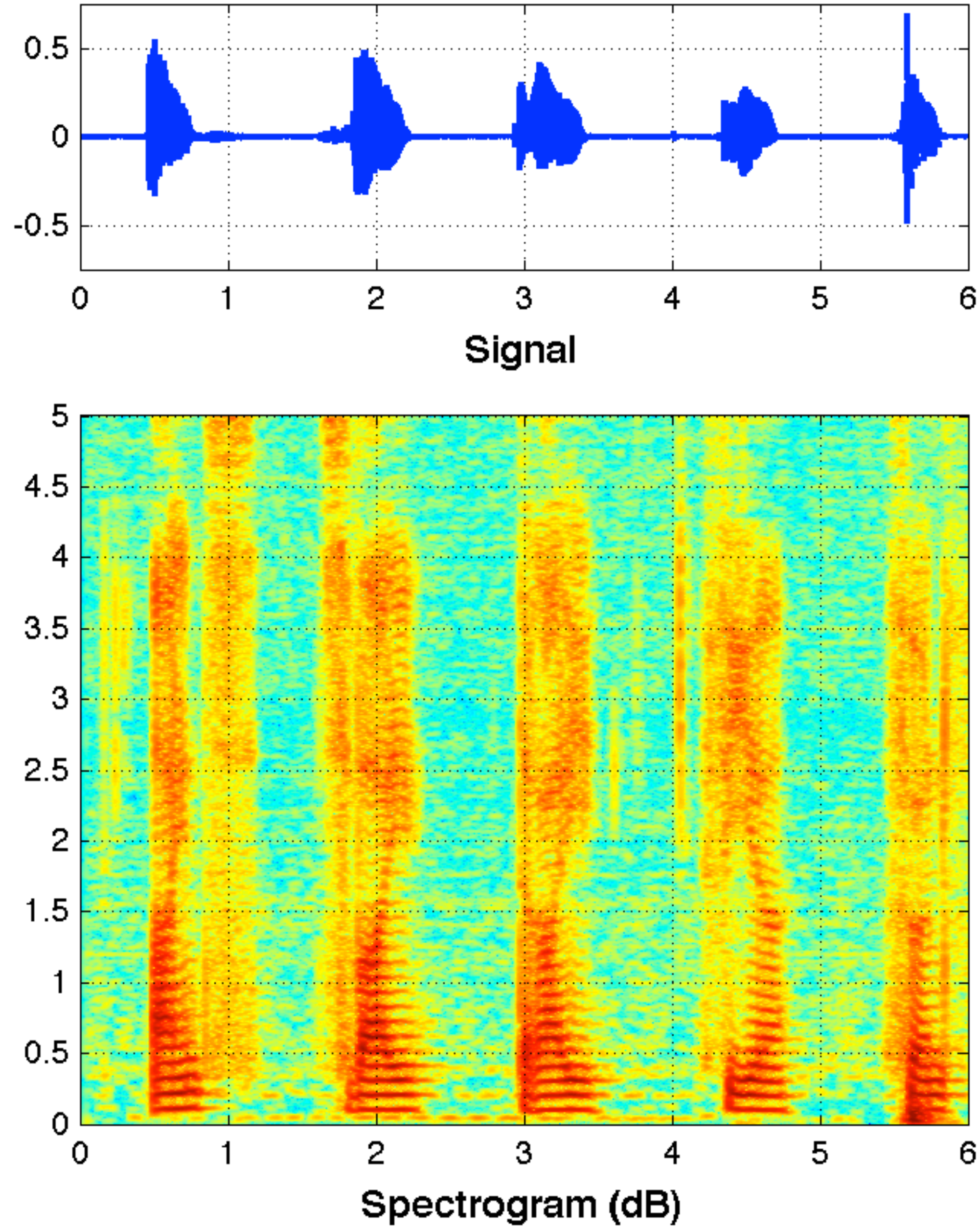

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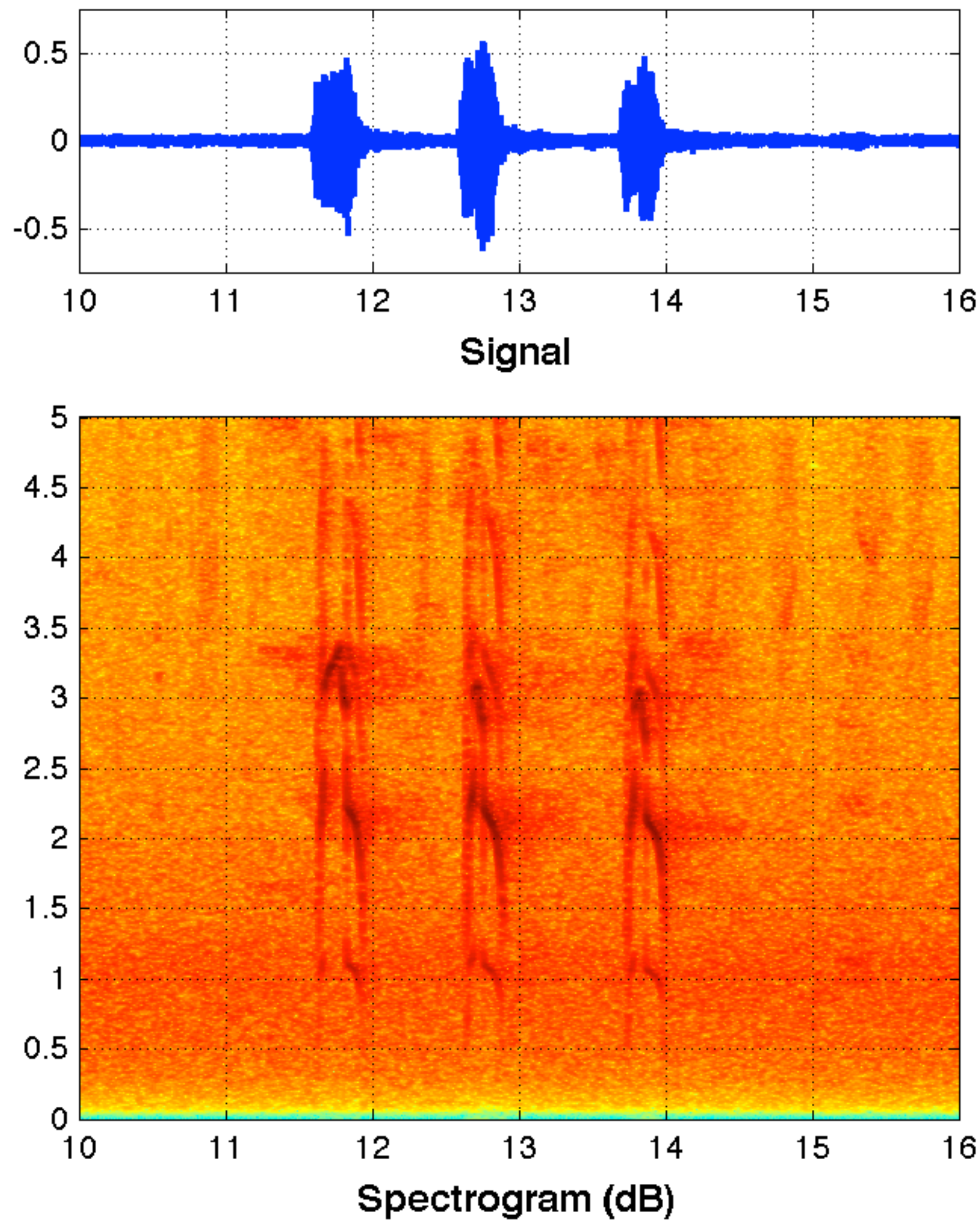

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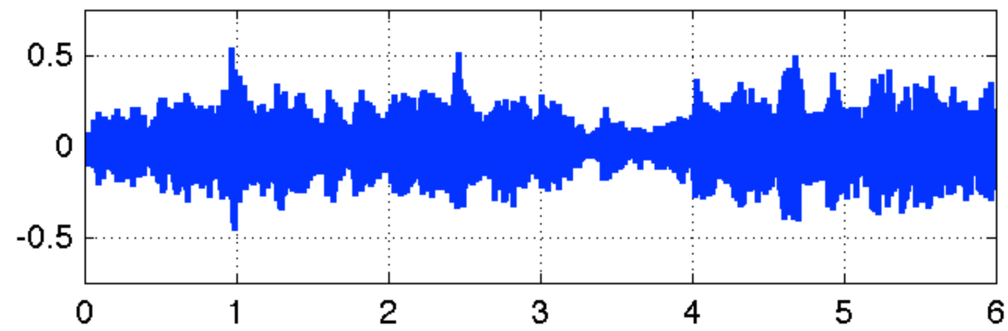
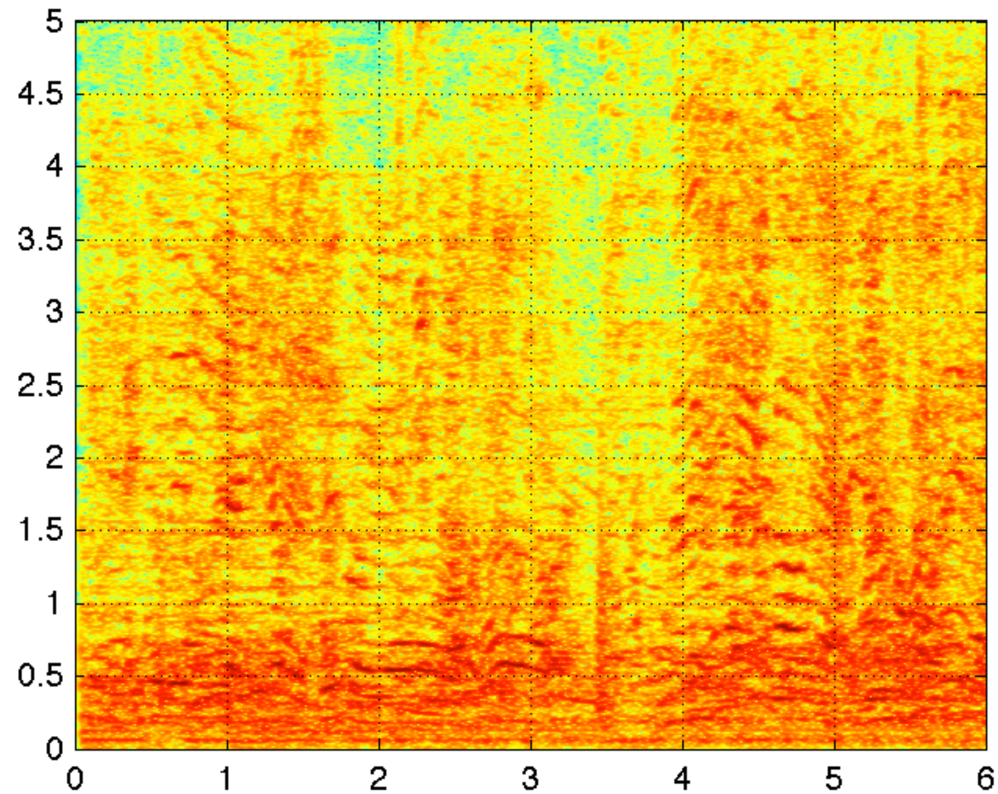


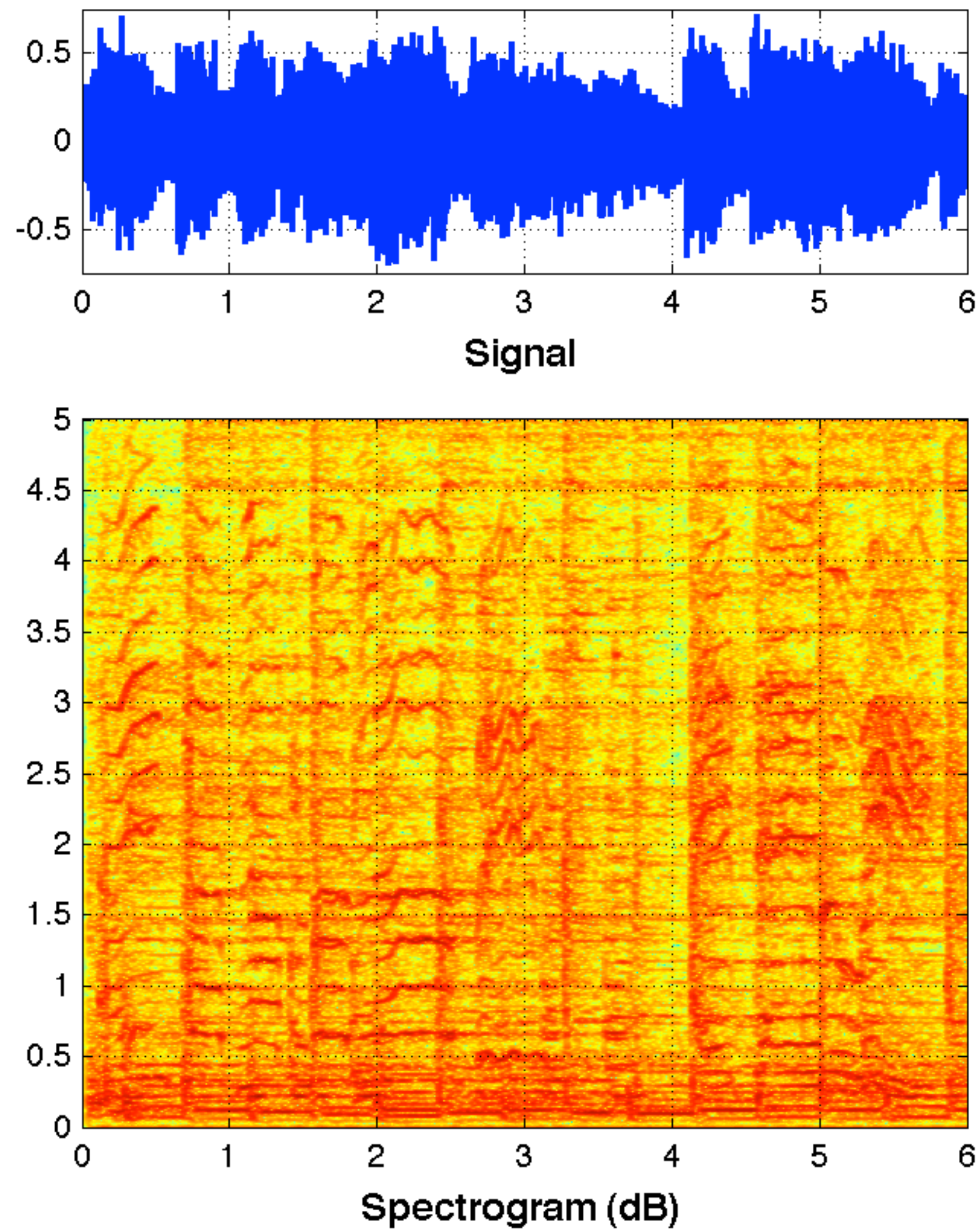
Signal



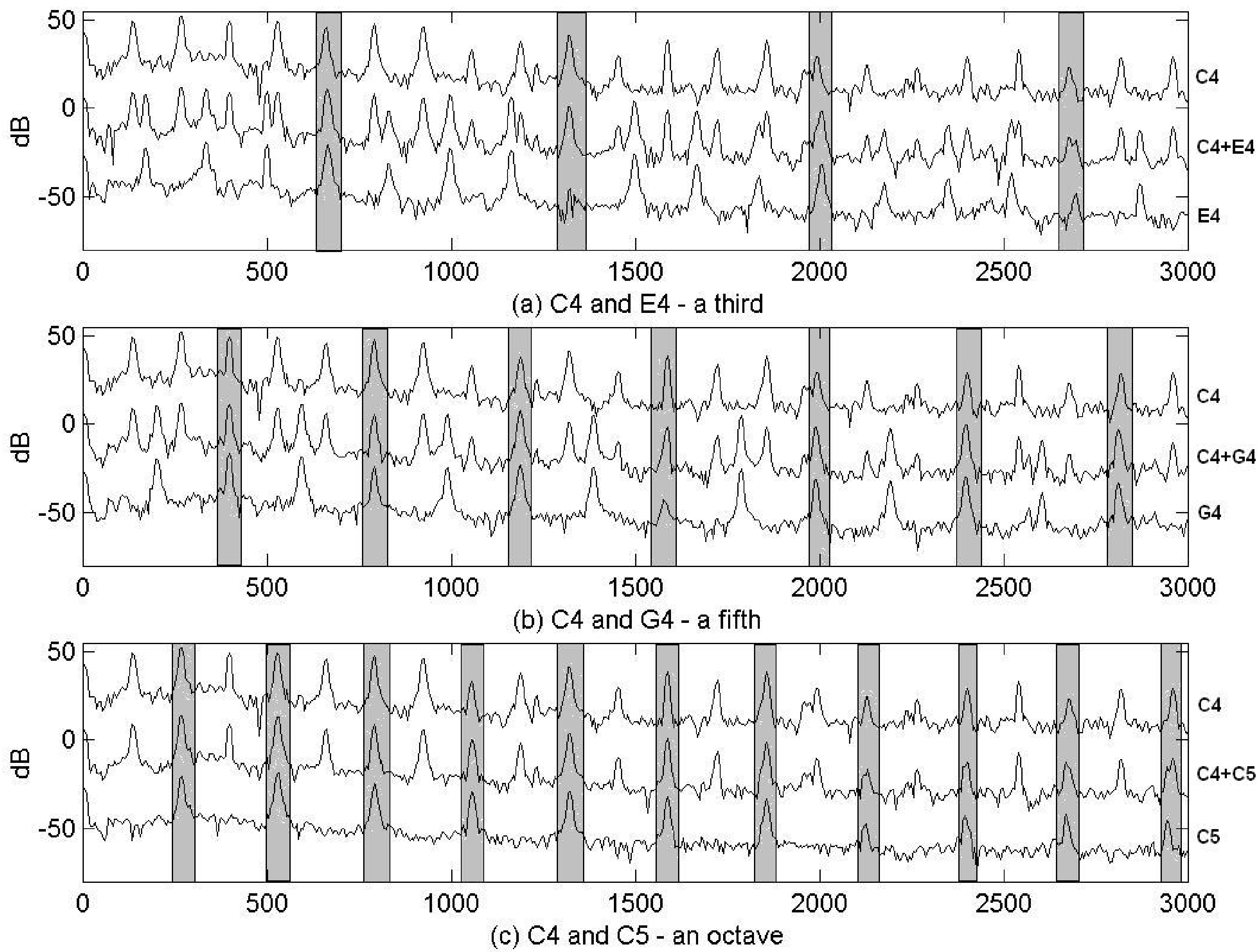
Spectrogram (dB)

Difficulties

- Quasi-periodicities, temporal variations
- Multiple periodicities associated with f0
- transients and noise
- Polyphonies: information overlap

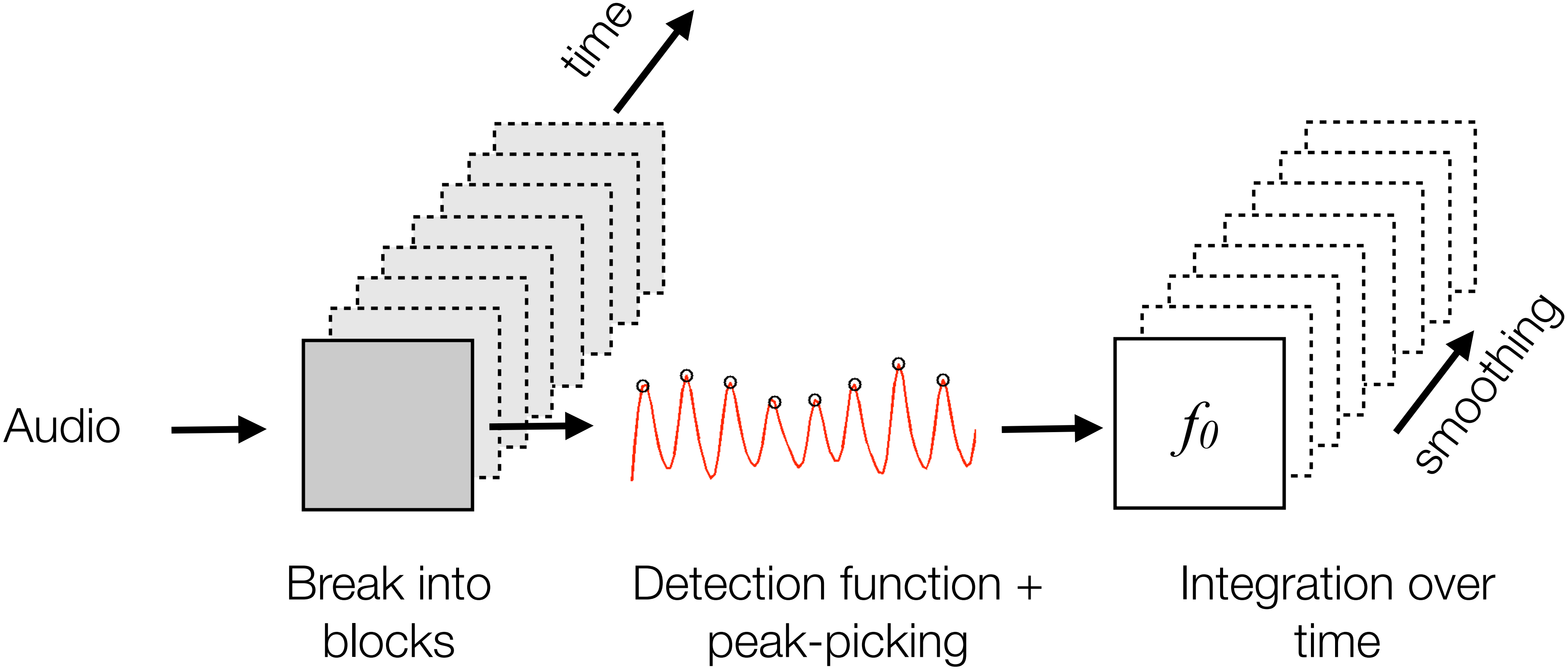


Overlap



50
0
-50
dB
C4
C4+E4
E4
0
500
1000
1500
2000
2500
3000
(a) C4 and E4 - a third
C4
C4+G4
G4
(b) C4 and G4 - a fifth
C4
C4+C5
C5
(c) C4 and C5 - an octave

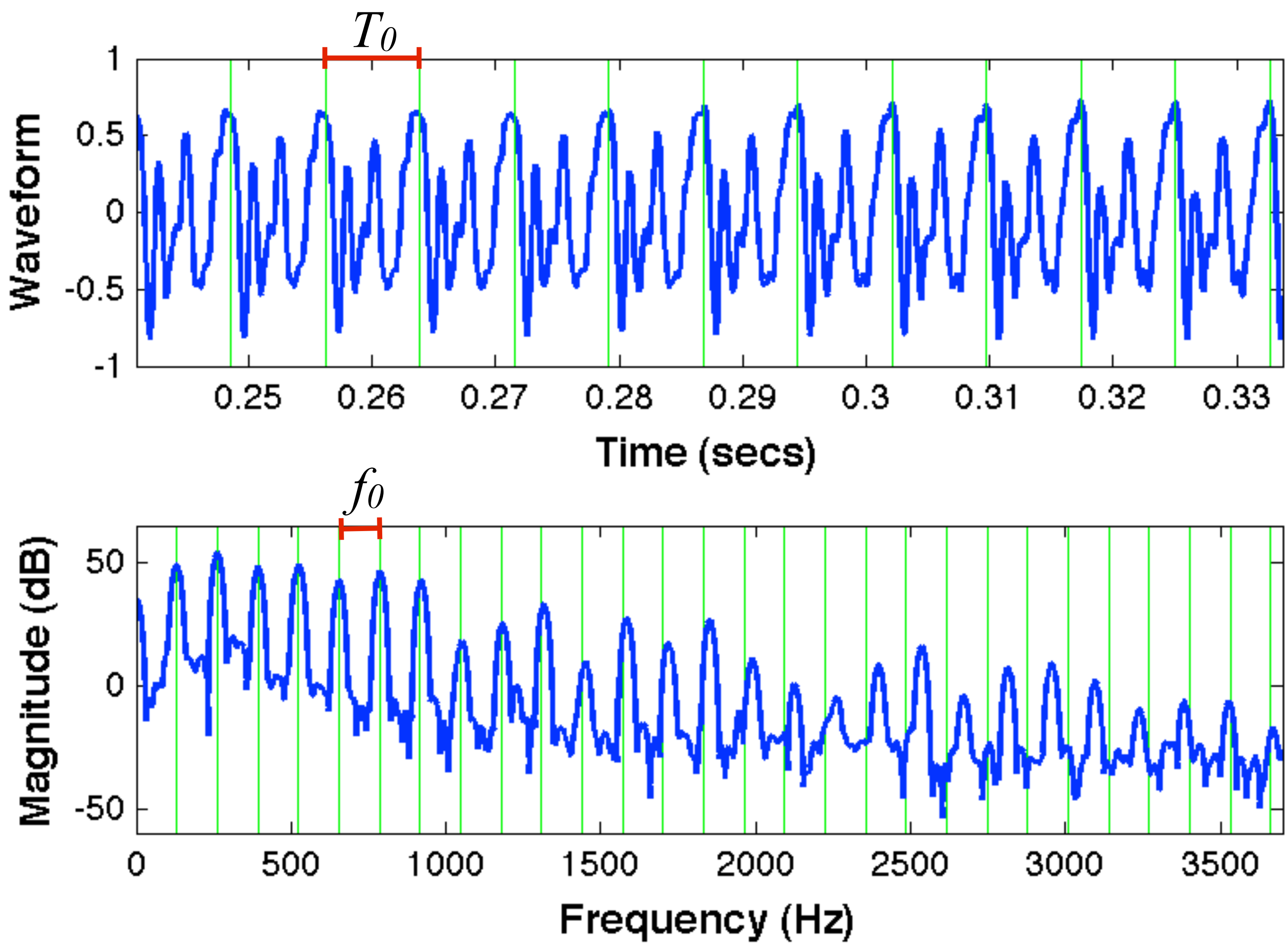
Architecture



Overview of Methods

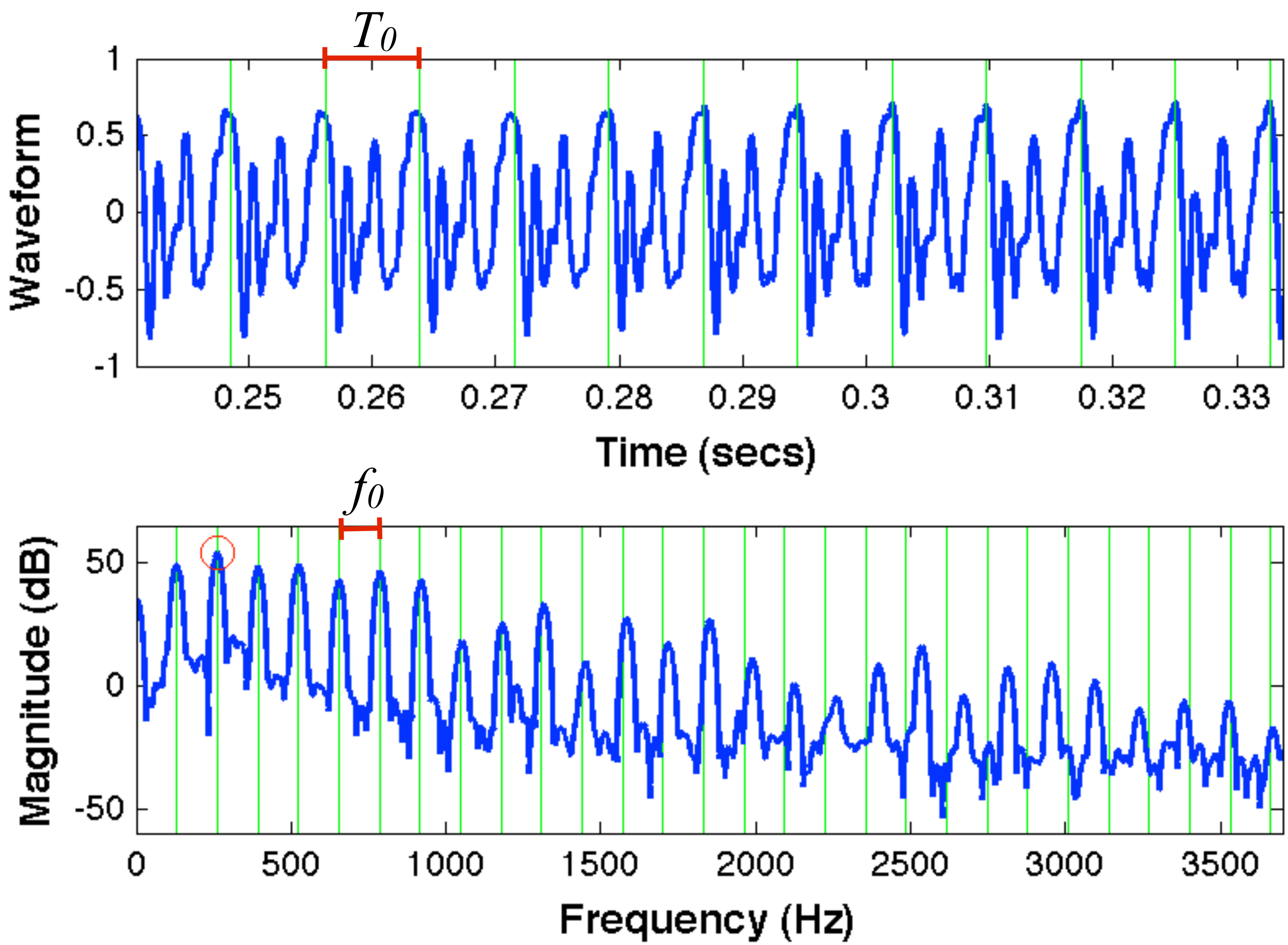
- DFT
- Autocorrelation
- Spectral Pattern Matching
- Cepstrum
- Spectral Autocorrelation
- YIN
- Auditory model

DFT

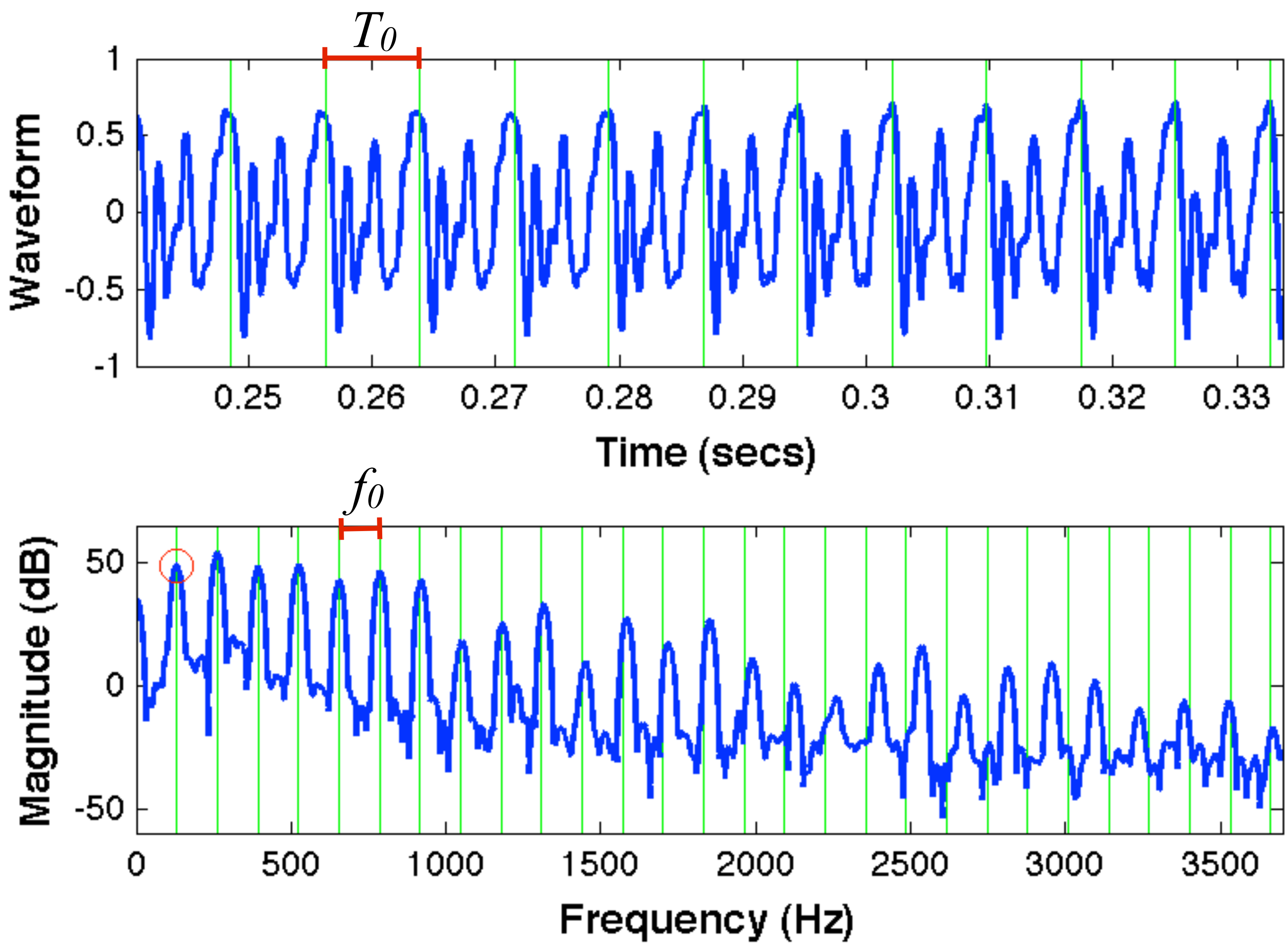


T_0
Waveform
1
0.5
0
-0.5
-1
0.25
0.26
0.27
0.28
0.29
0.3
0.31
0.32
0.33
Time (secs)
f_0
Magnitude (dB)
50
0
-50
0
500
1000
1500
2000
2500
3000
3500
Frequency (Hz)

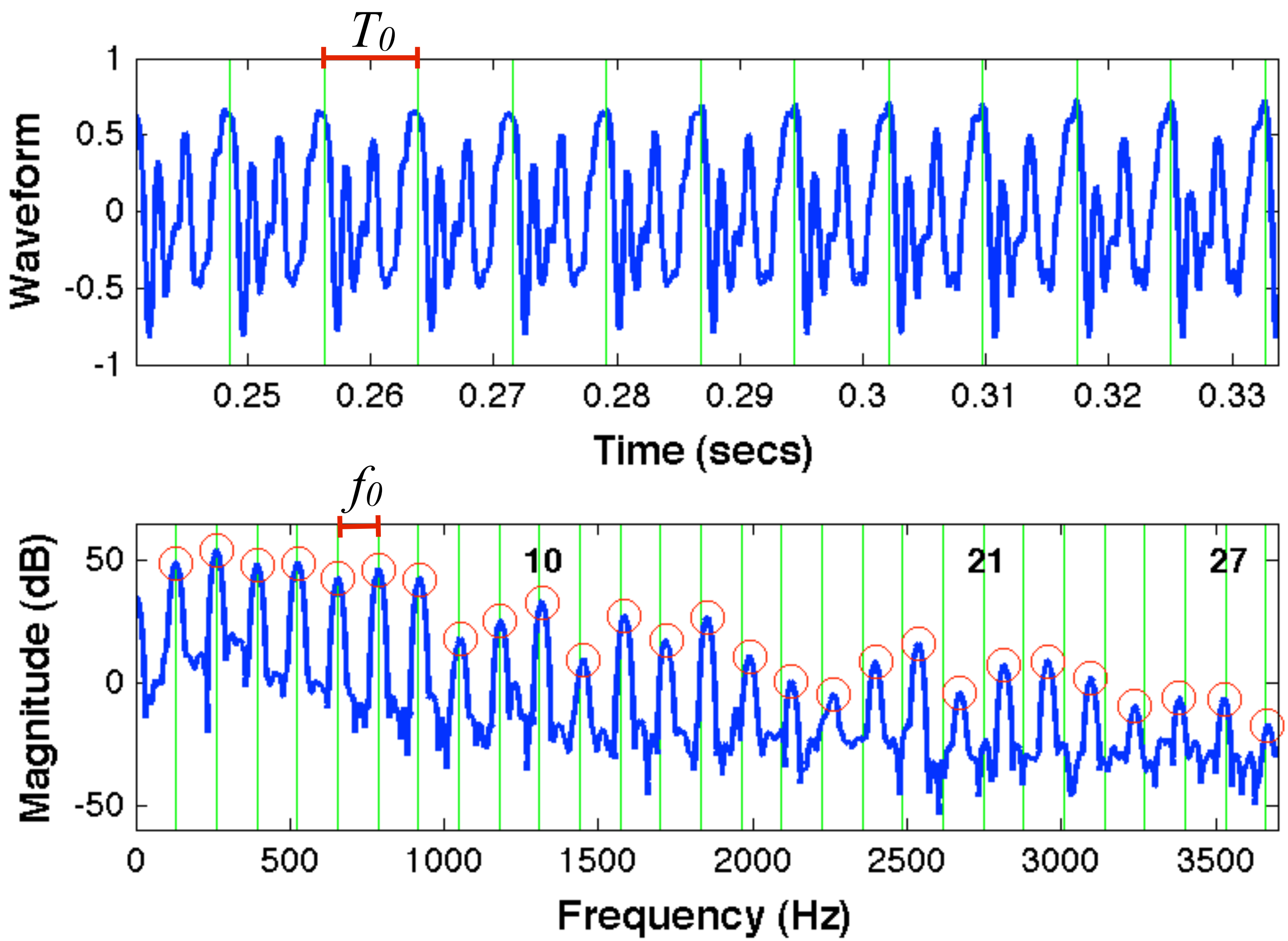
DFT



DFT


T0
1
0.5
0
-0.5
-1
Waveform
0.25
0.26
0.27
0.28
0.29
0.3
0.31
0.32
0.33
Time (secs)
f0
50
0
-50
Magnitude (dB)
0
500
1000
1500
2000
2500
3000
3500
Frequency (Hz)

DFT



T_0
Waveform
1
0.5
0
-0.5
-1
0.25
0.26
0.27
0.28
0.29
0.3
0.31
0.32
0.33
Time (secs)
f_0
Magnitude (dB)
50
0
-50
10
21
27
0
500
1000
1500
2000
2500
3000
3500
Frequency (Hz)

Autocorrelation

- Cross-product measures similarity across time
- Cross-correlation of two real-valued signals x and y:

$$r_{xy}(l) = \frac{1}{N} \sum_{n=0}^{N-1} x(n)y(n+l) \quad \leftarrow \text{modulo } N$$

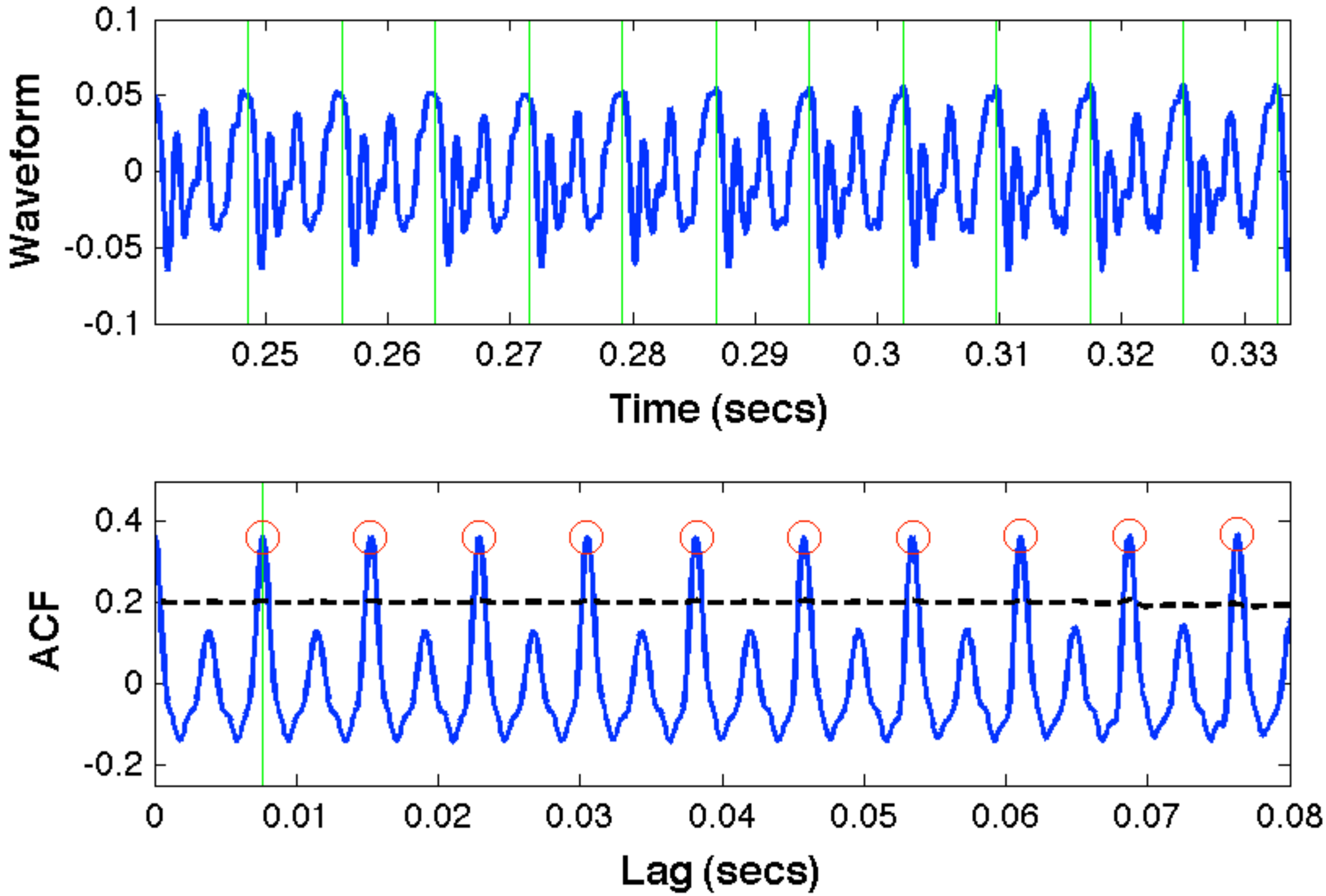
$$l = 0, 1, 2, \cdots, N-1$$

- Unbiased (short-term) Autocorrelation Function (ACF):

$$r_x(l) = \frac{1}{N-l} \sum_{n=0}^{N-1-l} x(n)x(n+l)$$

$$l = 0, 1, 2, \cdots, L-1$$

Autocorrelation



Autocorrelation

- The short-term ACF can also be computed as:

$$r_x(l) = \left(\frac{1}{N-l}\right) real(IFFT(|X|^2))$$

$$X \rightarrow FFT(x)$$

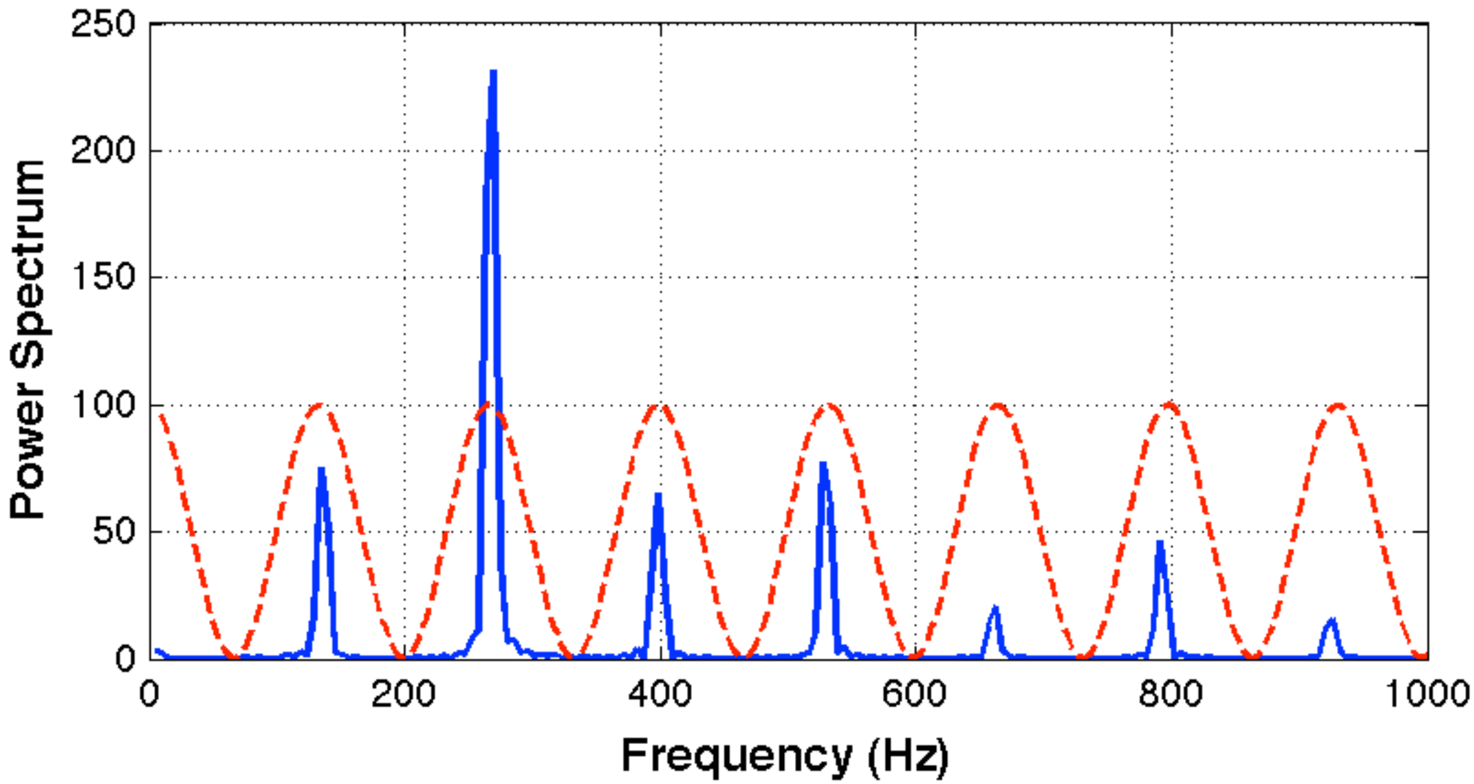
x zero-padded to next power of 2

after $(N+L)-1$

Autocorrelation

- This is equivalent to the following correlation:

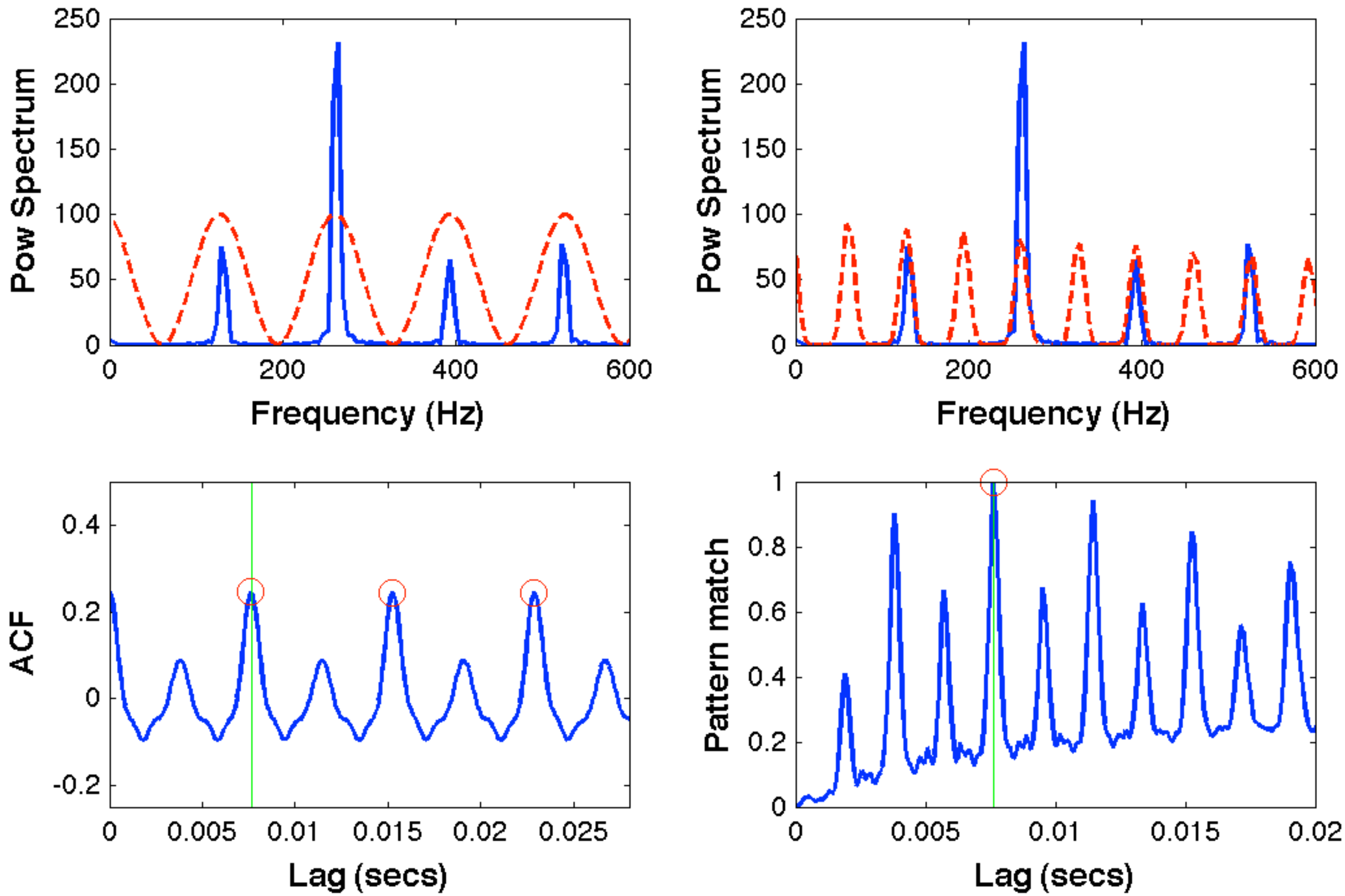
$$r_x(l) = \frac{1}{K-l} \sum_{k=0}^{K-1} cos(2\pi lk/K)|X(k)|^2$$



Pattern Matching

- Comb filtering is a common strategy
- Any other template that realistically fits the magnitude spectrum
- Templates can be specific to sound sources
- Matching strategies vary: correlation, likelihood, distance, etc.

Pattern Matching

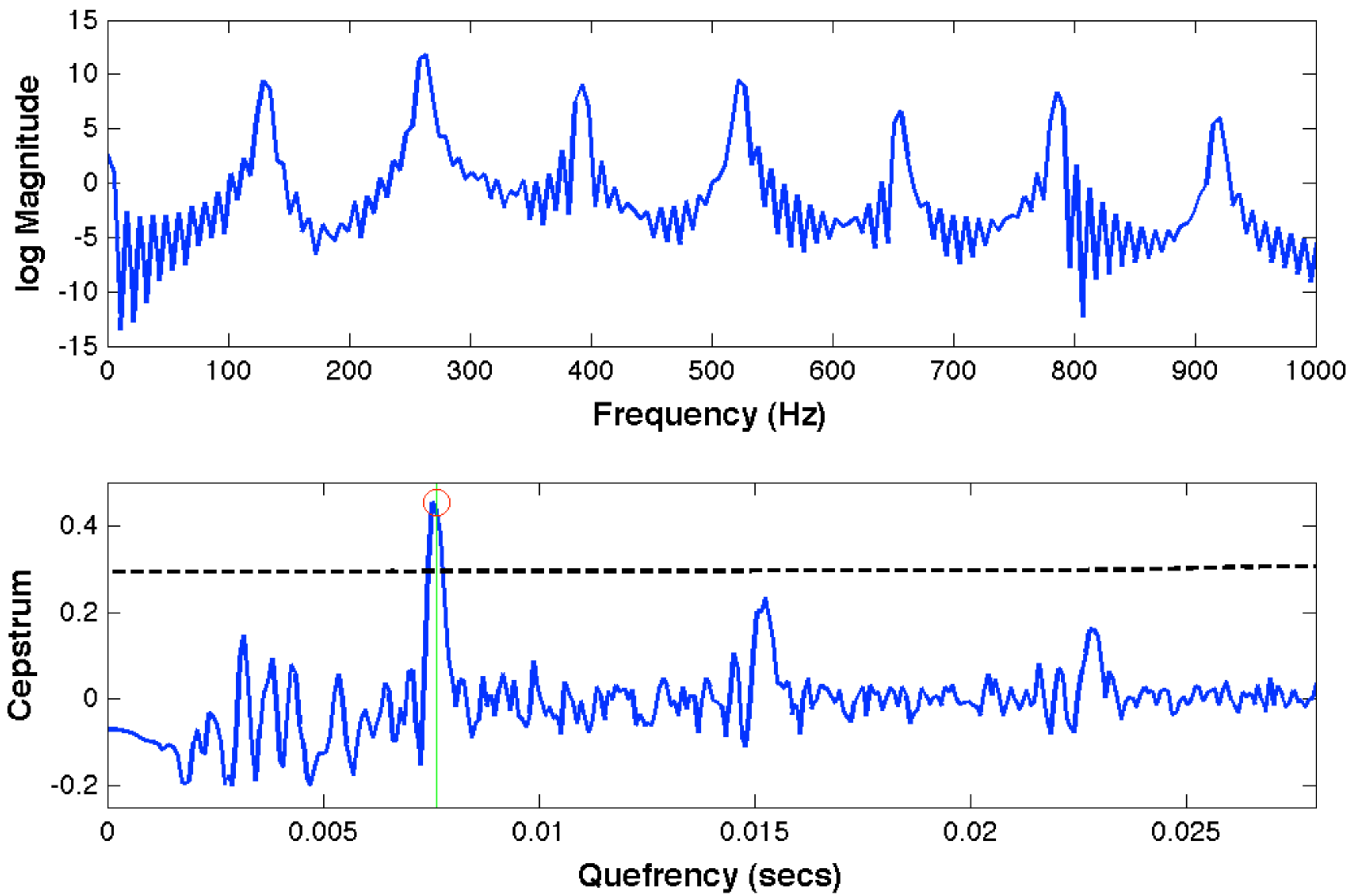


Cepstrum

- Treat the log magnitude spectrum as if it were a signal -> take its (I)DFT
- Measures rate of change across frequency bands (Bogert et al., 1963)
- Cepstrum -> Anagram of Spectrum (same for quefrency, liftering, etc)
- For a real-valued signal is defined as:

$$c_x(l) = real(IFFT(log(|FFT(x)|)))$$

Cepstrum


15
10
5
0
-5
-10
-15
log Magnitude
0
100
200
300
400
500
600
700
800
900
1000
Frequency (Hz)
0.4
0.2
0
-0.2
Cepstrum
0
0.005
0.01
0.015
0.02
0.025
Quefrency (secs)

Spectral ACF

- Spectral location -> sensitive to quasi-periodicities

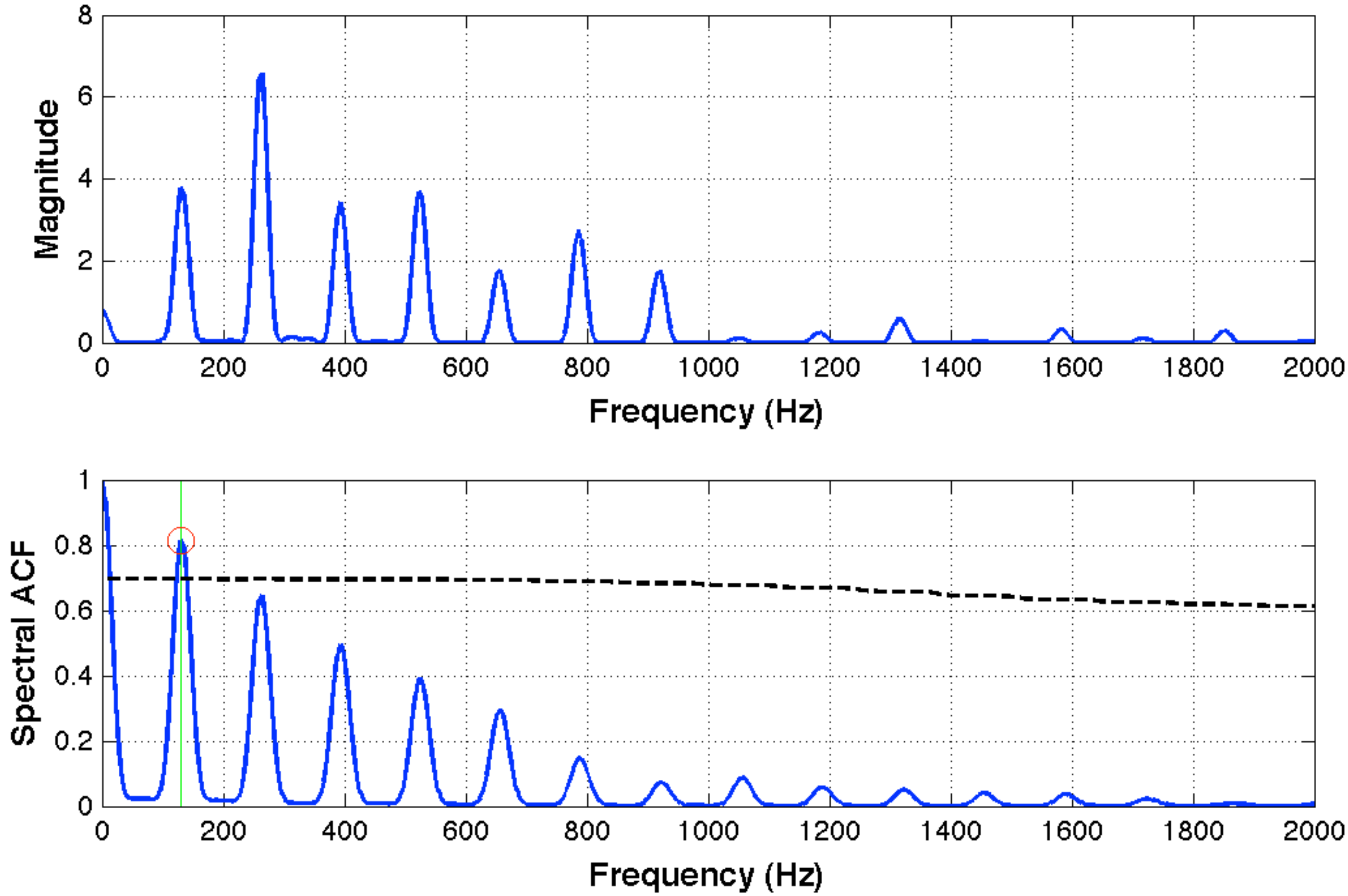
- (Quasi-)Periodic Spectrum, Spectral ACF.

$$r_X(l_f) = \frac{1}{N - l_f} \sum_{k=0}^{N-1-l_f} |X(k)||X(k+l_f)|$$

$$l_f = 0, 1, 2, \cdots, L-1$$

- Exploits intervalic information (more stable than locations of partials), while adding shift-invariance.

Spectral ACF



YIN

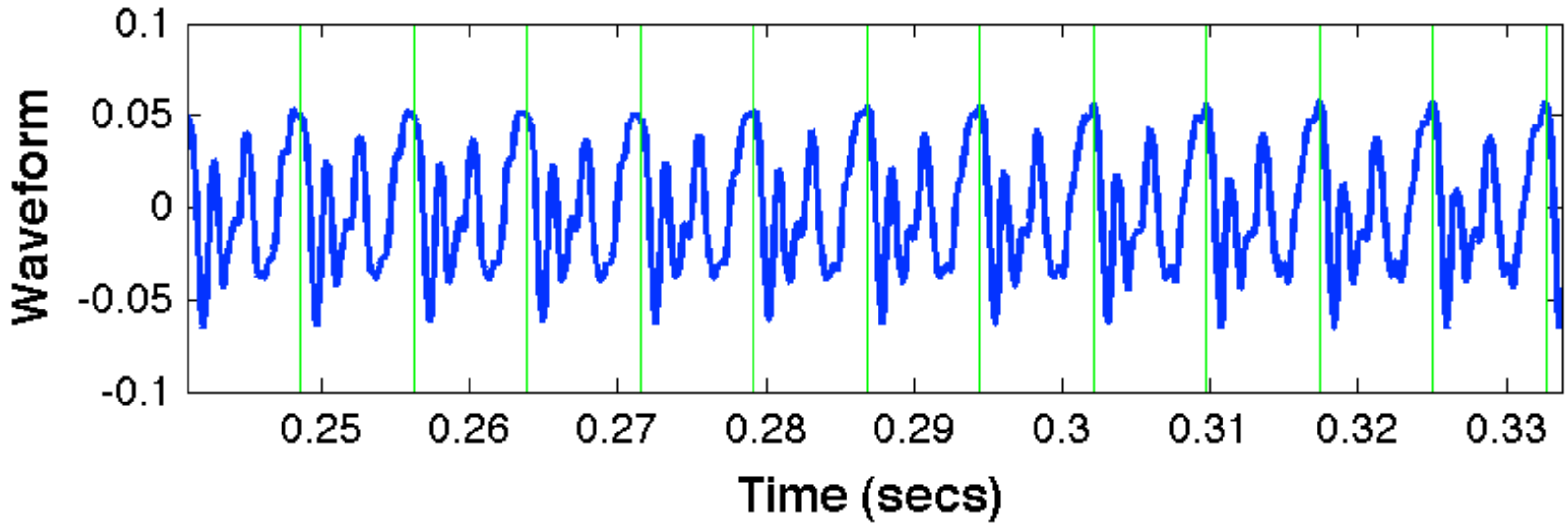
- Alternative to the ACF that uses the squared difference function (deCheveigne, 02):

$$d(l) = \sum_{n=0}^{N-1-l} \left(x(n) - x(n+l)\right)^2$$

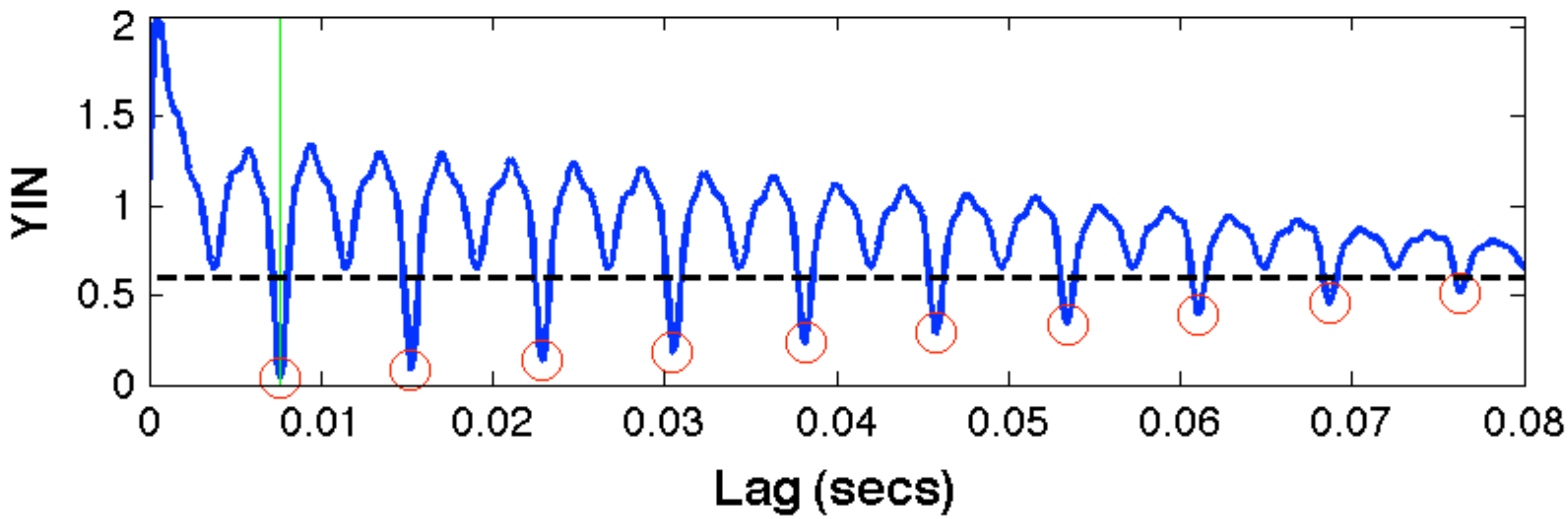
- For (quasi-)periodic signals, this functions cancel itself at $l = 0, l_0$ and its multiples. Zero-lag bias is avoided by normalizing as:

$$\hat{d}(l) = \begin{cases} 1 & l = 0 \\ d(l)/[(1/l)\sum_{u=1}^{l} d(u)] & \text{otherwise} \end{cases}$$

YIN

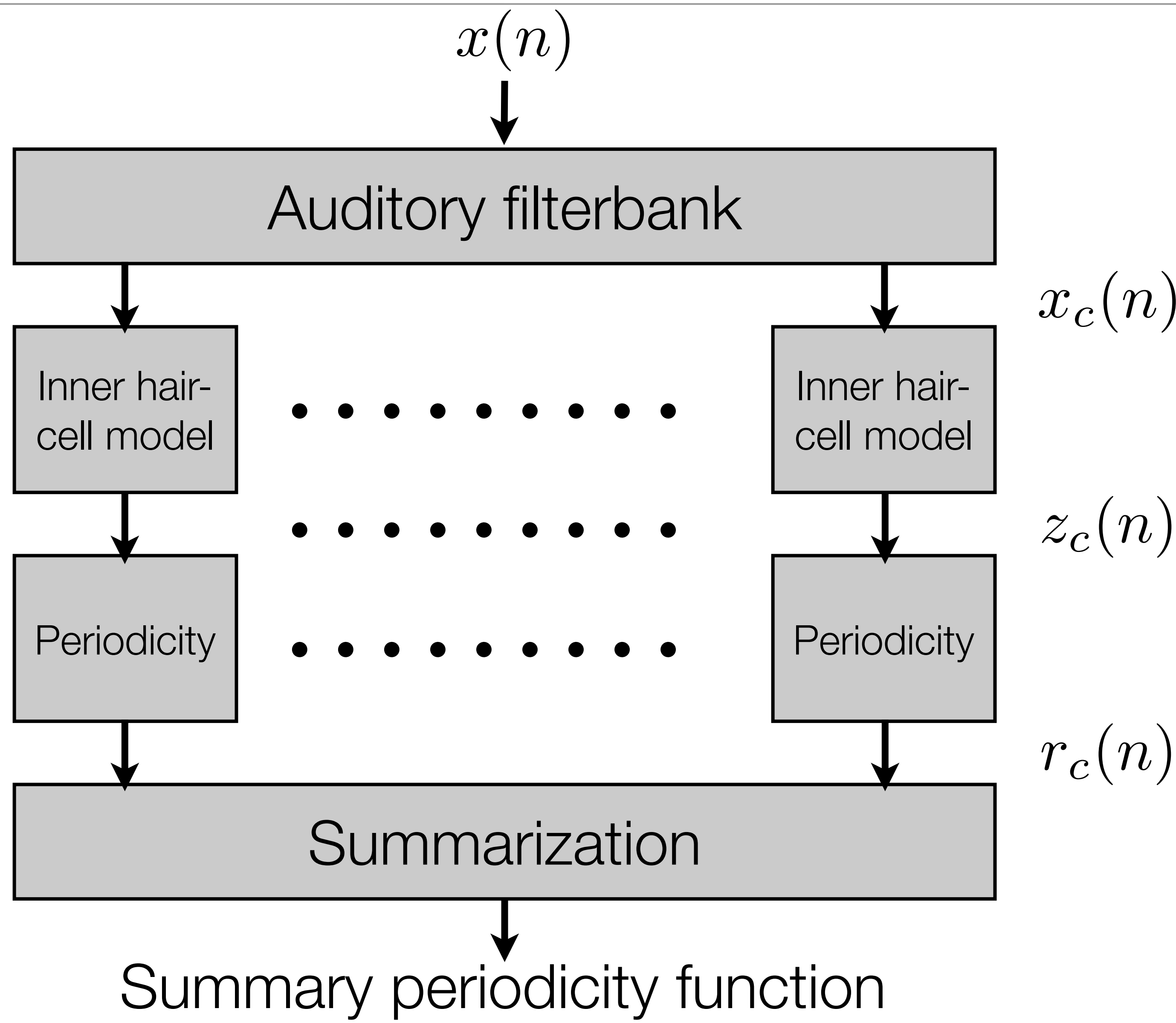


0.1
0.05
0
-0.05
-0.1
Waveform
0.25
0.26
0.27
0.28
0.29
0.3
0.31
0.32
0.33
Time (secs)



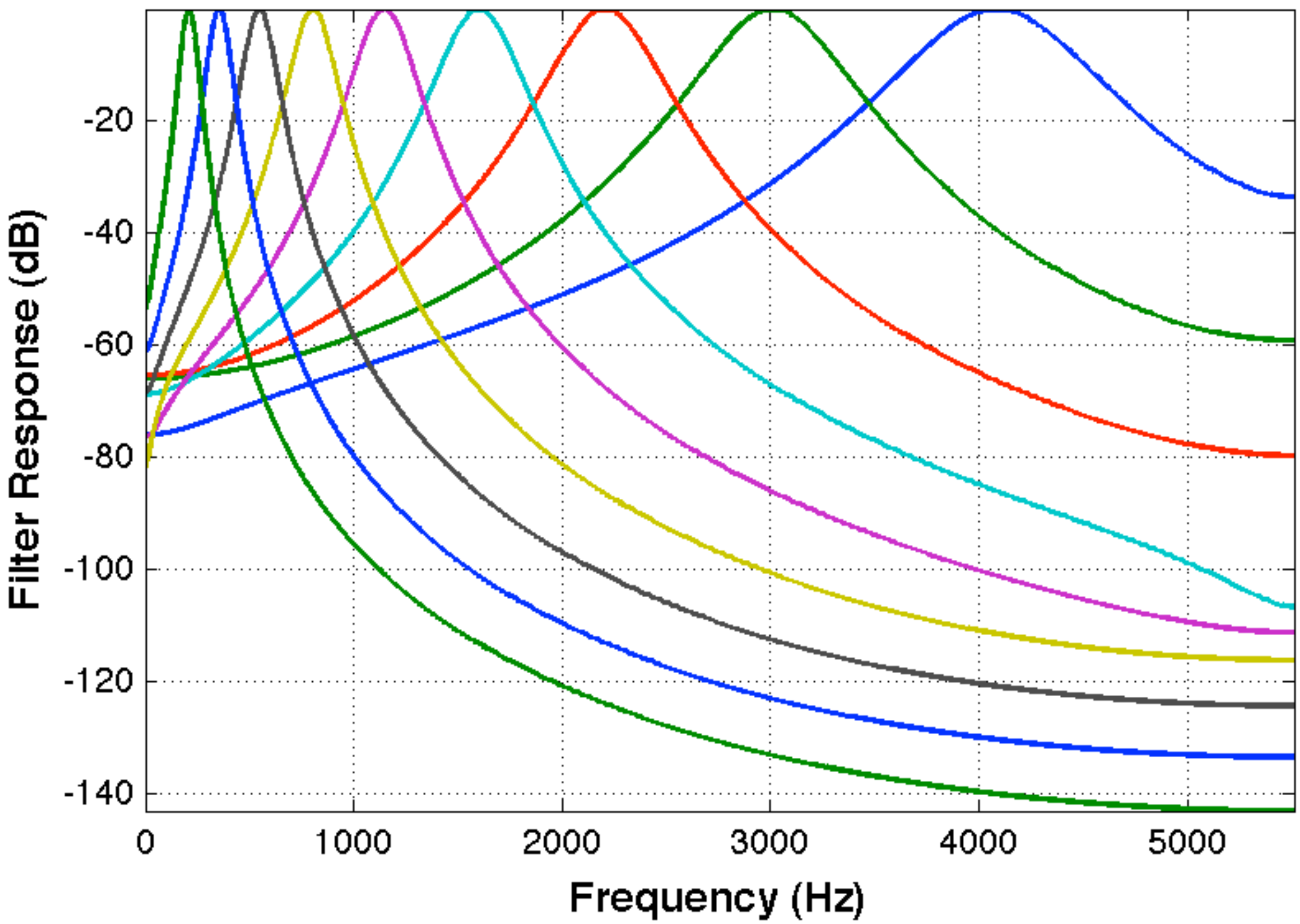
2
1.5
1
0.5
0
YIN
0
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
Lag (secs)

Auditory model



Auditory model

- Auditory filterbank: gammatone filters (Slaney, 93; Klapuri, 06):



Auditory model

The Equivalent Rectangular Bandwidths (ERB) of the filters:

$$b_c = 0.108 f_c + 24.7$$

$$f_c = 229 \times \left(10^{\psi/21.4} - 1\right)$$

$$\psi = \psi_{min} : (\psi_{max} - \psi_{min})/F : \psi_{max}$$

$$\psi_{min/max} = 21.4 \times \log_{10}(0.00437 f_{min/max} + 1)$$

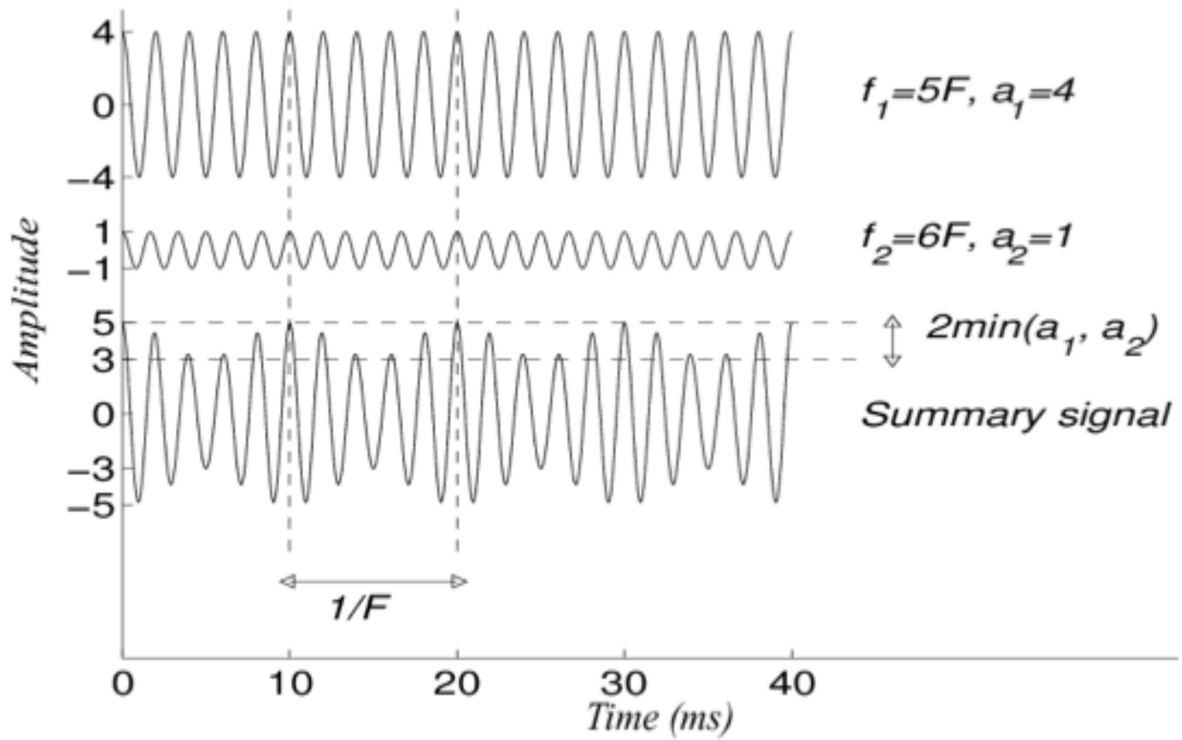
F = number of filters.

Auditory model

- Beating: interference between sounds of frequencies f_1 and f_2

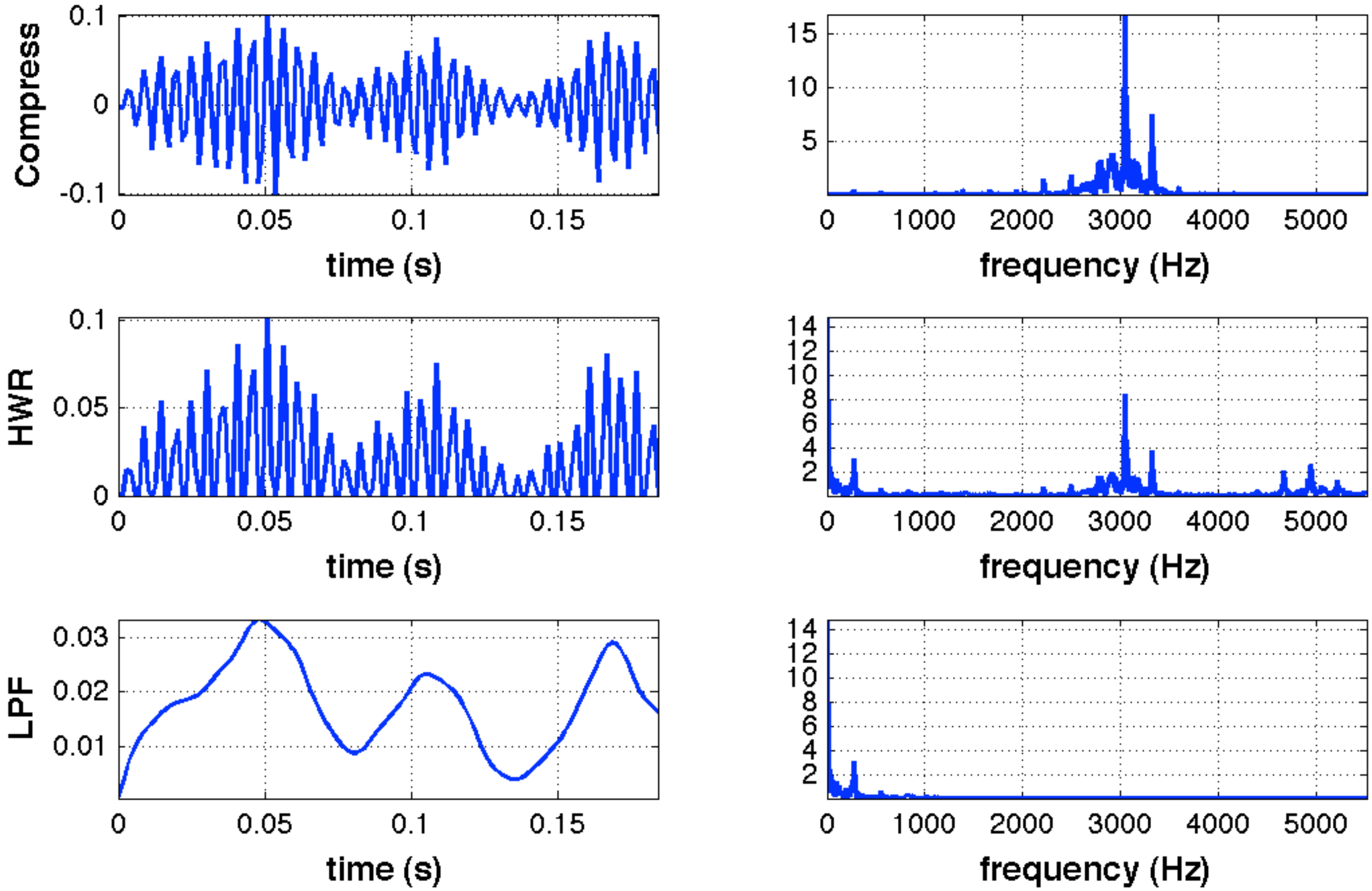
- Fluctuation of amplitude envelope of frequency $|f_2 - f_1|$

- The magnitude of the beating is determined by the smaller of the two amplitudes



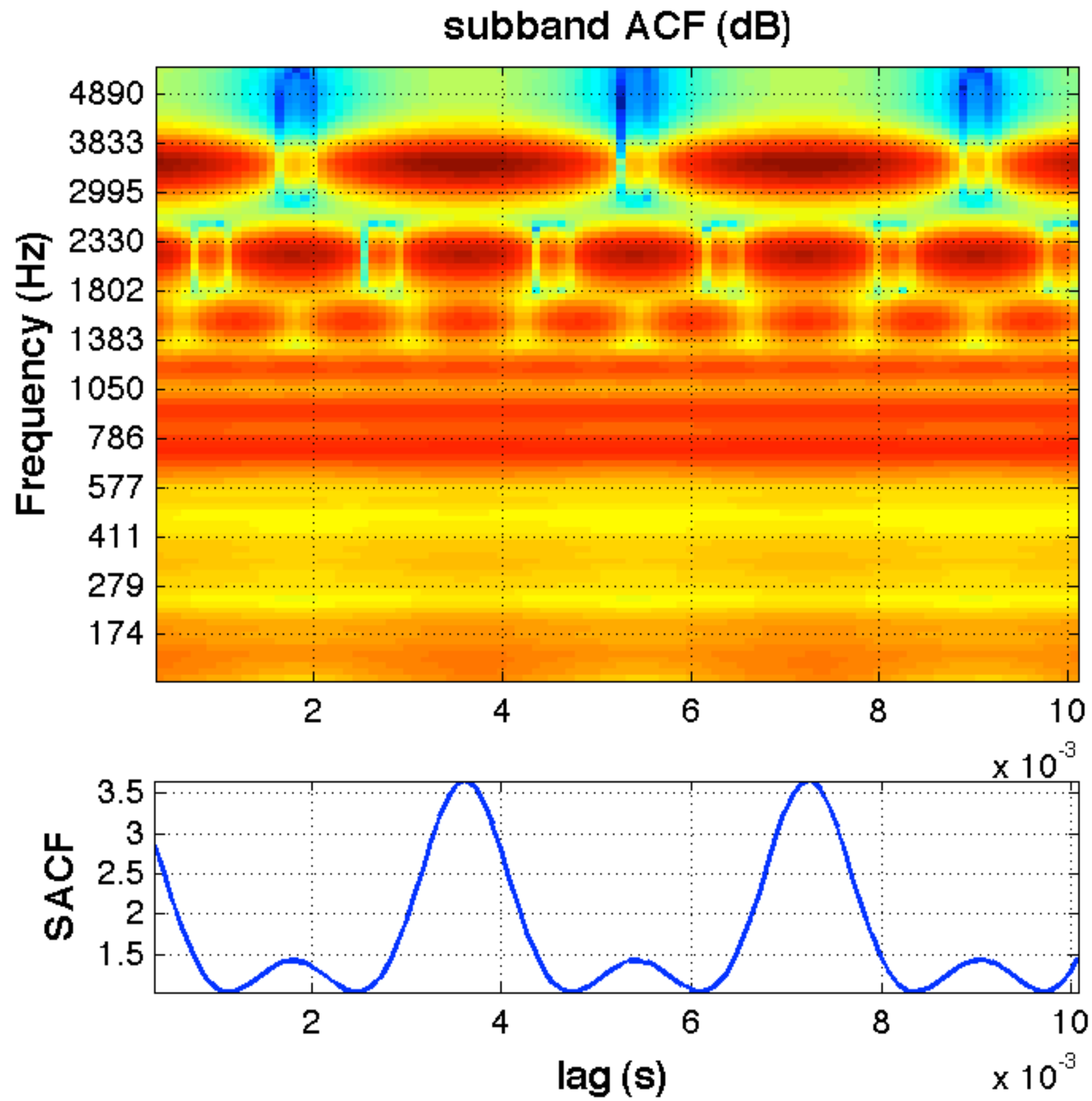
Auditory model

- Inner hair-cell (IHC) model:

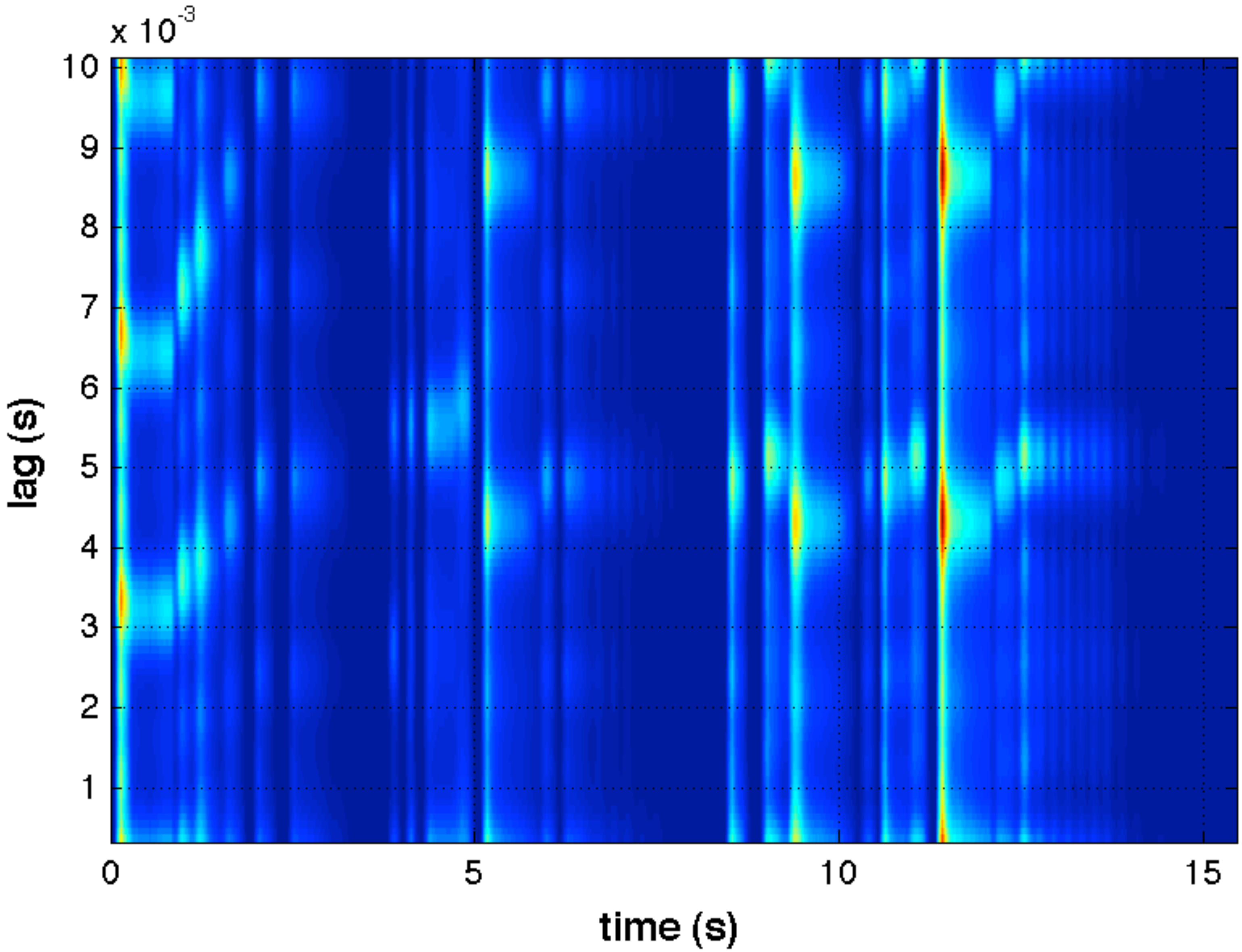


Auditory model

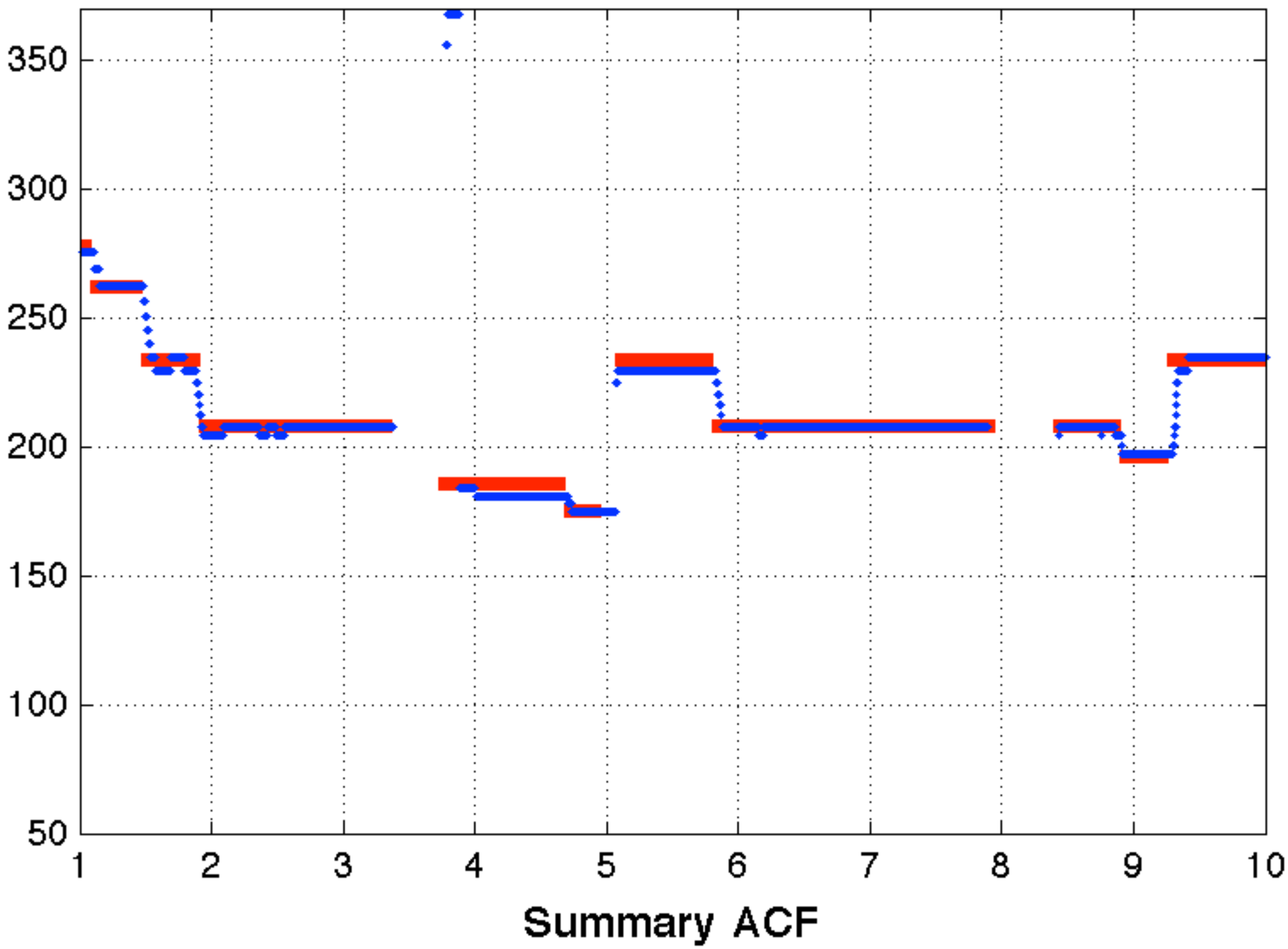
- Sub-band periodicity analysis using ACF
- Summing across channels (Summary ACF)
- Weighting of the channels changes the topology of the SACF



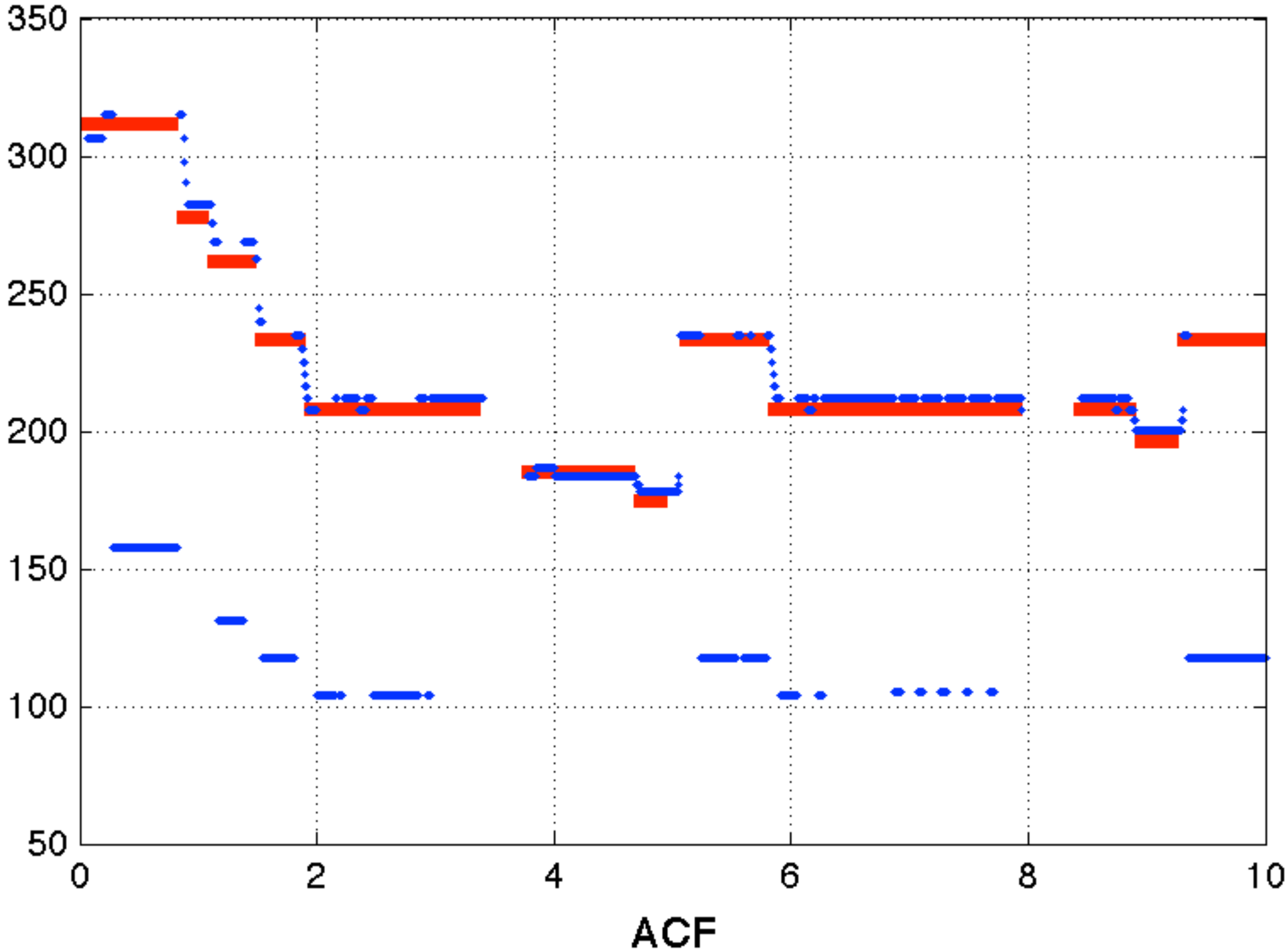
Auditory model



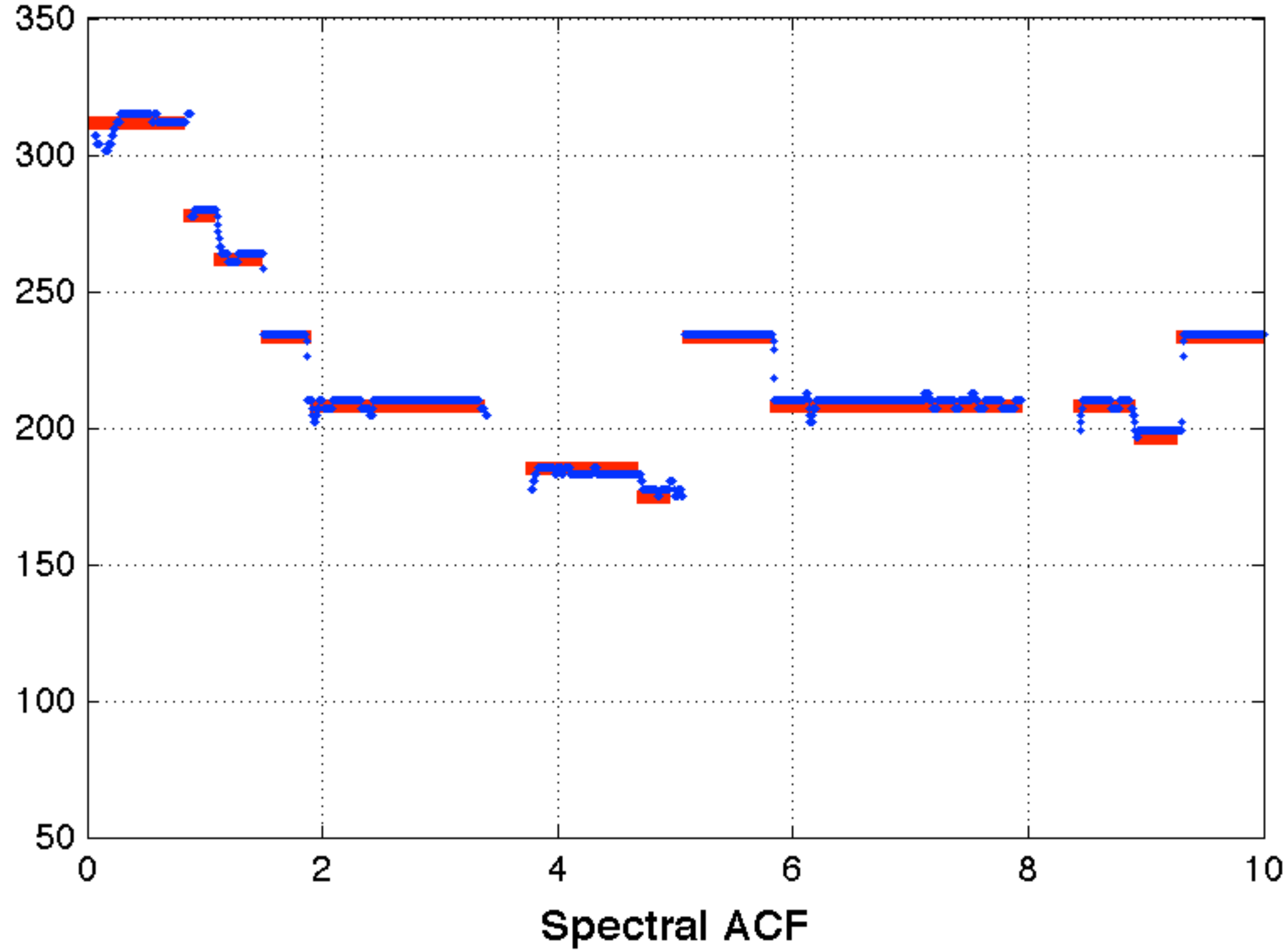
Comparing detection functions



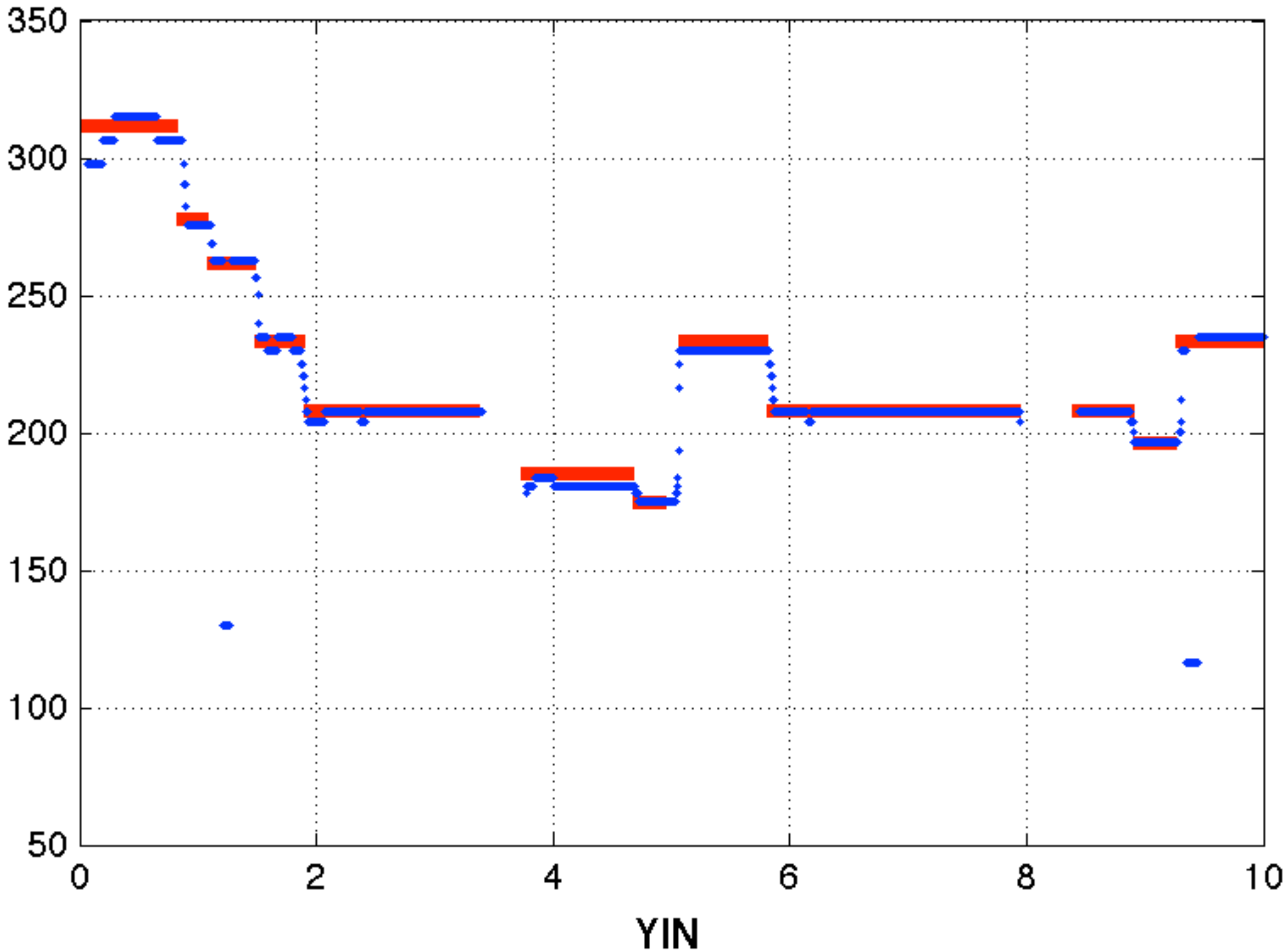
Comparing detection functions



Comparing detection functions



Comparing detection functions

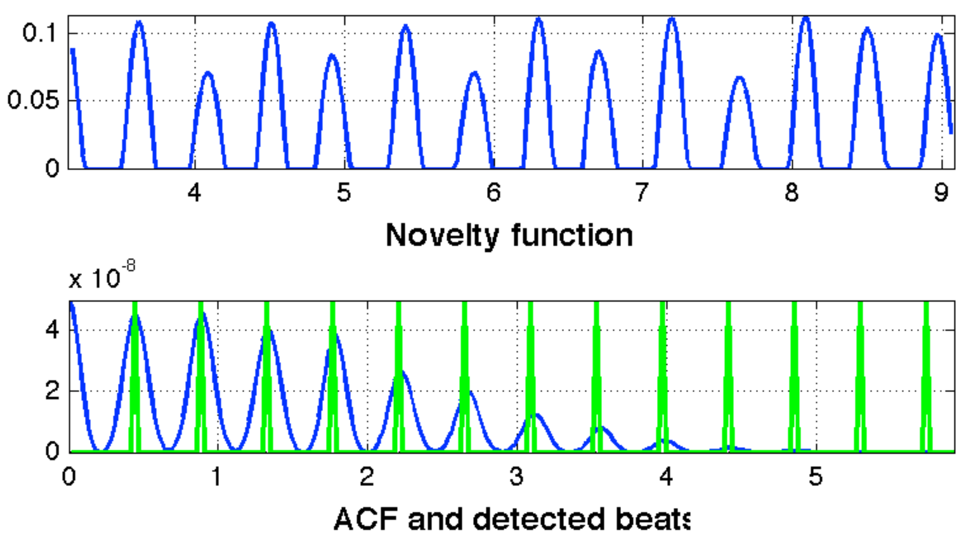


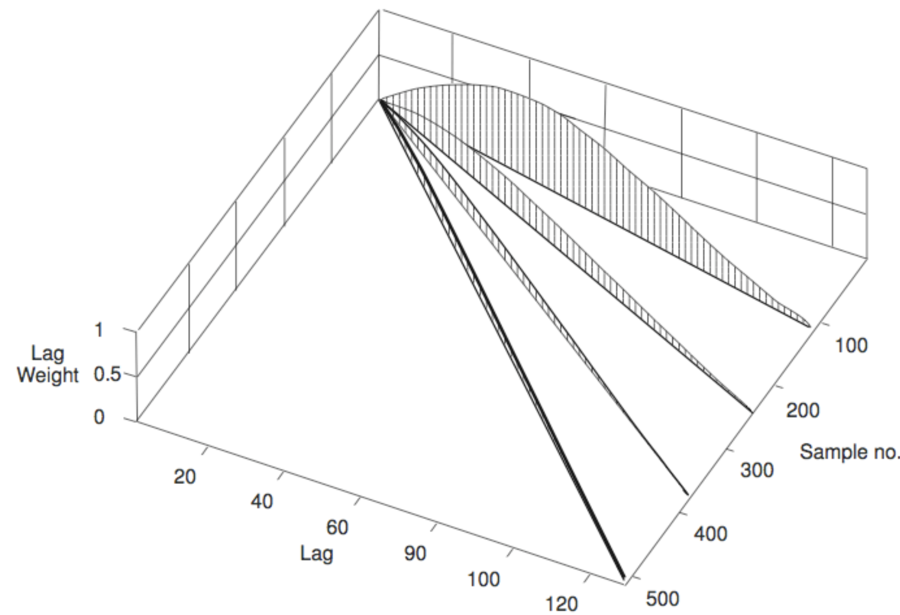
Tempo

- Tempo refers to the pace of a piece of music and is usually given in beats per minutes (BPM).
- Global quality vs time-varying local characteristic.
- Thus, in computational terms we differentiate between tempo estimation and tempo (beat) tracking.
- In tracking, beats are described by both their rate and phase.
- Vast literature: see, e.g. Hainsworth, 06; or Goto, 06 for reviews.

Tempo estimation and tracking (Davies, 05)

- Novelty function (NF): remove local mean + half-wave rectify
- Periodicity: dot multiply ACF of NF with a weighted comb filterbank



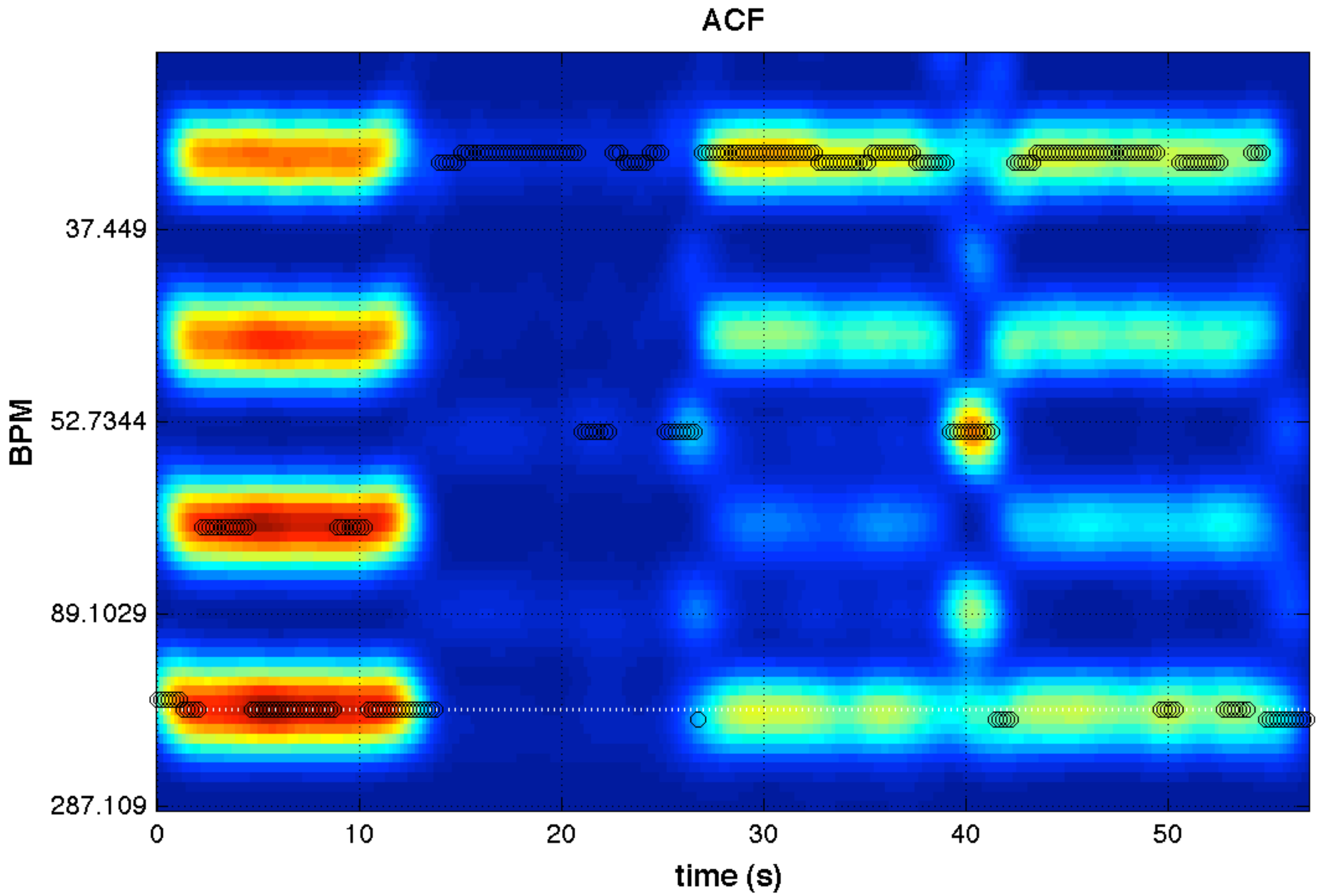


$$R_w(l) = (l/b^2)e^{\frac{-l^2}{2b^2}}$$

*From Davies and Plumbley, ICASSP 2005

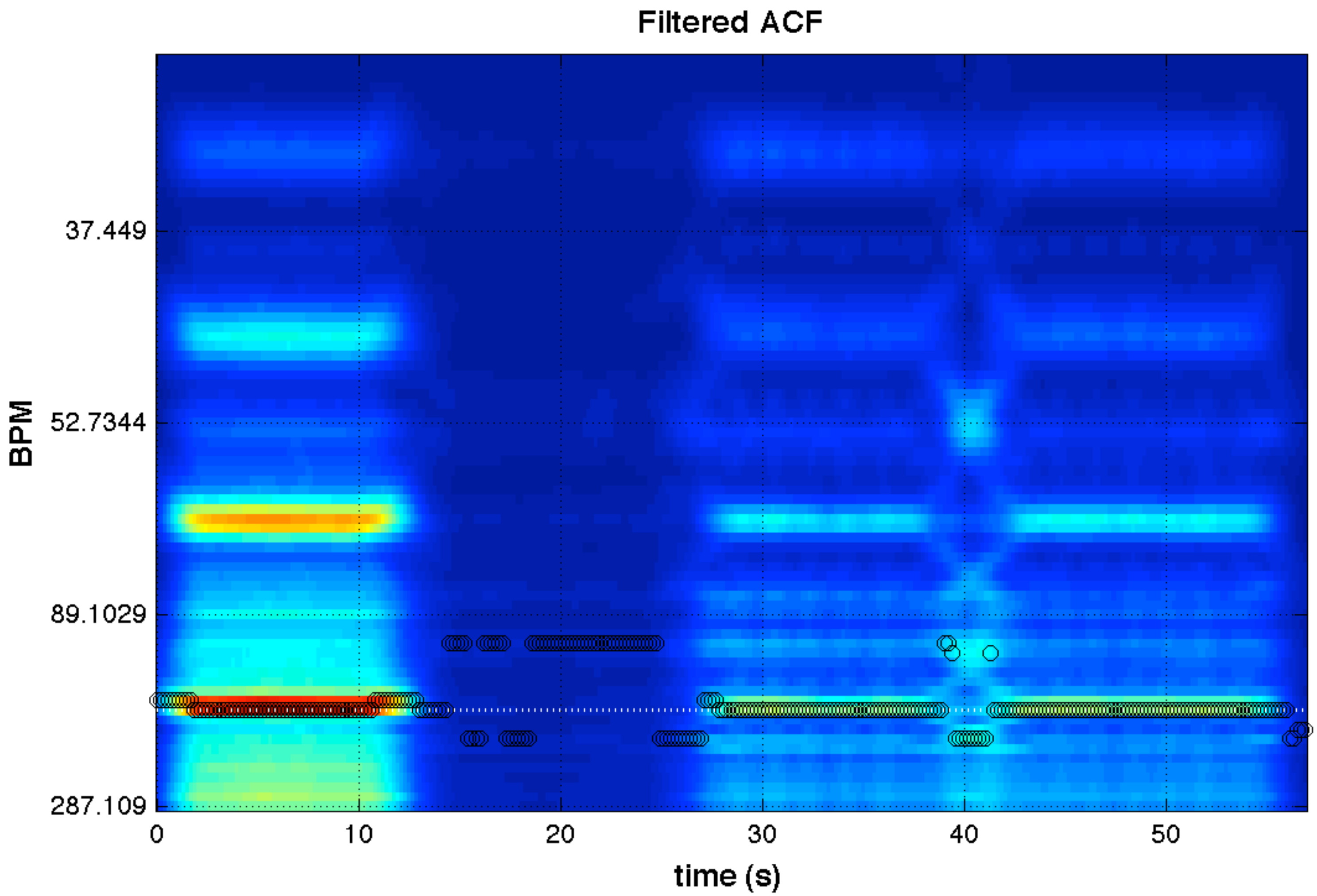
Tempo estimation and tracking (Davies, 05)

- Choose lag that maximizes the ACF

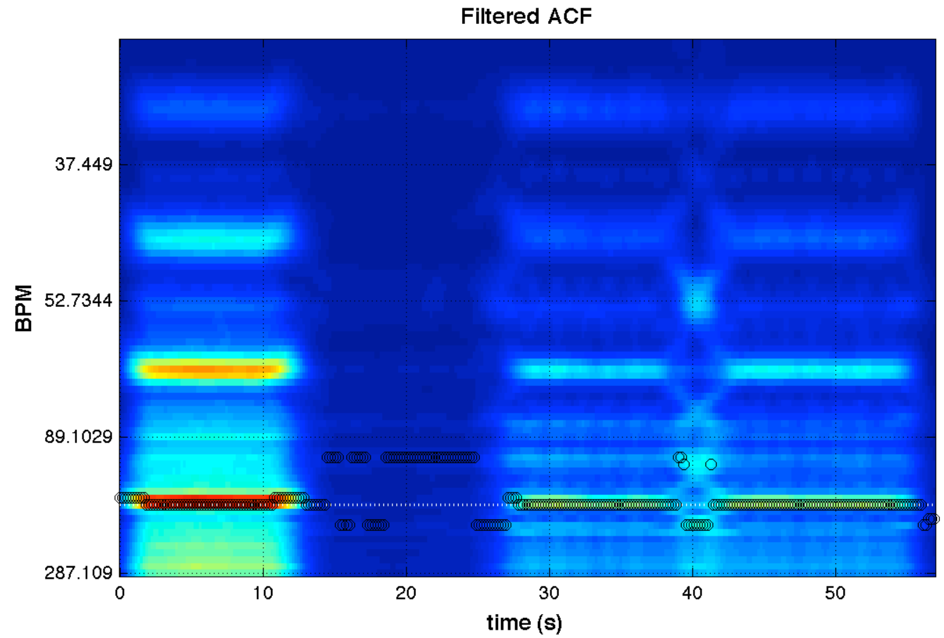


Tempo estimation and tracking (Davies, 05)

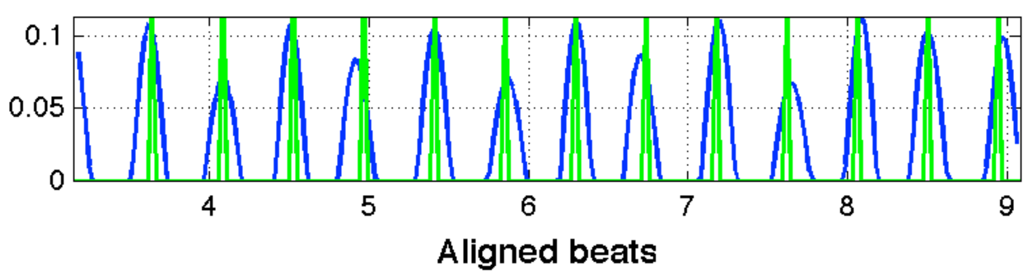
- Choose filter that maximizes the dot product

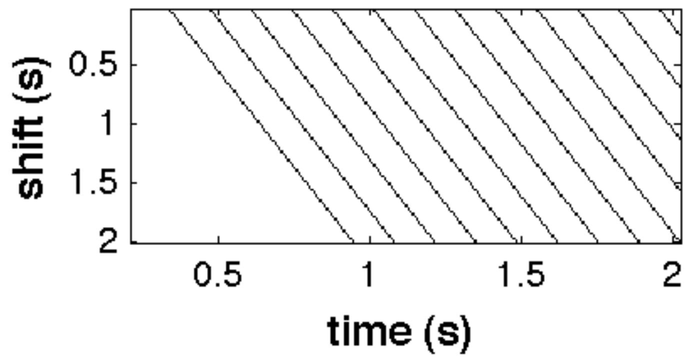


Tempo estimation and tracking (Davies, 05)



- Phase: dot multiply DF with shifted versions of selected comb filter





Tempo estimation and tracking (Grosche, 09)

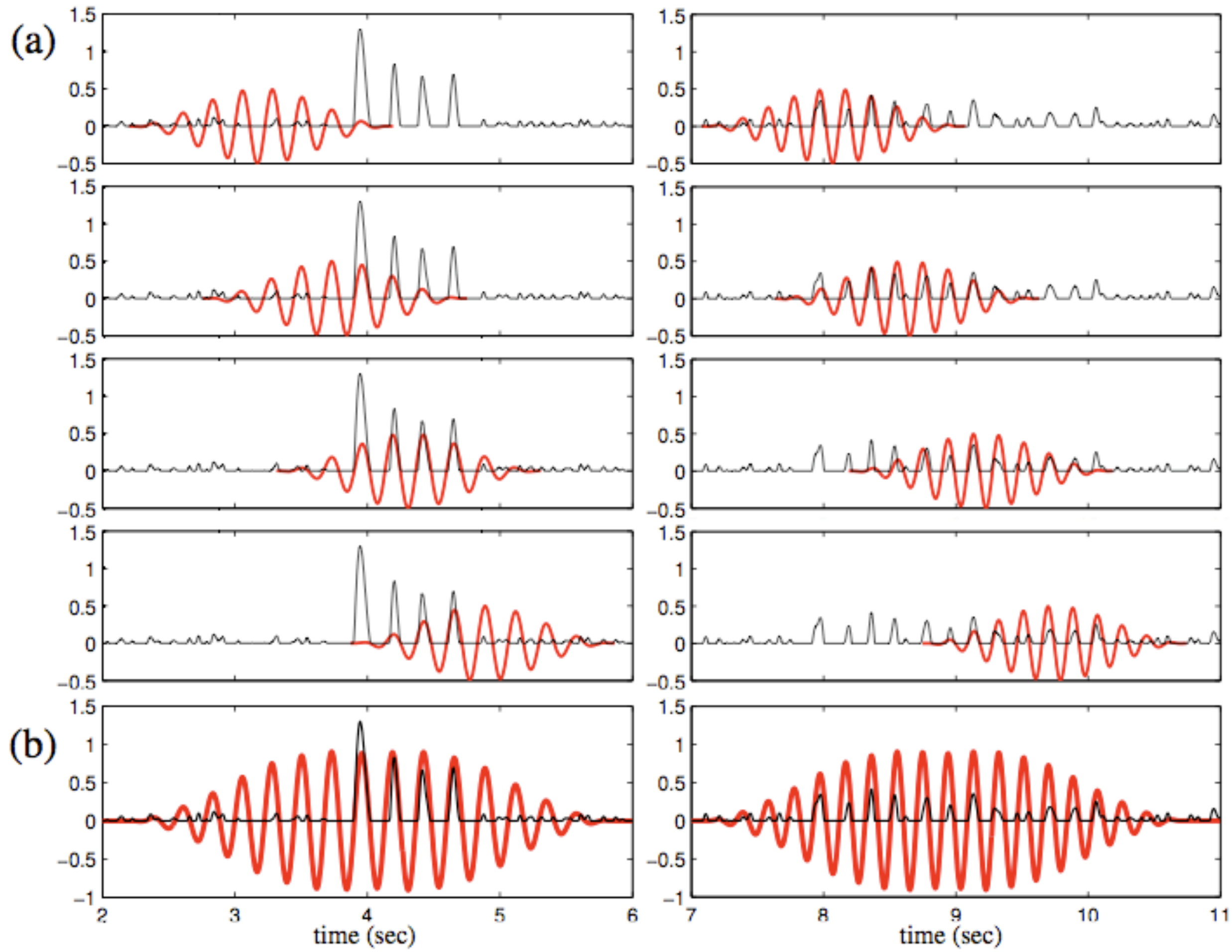
- DFT of novelty function γ(n) for frequencies: $\omega \in [30:600]/(60 \times \mathrm{fs}_\gamma)$
- Choose frequency that maximizes the magnitude spectrum at each frame
- Construct a sinusoidal kernel: $\kappa(m) = w(m-n)cos(2\pi(\hat{\omega}m - \hat{\varphi}))$
- In Grosche, 09 phase is computed as:

$$\hat{\varphi} = \frac{1}{2\pi}\arccos\left(\frac{Re(F(\hat{\omega},n))}{|F(\hat{\omega},n)|}\right)$$

- Alternatively, we can find the phase that maximizes the dot product of γ(n) with shifted versions of the kernel, as before.

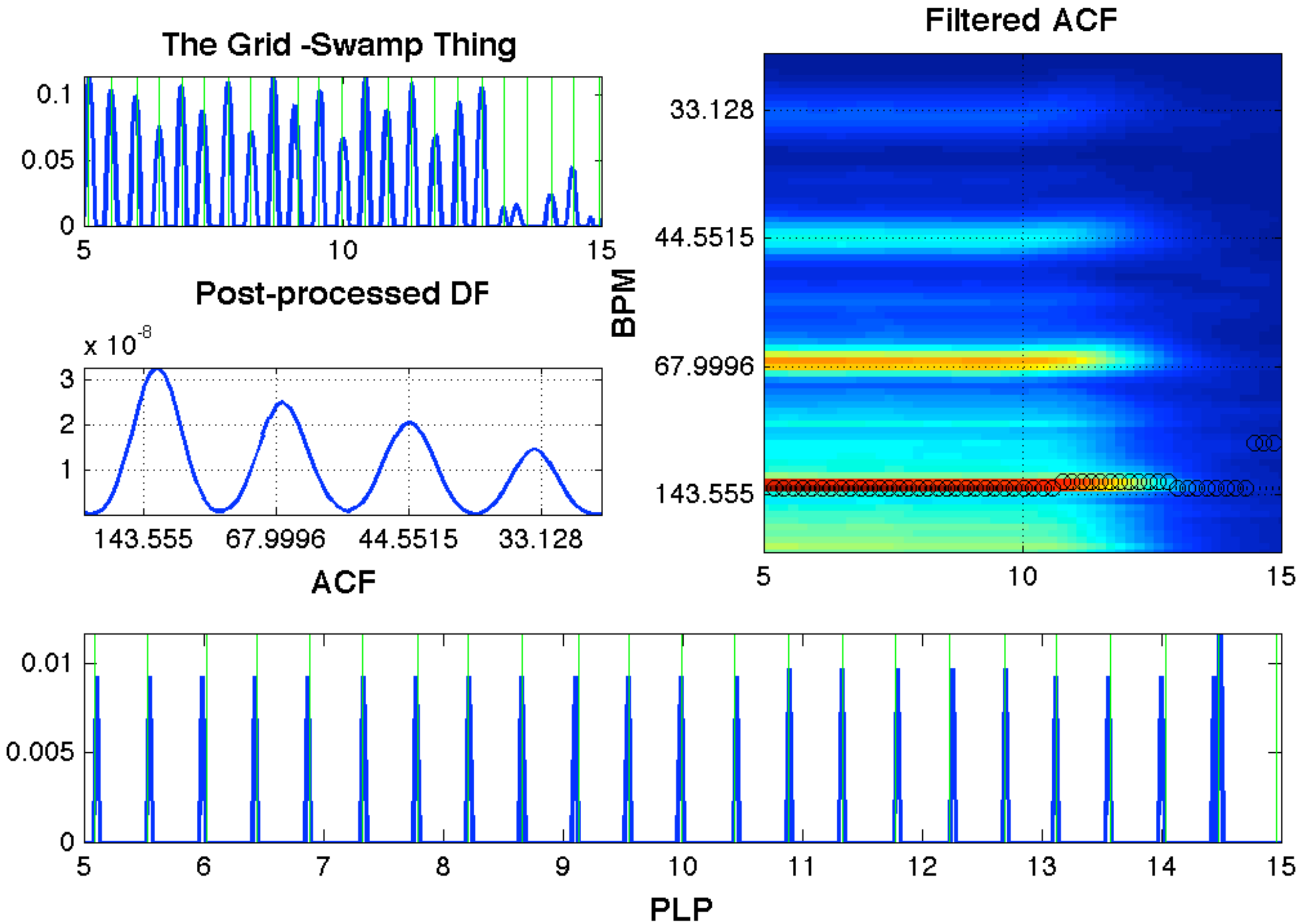
Tempo estimation and tracking (Grosche, 09)

- tracking function: Overlap-add of optimal local kernels + half-wave rectify

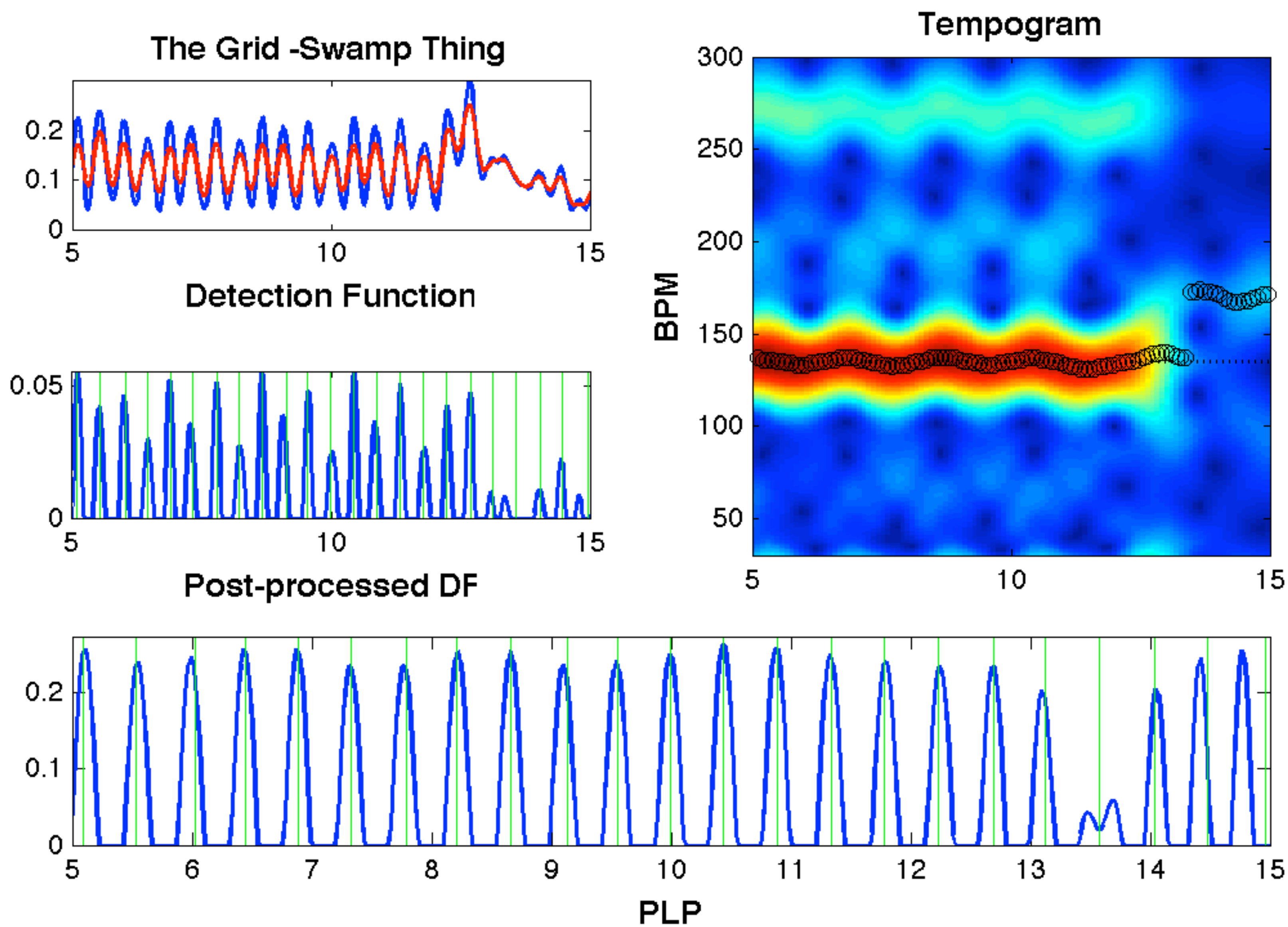


*From Grosche and Mueller, WASPAA 2009

Tempo estimation and tracking (Davies, 05)



Tempo estimation and tracking (Grosche, 09)



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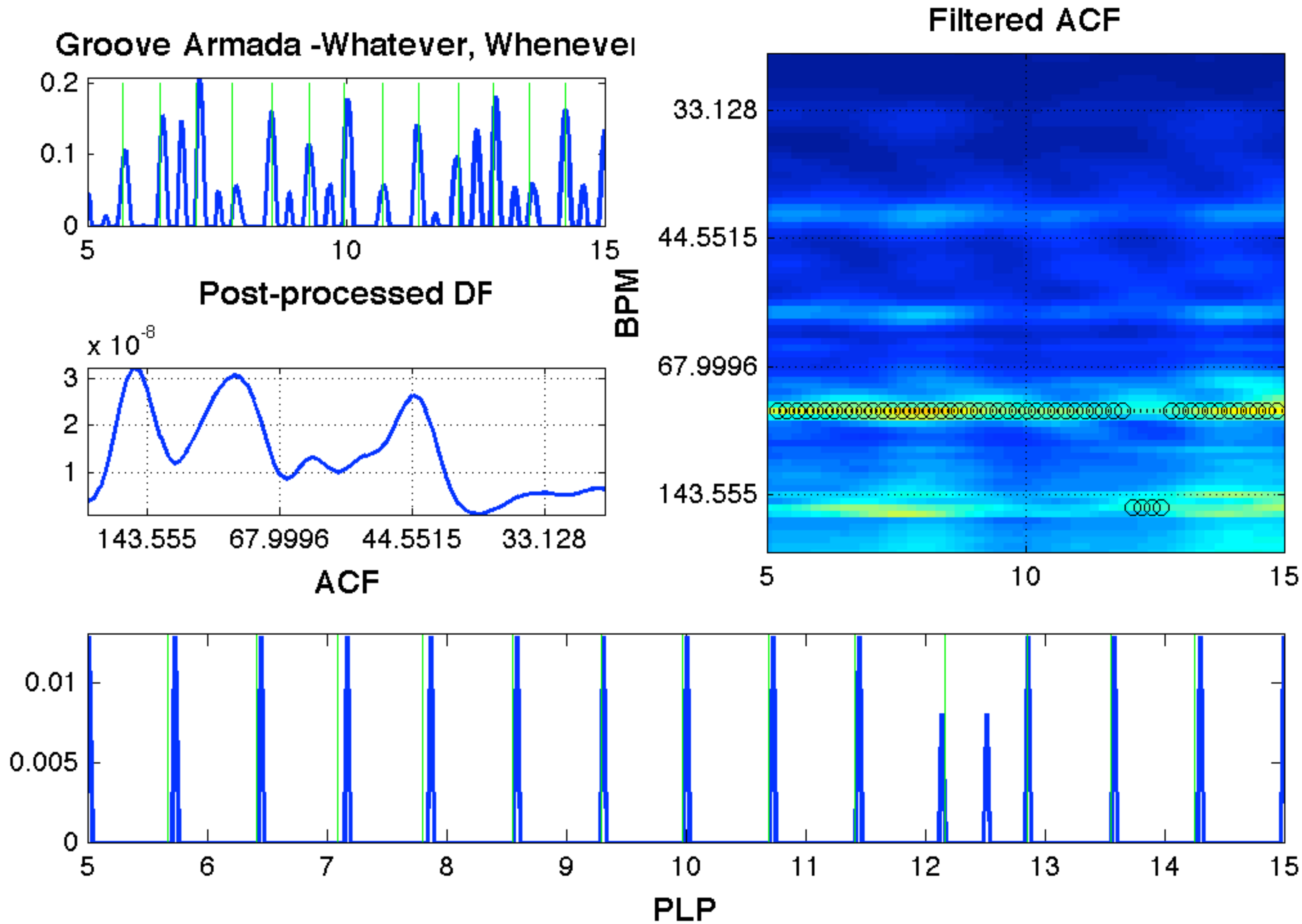

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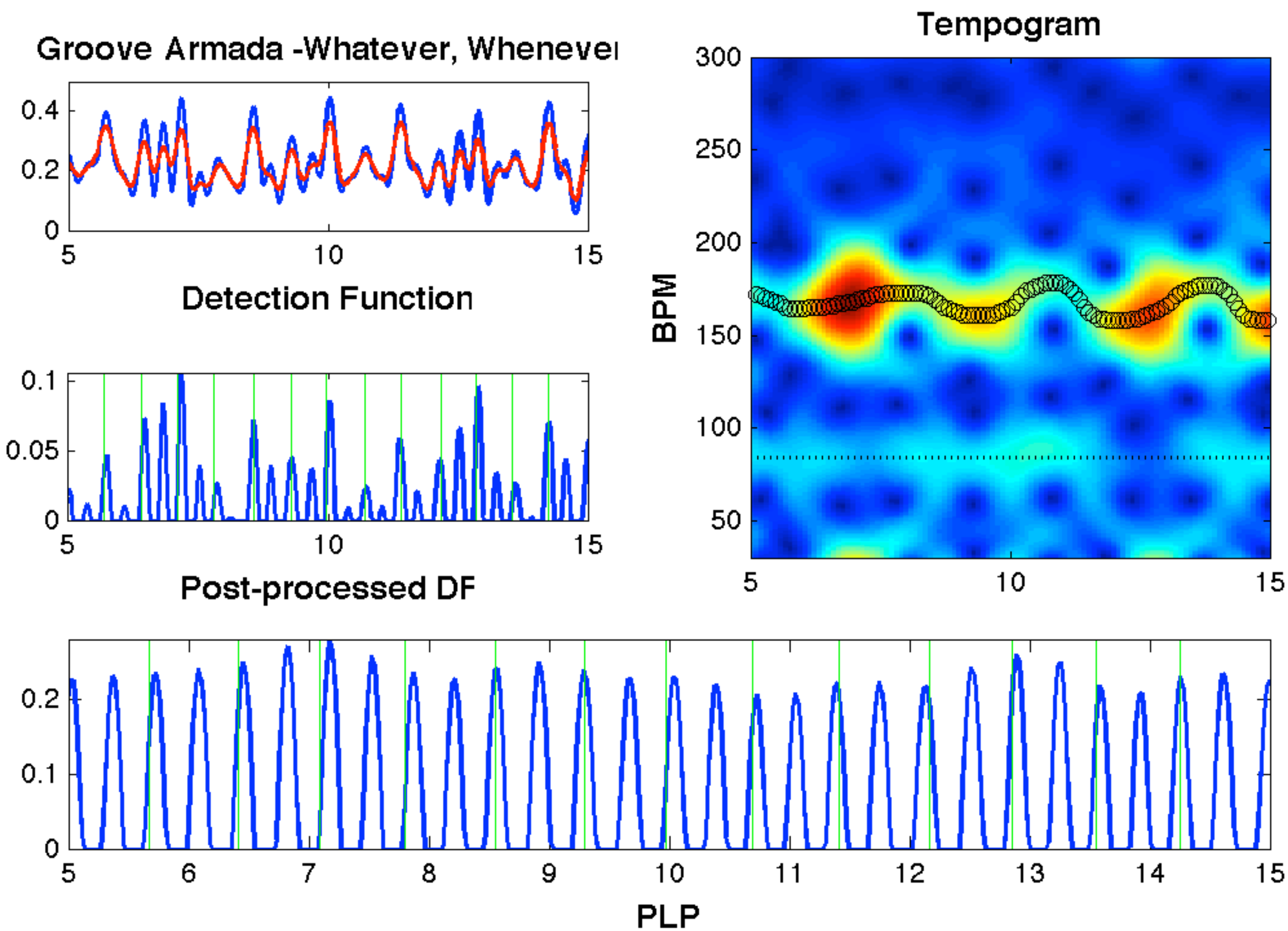

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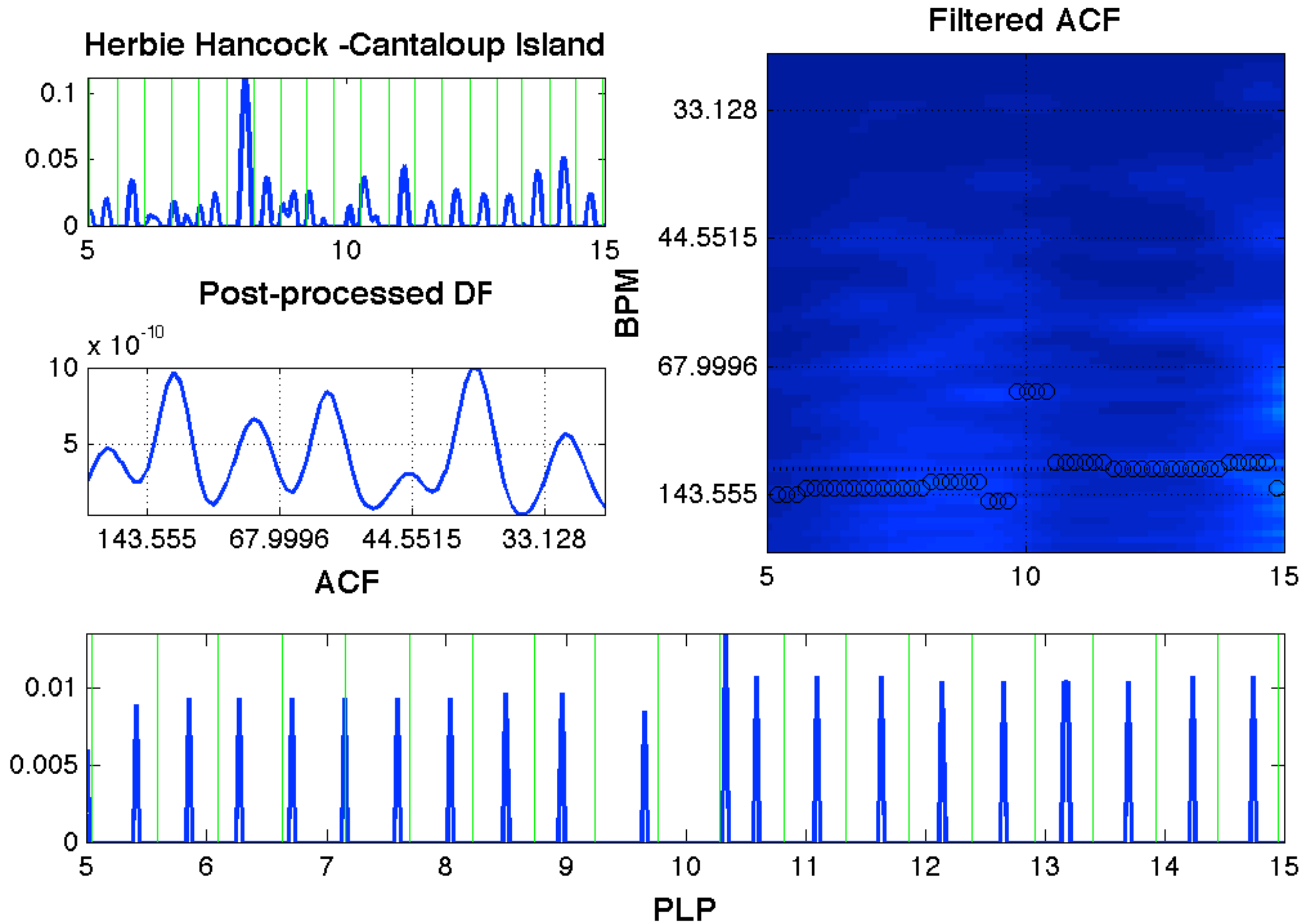

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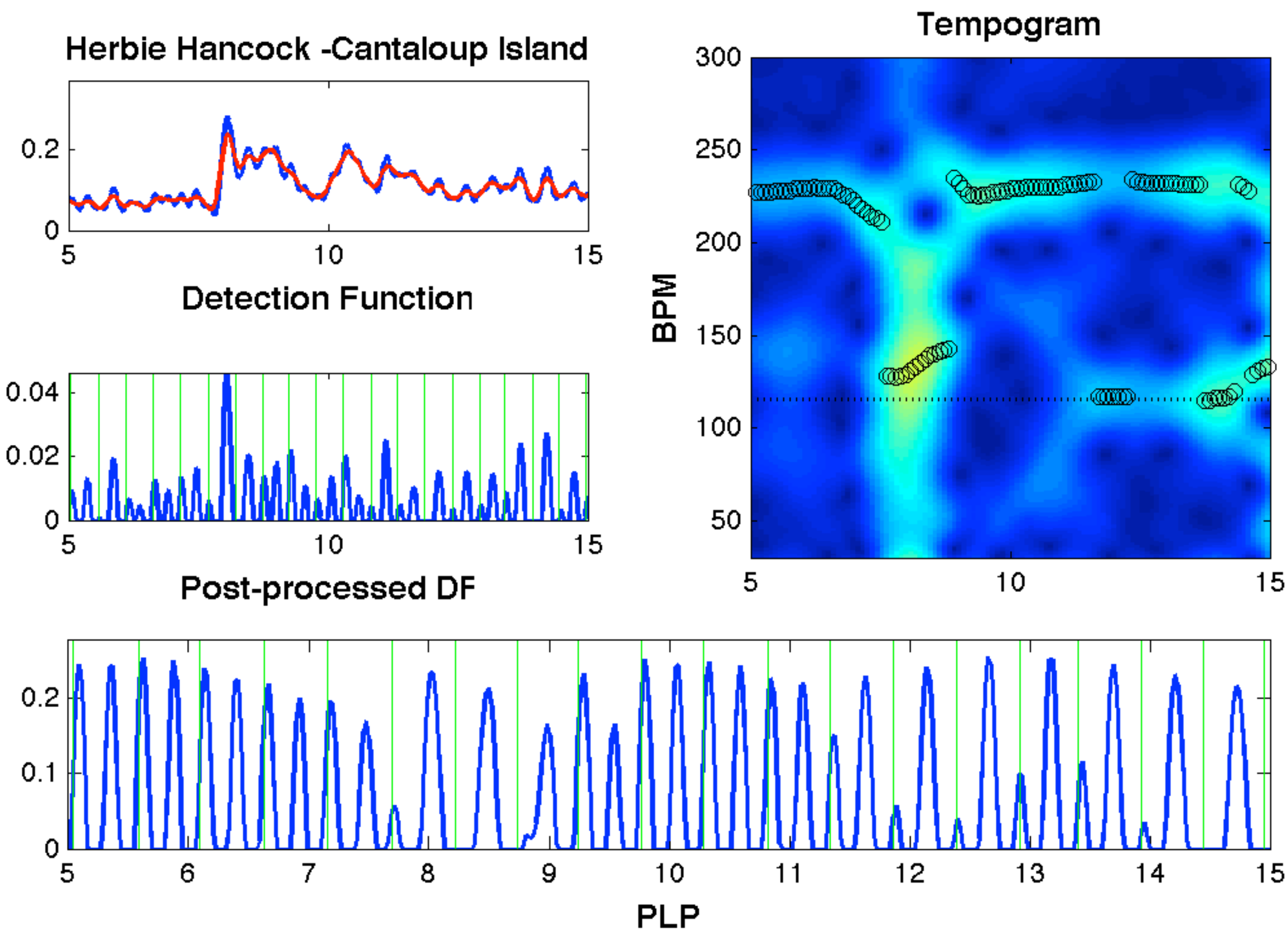

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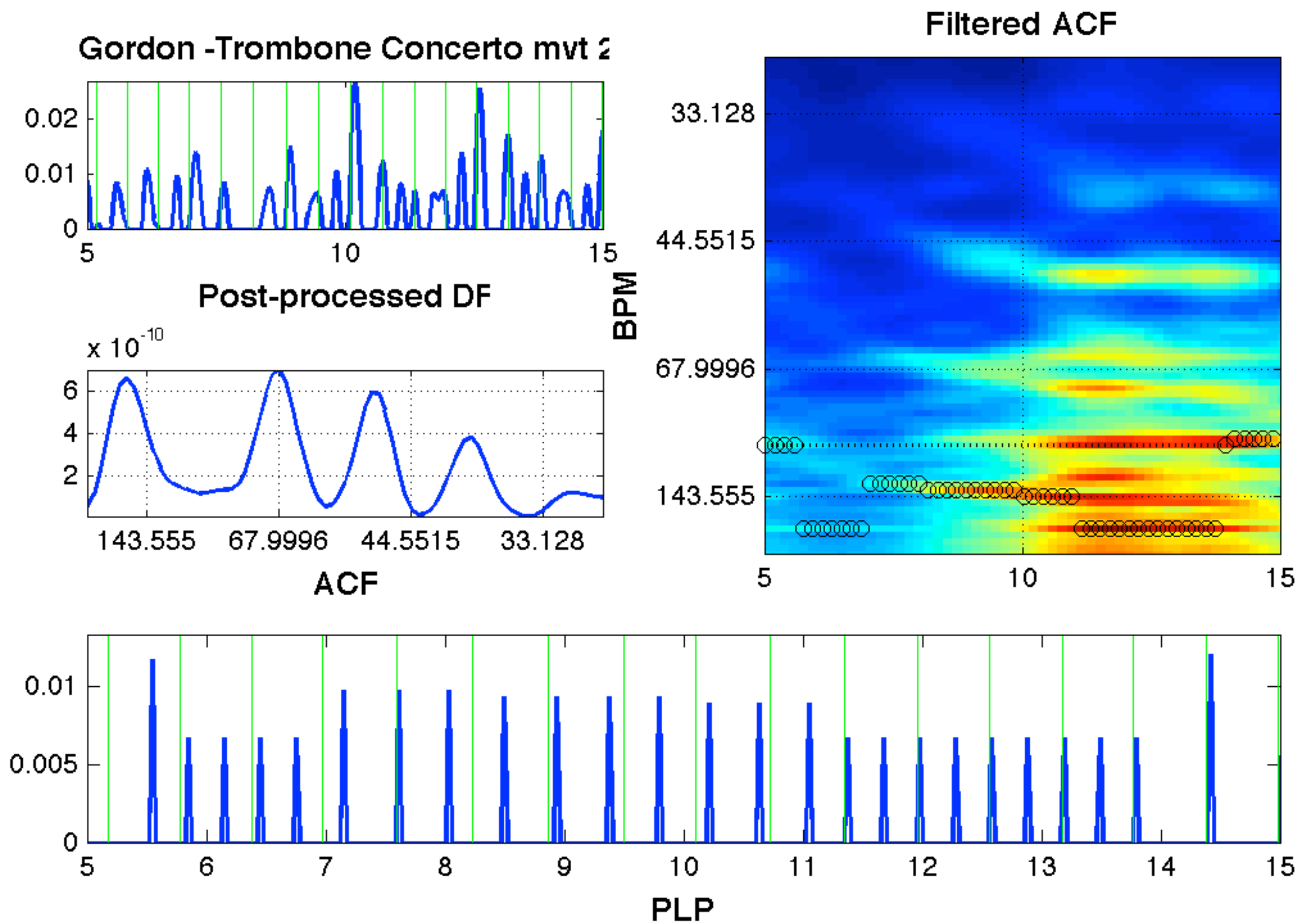
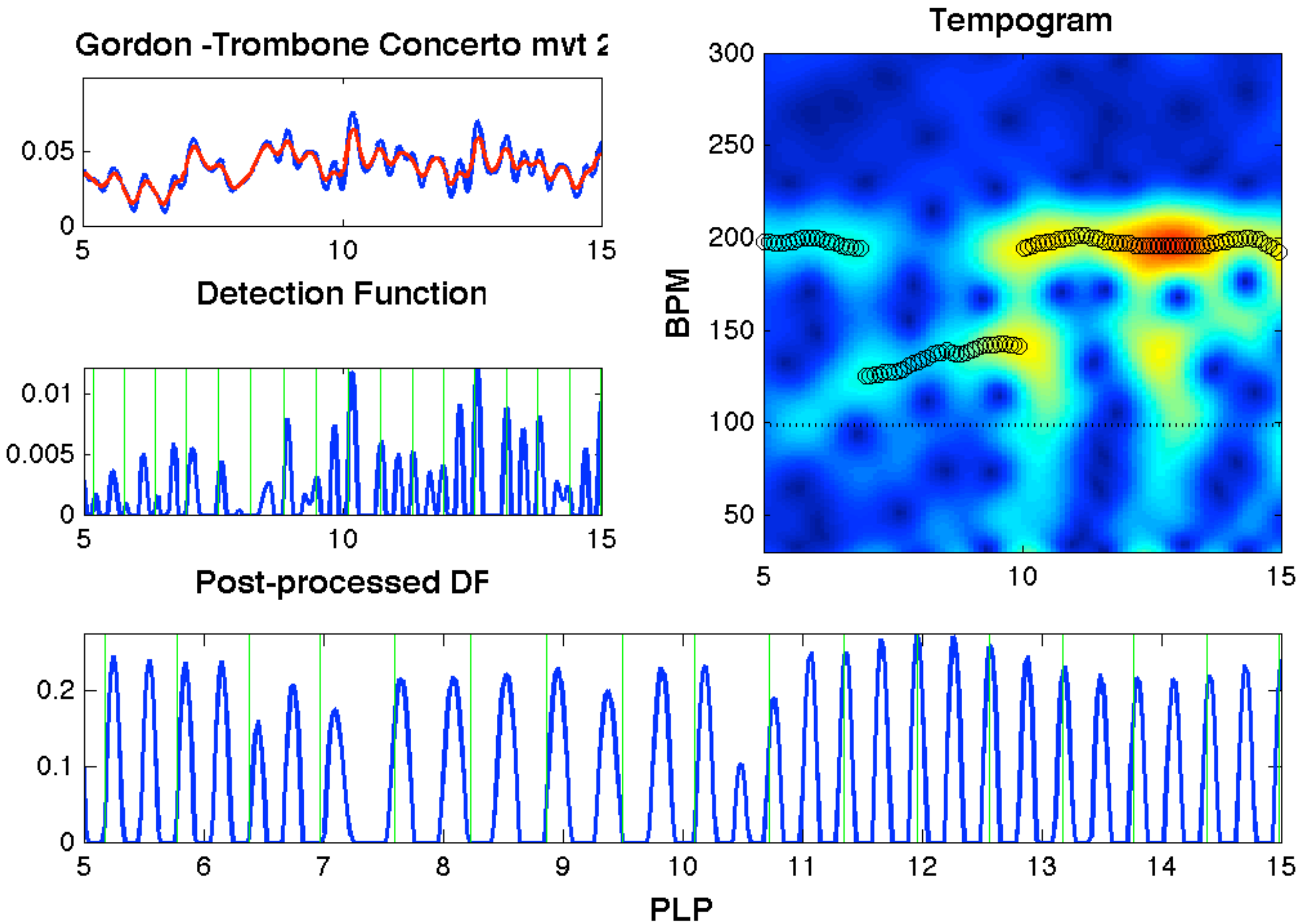


Tempo estimation and tracking (Grosche, 09)



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- This lecture borrows heavily from: Emmanuel Vincent’s lecture notes on pitch estimation (QMUL - Music Analysis and Synthesis); and from Anssi Klapuri’s lecture notes on F0 estimation and automatic music transcription (ISMIR 2004 Graduate School: http://ismir2004.ismir.net/graduate.html)