# Heading but not path or the tau-equalization strategy is used in the visual control of steering toward a goal

**Li Li**
Department of Psychology, The University of Hong Kong, Hong Kong SAR

**Joseph C. K. Cheng**
Department of Psychology, The University of Hong Kong, Hong Kong SAR

The visual strategies for the control of steering toward a goal include aligning one's instantaneous direction of travel (i.e., heading; J. J. Gibson, 1950) or the future path (J. P. Wann & D. K. Swapp, 2000) specified by optic flow with the target, equating the time to closure of the target-heading angle with the time to passage of the target (tau equalization, B. Fajen, 2001), or using the target egocentric direction and steering to center the target in the straight ahead or cancel the target optical drift (S. K. Rushton, J. M. Harris, M. Lloyd, & J. P. Wann, 1998). Supporting evidences for the use of these strategies in guiding steering or walking toward a goal were reported, but no consensus has been reached. In this study, by presenting participants with displays in which target egocentric direction was fixed and thus unavailable for steering to force participants to rely on information from optic flow for the control of self-motion, we systematically examined the use of the optic flow-based strategies in the visual control of steering toward a goal. We found that participants steered to align their heading with the target, supporting the use of the heading strategy. We found no evidence to support the use of the path or the tau-equalization strategy in the visual control of steering toward a goal.

Keywords: optic flow, heading, path, time to contact (tau), egocentric direction, locomotion control

Citation: Li, L., & Cheng, J. C. K. (2011). Heading but not path or the tau-equalization strategy is used in the visual control of steering toward a goal. *Journal of Vision, 11*(12):20, 1–12, http://www.journalofvision.org/content/11/12/20, doi:10.1167/11.12.20.

## Introduction

Successful steering toward a goal is important for human survival, and knowledge of this process can inform the design of biomimetic robots and unmanned vehicles. Since Gibson (1950) proposed that humans use the image motion of the environment on the retina experienced during locomotion (optic flow) to perceive and control self-motion in the world, much research has studied what information from optic flow humans use to steer/walk toward a fixed target in the environment. Among all the possible visual strategies that use information from optic flow to accomplish this task, three strategies, which are listed below, are supported by empirical data.

*1. Heading strategy*. When we travel on a straight path with no body, head, or eye rotation (pure translation), the focus of expansion (FOE) in the resulting radial flow pattern indicates our instantaneous direction of travel (i.e., heading). Previous psychophysical studies have shown that humans can locate the FOE in optic flow to accurately estimate heading (within 1° of visual angle) to support the precise control of self-motion in the world (e.g., Crowell & Banks, 1993; Warren, Morris, & Kalish, 1988). Thus, to steer toward a fixed target in the environment, we can align our heading indicated by the FOE in optic flow with the target (Figure 1a).

Under more complex but natural conditions such as when we travel on a curved path or when we rotate our body, head, or eye during traveling (translation and rotation), the rotation disrupts the radial flow pattern on the retina and shifts the FOE in retinal flow away from our heading direction. The process of extracting heading from retinal flow becomes complicated (Regan & Beverly, 1982). To determine whether humans can still accurately perceive heading under such circumstances, many studies examined heading perception during translation and rotation. While some studies reported that participants need extraretinal information to remove the rotation in the retinal flow field for accurate heading perception at high rotation rates (e.g., Banks, Ehrlich, Backus, & Crowell, 1996; Royden, Banks, & Crowell, 1992), other studies have reported that participants can estimate heading within 2° of visual angle at rotation rates up to 20°/s by relying on information solely from retinal flow regardless of whether the rotation in the flow field is due to simulated eye movement or path rotation (Cutting, Vishton, Flückiger, Baumberger, & Gerndt, 1997; Grigo & Lappe, 1999; Li, Chen, & Peng, 2009; Li, Sweet, & Stone, 2006a; Stone & Perrone, 1997; van den Berg, 1992). This indicates that during translation and rotation, given sufficient retinal flow information, we can still accurately perceive heading to support successful steering toward the target. Indeed, the use of heading in the control of locomotion on foot to

doi: 10.1167/11.12.20 Received June 17, 2011; published October 28, 2011 ISSN 1534-7362 © ARVO

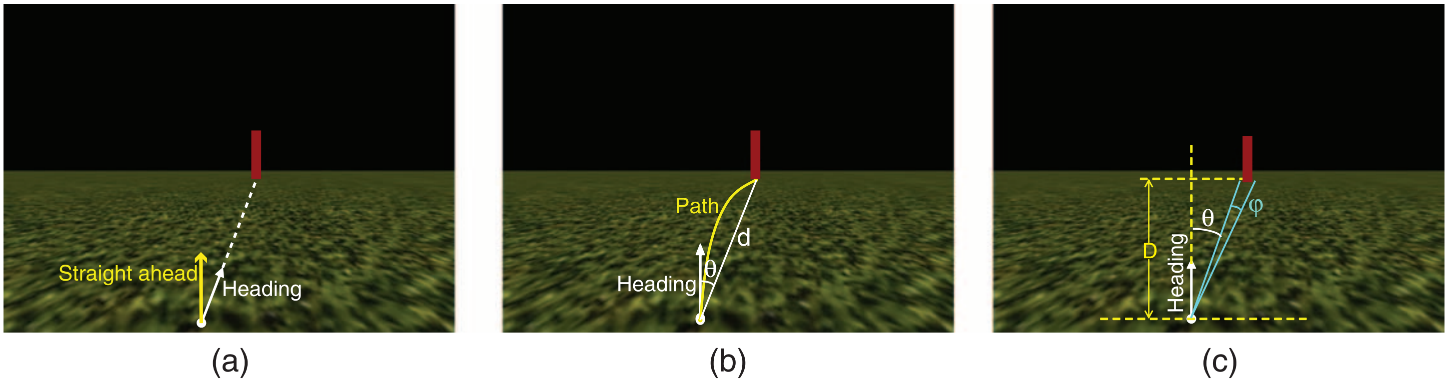


Figure 1. Illustrations of the three optic flow-based visual control strategies for steering toward a goal: (a) Align the heading specified by optic flow with the target, (b) align the future path with the target, and (c) equate the time to closure of the target-heading angle ($\tau_\theta$) with the time to passage of the target ($\tau_p$).

walk toward a goal has been supported by many previous studies (e.g., Harris & Carre, 2001; Warren, Kay, Zosh, Duchon, & Sahuc, 2001; Wood, Harvey, Young, Beedie, & Wilson, 2000).

2. *Path strategy*. It has been proposed that people can make use of the spatially integrated velocity vectors (field lines, Lee & Lishman, 1977) or the temporally integrated dot motion trajectories (flow lines, Kim & Turvey, 1999; Wann & Swapp, 2000) in the flow field to directly perceive their future trajectory of travel (path) through the environment without recovering heading from optic flow. Heading and path converge when one travels on a straight path but diverge when one travels on a curved path (see Figure 1 in Li et al., 2009). Mathematically, it has been shown that when observers fixate a target on their future path when traveling on a circular path, the flow lines of environmental points on the path are vertical, and observers can directly recover their path of forward travel by integrating all vertical flow lines in the flow field (Kim & Turvey, 1999; Wann & Swapp, 2000). Accordingly, Wann and Land (2000) proposed that people can use path, instead of heading, to guide their locomotion in the world. For example, in the case of steering toward a target, people can achieve the task by steering to change path curvature to a set value and hold it constant such that their future path would go through the target (Figure 1b). Mathematically, the required path curvature ($k_{\text{req}}$) is given by

$$k_{\text{req}} = \frac{2\sin\theta}{d}, \tag{1}$$

where $\theta$ is the target-heading angle and $d$ is the distance between the participant and the target (Land, 1998).

The supporting evidences for the use of the path strategy include the recent studies by Wilkie, Kountouriotis, Merat, and Wann (2010; Wilkie, Wann, & Allison, 2008). They found that when steering a bend in a driving simulator, participants directed their gaze on their future path at 1–2 s ahead for most of the time, suggesting that participants might be using the path strategy for locomotion control. However, this finding is in conflict with what was reported by Land and Lee (1994) that when steering a bend in real-world driving, drivers tend to look at the tangent point of the inside road edge. Furthermore, Kandil, Rotter, and Lappe (2009, 2010) also conducted real-world driving experiments and found that when steering a bend, instructing participants to look at the tangent point of the inside road edge improved both the accuracy and the steering stability in their driving performance. Even more recently, Li and Cheng (2011) reported that looking at a point on the future path does not help accurate path perception from optic flow. Observers estimate path curvature from the translation and rotation components in the flow field and recover their path of forward travel using their perceived heading as the anchoring reference direction. As unlike path perception, heading perception is robust and not affected by the source of rotation in the flow field, Li and Cheng proposed that heading rather than path supports the accurate control of self-motion in the world.

3. *Tau-equalization strategy*. In addition to the above two strategies, Fajen (2001) proposed that steering toward a goal can be achieved by steering to render the simultaneous closure (i.e., zeroing) of two gaps, the target-heading angle ($\theta$) and the distance between the participant and the target along the heading direction ($D$, Figure 1c). The time to closure of the target-heading angle, $\tau_\theta$, is optically specified by $\frac{\theta}{\dot{\theta}}$ (Lee, 1974, 1976). The optical specification of the time to closure of the distance between the observer and the target ($\frac{D}{\dot{D}}$), also termed as the time to passage ($\tau_p$, see Kaiser & Mowafy, 1993), is more complicated and given by

$$\tau_p = \left(\frac{\dot{\varphi}}{\sin\varphi} + \frac{\dot{\theta}}{\tan\left(\frac{\pi}{2} - \theta\right)}\right)^{-1}, \tag{2}$$

where $\varphi$ is the optical angle subtended by the contours of the target (Bootsma & Craig, 1999; Figure 1c). To steer to render the simultaneous closure of the two gaps is to equalize $\tau_\theta$ and $\tau_p$, i.e., the tau-equalization strategy. Given a known target position and a constant traveling speed, the use of the tau-equalization strategy computationally results in traveling on a curved path with decreasing path curvature toward the target. Fajen asked participants to steer toward a fixed target in the environment and compared the increase and decrease of the path curvature with the difference between $\tau_\theta$ and $\tau_p$ throughout the trial. He found that the change of the path curvature was consistent with the qualitative predictions of the tau-equalization strategy and concluded that participants used the tau-equalization strategy to steer toward a goal.

Note that in previous studies that examined the use of information from optic flow in the control of steering toward a goal, target egocentric direction cue is also available for steering. Rushton, Harris, Lloyd, and Wann (1998) proposed that humans can steer/walk toward a target using target egocentric direction without relying on information from optic flow (see also Harris & Rogers, 1999), consistent with an initial observation by Llewellyn (1971) that participants could steer toward a target by canceling the target drift on the image screen (i.e., the target optical drift). Specifically, as the straight ahead is the primary axis in egocentric space, Rushton and Harris (2004) theorized that in natural circumstances, centering the target in the straight ahead provides a direct straight line course to the target, while canceling the target optical drift to keep it at a fixed egocentric direction other than the straight ahead results in traveling on equal-angular spirals toward the target (see also Rushton, Wen, & Allison, 2002).

Given that target egocentric direction is the first cue available in the guidance of locomotion before participants initiate self-motion (Rushton et al., 1998; Warren et al., 2001), participants in previous studies that examined the use of optic flow-based control strategies could have used target egocentric direction to perform the steering task, which most of these studies failed to analyze (see Harris & Rogers, 1999). In the presence of the target egocentric direction cue, it is difficult to determine whether participants rely on optic flow for the control of locomotion, which makes it even more difficult to conclude what information from optic flow participants use for the control of self-motion. To examine which optic flow-based strategy participants use to steer toward a goal, it is therefore important to generate displays to make target egocentric direction unavailable for steering and force participants to use optic flow to perform the task.

The current study addressed this problem and systematically examined the use of optic flow-based strategies in visual control of steering toward a goal. We aimed to resolve the controversy in the field on what information from optic flow people use to steer toward a target. Specifically, the display simulated a participant walking over a ground plane. A target was placed at 25 m in distance and 20° away from the participant's initial heading direction. In each 10-s trial, participants were instructed to use the joystick to change the curvature of the path of their forward travel to steer toward the target. The camera in the computer program for the display (i.e., the participant's simulated gaze direction) always pointed at the target. The target position was thus fixed on the screen. As participants were seated such that the center of the screen corresponded to their straight ahead, the target egocentric direction was fixed and unavailable for steering, which allowed us to examine whether participants use the heading, the path, or the tau-equalization strategy to steer toward the target. This technique has been used in previous studies that investigated the information in retinal flow that people use for steering during simulated observer rotation (Li & Warren, 2002; Rushton, Harris, & Wann, 1999). We tested both a sparse random-dot and a dense textured ground display (Figures 3a and 3b) to examine whether the steering performance changes when dense optic flow information is available.

The logic of the study was given as follows: If participants used the heading strategy to steer toward the target, they would steer to align their heading with the target. As a consequence, heading error (i.e., the target-heading angle) should get smaller and close to zero relatively rapidly. In contrast, if participants used the path strategy, they would steer to adjust path curvature to a set value such that their future path would go through the target, in which case heading error would decrease at a roughly constant rate and not get close to zero until the very end of the trial. If participants used the tau-equalization strategy, they would steer so as to equalize the time to closure of the target-heading angle ($\tau_\theta$) and the time to passage of the target ($\tau_p$) regardless of the change of heading error and path curvature. Last, given that heading perception during translation and rotation improves with a dense flow field especially at high rotation rates (e.g., Cutting et al., 1997; Li & Warren, 2000), if participants used the heading strategy to steer toward the target, the control performance should be better for the textured than the random-dot ground display, assuming that performance did not reach ceiling in the random-dot ground display.

# Methods

## Participants

Fourteen students and staff (12 naive as to the specific goals of the study; 7 males, 7 females) between the age of 20 and 38 at the University of Hong Kong participated in the experiment. All had normal or corrected-to-normal vision and provided informed consent in accordance with

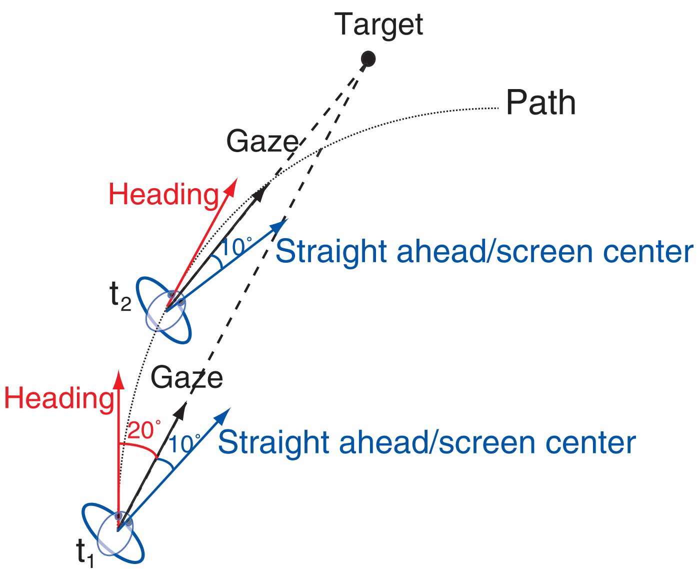


Figure 2. A bird's eye view of the experimental setup with the target placed at 20° to the right of the initial heading. The participant's simulated/virtual gaze direction (indicated by the dashed line) always pointed at the target, and the target appeared at 10° to the left of the center of the screen that was calibrated to correspond to the participant's straight ahead. The target position was thus fixed on the screen and so was the target egocentric direction with respect to the participant's straight ahead.

guidelines from the University of Hong Kong Human Research Ethics Committee. Two naive participants (females) showed more than three times larger variance in their control performance than the other participants and were thus excluded from further data analyses.

## Visual stimuli and control

The display simulated a participant walking over a ground plane (depth range: 1.41–100 m) at the fast walking pace of 2 m/s. At the beginning of a trial, a red post target (1.3°W × 6.9°H) was placed at 25 m in distance and 20° to the left or right of the participant's initial heading. This target appeared at 10° away from the center of the screen. Participants were seated such that the center of the screen corresponded to their straight ahead. The target position offset from the center of the screen (i.e., the participant's straight ahead) was on the opposite side of the initial target-heading angle, e.g., if the target was at 20° to the right of the initial heading, the target would appear at 10° to the left of the center of the screen (Figure 2). This was to dissociate the target direction relative to the participant's initial heading from the target egocentric direction relative to the participant's straight ahead. The participant's simulated gaze direction (i.e., the "camera" in the computer program for the display) always pointed to the target (Figure 2), thus the target position on the screen (i.e., the target egocentric direction) was fixed throughout the trial. The target naturally expanded when the participant approached it to allow the use of the tau-equalization strategy. The expansion rate of the target ranged from 0.1°/s to 2.4°/s (average: 0.5°/s) and was well above the reported looming detection threshold (0.063°/s, see Cavallo & Laurent, 1988; Hoffmann, 1994; Lee, 1976; Wann, Poulter, & Purcell, 2011).

Participants were asked to use the joystick (B&G Systems, FlyBox) to steer toward the red post target. The control dynamics of the joystick was similar to that of the steering wheel of a car, i.e., the displacement of the joystick was proportional to path curvature. The joystick displacement was sampled at 60 Hz (i.e., every frame of the display). The measured system feedback delay was 50 ms, which is a small fraction of human reaction time. The joystick displacement values ranged from −1 to 1, corresponding to peak path rotation rates of ±20°/s.

Two types of displays were tested: (1) a random-dot ground in which the ground plane was composed of 100 white dots (0.5° in diameter, luminance contrast +99%) uniformly distributed on the ground (Figure 3a) and (2) a textured ground in which the ground plane was mapped with a green multi-scale texture with a power spectrum of 1/*f*

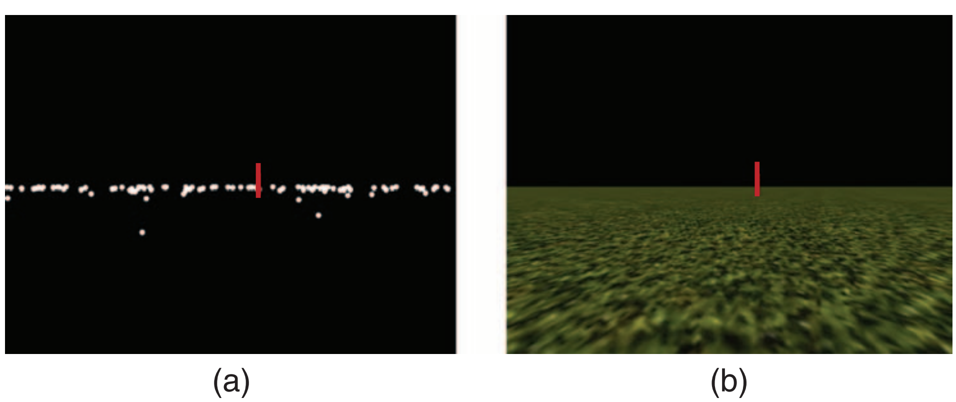


Figure 3. Illustrations of the two display conditions: (a) A random-dot ground and (b) a textured ground. The post is at 10° to the right of the center of the display and 20° to the right of the initial heading.

(maximum luminance contrast +99%, Figure 3b). The former provided a sparse flow field and the latter provided a dense flow field. The number of visible dots per frame in the random-dot ground display was kept constant throughout the trial, i.e., if a certain number of dots moved outside of the field of view in one frame, the same number of dots were regenerated in that frame in such a way that the dot distribution on the ground remained uniform. The background sky was black in both display conditions.

The visual stimuli were generated on a Dell Precision Workstation 670n with an NVIDIA Quadro FX 1800 graphics card at 60 Hz. They were rear-projected on a large screen (110°H × 94°V) with an Epson EMP-9300 LCD projector (native resolution: 1400 × 1050 pixels, refresh rate: 60 Hz) in a light-excluded viewing booth. The screen edges were covered with matte black cloth to minimize their visibility. The simulated eye height in the display was 1.51 m corresponding to the eye height of a participant sitting on a high chair at 0.56 m away from the center of the screen. Participants viewed the display monocularly with their dominant eye and with their head stabilized by a chin rest. Before the experiment started, the participant's cyclopean eye and midline of the body (i.e., the participant's straight ahead) were calibrated to be aligned with the center of the screen.

## Procedure

On each trial, participants were instructed to imagine that they were walking over a ground plane and their task was to use the joystick to steer toward the red post. Participants were informed that the control dynamics of the joystick was similar to that of the steering wheel of a car. The first frame was displayed until participants pulled the trigger to start each 10-s trial. Participants could freely move their eyes when looking at the displays. The time series of the participant's position in the virtual world was recorded for further analysis.

The experiment was composed of two blocks for the two display conditions. Each block contained 30 randomized trials (15 trials × 2 target directions) for each display condition, and the testing order of the display conditions was counterbalanced between participants. Before the experiment started, participants received 12 randomized practice trials (3 trials × 2 target directions × 2 display types) to make sure that they understood the instructions and were familiarized with the joystick control dynamics. No feedback was given during the practice or the data collection trials. The experiment lasted less than 30 min.

# Results

If the participant's end position was on the opposite side of the target in a given trial, an indication that the participant did not steer toward the target in that specific trial, we excluded this trial from the data analysis. This resulted in the exclusion of only four out of 720 trials (60 trials × 12 participants) in total in the data analysis. To evaluate the use of the heading, path, and tau-equalization strategies in the control of steering toward the target, we computed the time series of heading error (i.e., the target-heading angle), path curvature (i.e., the inverse of path radius), and $\tau_\theta$ and $\tau_p$ from the recorded time series of the participant's position in the virtual world and the given constant walking speed (2 m/s). Given the mirror-image performance in the left and right target direction conditions as illustrated by the recorded time series of individual participant's position for these two conditions (Figure 4), we collapsed the performance data across the two target directions.

## Heading error profile

If participants used the heading strategy and steered to align their heading with the target, heading error would start at the initial target offset angle of 20° and increase slightly until participants initiated the control response. Then, as soon as participants started to respond to the input heading error and minimize it, heading error would quickly converge to zero. Figure 5a shows the simplified performance prediction assuming participants used this heading control strategy to steer toward the target. In comparison, Figure 5b plots the time series of heading error performance data for the random-dot and textured ground display conditions, computed from the recorded time series of the participant's position averaged across 12 participants. Positive heading errors represent understeering and negative heading errors represent oversteering. Indeed, heading error starts at 20° and then quickly gets smaller, reaching steady-state performance at 4–5 s. In fact, participants corrected 90% of the initial heading error within the first 5 s in 81% of the trials. This is consistent with the use of the heading strategy that requires participants to minimize the input heading error to steer toward the target. The heading error profile for the random-dot ground display appears to lag behind that for the textured ground display, suggesting that participants initiated faster control responses when the display provided a dense flow field.

To examine whether the initiation and accuracy of the control response changed with display condition, we analyzed the time delay of the control response, indicated by the peak of the heading error profile at the beginning of the trial, and the final heading error averaged across the last 1 s of the trial. Figure 6 plots the time delay and final heading error against display condition for each participant, computed from the recorded time series of individual participant's position averaged across the left and right target directions in each display condition. A paired *t*-test revealed that the time delay for the random-dot

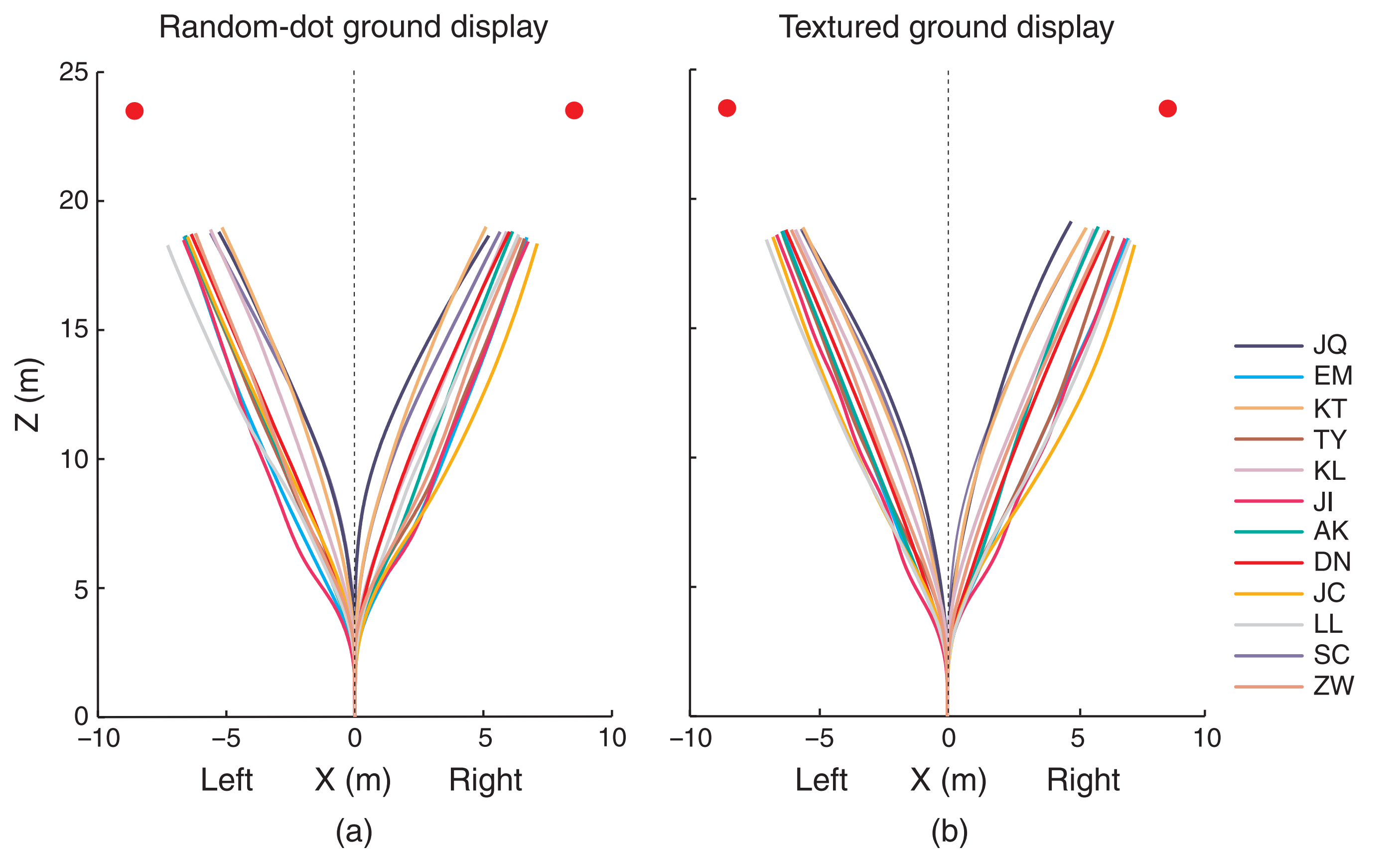


Figure 4. The recorded time series of individual participant's position averaged over 15 trials in the left or right target direction condition for (a) the random-dot and (b) the textured ground display conditions. The red dot indicates the target position.

ground display (mean ± SE: 950 ms ± 140 ms) was significantly larger than that for the textured ground display (760 ms ± 90 ms), with $t(11) = 2.23$, $p < 0.05$, indicating that on average participants initiated 20% faster control responses when the display contained a dense flow field. On the other hand, the final heading errors for the random-dot ground display (2.08° ± 1.04°) and the textured ground display (2.03° ± 0.92°) were not significantly

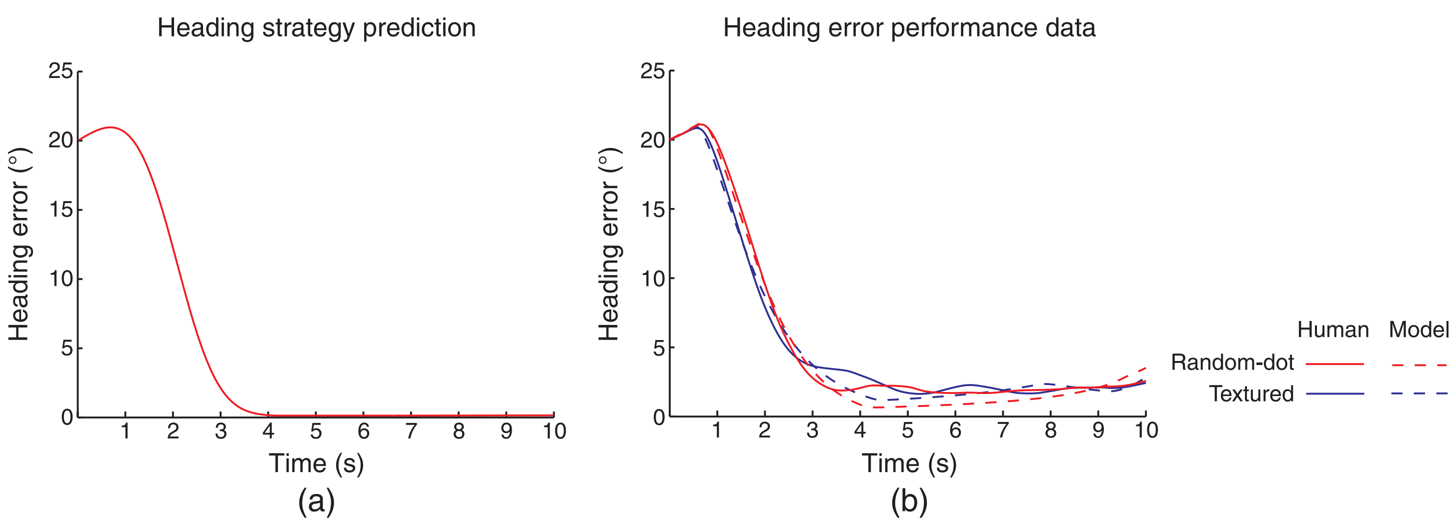


Figure 5. (a) The predicted heading error profile for the heading strategy and (b) the time series of heading error performance data computed from the recorded time series of the participant's position averaged across 12 participants for the random-dot and textured ground displays. The dashed lines in (b) plot the model simulation of a control system that minimizes the input heading error with a second-order lag.

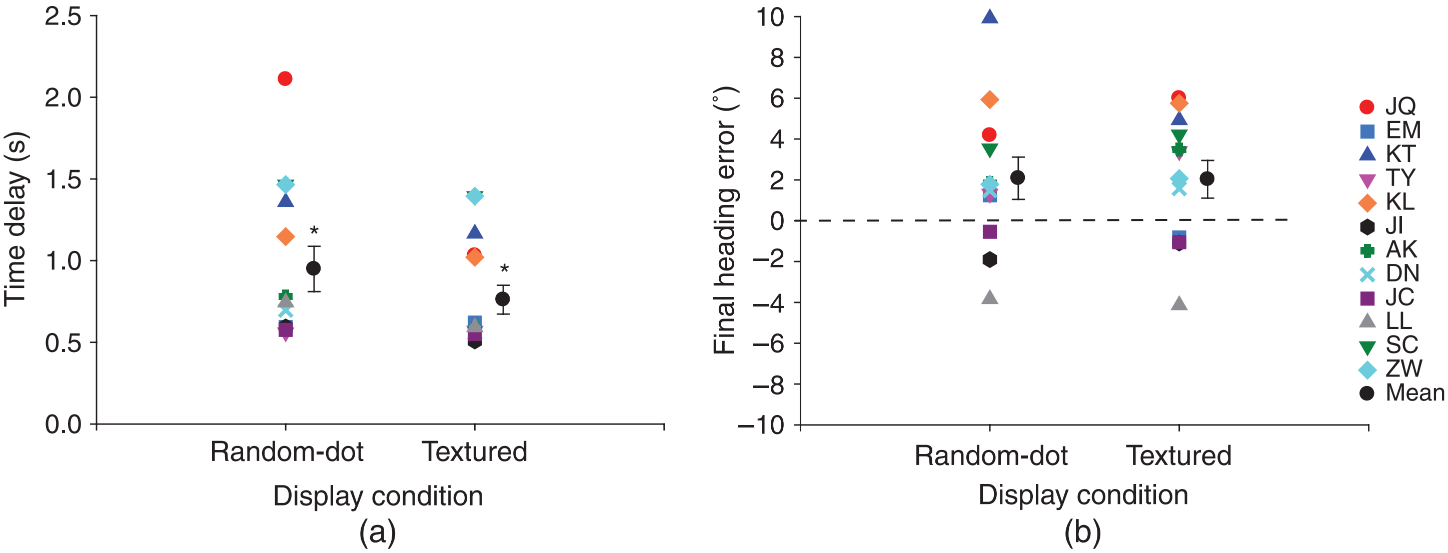


Figure 6. (a) Time delay and (b) final heading error against display condition for each participant along with the means averaged across participants. Error bars for the means are SEs across 12 participants. Asterisk indicates the statistical significance of a paired *t*-test ($p < 0.05$). The dashed line at 0° indicates the perfect steering performance.

different from each other. Separate *t*-tests revealed that the mean final heading errors for the two display conditions were borderline significantly larger than zero ($t(11) = 2.00$, $p = 0.07$ and $t(11) = 2.20$, $p = 0.0501$, for the random-dot and the textured ground displays, respectively). As the size of the target at 4 s when steady-state control performance starts is 2° in width, the about 2° understeering final heading error observed for both display conditions indicates that even with a sparse flow field provided by the random-dot ground, participants could still accurately align their heading with the target. The 2° understeering final heading error is also consistent with the heading perception threshold for traveling on a curved path with large path curvatures (Li et al., 2006a).

## Path curvature profile

If participants used the path strategy to steer toward the target, they would steer to increase the path curvature to a set value and then hold it constant to align their optimal future path with the target. Such control would be the most efficient and require a minimum amount of steering correction (Wann & Land, 2000). Figure 7a plots the simplified path curvature predictions assuming participants used this path curvature control strategy and generated a time-delayed response to the initial path curvature demand. In comparison, Figure 7b plots the time series of path curvature performance data for the two display conditions, computed from the recorded time series of the participant's position averaged across 12 participants. Positive path curvatures indicate path curvature in the target direction and negative path curvatures indicate the opposite. As the target is placed at 20° away from the participant's initial heading, the negative path curvature at the beginning of the trial especially for the random-dot ground display is consistent with the fact that participants initially traveled away from the target before they made any control adjustment. Participants then quickly adjusted the path curvature to steer toward the target.

From the same averaged time series of the participant's position, we computed the required path curvature ($k_{\text{req}}$) at each moment in time that would lead the participant to the target for the two display conditions (dashed lines in Figure 7b). For both display conditions, the actual path curvature and the required path curvature do not converge until they are both close to zero at about 3 s, indicating that participants did not follow a smoothly curved path to the target by setting and holding a constant curvature as shown in Figure 7a. Instead, the quick zeroing of the path curvature indicates that participants tried to steer toward the target via a straight path by aligning their heading with the target. This is contrary to the use of the path strategy and supports the use of the heading strategy in the control of steering toward a goal.

## $\tau_\theta$ and $\tau_p$ profile

If participants used the tau-equalization strategy to steer toward the target, they would steer to equalize the time to closure of the target-heading angle ($\tau_\theta$) with the time to passage of the target ($\tau_p$) when approaching the target. Accordingly, both $\tau_\theta$ and $\tau_p$ should decrease smoothly with time as shown in Figure 8a. Figure 8b plots the time series of $\tau_\theta$ and $\tau_p$ performance data, again computed from the recorded time series of the participant's position averaged across 12 participants. Positive $\tau$ indicates that

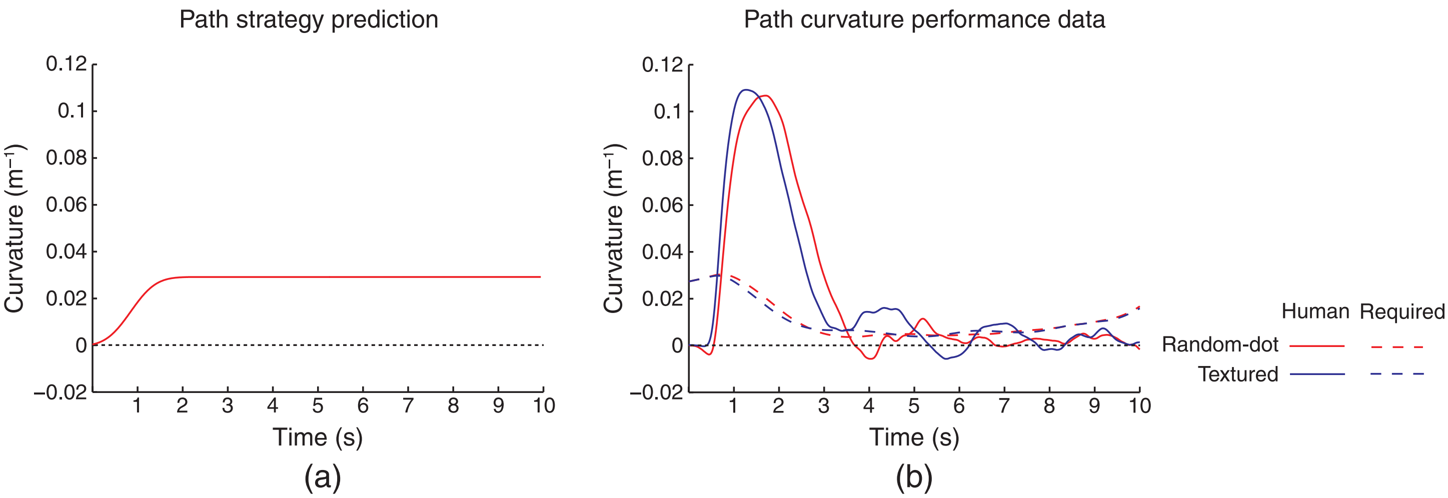


Figure 7. (a) The predicted path curvature profile for the path strategy and (b) the time series of path curvature performance data computed from the recorded time series of the participant's position averaged across 12 participants for the random-dot and textured ground displays. The dashed lines in (b) plot the required path curvature for the path to go through the target computed from the same averaged time series of the participant's position.

participants steered to close the relevant gap and negative $\tau$ indicates that participants steered to enlarge the gap. $\tau_\mathrm{p}$ starts at about 12 s and decreases to close to zero at the end of the trial, indicating that participants steered to shorten their distance to the target throughout the trial. In contrast, $\tau_\theta$ starts at a negative value, consistent with the initial increase in the heading error (i.e., the initial enlarging of the target-heading angle, Figure 5b), then increases and decreases abruptly, finally converging close to zero. $\tau_\theta$ nevertheless spikes again after 3 s, which continues until the end of the trial. For both display conditions, the profile of $\tau_\mathrm{p}$ shows a continual decrease and is in sharp contrast with the unstable profile of $\tau_\theta$. This indicates that participants did not try to equalize $\tau_\theta$ with $\tau_\mathrm{p}$ to travel on a curved path with changing curvature to the target (Fajen, 2001). On the contrary, the profiles of $\tau_\theta$ and $\tau_\mathrm{p}$ are consistent with the use of the heading strategy to steer toward the target via a straight path. Specifically, given $\tau_\theta = \frac{\theta}{\dot{\theta}}$ (Lee, 1974, 1976), where $\theta$ is the target-heading angle (i.e., heading error), the spikes of $\tau_\theta$ after 3 s are due to small changes in heading error (i.e., small $\dot{\theta}$) when participants completed heading adjustment and were traveling on a straight path to the target.

## Modeling

In summary, the heading error, path curvature, and $\tau_\theta$ and $\tau_\mathrm{p}$ performance data all support that participants used the continuous visual feedback of heading error to align

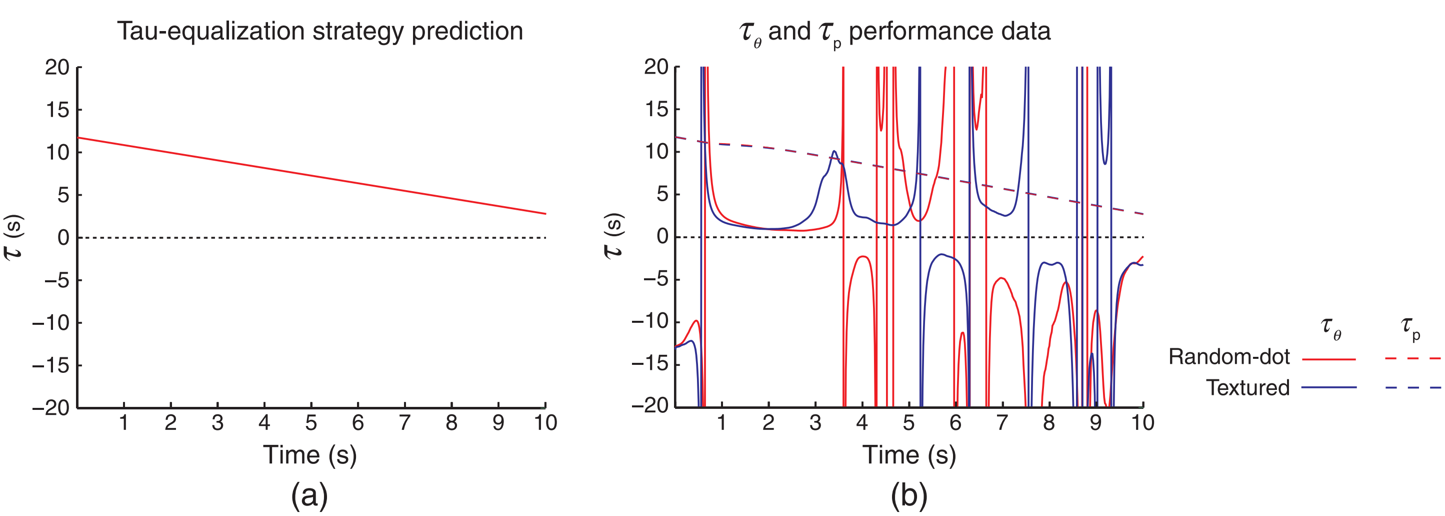


Figure 8. (a) The predicted tau ($\tau$) profile for the tau-equalization strategy and (b) the time series of $\tau_\theta$ and $\tau_\mathrm{p}$ performance data computed from the recorded time series of the participant's position averaged across 12 participants for the random-dot and textured ground displays.

their heading with the target. As human control compensation in such a closed-loop control task typically displays a second-order lag behind the input error signal due to physical limitations (Li, Stone, & Chen, 2011; Li, Sweet, & Stone, 2005, 2006b; McRuer, Graham, Krendel, & Reisener, 1965; McRuer & Krendel, 1974; Wickens, 1986), we simulated participants' heading control performance as if they were responding to the input heading error and generating control responses to minimize heading error through changing path curvature with a second-order lag. The transfer function of participants ($Y_p$) is given by

$$Y_p = \frac{Ke^{-s\tau}}{s^2/\omega_n^2 + 2s\zeta/\omega_n + 1}, \quad (3)$$

where $K$ represents the gain in the control compensation, $\tau$ represents the sum of perceptual and neuromotor delays that specify the participant's reaction time to the input heading error, $\omega_n$ and $\zeta$ represent the fixed second-order response dynamics of the participant independent of the input heading error, and $s$ is the Laplace transform variable.

The dashed lines in Figure 5b show the simulation results for the two display conditions. As on average participants corrected 81% of the heading errors within the first 5 s, we determined the parameters of the transfer function by a least-squares best fit to the heading error performance data of the first 5 s. Given that the width of the target was 2° at 4 s, the input heading error was considered zero when it was smaller than 2° after 4 s. We fixed $K$, $\omega_n$, and $\zeta$ across the two display conditions such that there were in total five parameter values to fit 600 data points (5 s × 60 Hz × 2 display conditions). The Pearson correlation coefficients between the simulation and the performance data are 0.996 and 0.995 for the random-dot and the textured ground displays, respectively, indicating that 99.1% of the variance in the heading error performance data can be accounted for by the model. The fitted model parameter $\tau$ was 594 ms for the random-dot ground display and 469 ms for the textured ground display. This indicates that participants responded 21% faster to the input heading error for the textured than the random-dot ground display, consistent with the observed time delay data described above.

## Discussion

Combining the results, we conclude that when target egocentric direction is not available and participants have to use information from optic flow for steering, they steer toward a target by increasing path curvature to a maximum to quickly minimize heading error, then decreasing curvature back down to near zero to smoothly converge their heading to the target direction. Our findings thus support the theory proposed by Gibson (1950) that people steer toward a goal by aligning their heading specified by optic flow with the goal. As perceiving heading from optic flow requires 300–430 ms processing time (Crowell, Royden, Banks, Swenson, & Sekuler, 1990; Hooge, Beintema, & van den Berg, 1999) and, in the current study, the target is placed at 20° away from the participant's initial heading, the initial increase of heading error and the negative path curvature are consistent with the fact that participants initially traveled away from the target, and as soon as optic flow kicked in, they started controlling heading to smoothly converge it to the target direction. The time delay from the performance data and the reaction time from the model simulation both show that participants generated about 20% faster control responses for the textured ground display that provided a dense flow field than for the random-dot ground display. The larger response delay observed for the random-dot ground display explains the larger initial increase of heading error and the larger negative path curvature observed for the random-dot than the textured ground display. The modeling of the heading control response with a second-order lag describes the heading error performance data almost perfectly, indicating that heading can be controlled online without any explicit knowledge of the world. This is in line with the rationale of the behavioral dynamics model proposed by Fajen and Warren (2003), with our model suggesting an alternative and simple way to model the walking data in their study.

The observed final approach to the target along a nearly straight path is contrary to that expected from the path curvature control strategy whereby participants would align their future path with the target by setting and holding a constant curvature (Wann & Land, 2000; Wann & Swapp, 2000). The use of heading but not path in the control of steering toward a goal is consistent with our recent findings showing that for traveling on a curved path, while path perception is accurate only when the rotation in retinal flow corresponds to path rotation, heading perception is accurate regardless of the source of rotation in the flow field. This suggests that heading rather than path provides a more robust source of information for online control of steering (Li & Cheng, 2011). Furthermore, in comparison with the studies that found that participants directed their gaze at a point on their future path at 1–2 s ahead when steering around a bend in a driving simulator (Wilkie et al., 2010, 2008), we conducted a separate experiment in which we measured the eye movement of seven participants when they steered toward the target with the textured ground display. We found that on average participants looked at the red post target for 80% (*SD*: 6%) of the time throughout the trial, indicating that people tend to look at the target but not a point on their future path when steering toward the target.

The final nearly straight path to the target is also contrary to that expected from the tau-equalization strategy whereby participants would steer to equalize the time to closure of the target-heading angle ($\tau_\theta$) with the time to passage of the target ($\tau_p$) and travel on a curved path with changing curvature to the target (Fajen, 2001). The differences in the findings from the current study and the study by Fajen (2001) can be explained from two aspects. First, instead of quantitatively computing $\tau_\theta$ and $\tau_p$ to directly compare their values, Fajen compared the change of path curvature with the qualitative predictions of the tau-equalization strategy, i.e., when $0 < \tau_\theta < \tau_p$, participants do not have to decrease path curvature, but when $\tau_\theta > \tau_p$, participants must increase curvature. This qualitative comparison is a weaker test than our analysis of the quantitative values of $\tau_\theta$ and $\tau_p$ to directly examine whether the tau-equalization strategy is used for steering throughout the trial. Second, in Fajen's study, the target drifted on the screen, thus the target egocentric direction relative to the participant's straight ahead changed during steering. Accordingly, participants could have steered to cancel the target optical drift or center the target in their straight ahead, which the study failed to analyze. In the current experiment, we carefully controlled the target egocentric direction to make sure that participants could not steer toward the target using this cue, and the data indicate that participants did not steer to equalize $\tau_\theta$ and $\tau_p$ at any moment during the course of the trial. We thus conclude that people do not use the tau-equalization strategy to steer toward a goal.

Although our data support the use of heading specified by optic flow in the control of steering toward a goal and are consistent with the findings from previous studies that reported the use of optic flow in the control of walking toward a goal (e.g., Harris & Carre, 2001; Warren et al., 2001; Wood et al., 2000), we would like to emphasize that we found such results when target egocentric direction is unavailable for steering and participants have to rely on information from optic flow to perform the steering task. While the target egocentric direction cue can be easily made unavailable for steering on a driving or a flying simulator or a video game interface through making the virtual gaze direction (i.e., the camera direction in the virtual world) always point at the target as we did in the current study, in the real world, this cue can hardly be made unavailable. This is due to the fact that in the real world, target egocentric direction is available instantaneously from the retinal position of a target relative to the midline of the body (i.e., the straight ahead). As we maintain our gaze direction on the target while moving in the world, we generate eye and/or head movements. The extraretinal information of eye and head movements informs the brain about the change in target egocentric direction even when the target stays in a constant position on the retina. The question of whether people still rely on heading specified by optic flow or switch to use target egocentric direction when such a cue is available for the control of steering/walking toward a goal remains unanswered and offers interesting perspectives for future research.

## Acknowledgments

This study was supported by a grant from the Research Grants Council of Hong Kong (HKU 7480/10H) to L. Li. We thank Lee Stone for his helpful discussion, Gordon Wong, Diederick Niehorster, and Willie Xiang for their assistance in data collection, and three anonymous reviewers and Diederick Niehorster for their helpful comments on a previous draft.

Commercial relationships: none.
Corresponding author: Li Li.
Email: lili@hku.hk.
Address: Department of Psychology, The University of Hong Kong, Pokfulam, Hong Kong SAR.

## References

Banks, M. S., Ehrlich, S. M., Backus, B. T., & Crowell, J. A. (1996). Estimating heading during real and simulated eye movements. *Vision Research, 36,* 431–443.

Bootsma, R. J., & Craig, C. M. (1999). *Information in movement, information for movement*. Paper presented at the 10th International Conference on Perception and Action, Edinburgh, Scotland.

Cavallo, V., & Laurent, M. (1988). Visual information and skill level in time-to-collision estimation. *Perception, 17,* 623–632.

Crowell, J. A., & Banks, M. S. (1993). Perceiving heading with different retinal regions and types of optic flow. *Perception & Psychophysics, 53,* 325–337.

Crowell, J. A., Royden, C. S., Banks, M. S., Swenson, K. H., & Sekuler, A. B. (1990). Optic flow and heading judgments [Abstract]. *Investigative Ophthalmology and Vision Science (Supplement), 31,* 522.

Cutting, J. E., Vishton, P. M., Flückiger, M., Baumberger, B., & Gerndt, J. D. (1997). Heading and path information from retinal flow in naturalistic environments. *Perception & Psychophysics, 59,* 426–441.

Fajen, B. (2001). Steering toward a goal by equalizing taus. *Journal of Experimental Psychology: Human Perception and Performance, 27,* 953–968.

Fajen, B. R., & Warren, W. H. (2003). Behavioral dynamics, of steering, obstacle avoidance, and route selection. *Journal of Experimental Psychology: Human Perception and Performance, 29,* 343–362.

Gibson, J. J. (1950). *The perception of the visual world*. Boston: Houghton Mifflin.

Grigo, A., & Lappe, M. (1999). Dynamical use of different sources of information in heading judgments from retinal flow. *Journal of the Optical Society of America A, 16,* 2079–2091.

Harris, J. M., & Rogers, B. J. (1999). Going against the flow. *Trends in Cognitive Science, 3,* 449–450.

Harris, M. G., & Carre, G. (2001). Is optic flow used to guide walking while wearing a displacing prism? *Perception, 30,* 811–818.

Hoffmann, E. R. (1994). Estimation of time to vehicle arrival—Effects of age on use of available information. *Perception, 23,* 947–955.

Hooge, I. T. C., Beintema, J. A., & van den Berg, A. V. (1999). Visual search of heading direction. *Experimental Brain Research, 129,* 615–628.

Kaiser, M. K., & Mowafy, L. (1993). Optical specification of time-to-passage: Observers' sensitivity to global tau. *Journal of Experimental Psychology: Human Perception and Performance, 19,* 1028–1040.

Kandil, F. I., Rotter, A., & Lappe, M. (2009). Driving is smoother and more stable when using the tangent point. *Journal of Vision, 9*(1):11, 1–11, http://www.journalofvision.org/content/9/1/11, doi:10.1167/9.1.11. [PubMed] [Article]

Kandil, F. I., Rotter, A., & Lappe, M. (2010). Car drivers attend to different gaze targets when negotiating closed vs. open bends. *Journal of Vision, 10*(4):24, 1–11, http://www.journalofvision.org/content/10/4/24, doi:10.1167/10.4.24. [PubMed] [Article]

Kim, N. G., & Turvey, M. T. (1999). Eye movements and a rule for perceiving direction of heading. *Ecological Psychology, 11,* 233–248.

Land, M. F. (1998). The visual control of steering. In L. R. Harris & M. Jenkins (Eds.), *Vision and action* (pp. 163–180). Cambridge, UK: Cambridge University Press.

Land, M. F., & Lee, D. N. (1994). Where we look when we steer. *Nature, 369,* 742–744.

Lee, D. N. (1974). Visual information during locomotion. In R. B. McLeod & H. Pick (Eds.), *Perception: Essays in honor of J. J. Gibson* (pp. 250–267). Ithaca, NY: Cornell University Press.

Lee, D. N. (1976). A theory of visual control of breaking based on information about time-to-collision. *Perception, 5,* 437–459.

Lee, D. N., & Lishman, R. (1977). Visual control of locomotion. *Scandinavian Journal of Psychology, 18,* 224–230.

Li, L., Chen, J., & Peng, X. (2009). Influence of visual path information on human heading perception during rotation. *Journal of Vision, 9*(3):29, 1–14, http://www.journalofvision.org/content/9/3/29, doi:10.1167/9.3.29. [PubMed] [Article]

Li, L., & Cheng, J. C. K. (2011). Perceiving path from optic flow. *Journal of Vision, 11*(1):22, 1–15, http://www.journalofvision.org/content/11/1/22, doi:10.1167/11.1.22. [PubMed] [Article]

Li, L., Stone, L. S., & Chen, J. (2011). Influence of optic-flow information beyond the velocity field on the active control of heading. *Journal of Vision, 11*(4):9, 1–16, http://www.journalofvision.org/content/11/4/9, doi:10.1167/11.4.9. [PubMed] [Article]

Li, L., Sweet, B. T., & Stone, L. S. (2005). Effect of contrast on the active control of a moving line. *Journal of Neurophysiology, 93,* 2873–2886.

Li, L., Sweet, B. T., & Stone, L. S. (2006a). Humans can perceive heading without visual path information. *Journal of Vision, 6*(9):2, 874–881, http://www.journalofvision.org/content/6/9/2, doi:10.1167/6.9.2. [PubMed] [Article]

Li, L., Sweet, B. T., & Stone, L. S. (2006b). Active control with an isoluminant display. *IEEE Transactions on Systems, Man, and Cybernetics, Part A: Systems and Humans, 36,* 1124–1134.

Li, L., & Warren, W. H. (2000). Perception of heading during rotation: Sufficiency of dense motion parallax and reference objects. *Vision Research, 40,* 3873–3894.

Li, L., & Warren, W. H. (2002). Retinal flow is sufficient for steering during observer rotation. *Psychological Science, 13,* 485–491.

Llewellyn, K. R. (1971). Visual guidance of locomotion. *Journal of Experimental Psychology, 91,* 245–261.

McRuer, D. T., Graham, D., Krendel, E. S., & Reisener, W. (1965). *Human pilot dynamics in compensatory systems* (Wright-Patterson Air Force Base Technical Report, AFFDL-TR-65-15). Ohio, USA: Wright-Patterson Air Force Base.

McRuer, D. T., & Krendel, E. S. (1974). *Mathematical models of human pilot behavior* (NASA Technical Report, AGARD-AG-188). USA: The National Aeronautics and Space Administration.

Regan, D., & Beverly, K. I. (1982). How do we avoid confounding the direction we are looking and the direction we are moving? *Science, 215,* 194–196.

Royden, C. S., Banks, M. S., & Crowell, J. A. (1992). The perception of heading during eye movements. *Nature, 360,* 583–585.

Rushton, S. K., & Harris, J. M. (2004). The utility of not changing direction and the visual guidance of locomotion. In V. Lucia, S. A. Beardsley, & S. K. Rushton (Eds.), *Optic flow and beyond* (pp. 363–381). Dordrecht, The Netherlands: Kluwer Academic Publishers.

Rushton, S. K., Harris, J. M., Lloyd, M., & Wann, J. P. (1998). Guidance of locomotion on foot uses perceived target location rather than optic flow. *Current Biology, 8,* 1191–1194.

Rushton, S. K., Harris, J. M., & Wann, J. P. (1999). Steering, optic flow, and the respective importance of depth and retinal motion distribution. *Perception, 28,* 255–266.

Rushton, S. K., Wen, J., & Allison, R. S. (2002). Egocentric direction and the visual guidance of robot locomotion: Background, theory, and implementation. In H. H. Bülthoff, S.-W. Lee, T. A. Poggio, & C. Wallraven (Eds.), *Biologically motivated computer vision: Lecture notes in computer science* (pp. 576–591). Verlag, Germany: Springer.

Stone, L. S., & Perrone, J. A. (1997). Human heading estimation during visually simulated curvilinear motion. *Vision Research, 37,* 573–590.

van den Berg, A. V. (1992). Robustness of perception of heading from optic Flow. *Vision Research, 32,* 1285–1296.

Wann, J., & Land, M. (2000). Steering with or without the flow: Is the retrieval of heading necessary? *Trends in Cognitive Sciences, 4,* 319–324.

Wann, J. P., Poulter, D. R., & Purcell, C. (2011). Reduced sensitivity to visual looming inflates the risk posed by speeding vehicles when children try to cross the road. *Psychological Science, 22,* 429–434.

Wann, J. P., & Swapp, D. K. (2000). Why you should look where you are going. *Nature Neuroscience, 3,* 647–648.

Warren, W. H., Kay, B. A., Zosh, W. D., Duchon, A. P., & Sahuc, S. (2001). Optic flow is used to control human walking. *Nature Neuroscience, 4,* 213–216.

Warren, W. H., Morris, M. W., & Kalish, M. (1988). Perception of translation heading from optic flow. *Journal of Experimental Psychology: Human Perception and Performance, 14,* 646–660.

Wickens, C. (1986). The effects of control dynamics on performance. In K. Boff, L. Kaufman, & J. Thomas (Eds.), *Handbook of perception and human performance: Cognitive processes and performance* (vol. II, pp. 39-1 to 39-60). New York: John Wiley & Sons.

Wilkie, R. M., Kountouriotis, G. K., Merat, N., & Wann, J. P. (2010). Using vision to control locomotion: Looking where you want to go. *Experimental Brain Research, 204,* 539–547.

Wilkie, R. M., Wann, J. P., & Allison, R. S. (2008). Active gaze, visual look-ahead, and locomotor control. *Journal of Experimental Psychology: Human Perception and Performance, 34,* 1150–1164.

Wood, R. M., Harvey, M. A., Young, C. E., Beedie, A., & Wilson, T. (2000). Weighting to go with the flow? *Current Biology, 10,* R545–546.