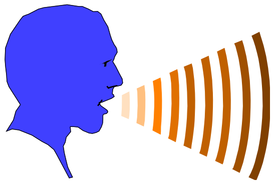

Image and Video Processing

Background Modeling and Camera Motion Estimation

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Last Lecture

- 2-D motion vs. optical flow
- Optical flow equation and ambiguity in motion estimation
- General methodologies in motion estimation
  - Motion representation
  - Motion estimation criterion
  - Optimization methods
- Lucas-Kanade Flow Estimation Method and KLT tracker
- Block Matching Algorithm
  - EBMA algorithm
  - Half-pel EBMA
  - Hierarchical EBMA (HBMA)
- Deformable image registration

Summary 1: General Methodology

- What causes 2D motion?
  - Object motion projected to 2D
  - Camera motion
  - Optical flow vs. true 2D motion
- Constraints for 2D motion
  - Optical flow equation
  - Derived from constant intensity and small motion assumption
  - Ambiguity in motion estimation
- Estimation criterion:
  - DFD (constant intensity)
  - OF (constant intensity+small motion)
  - Regularization (motion smoothness or other prior knowledge)
    - not required
- Search method:
  - Exhaustive search, gradient-descent, multi-resolution
  - Least squares solution under optical flow equation

Summary 2: Motion Estimation Methods

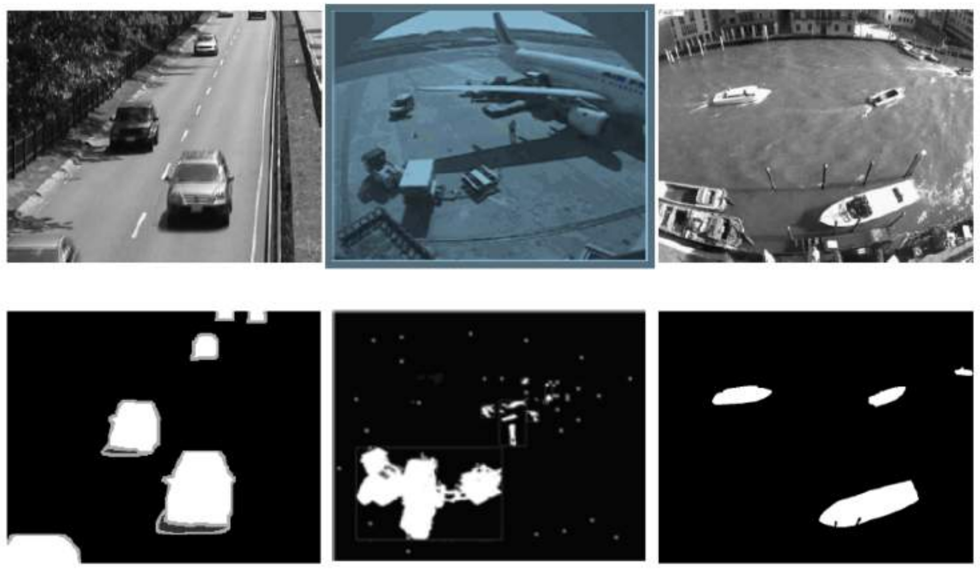
- Pixel-based motion estimation (also known as optical flow estimation)
  - Most accurate representation, but also most costly to estimate
  - Need to put additional constraints on motion of neighboring pixels
  - Lucas-Kanade method
    - Assuming motion in the neighborhood is the same
    - Using Taylor expansion
  - How to handle large motion: iterative refinement, multiresolution
  - KLT tracker: apply LK method on feature points only
  - Automatically yield fractional accuracy
- Block-based motion estimation, assuming each block has a constant motion
  - Good trade-off between accuracy and speed
  - EBMA and its fast but suboptimal variant is widely used in video coding for motion-compensated temporal prediction.
  - HBMA can not only reduce computation but also yield physically more correct motion estimates

This Lecture

- Background modeling and moving object detection
  - Low rank+sparse decomposition (RPCA)
- Object tracking
- Camera motion estimation
- Object detection under camera motion
- Video shot segmentation
- Video stabilization

Background Modeling and Object Detection

- Main applications:
  - Visual surveillance (Road, Airport, Parking lot, In home security,…)
  - Activity pattern discovery: e.g. # of people, # of cars
- In most applications, we want to detect the moving objects
- In some applications, we want to form a complete background from video frames



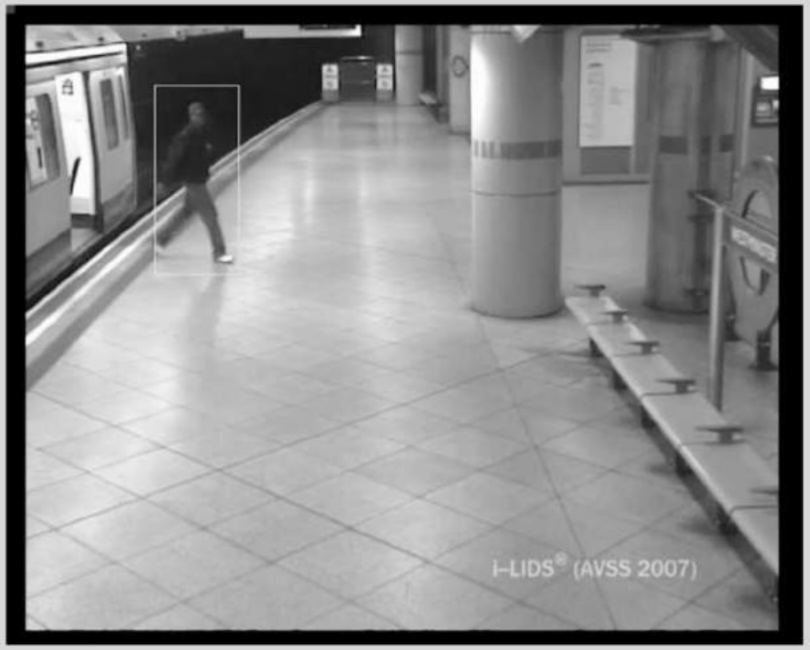
Moving Object Detection

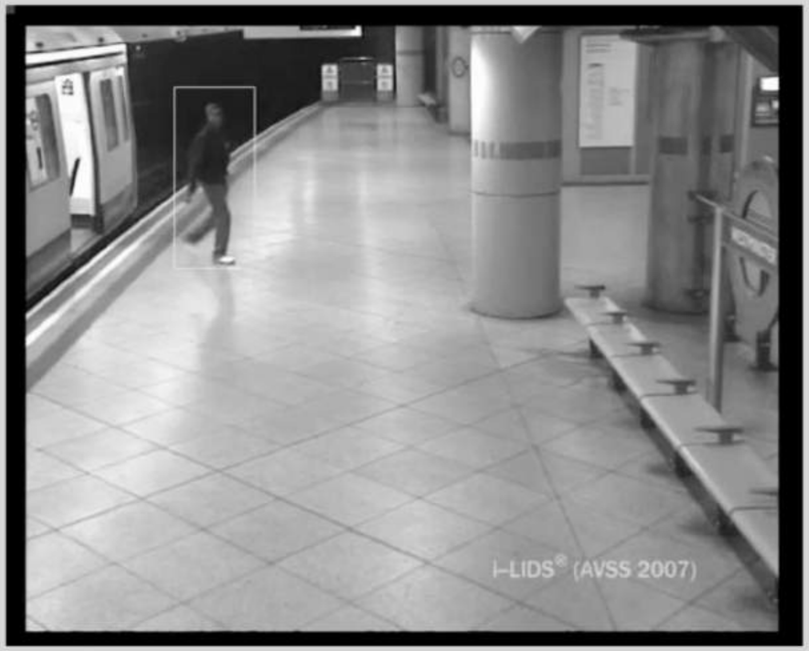
- Simple idea
  - Assuming background is stationary, color changes only at moving regions
  - Take difference between two frames, detect pixels with large difference.
  - Post processing is needed to form smooth, connected foreground regions

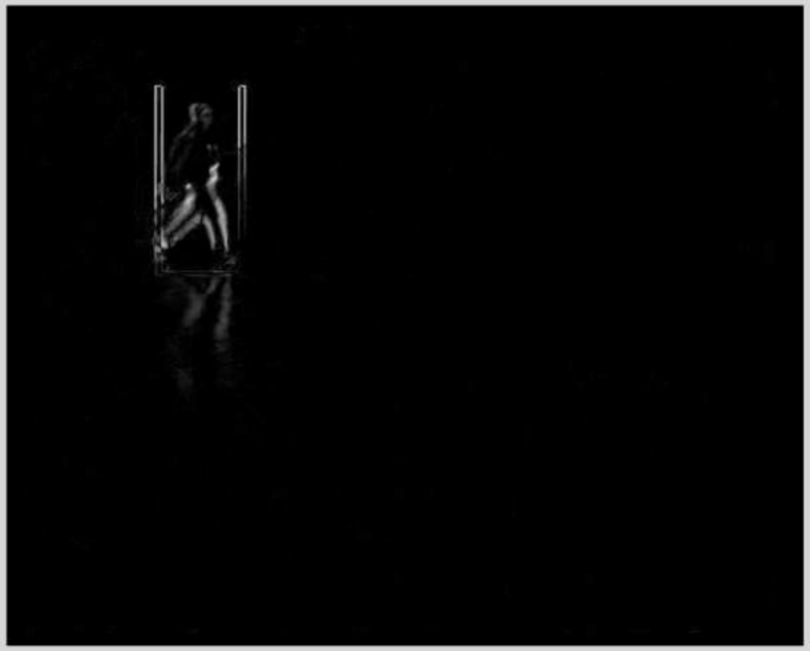
Moving object detection by examining frame difference

- Frame at t: $f(x,y,t)$
- Frame Difference at t: $e(x,y,t)=|f(x,y,t)-f(x,y,t-1)|$
- Thresholding the difference to highlight pixels with large change
- Postprocessing
  - Remove isolated foreground pixels due to false detection
  - Find a connected blob covering the foreground pixels (blob detection, connectivity analysis, and other tools in openCV/Matlab)
  - Alternative: Put a bounding box covering all detected foreground pixels after removing isolated pixels

- Show example

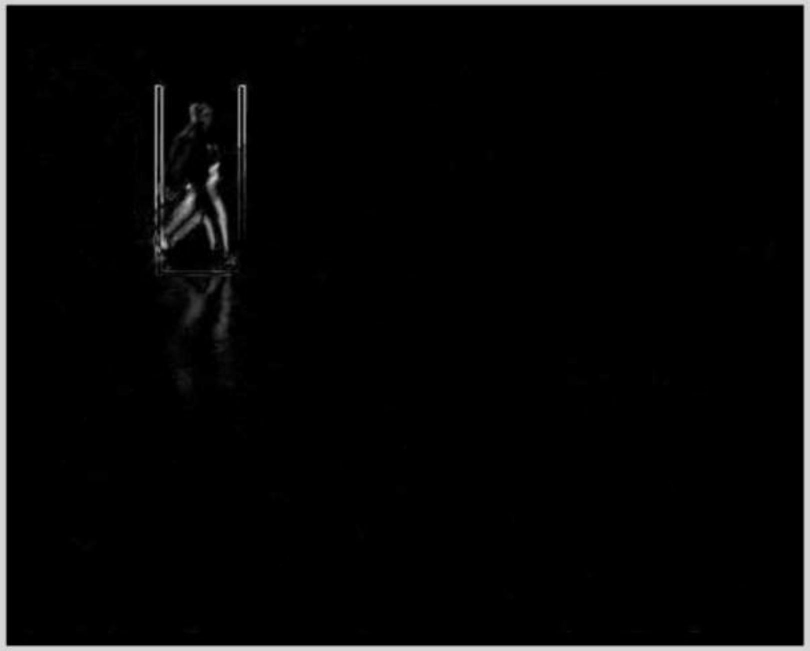


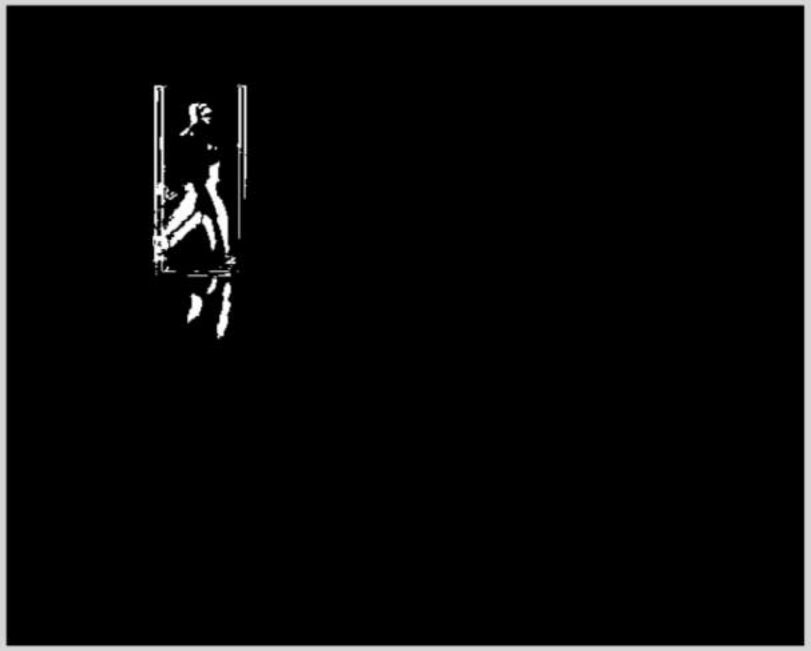


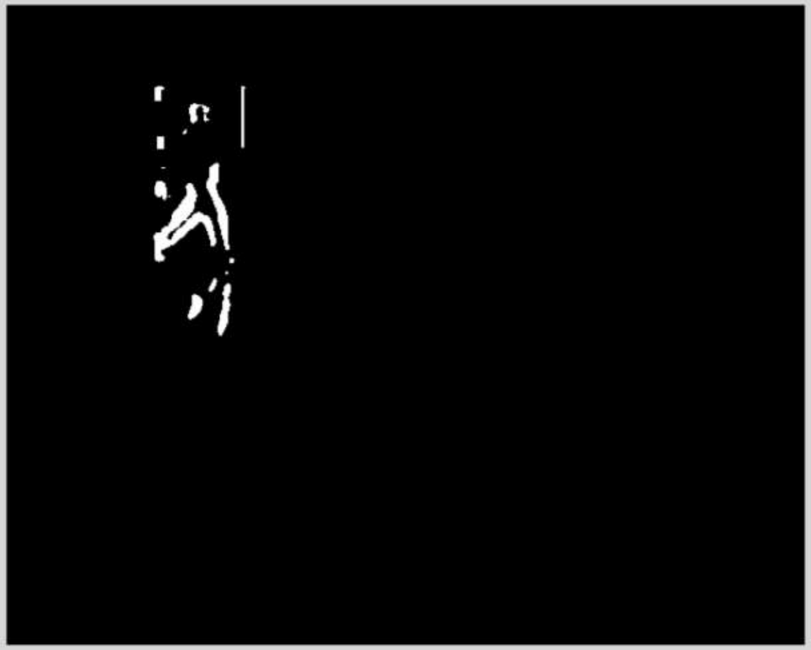


```
>> img1=imread('frame31.jpg');
>> img2=imread('frame32.jpg');
>> img1=rgb2gray(img1);
>> img2=rgb2gray(img2);
>> img1=int16(img1);
>> img2=int16(img2);
>> diff=abs(img1-img2);

>> figure(1),imshow(img1,[])
>> figure(2),imshow(img2,[])
>> figure(3),imshow(diff,[])
```







```
>> figure(3),imshow(diff,[])
>> max(max(diff))
>> diffT=(diff>20);
>> figure(4),imshow(diffT,[])
>> diffTM=medfilt2(diffT,[5 5]);
>> figure(5),imshow(diffTM,[])
```

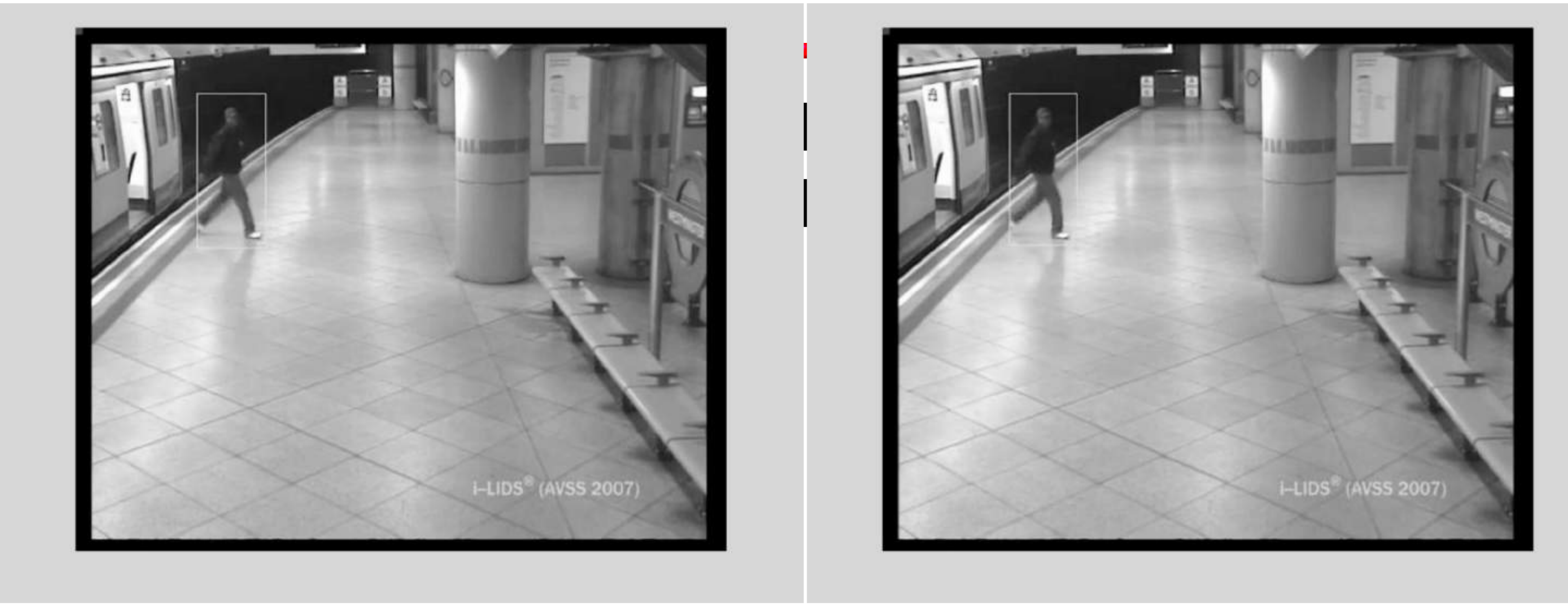
Problem with frame difference

- The background may not be stationary
  - Tree leaf motion
  - Lighting change
  - Camera motion
- More advanced methods are needed to compensate for such changes
  - Background modeling (later)

Object Tracking

- Suppose you identified a person or an object in one frame, and you want to find how does it move in the subsequent frames.
- How do you do that?
- Simple approach:
  - If you put a bounding box over the person, then the color pattern within the bounding box (template block) should not change much even if the box is moving over time
  - We can find how does the box move by searching for a same sized box with similar color pattern in successive frames – Block Matching (also known as template matching)

Example of Object Tracking



```
>> figure(1),imshow(img1,[])
>> figure(2),imshow(img2,[])
>> x0=112,y0=59,x1=175,y1=202
>> Rx=24,Ry=10
>> template=img1(y0:y1,x0:x1);
>> [xm,ym,matchblock]=EBMA(template,img2,x0,y0,Rx,Ry);
>> xm, ym
```

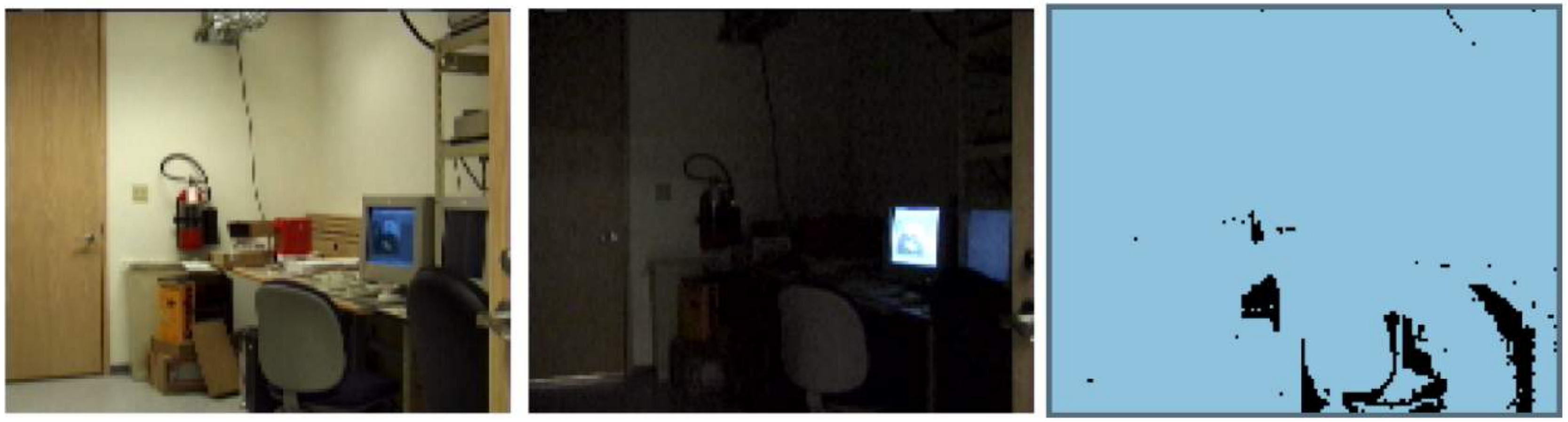
Problems with Block Matching

- The shape and appearance of the object may change if the motion is not just a shift
  - Search for the parameters of possible object motion to minimize the intensity difference after motion compensation in the object region. (See following for global motion estimation)
- Different parts of the object may move differently
- Some parts may disappear, new parts may appear (occlusion issues)
- More sophisticated algorithms are needed to solve these challenges (outside the scope of this lecture)
- However, when two frames are very close in time (e.g under high frame rate), the movement of most objects are small and simple block matching can work quite well

Challenges for Background Modeling

- Illumination change (background looks different at different times of the day, under different weathers)
- Camera jitter causes changes in the background
- Shadow effect (Shadow is slowly moving, may be falsely considered moving object without proper treatment)
- Stationary foreground objects (e.g. parked cars) are hard to differentiate from background

Challenge due to Illumination Changes

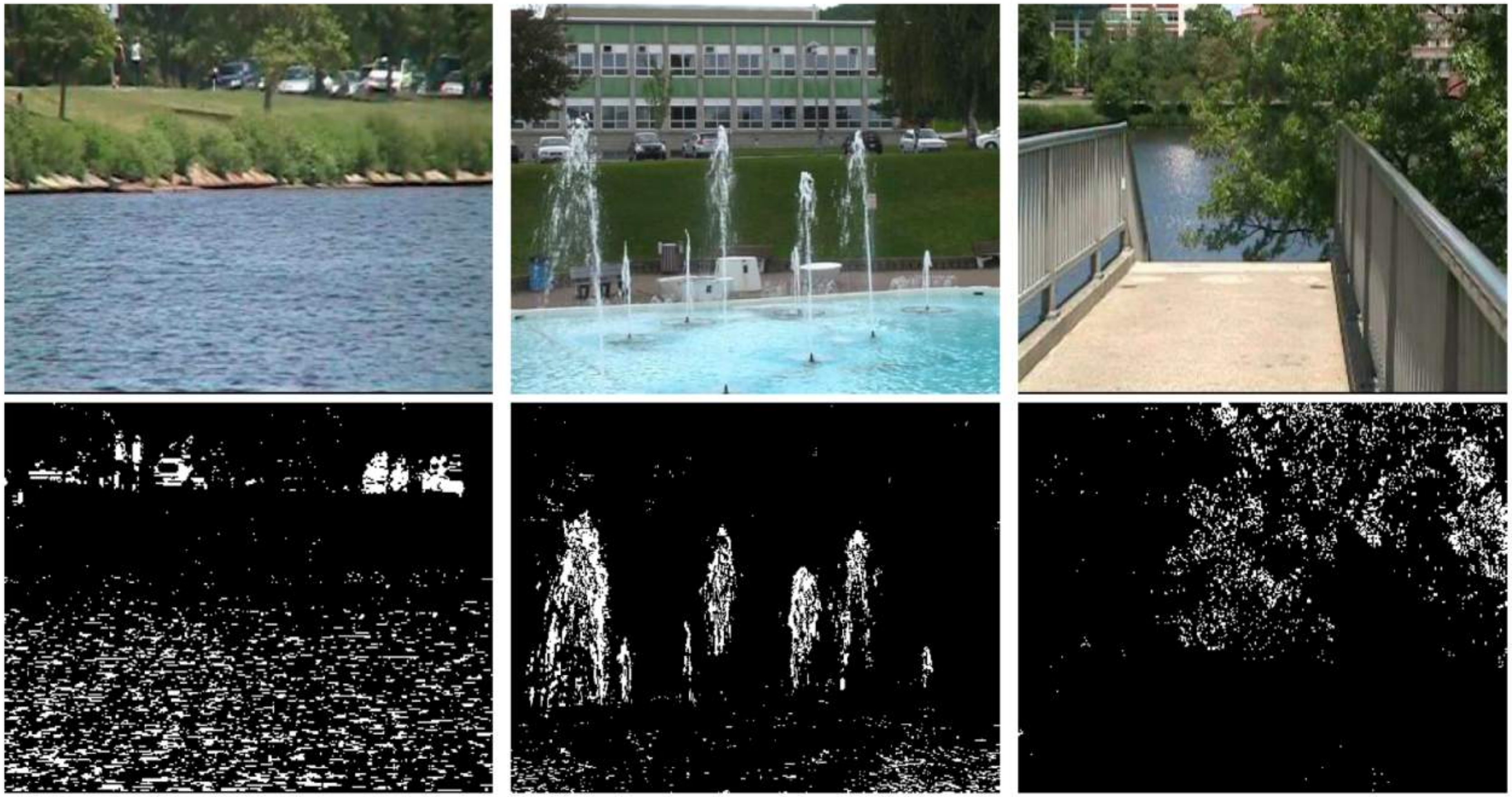


a) Light-on b) Light-off c) Foreground mask

Foreground detected using the MoG algorithm

From: T. Bouwmans, F. El-Baf, and B. Vachon. Background modeling using mixture of Gaussians for foreground detection: A survey. Recent Patents on Computer Science , 1(3):219–237, November 2008.

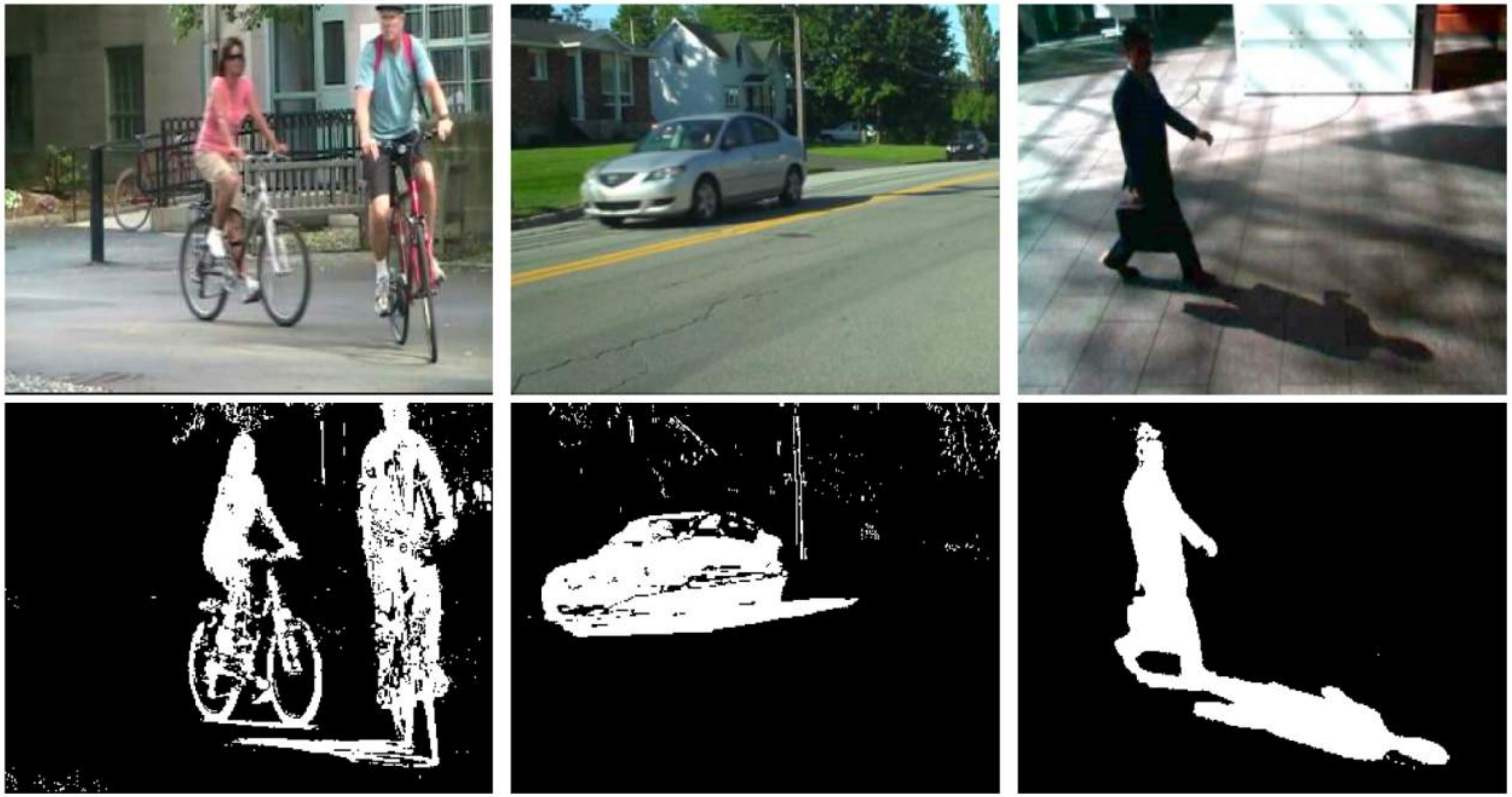
Challenge due to Dynamic Background



Foreground detected using the MoG algorithm

From: T. Bouwmans, F. El-Baf, and B. Vachon. Background modeling using mixture of Gaussians for foreground detection: A survey. Recent Patents on Computer Science , 1(3):219–237, November 2008.

Challenge due to Shadows



Foreground detected using the MoG algorithm

From: T. Bouwmans, F. El-Baf, and B. Vachon. Background modeling using mixture of Gaussians for foreground detection: A survey. Recent Patents on Computer Science , 1(3):219–237, November 2008.

Background Modeling

- Simple method
  - Averaging all frames (hoping moving objects will be averaged out over a long period of time)
  - Work well when the camera is stationary and illumination is nearly constant, and you can average many many frames
- Recursive update
  - The background up to the previous frame B_{t-1}
  - Given a new frame F_t, form difference $D(x,y)=F_t(x,y)-B_{t-1}(x,y)$
  - If $|D(x,y)| < T$, assign (x,y) to Background, use F(x,y) to update $B_{t-1}(x,y)$.
  - $B_t(x,y)=(1-a)\, B_{t-1}(x,y)+ aF_t(x,y)$
- More sophisticated method
  - Modeling the colors at each pixel using a Gaussian mixture model (GMM) (aka mixture of Gaussian or MoG)
    - Recursively update the GMM parameter at each pixel
  - Robust PCA

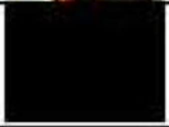
Background Modeling using GMM

- Initial paper:
  - Stauffer, Chris, and W. Eric L. Grimson. "Adaptive background mixture models for real-time tracking." *Computer Vision and Pattern Recognition, 1999. IEEE Computer Society Conference on.*. Vol. 2. IEEE, 1999.
  - http://www.ai.mit.edu/projects/vsam/Publications/stauffer_cvpr98_track.pdf

- A good review:
  - T. Bouwmans, F. El-Baf, and B. Vachon. Background modeling using mixture of Gaussians for foreground detection: A survey. Recent Patents on Computer Science , 1(3):219–237, November 2008.

- Not required for this class

Foreground Detection using GMM-based Approaches

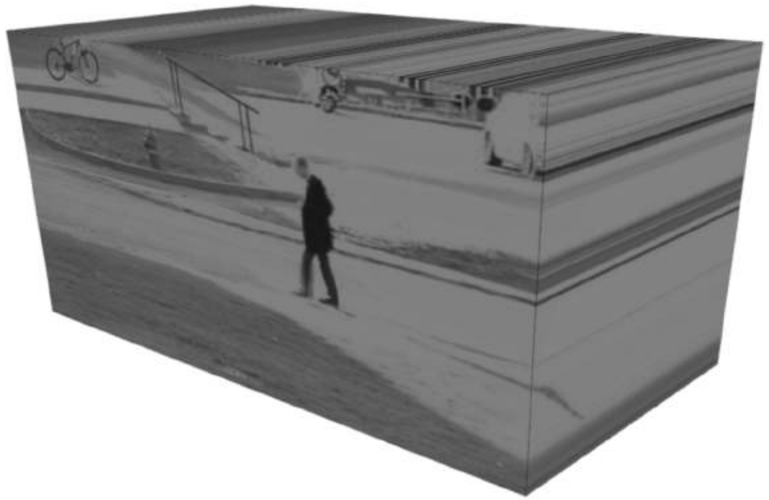
Sequence	MO	TD	LS	WT	C	B	FA
Test image							
Ground Truth							
Stauffer *et al.* [1] **MOG**							
White *et al.* [138] **MOG with PSO**							
Setiawan *et al.* [118] **MOG using IHLS**							
Wang *et al.* [70] **Improved MOG**							
Schindler *et al.* [153] **MOG with MRF**							
Cristani *et al.* [43] **S-TAPMOG**	-	-	-				
Cristani *et al.* [92] **ASTNA**	-	-	-				

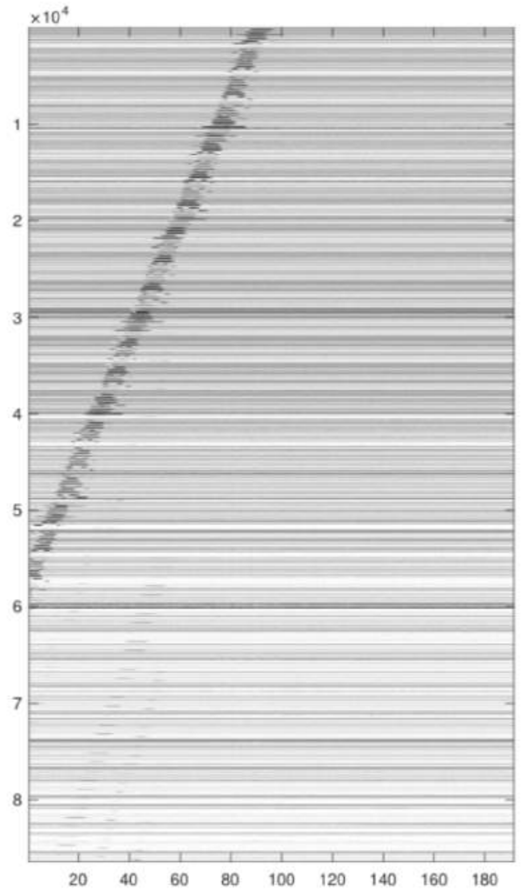
From: T. Bouwmans, F. El-Baf, and B. Vachon. Background modeling using mixture of Gaussians for foreground detection: A survey. Recent Patents on Computer Science , 1(3):219–237, November 2008.

Background Separation Using Low Rank + Sparse Decomposition

- Main idea
  - Reorder pixels in each frame into a vector
  - Put successive frame vectors as columns of a matrix **M**
  - If all the frames are the same (stationary), then all columns will be the same, the matrix has rank 1
  - If all the frames only differ by a scale factor (e.g. due to illumination change), the matrix still has rank 1
  - If all the frames vary from each other slightly, generally the matrix has a low rank (each frame is a linear combination of a few other frames)
  - If there is a moving object in the scene, the matrix may not be low rank any more
  - Generally, M may be decomposed into a low rank matrix L (corresponding to slowly changing background) and a sparse matrix S (corresponding to moving foreground, occupying only a sparse set of pixels)

Stacking Video frames in a Matrix





How to find L and S from M?

M = L + S
L: Background, low rank
S: Moving foreground, sparse

Rank of a Matrix and Singular Vector Decomposition (SVD)

- Rank = # of independent columns (or rows) in a matrix
- Rank = # non-zero singular values of the matrix

- SVD: any matrix can be decomposed as

$$X = U\Lambda V^T = \sum_{r=1}^{R} \lambda_r \boldsymbol{u}_r \boldsymbol{v}_r^T$$

Low-rank+Sparse Decomposition

- Given M, determine L and S, so that
  - M=L+S, L is low rank, S is sparse (Small L0 norm)
- Mathematical formulation
  - $\min\ Rank(L) + \lambda \|S\|_0$, subj to $L + S = M$. (Original problem)
  - Hard to solve!
- Wright et al proved, under some conditions and for a suitably chosen λ, the above problem is equivalent to
  - $\min \|L\|_* + \lambda \|S\|_1$, subj to $L + S = M$. (Convex relaxed problem)
  - $\|L\|_*$ is the Nuclear Norm of L (Sum of singular values of L)
  - Convex problem, and can be solved through ADMM (iterative SVD and soft thresholding)
- L consists of principle components of M, robust to outliers S.
- Candès, E. J., Li, X., Ma, Y., & Wright, J. (2011). Robust principal component analysis?. *Journal of the ACM (JACM)*, *58*(3), 11.
- Wright, J., Ganesh, A., Rao, S., Peng, Y., & Ma, Y. (2009). Robust principal component analysis: Exact recovery of corrupted low-rank matrices via convex optimization. In *Advances in neural information processing systems* (pp. 2080-2088).
- https://sites.google.com/site/backgroundsubtraction/available-implementation/recent-background-modeling/background-modeling-via-rpca

Principle Component Analysis (PCA)

- Original formulation of PCA
  - Given observation vectors xi, form matrix X=[x_1, x_2,…, x_N]
  - Covariance matrix C=X X^T
  - Principle components = Eigenvectors of C: $Cu_i = \lambda_i \, u_i$, X X^T ui = li ui
  - SVD of X: $X = USV^T$
  - $C = X\,X^T = USV^T\,VSU^T = US^2U^T, CU = US^2$
  - Principle components can be found using SVD: u_i is eigenvector with eigenvalue s_i^2
- Another interpretation of PCA
  - Finding a low rank approximation of X with minimal L2 error
  - $\min \|X - L\|_2$, subj to $\mathrm{rank}(L) = K$
  - L=SVD of X with K largest singular values:
    - $X = USV^T$ -> $\mathrm{L} = US_KV^T$

Robust PCA (RPCA)

- PCA: $\min \|X - L\|_2$, subj to $\mathrm{rank}(L) = K$
  - the principle components are greatly affected by outliers (noise with large values)

- Robust PCA: $\min \ Rank(L) + \lambda\|S\|_0$, subj to $L + S = X$
  - S represent “outliers”, which occur rarely but can be large
  - RPCA=Low Rank+Sparse Decomposition!

- Under mild conditions, RPCA is equivalent to solve

$$\min\|L\|_* + \lambda\|S\|_1, \text{ subj to } L + S = M.$$

  - known as Principle Component Pursuit or PCP
  - Can be solved using ADMM

Alternating direction method of multipliers (ADMM, Review)

- ADMM problem form (with f, g convex)

$$\begin{array}{ll} \text{minimize} & f(x) + g(z) \\ \text{subject to} & Ax + Bz = c \end{array}$$

Typical use case:
f(x) is quadratic in x
g(z) contains L1 norm
B= diagonal

  – two sets of variables, with separable objective

Can be grouped as \rho/2 ||y/rho+ (Ax+Bz-c)||^2 by completing square

- $L_\rho(x,z,y) = f(x) + g(z) + \underline{y^T(Ax+Bz-c) + (\rho/2)\|Ax+Bz-c\|_2^2}$

- ADMM:

Minimizing a quadratic problem, with closed-form solution

$$\begin{array}{lll} x^{k+1} & := & \operatorname{argmin}_x L_\rho(x, z^k, y^k) \quad \textit{// x-minimization} \\ z^{k+1} & := & \operatorname{argmin}_z L_\rho(x^{k+1}, z, y^k) \quad \textit{// z-minimization} \\ y^{k+1} & := & y^k + \rho(Ax^{k+1} + Bz^{k+1} - c) \quad \textit{// dual update} \end{array}$$

Soft thresholding if B=diagonal

Croped from: https://web.stanford.edu/~boyd/papers/pdf/admm_slides.pdf

ADMM Solution of L+S

- Original problem: $\begin{array}{ll}\text{minimize} & \|L\|_* + \lambda\|S\|_1 \\ \text{subject to} & L + S = M\end{array}$
- Augmented Lagrangian:

$$l(L, S, Y) = \|L\|_* + \lambda\|S\|_1 + \langle Y, M - L - S\rangle + \frac{\mu}{2}\|M - L - S\|_F^2$$

- ADMM: Solve L and S alternatingly, each with closed form solution

L-minimization: $\arg\min_L l(L, S, Y) = \mathcal{D}_{1/\mu}(M - S + \mu^{-1}Y)$

$$\mathcal{D}_\tau(X) = U\mathcal{S}_\tau(\Sigma)V^*, \text{ where } X = U\Sigma V^*$$

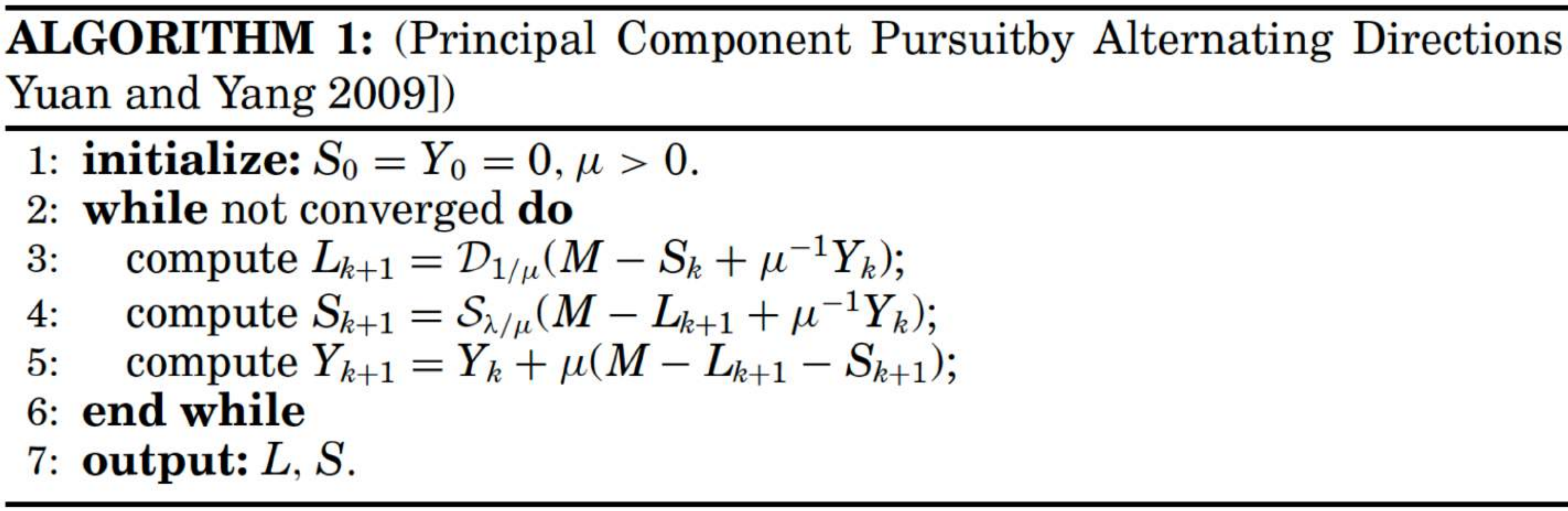
Find the SVD of $X = M - S + \mu^{-1}Y$,
soft hresholding singular values with threshold μ^{-1}

S-minimization: $\arg\min_S l(L, S, Y) = \mathcal{S}_{\lambda/\mu}(M - L + \mu^{-1}Y)$

$$\mathcal{S}_\tau[x] = \text{sgn}(x)\max(|x| - \tau, 0)$$

From: Candès, E. J., Li, X., Ma, Y., & Wright, J. (2011). Robust principal component analysis?. *Journal of the ACM (JACM)*, *58*(3), 11.

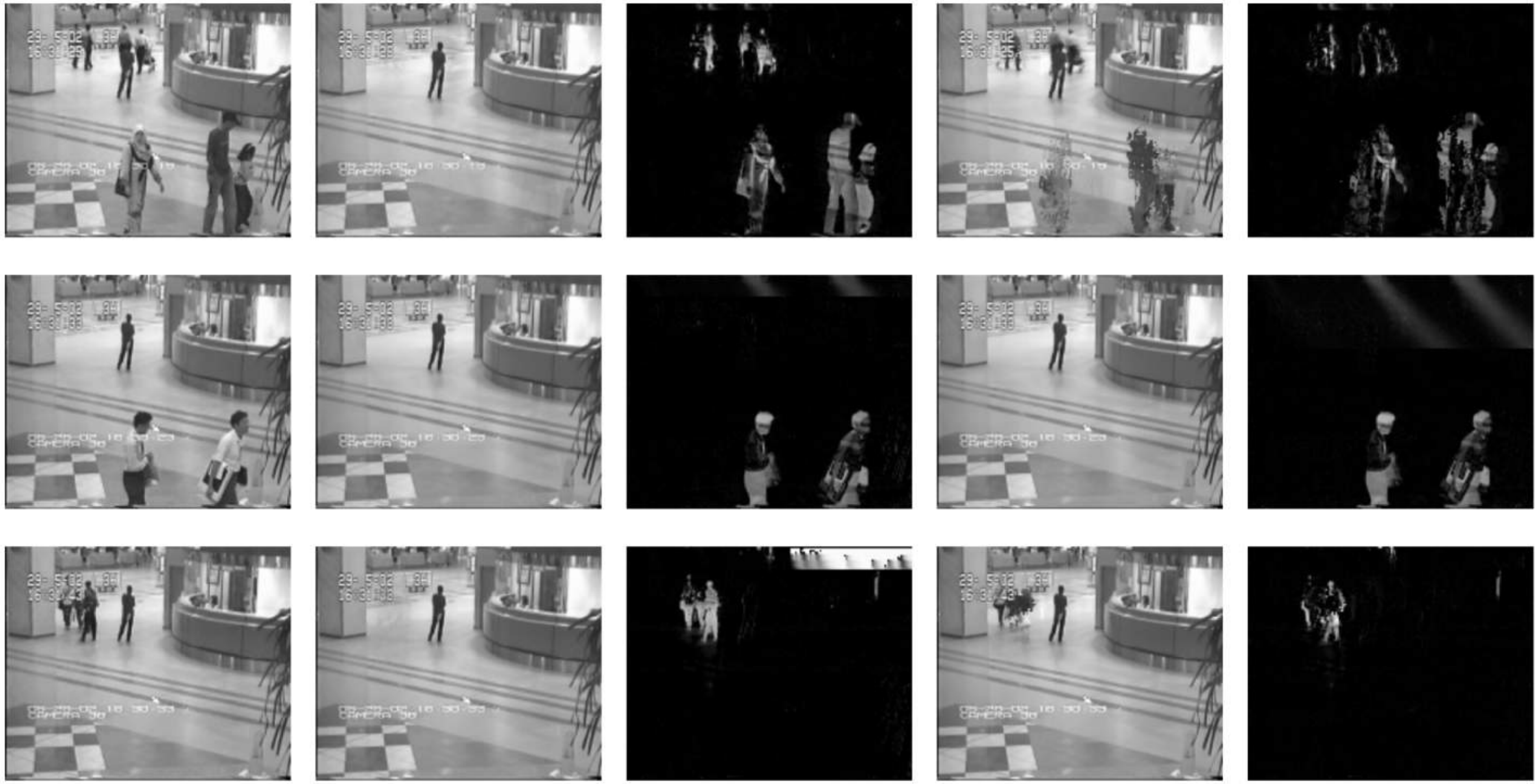
L+S Using ADMM



ALGORITHM 1: (Principal Component Pursuitby Alternating Directions
Yuan and Yang 2009])

```
1: initialize: $S_0 = Y_0 = 0$, $\mu > 0$.
2: while not converged do
3:     compute $L_{k+1} = \mathcal{D}_{1/\mu}(M - S_k + \mu^{-1} Y_k)$;
4:     compute $S_{k+1} = \mathcal{S}_{\lambda/\mu}(M - L_{k+1} + \mu^{-1} Y_k)$;
5:     compute $Y_{k+1} = Y_k + \mu(M - L_{k+1} - S_{k+1})$;
6: end while
7: output: $L$, $S$.
```

From: Candès, E. J., Li, X., Ma, Y., & Wright, J. (2011). Robust principal component analysis?. *Journal of the ACM (JACM)*, *58*(3), 11.


(a) Original frames (b) Low-rank $\hat{L}$ (c) Sparse $\hat{S}$ (d) Low-rank $\hat{L}$ (e) Sparse $\hat{S}$

Convex optimization (this work) Alternating minimization [47]

From: Candès, E. J., Li, X., Ma, Y., & Wright, J. (2011). Robust principal component analysis?. *Journal of the ACM (JACM)*, *58*(3), 11.

Variations of RPCA (not required)

- Complexity issue:
  - ADMM requires solving SVD in each iteration
  - Many fast algorithms have been developed
- Robustness to noise:
  - Beyond the moving object, variations in the measurement matrix M may be due to camera noise, dynamic background (moving tree leaves, ocean waves, etc)
  - Requiring M=L+S exactly may not be appropriate
  - Stable PCP:
    - M= L+S+E, where E represents small random variations
    - $\min \|L\|_* + \lambda \|S\|_1$, subj to $\|M - L - S\|_F \leq \delta$
  - Alternate formulation
    - $\min \|L\|_* + \lambda_1 \|S\|_1 + \lambda_2 \|M - L - S\|_F^2$

- T. Bouwmans, E. Zahzah, "Robust PCA via Principal Component Pursuit: A Review for a Comparative Evaluation in Video Surveillance", Special Issue on Background Models Challenge, Computer Vision and Image Understanding, CVIU 2014, Volume 122, pages 22–34, May 2014. [pdf]

Object Tracking by Template Matching

- Suppose you identified a person or an object in one frame, and you want to find how does it move in the subsequent frames.
- How do you do that?
- Simple approach:
  - If you put a bounding box over the person, then the color pattern within the bounding box (template block) should not change much even if the box is moving over time
  - We can find how does the box move by searching for a same sized box with similar color pattern in successive frames – Block Matching (also known as template matching)

Other Alternatives for Object Tracking

- Previous approach
  - Detect in one frame, track in future frames
  - Often loose track after a while
- Alternative approach
  - Detect possible objects in each frame
  - Link (associate) objects across frames based on their visual similarity
- Yet another alternative
  - Detect feature points in each frame
  - Link features across frames (e.g. KLT tracker) to form “tracklets”
  - Merge tracklets belonging to the same object
    - Motion consistency, spatial adjacency

Robust Vehicle Tracking for Urban Traffic Videos at Intersections

Li C., Chiang A., Dobler G., Wang Y., Xie K., Ozbay K., Ghandehari M., Zhou J., Wang D.
Center for Urban Science + Progress (CUSP), New York University
Department of Electrical and Computer Engineering, New York University
Paper ID 96

Introduction

1. A robust system to **automatically extract vehicle trajectory** data from video data obtained by existing traffic cameras from the New York City Department of Transportation (**NYCDOT**).
2. Automatic trajectory information will be used to develop **realistic surrogate safety measures** to both identify high risk locations and assess implemented safety improvements.

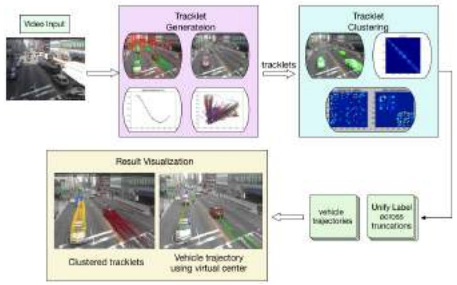


Figure 1: A representation of the steps in our vehicle tracking system.

KLT Tracklet Generation

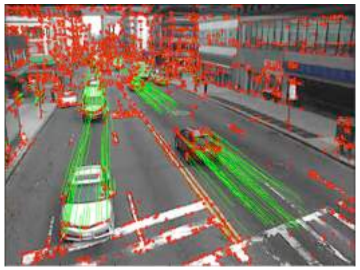


Figure 2: The KLT tracking result shows the effects of perspective as well as the stationary points which are to be filtered out as non-vehicle tracklets in subsequent steps. Feature points are shown in red and tracklets in green.

Motivation and Objectives

We develop a robust, **unsupervised** vehicle tracking system for videos of very congested road intersections in urban environments. Raw tracklets from the standard **Kanade-Lucas-Tomasi**(KLT) tracking algorithm are treated as sample points and grouped to form different vehicle candidates. Each tracklet is described by multiple features including position, velocity, and a foreground score derived from **robust PCA** background subtraction. By considering each tracklet as a node in a graph, we build the adjacency matrix for the graph based on the feature similarity between the tracklets and group these tracklets using spectral embedding and **Dirichelet Process Gaussian Mixture Models**. The proposed system yields excellent performance for traffic videos captured in urban environments and highways.

Challenges from Urban Street Level Videos

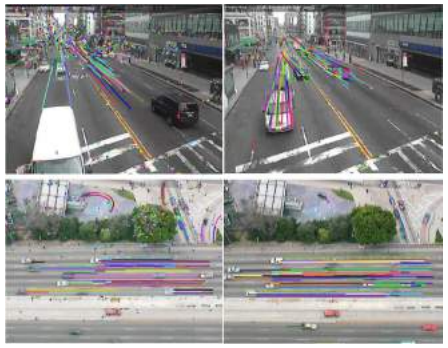


Figure 3: Tracklets are shown before (*left*) and after (*right*) filtering and smoothing. Note that the filtering process removes the majority of stationary points on building corners and street paint.

1. High degree of **partial occlusions** in dense traffic
2. Vehicles have **deformable appearances** due to viewing angles as opposed to the high bird's eye view
3. Traffic lights at the intersection lead to vehicle **stop-and-go** conditions
4. NYC DOT surveillance videos have low resolution (480×640 pixels), frequent illumination changes among frames

Tracklet Clustering

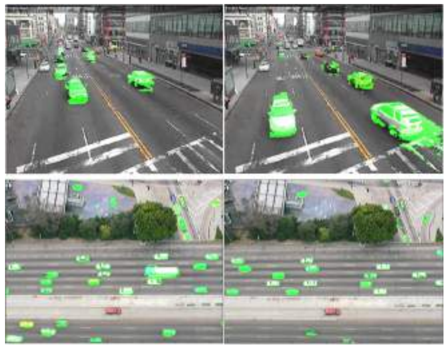


Figure 4: The results of the rPCA foreground/background separation are shown for both street-level NYCDOT (*top*) and NGSIM (*bottom*) video.

1. The **adjacency matrix** between two tracklets A_{ij} is defined as,

$$\ln A_{ij} = -\vec{w} \bullet \vec{f}_{ij}, \tag{1}$$

1. $\vec{w}$ is a weight vector for each feature in $\vec{f}_{ij} \equiv (x_{ij,\max}, v_{x,ij,\max}, v_{y,ij,\max}, b_{\text{cen},ij,\max})$.
2. $x_{ij,\max} \equiv \max_k |\vec{x}_i(t_k) - \vec{x}_j(t_k)|$ is the maximum **positional separation** along the tracklet
3. $v_{x,ij,\max} \equiv \max_k |v_{x,i}(t_k) - v_{x,j}(t_k)|$ and $v_{y,ij,\max} \equiv \max_k |v_{y,i}(t_k) - v_{y,j}(t_k)|$ are the maximum **velocity separations** in two dimensions along the tracklet
4. $b_{\text{cen},ij,\max} \equiv \max_k |\vec{b}_{\text{cen},i}(t_k) - \vec{b}_{\text{cen},j}(t_k)|$ is the maximum **separation of the center of mass of the blob labels**

Result

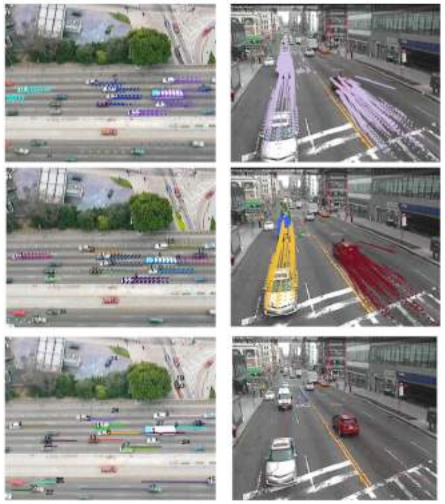


Figure 5: The initial groups from thresholding adjacency matrix (*top*) and the clustered results for all KLT tracklets(*middle*) are shown for NGSIM (*left*) and NYCDOT (*right*) videos. The final extracted trajectories after spectral clustering and DPGMM are shown in the (*bottom*) panels.

1. **Perspective Transformation**: Warping the non-parallel trajectories into parallel
2. **Hard thresholding** A_{ij} is thresholded according to $x_{ij,\max} \leq d$ to form connected components which are never separated by more than a distance d.
3. **Spectral embedding** and **Dirichlet Processs Gaussian Mixture Model** (DPGMM) to identify the number of clusters automatically and label each tracklet.

Contact: Chenge Li, *chenge.li@nyu.edu*

Figure 2: The KLT tracking result shows the effects of perspective as well as the stationary points which are to be filtered out as non-vehicle tracklets in subsequent steps. Feature points are shown in red and tracklets in green.

3. Traffic lights at the intersection lead to vehicle **stop-and-go** conditions
4. NYC DOT surveillance videos have low resolution (480×640 pixels), frequent illumination changes among frames

1. $\vec{w}$ is a weight vector for each feature in $\vec{f}_{ij} \equiv (x_{ij,\max}, v_{x,ij,\max}, v_{y,ij,\max}, b_{\text{cen},ij,\max})$.
2. $x_{ij,\max} \equiv \max_k |\vec{x}_i(t_k) - \vec{x}_j(t_k)|$ is the maximum **positional separation** along the tracklet
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4. $b_{\text{cen},ij,\max} \equiv \max_k |\vec{b}_{\text{cen},i}(t_k) - \vec{b}_{\text{cen},j}(t_k)|$ is the maximum **separation of the center of mass of the blob labels**

separated by more than a distance d.
3. **Spectral embedding** and **Dirichlet Processs Gaussian Mixture Model** (DPGMM) to identify the number of clusters automatically and label each tracklet.

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Challenges with Object Tracking

- The shape and appearance of the object may change if the motion is not just a shift
  - Search for the parameters of possible object motion to minimize the intensity difference after motion compensation in the object region. (See following for global motion estimation)
- Different parts of the object may move differently
- Some parts may disappear, new parts may appear (occlusion issues)
- Many advanced algorithms have been developed to solve these challenges (outside the scope of this lecture)
- Good references:
  - Yilmaz, Alper, Omar Javed, and Mubarak Shah. "Object tracking: A survey." *Acm computing surveys (CSUR)* 38.4 (2006): 13. http://7xq232.com1.z0.glb.clouddn.com/talk/2013.12.20-Student.Workshop.pdf
  - Wu, Yi, Jongwoo Lim, and Ming-Hsuan Yang. "Online object tracking: A benchmark." *Proceedings of the IEEE conference on computer vision and pattern recognition*. 2013. http://www.cv-foundation.org/openaccess/content_cvpr_2013/papers/Wu_Online_Object_Tracking_2013_CVPR_paper.pdf

Deep Learning for Object Tracking

- Many possible approaches
- Held, David, Sebastian Thrun, and Silvio Savarese. "Learning to track at 100 fps with deep regression networks." *European Conference on Computer Vision*. Springer, Cham, 2016. https://arxiv.org/pdf/1604.01802, http://davheld.github.io/GOTURN/GOTURN.html
- Bertinetto, Luca, Jack Valmadre, Joao F. Henriques, Andrea Vedaldi, and Philip HS Torr. "Fully-convolutional siamese networks for object tracking." In *European conference on computer vision*, pp. 850-865. Springer, Cham, 2016. https://arxiv.org/pdf/1606.09549.pdf

- Work at NYU Video Lab
- Not required

GOTURN

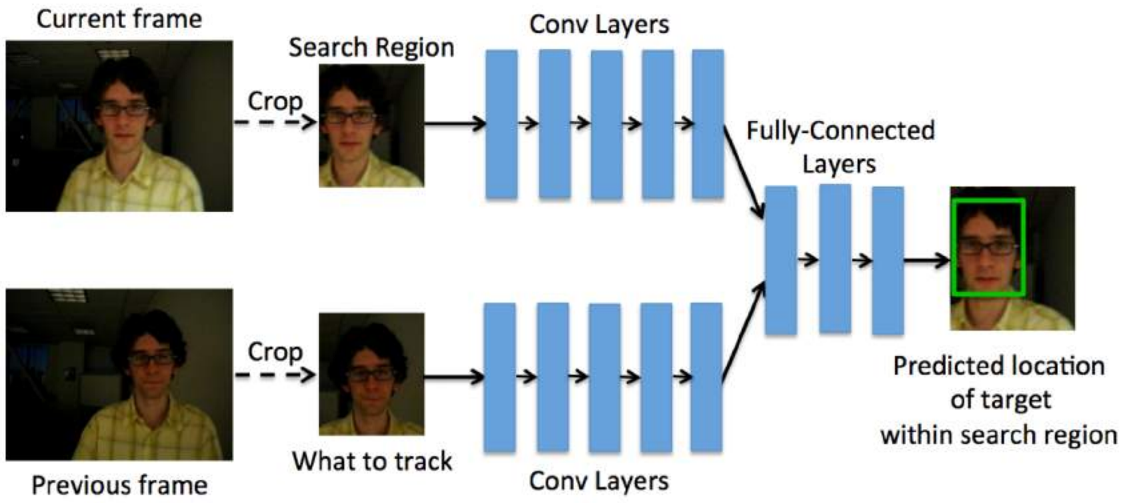
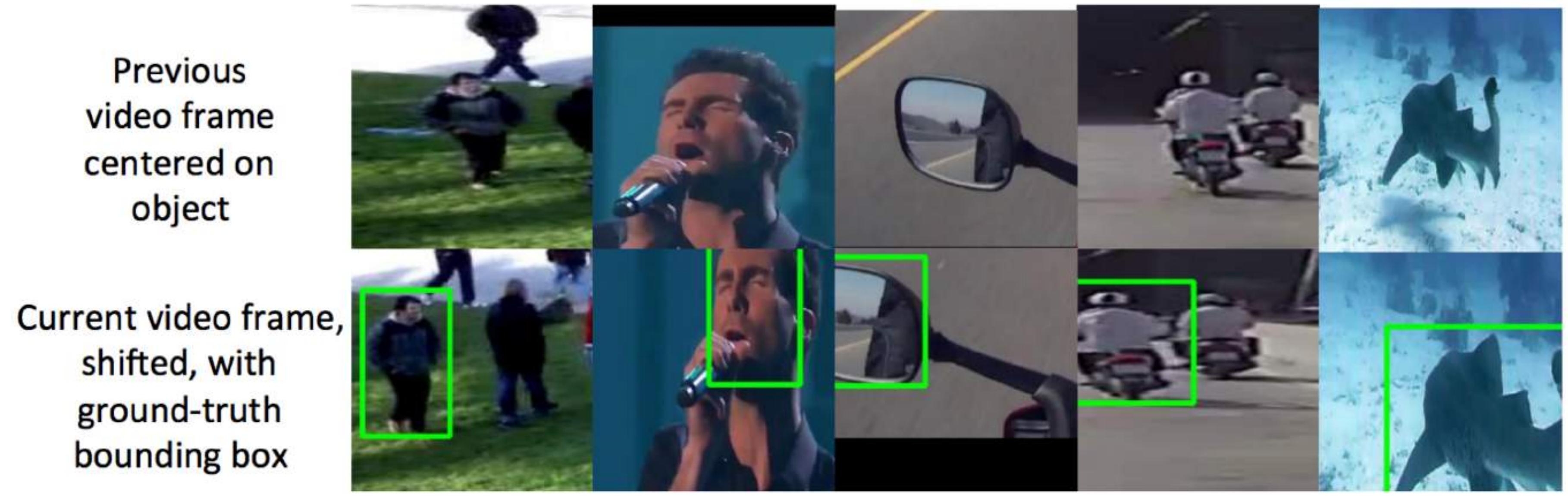


Fig. 2. Our network architecture for tracking. We input to the network a search region from the current frame and a target from the previous frame. The network learns to compare these crops to find the target object in the current image

Held, David, Sebastian Thrun, and Silvio Savarese. "Learning to track at 100 fps with deep regression networks." *European Conference on Computer Vision*. Springer, Cham, 2016. https://arxiv.org/pdf/1604.01802,

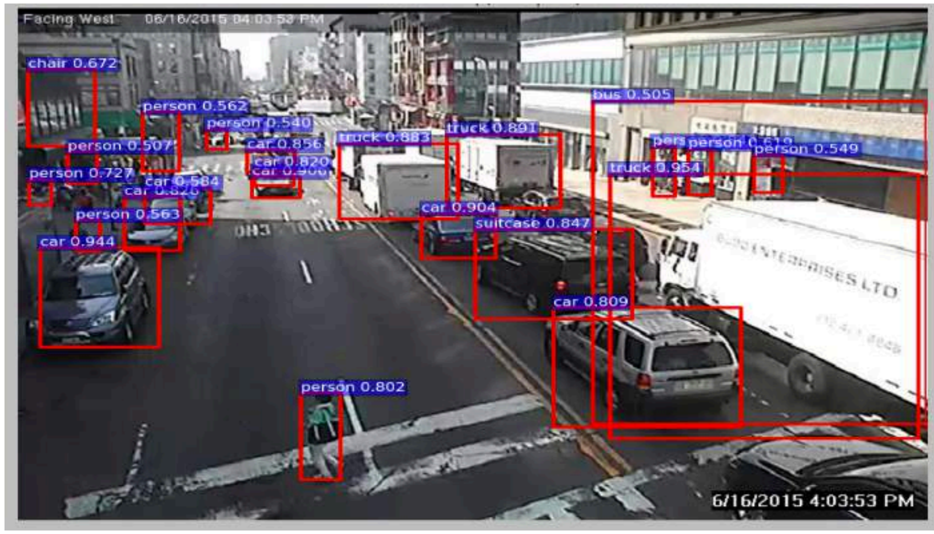

Previous video frame centered on object
Current video frame, shifted, with ground-truth bounding box

Held, David, Sebastian Thrun, and Silvio Savarese. "Learning to track at 100 fps with deep regression networks." *European Conference on Computer Vision*. Springer, Cham, 2016. https://arxiv.org/pdf/1604.01802,

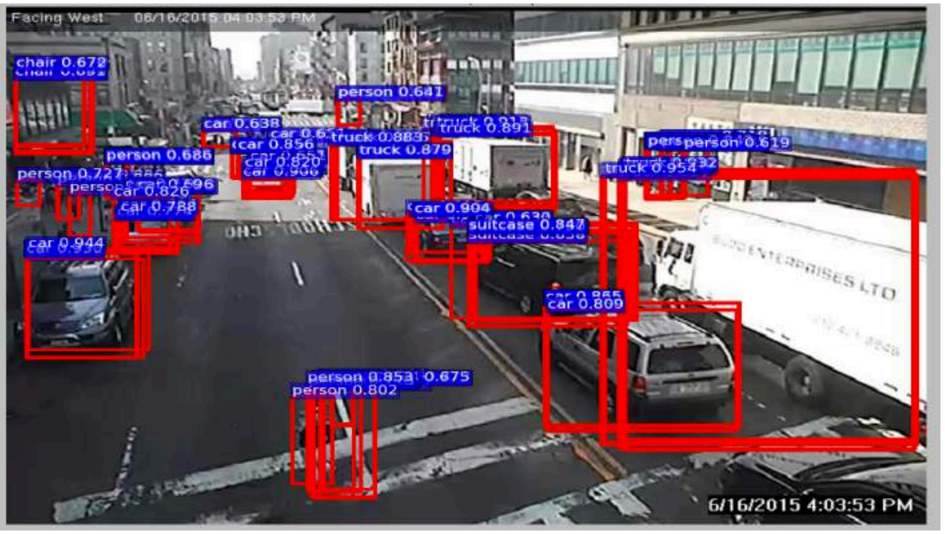
Multiple Object Tracking

- Detect and track multiple objects
- Conventional approach
  - Detect individual objects in each frame, then associate corresponding objects (Detect and then track)
- Work at NYU video lab
  - Detect a “tube” in a video segment that contains the object in successive frames
  - Chenge Li, Gregory Dobler, Xin Feng, Yao Wang “TrackNet: Simultaneous Object Detection and Tracking and Its Application in Traffic Video Analysis“.

Tracking by Detection in Individual Frames (Conventional Approach)

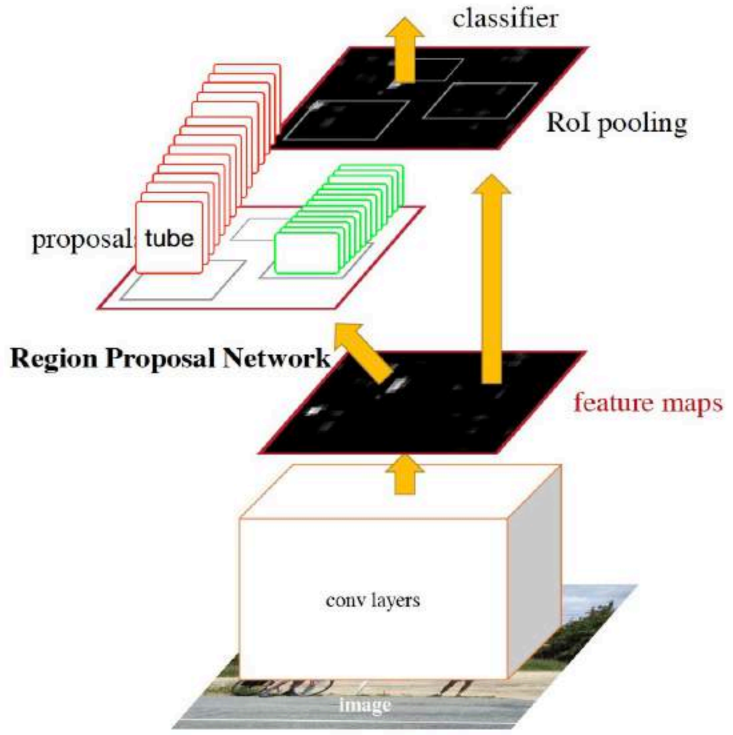


Single frame object detection
(results of faster-region-CNN on one frame)

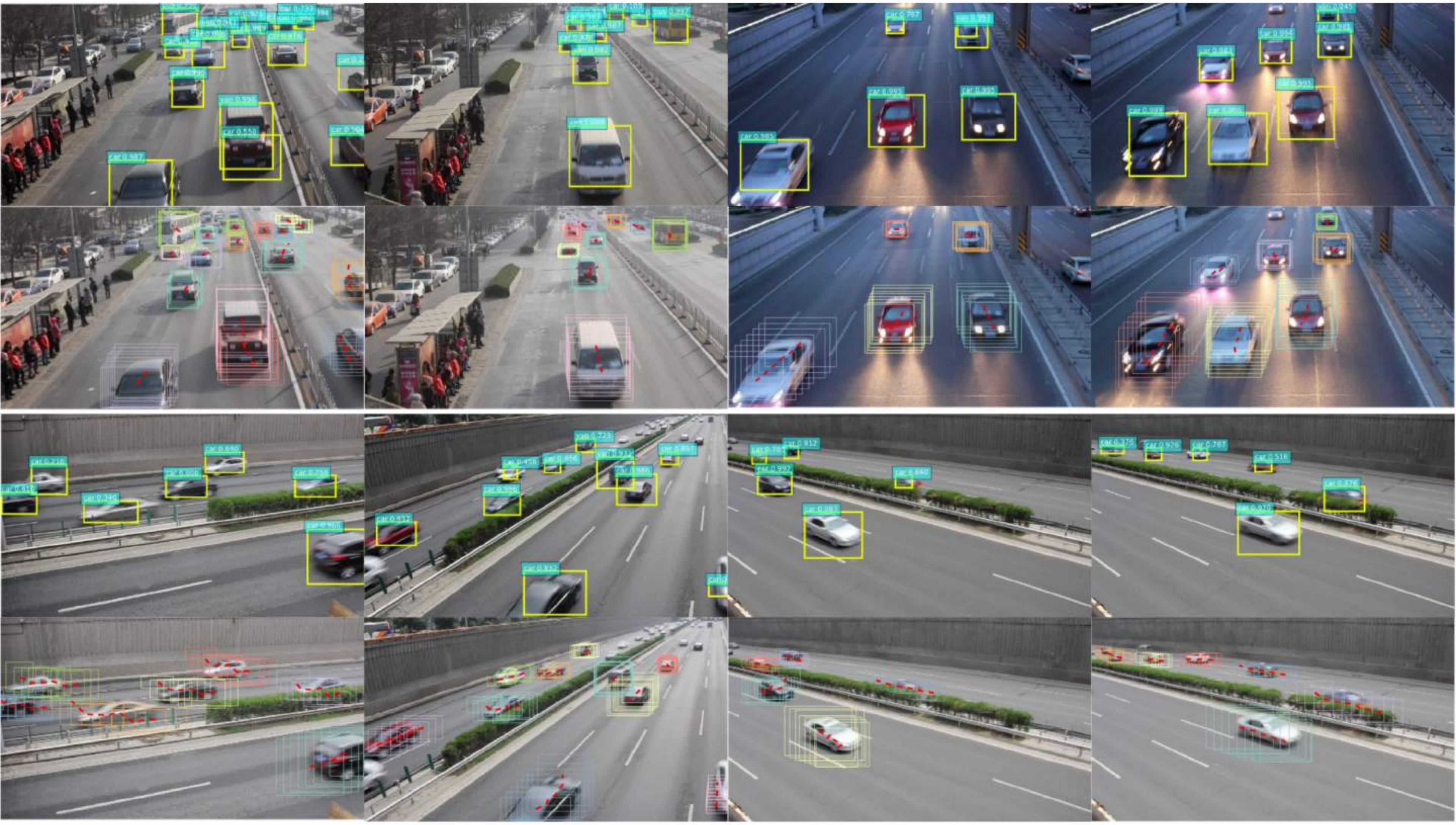


Multiple frame object detection
(results of applying faster-region-CNN on each frame. Need to determine which boxes in different frames correspond to the same object)

Extending Region-CNN For Moving Object Detection in Video

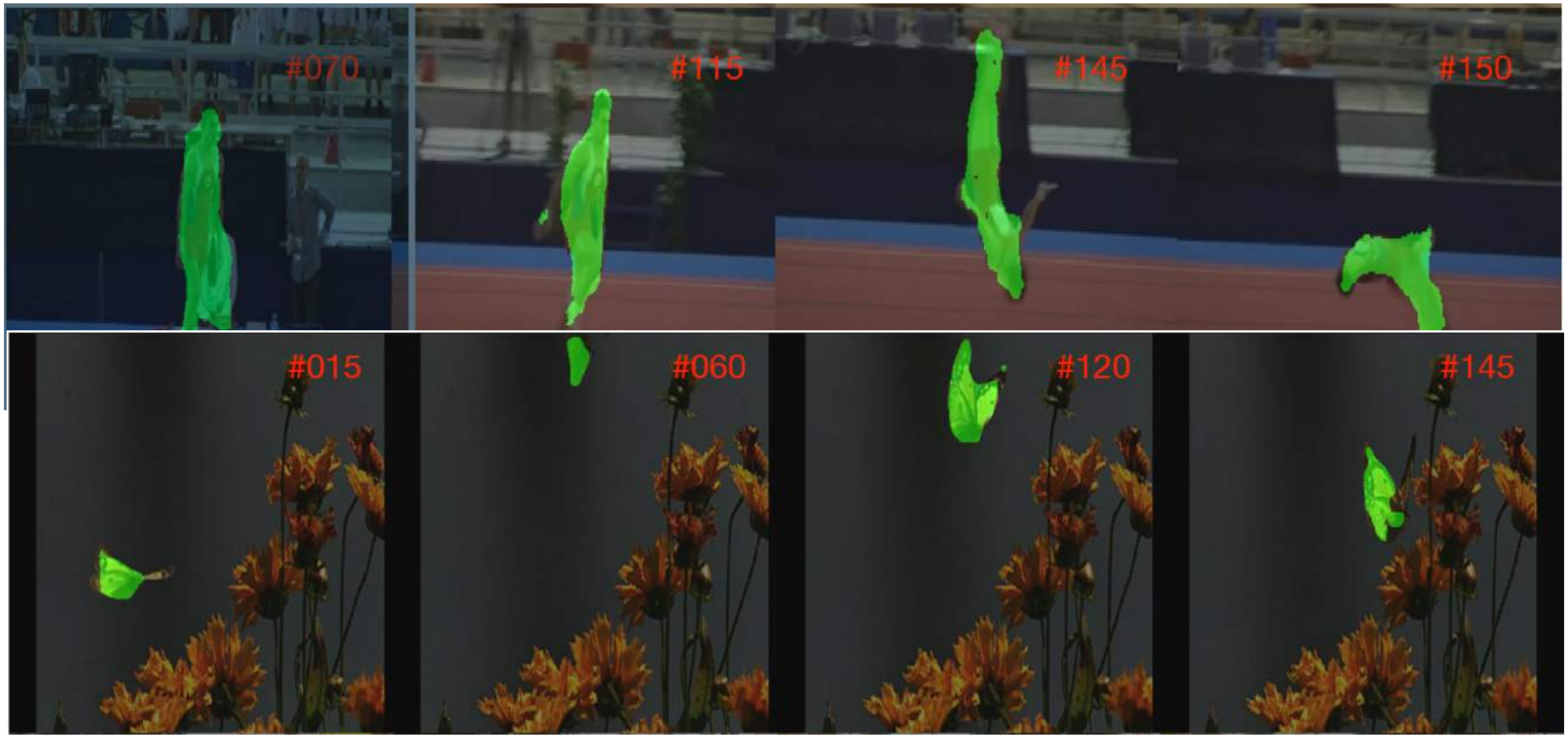


- Consider a video segment consisting of multiple frames
- Detecting a tube bounding each moving object
- Use 3D and 2D convolution for feature extraction (C3D and VGG)
- Generate object proposals (bounding tubes of various sizes and orientations)
- Refine proposals and classify each detected tube (car, van, bus, pedestrian, …)



Visual Results

Pixel-Wise Object Tracking



Proposed Framework

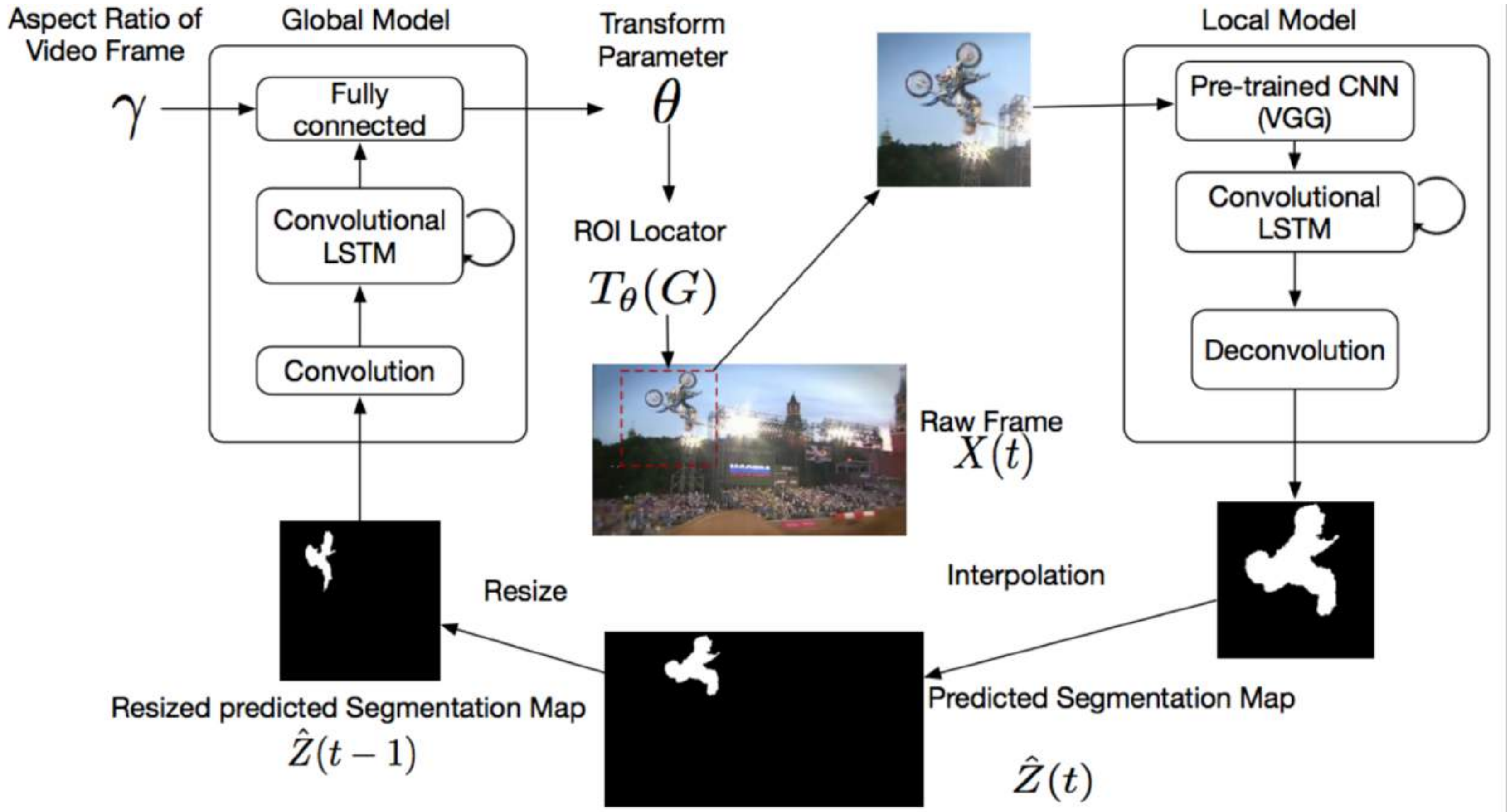
- **Two stage tracking:**

- The global model predicts a region of interests (RoI) in the new frame based on the segmentation masks of past frames.
  - Employs a convolutional LSTM structure to generate the latent feature characterizing the object motion.

- The local model segments the RoI to identify pixels belonging to the object.
  - Also uses a convolutional LSTM structure whose memory state evolves with object appearance.

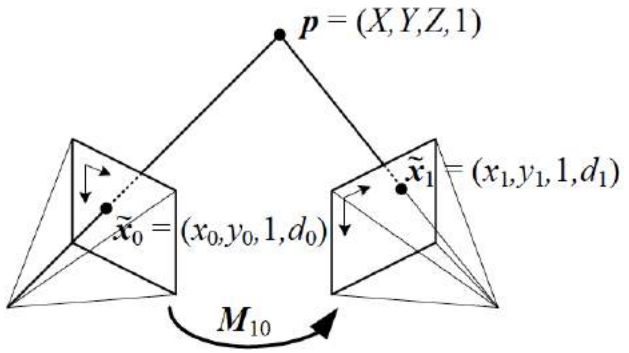
- The two-stage framework is robust to significant appearance shift, occlusion, and large motion and varying object sizes.

- Yilin Song, Chenge Li, Yao Wang "Pixel-wise object tracking", Initial version: Nov. 2017, Last updated: July 2018.


Aspect Ratio of Video Frame
γ
Global Model
Fully connected
Convolutional LSTM
Convolution
Transform Parameter
θ
ROI Locator
$T_\theta(G)$
Raw Frame
$X(t)$
Local Model
Pre-trained CNN (VGG)
Convolutional LSTM
Deconvolution
Interpolation
Resize
Resized predicted Segmentation Map
$\hat{Z}(t-1)$
Predicted Segmentation Map
$\hat{Z}(t)$

Camera Motion Model: How are two images related?

- If two images F and G are taken of the same scene from different view points, they are related by a geometric mapping or transformation



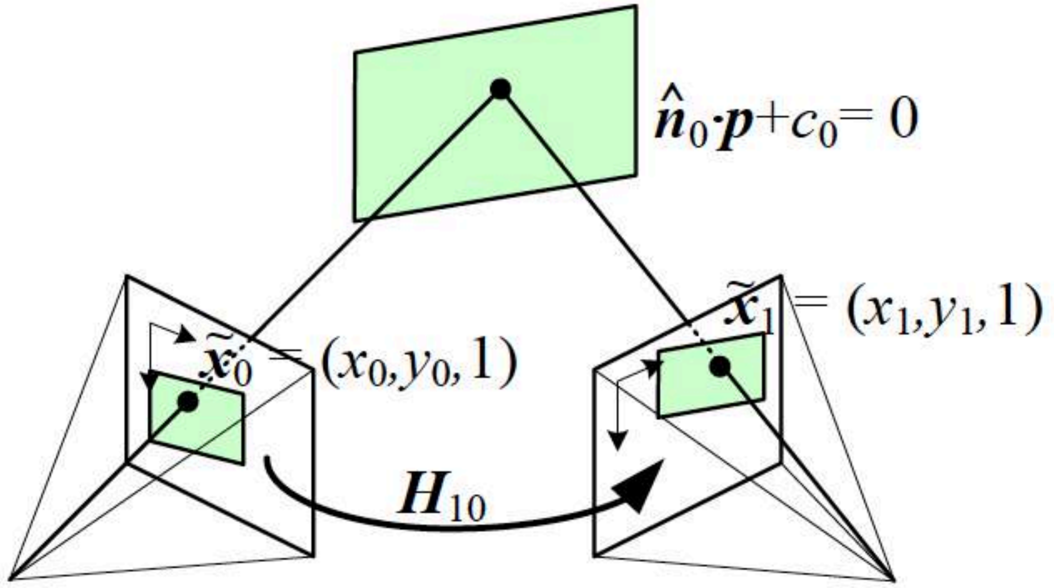
$p = (X, Y, Z, 1)$
$\tilde{x}_1 = (x_1, y_1, 1, d_1)$
$\tilde{x}_0 = (x_0, y_0, 1, d_0)$
M_{10}

$\mathbf{x}_0$ in F corresponds to $\mathbf{x}_1$ in G

Mapping function:
$\mathbf{x}_1 = \mathbf{h}(\mathbf{x}_0)$
or
$x_1 = h_x(x_0, y_0)$, $y_1 = h_y(x_0, y_0)$

- What determines the mapping function?
  - Need to know camera 3D->2D projection geometry
  - Need to know how to model camera motion

Planar Homography: Mapping for Points on the Same Plane (from Previous Lecture)


$\hat{n}_0 \cdot p + c_0 = 0$
$\tilde{x}_0 = (x_0, y_0, 1)$
$\tilde{x}_1 = (x_1, y_1, 1)$
H_{10}

Let the plane be represented by Z=aX+bY+c, previous general relation becomes

$$x_1 = \frac{a_0 + a_1 x_0 + a_2 y_0}{c_0 + c_1 x_0 + c_2 y_0}, \quad y_1 = \frac{b_0 + b_1 x_0 + b_2 y_0}{c_0 + c_1 x_0 + c_2 y_0}$$

The parameters depend on plane parameters (a,b,c) and camera parameters (F, R, T)

Note there is inherent scale ambiguity (unless the 3D position of a point is known!). Often we set $c_0 = 1$

Using homogeneous coordinate:

$$\begin{bmatrix} x_1 \\ y_1 \\ w \end{bmatrix} = \begin{bmatrix} a_1 & a_2 & a_0 \\ b_1 & b_2 & b_0 \\ c_1 & c_2 & c_0 \end{bmatrix} \begin{bmatrix} x_0 \\ y_0 \\ 1 \end{bmatrix}$$

$$\tilde{\mathbf{x}}_1 = \mathbf{H}\tilde{\mathbf{x}}_0$$

Approximation of Projective Mapping by Affine and Bilinear Model

- Affine (6 parameters):

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} a_0 + a_1 u + a_2 v \\ b_0 + b_1 u + b_2 v \end{bmatrix}$$

$$\begin{bmatrix} \tilde{x} \\ \tilde{y} \\ w \end{bmatrix} = \begin{bmatrix} a_1 & a_2 & a_0 \\ b_1 & b_2 & b_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u \\ v \\ 1 \end{bmatrix}$$

  - Affine model sufficiently capture mapping due to in-plane camera motion (scaling, roll and translation in x,y only)
  - Also known as affine homography

- Bilinear (8 parameters):

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} a_0 + a_1 u + a_2 v + a_3 uv \\ b_0 + b_1 u + b_2 v + b_3 uv \end{bmatrix}$$

Camera Motion Estimation

- Feature-based vs. intensity-based
  - Feature-based: First determine some corresponding feature points (using feature detector and descriptor) in both images, then try to fit the correspondences into a chosen mapping model (covered previously)
    - Least squares
    - Robust fitting: RANSAC
  - Intensity-based: Directly determine the motion field (or motion parameters) so that the intensities of corresponding pixels match (focus in this lecture)
- Direct vs. indirect estimation under intensity-based approach
  - Direct: Directly finding the motion parameters
  - Indirect: First find dense motion field, then fit the motion field to a chosen motion model
- Robust estimator
  - Using squared error (L2 norm) make the resulting estimate sensitive to outliers (pixels which do not follow the camera motion, such as those corresponding to moving objects)
  - Use L1 or L0 norm to minimize the impact of outliers

Direct Estimation

- Parameterize the DFD error in terms of the motion parameters, and estimate these parameters by minimizing the DFD error

$$E_{\mathrm{DFD}} = \sum_{n \in \mathcal{N}} w_n |\psi_2(\mathbf{x}_n + \mathbf{d}(\mathbf{x}_n; \mathbf{a})) - \psi_1(\mathbf{x}_n)|^p$$

Weighting w_n coefficients depend on the importance of pixel $\mathbf{x}_n$.

Ex: Affine motion:

$$\begin{bmatrix} d_x(\mathbf{x}_n;\mathbf{a}) \\ d_y(\mathbf{x}_n;\mathbf{a}) \end{bmatrix} = \begin{bmatrix} a_0 + a_1 x_n + a_2 y_n \\ b_0 + b_1 x_n + b_2 y_n \end{bmatrix}, \quad \mathbf{a} = [a_0, a_1, a_2, b_0, b_1, b_2]^T$$

Exhaustive search or gradient descent method can be used to find **a** that minimizes E_{DFD}

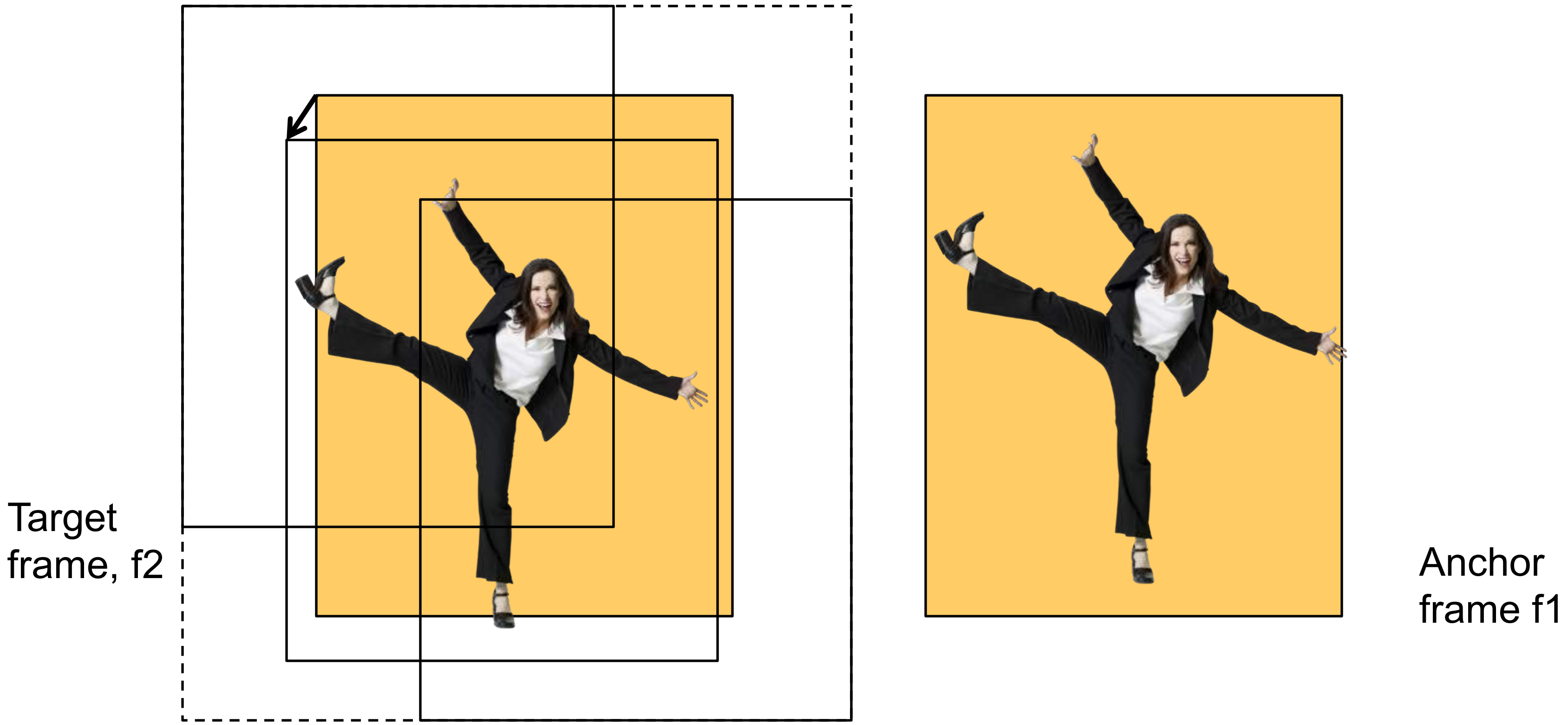
When the motion is small, can also minimize the error to the optical flow equation at each pixel, E_{OF}

Global Translation Estimation

- Simple global motion: Every pixel is moved by the same amount due to camera in-plane shift (global translation)
- How to find the global translation?
- Exhaustive search: Applying EBMA to the entire frame
  - Find the shift between the anchor frame and the target frame so that the matching error is minimal
  - Integer or fractional pel search
  - Matching error should be calculated over overlapping pixels only, and the error should be normalized by the number of pixels in the overlapping area (average error / pixel)
- When the global translation is known to be small, the shift can be determined by solving an equation derived from optical flow constraint

Estimating Global Shift

- Think of the whole anchor frame f1 as a block
- Find the match of f1 in the target frame f2 by evaluating matching error with all possible shifts
- =EBMA using the whole anchor frame as the template!



Sample MATLAB Code

```
function [dx,dy]=GlobalMVEstimation1(f2, f1,Rx, Ry)
%finding global MV of f2 with respect to f1
%Rx and Ry are search range (maximum possible absolute shift)

[H,W]=size(f);[Hr,Wr]=size(fr);Maxerror=255; dx=0;dy=0;
for (k=-Ry:Ry, l=-Rx:Rx) %try all possible shifts
        error=0;count=0;
        for (m1=1:Hr,n1=1:Wr)
                m2=m1+k;n2=n1+l;
                if ((m2>0) & (m2 <=H) &( n2>0) &( n2<=W))
                        count+=1;
                        error += abs(f1(m1,n1)-f2(m2,n2);
                end
        end
        error=error/count;
        if (error<maxerror)
                dy=k,dx=l,maxerror=error;
        end
end
%This script is not very efficient. How do you improve it by not looping through m1,n1 and
checking "if ..."
```

Alternate Faster Implementation

```
function [dx,dy]=GlobalMVEstimation2(f2, f1,Rx, Ry)
%finding global MV of f2 with respect to f1
%Rx and Ry are search range (maximum possible absolute shift)

[H,W]=size(f2);[Hr,Wr]=size(f1);Maxerror=255; dx=0;dy=0;
for (k=-Ry:Ry, l=-Rx:Rx) %try all possible shifts
          error=0;count=0;
          mb=…, me=…, nb=…, ne=…
          count=(me-mb+1)*(nb-ne+1);
%mb,me,nb,ne should be determined so that mb>=1 & mb+k>=1, similarly me<=Hr &
me+k<=H. Similarly for nb,ne
          error=sum(sum(abs(f1(mb:me,nb:ne)-f2(mb+k:me+k,nb+l:be+l))))/count;
          if (error<maxerror)
                    dy=k,dx=l,maxerror=error;
          end
end

%Hint:  To satisfy mb>=1 & mb+k>=1, we can set mb=max(1,1-k)
```

Global Affine Transformation

- Affine mapping is a good approximation of the global motion due to camera motion, especially for far-away view
- Global Affine Transformation (6 parameters)

$$\psi_1(x,y)=\psi_2(x+d_x(x,y),y+d_y(x,y))=\psi_2(x+\mathbf{A}(x,y)\mathbf{a},y+\mathbf{A}(x,y)\mathbf{b})$$

$$\begin{bmatrix} d_x(x,y) \\ d_y(x,y) \end{bmatrix}=\begin{bmatrix} a_0+a_1x+a_2y \\ b_0+b_1x+b_2y \end{bmatrix}=\begin{bmatrix} \mathbf{A}(x,y) & 0 \\ 0 & \mathbf{A}(x,y) \end{bmatrix}\begin{bmatrix} \mathbf{a} \\ \mathbf{b} \end{bmatrix}$$

$$\mathbf{A}(x,y)=\begin{bmatrix} 1 & x & y \end{bmatrix},\mathbf{a}=\begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix},\mathbf{b}=\begin{bmatrix} b_0 \\ b_1 \\ b_2 \end{bmatrix}$$

- Special cases:
  - Translation only: $a_0 = x\text{-direction shift}, a_1=0, a_2=0$

    $\mathrm{b}_0 = y\text{-direction shift}, a_1=0, a_2=0$

Direct Estimation of Affine Motion: Minimizing DFD Error

- Parameterize the DFD error in terms of the motion parameters, and estimate these parameters by minimizing the DFD error

$$E_{\text{DFD}} = \sum_{n \in \mathcal{N}} w_n |\psi_2(\mathbf{x}_n + \mathbf{d}(\mathbf{x}_n; \mathbf{a})) - \psi_1(\mathbf{x}_n)|^p$$

Weighting w_n coefficients depend on the importance of pixel $\mathbf{x}_n$.

Ex: Affine motion:
$$\begin{bmatrix} d_x(\mathbf{x}_n;\mathbf{a}) \\ d_y(\mathbf{x}_n;\mathbf{a}) \end{bmatrix} = \begin{bmatrix} a_0 + a_1 x_n + a_2 y_n \\ b_0 + b_1 x_n + b_2 y_n \end{bmatrix} = \mathbf{A}(\mathbf{x}_n)\mathbf{a}$$

$$\mathbf{A}(\mathbf{x}_n) = \begin{bmatrix} 1 & x_n & y_n & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & x_n & y_n \end{bmatrix}$$

$$\mathbf{a} = [a_0, a_1, a_2, b_0, b_1, b_2]^T$$

Exhaustive search of all 6 parameters is computationally prohibitive!
Using gradient descent method.

Direct Estimation of Affine Motion Using Gradient Descent Method

$$E_{\mathrm{DFD}}(\mathbf{a},\mathbf{b}) = \frac{1}{2}\sum_{\mathbf{x}\in B(\mathbf{x}_n)} \left|\psi_2(x+\mathbf{A}(x,y)\mathbf{a}, y+\mathbf{A}(x,y)\mathbf{b}) - \psi_1(x,y)\right|^2 \to \min$$

$$\mathbf{A}(x,y) = \begin{bmatrix} 1 & x & y \end{bmatrix}, \mathbf{a}^T = \begin{bmatrix} a_0 & a_1 & a_2 \end{bmatrix}, \mathbf{b}^T = \begin{bmatrix} b_0 & b_1 & b_2 \end{bmatrix}$$

B refers to whole frame. Should make frame center has coordinates x=0,y=0

$$\frac{\partial E}{\partial \mathbf{a}} = \begin{bmatrix} \frac{\partial E}{\partial a_0} \\ \frac{\partial E}{\partial a_1} \\ \frac{\partial E}{\partial a_2} \end{bmatrix} = \sum_{(x,y)\in B} e(x,y) \left.\frac{\partial \psi_2}{\partial x}\right|_{(x+\mathbf{A}(x,y)\mathbf{a},y+\mathbf{A}(x,y)\mathbf{b})} \mathbf{A}(x,y)^T = \begin{bmatrix} \sum_{(x,y)\in B} e(x,y) G_x(x+a_0+a_1x+a_2y, y+b_0+b_1x+b_2y) \\ \sum_{(x,y)\in B} e(x,y) G_x(x+a_0+a_1x+a_2y, y+b_0+b_1x+b_2y)x \\ \sum_{(x,y)\in B} e(x,y) G_x(x+a_0+a_1x+a_2y, y+b_0+b_1x+b_2y)y \end{bmatrix}$$

$$\frac{\partial E}{\partial \mathbf{b}} = \begin{bmatrix} \frac{\partial E}{\partial b_0} \\ \frac{\partial E}{\partial b_1} \\ \frac{\partial E}{\partial b_2} \end{bmatrix} = \sum_{(x,y)\in B} e(x,y) \left.\frac{\partial \psi_2}{\partial y}\right|_{(x+\mathbf{A}(x,y)\mathbf{a},y+\mathbf{A}(x,y)\mathbf{b})} \mathbf{A}(x,y)^T = \begin{bmatrix} \sum_{(x,y)\in B} e(x,y) G_y(x+a_0+a_1x+a_2y, y+b_0+b_1x+b_2y) \\ \sum_{(x,y)\in B} e(x,y) G_y(x+a_0+a_1x+a_2y, y+b_0+b_1x+b_2y)x \\ \sum_{(x,y)\in B} e(x,y) G_y(x+a_0+a_1x+a_2y, y+b_0+b_1x+b_2y)y \end{bmatrix}$$

$e(x,y) = \psi_2(x+\mathbf{A}(x,y)\mathbf{a}, y+\mathbf{A}(x,y)\mathbf{b}) - \psi_1(x,y)$: Current prediction error image;

$G_x(x,y) = \frac{\partial \psi_2}{\partial x}(x,y)$: Gradient image in x-direction; $G_y(x,y) = \frac{\partial \psi_2}{\partial y}(x,y)$: Gradient image in y-direction

Implementation Details: Gradient Vector Calculation

First order gradient descent (starting from some initial condition):

At $(l+1)$-th iteration:

$$\mathbf{a}^{(l+1)} = \mathbf{a}^{(l)} - \alpha \left.\frac{\partial E}{\partial \mathbf{a}}\right|_{\mathbf{a}^{(l)},\mathbf{b}^{(l)}}, \mathbf{b}^{(l+1)} = \mathbf{b}^{(l)} - \alpha \left.\frac{\partial E}{\partial \mathbf{b}}\right|_{\mathbf{a}^{(l)},\mathbf{b}^{(l)}}$$

First calculate the gradient images of $\psi_2(x,y)$, to generate original gradient image $G_x(x,y), G_y(x,y)$

Find an initial solution, $\mathbf{a}^{(0)}, \mathbf{b}^{(0)}$.

At end of (l)-th iteration, you have $[a_0, a_1, a_2]^T = \mathbf{a}^{(l)}, [b_0, b_1, b_2]^T = \mathbf{b}^{(l)}$.

Determine the predicted image:

$$\psi_p^{(l)}(x,y) = \psi_2(x + a_0 + a_1 x + a_2 y, y + b_0 + b_1 x + b_2 y) = warp(\psi_2(x,y), a_0, a_1, a_2, b_0, b_1, b_2)$$

Determine prediction error image: $e^{(l)}(x,y) = \psi_p^{(l)}(x,y) - \psi_1(x,y)$

Determine the shifted gradient images:

$$G_x^{(l)}(x,y) = warp(G_x(x,y), a_0, a_1, a_2, b_0, b_1, b_2); \quad G_y^{(l)}(x,y) = warp(G_y(x,y), a_0, a_1, a_2, b_0, b_1, b_2)$$

Compute the gradient vectors, using $\frac{\partial E}{\partial \mathbf{a}} = \begin{bmatrix} \sum_{(x,y)\in B} e^{(l)}(x,y) G_x^{(l)}(x,y) \\ \sum_{(x,y)\in B} e^{(l)}(x,y) G_x^{(l)}(x,y) x \\ \sum_{(x,y)\in B} e^{(l)}(x,y) G_x^{(l)}(x,y) y \end{bmatrix}, \quad \frac{\partial E}{\partial \mathbf{b}} = \begin{bmatrix} \sum_{(x,y)\in B} e^{(l)}(x,y) G_y^{(l)}(x,y) \\ \sum_{(x,y)\in B} e^{(l)}(x,y) G_y^{(l)}(x,y) x \\ \sum_{(x,y)\in B} e^{(l)}(x,y) G_y^{(l)}(x,y) y \end{bmatrix}$

Implementation Details: Gradient Image Calculation

- Simple implementation (using centered difference)

$$G_x(x,y)=(\psi_x(x+1,y)-\psi_x(x-1,y))/2$$
$$G_y(x,y)=(\psi_y(x,y+1)-\psi_y(x,y-1))/2$$

- Convolves with filters that approximate the gradient operation

$$G_x(x,y)=\psi(x,y)*H_x(x,y),\quad G_y(x,y)=\psi(x,y)*H_y(x,y),$$

  - Sobel operator $H_x=\frac{1}{4}\begin{bmatrix}-1 & 0 & 1\\ -2 & 0 & 2\\ -1 & 0 & 1\end{bmatrix}, H_y=\frac{1}{4}\begin{bmatrix}-1 & -2 & -1\\ 0 & 0 & 0\\ 1 & 2 & 1\end{bmatrix}$

  - Derivative of Gaussian filters (see previous lecture notes)

How to determine the initial solution?

- If the anticipated rotation is small, may assume only translation is present. Estimate the translation parameters using the global translation estimation algorithm.

$$a_0 = \text{translation in x}, a_1 = 0, a_2 = 0;$$
$$b_0 = \text{translation in y}, b_1 = 0, b_2 = 0.$$

- If the anticipated translation is also small, can assume

$$a_0 = 0, a_1 = 0, a_2 = 0;$$
$$b_0 = 0, b_1 = 0, b_2 = 0.$$

When to stop the iteration?

- When the energy functional being minimized stop decreases
- Energy function at (l+1) iteration

$$E^{(l+1)} = \sum \left(e^{(l+1)}(x,y)\right)^2$$

- At end of (l+1) iteration, check

$$\frac{\left(E^{(l+1)} - E^{(l)}\right)}{E^{(l)}} < T?$$

Solving Affine Mapping Using Optical Flow Constraint

Optical Flow Equation: $\frac{\partial\psi_2}{\partial x}d_x + \frac{\partial\psi_2}{\partial y}d_y + \psi_2(x,y) - \psi_1(x,y) =>$

$$\frac{\partial\psi_2}{\partial x}\mathbf{A}(x,y)\mathbf{a} + \frac{\partial\psi_2}{\partial y}\mathbf{A}(x,y)\mathbf{b} + \psi_2(x,y) - \psi_1(x,y) = 0$$

We can find **a**,**b** by minimizing the following energy cost:

$$E_{OF}(\mathbf{a},\mathbf{b}) = \frac{1}{2}\sum_{\mathbf{x}\in B}\left|\frac{\partial\psi_2}{\partial x}\mathbf{A}(x,y)\mathbf{a} + \frac{\partial\psi_2}{\partial y}\mathbf{A}(x,y)\mathbf{b} + \psi_2(x,y) - \psi_1(x,y)\right|^2 \to \min$$

Set:

$$\frac{\partial E}{\partial\mathbf{a}} = \sum_{\mathbf{x}\in B}\left(\frac{\partial\psi_2}{\partial x}\mathbf{A}(x,y)\mathbf{a} + \frac{\partial\psi_2}{\partial y}\mathbf{A}(x,y)\mathbf{b} + \psi_2(x,y) - \psi_1(x,y)\right)\frac{\partial\psi_2}{\partial x}\mathbf{A}(x,y)^T$$

$$= \sum_{\mathbf{x}\in B}\left(\frac{\partial\psi_2}{\partial x}\frac{\partial\psi_2}{\partial x}\mathbf{A}(x,y)^T\mathbf{A}(x,y)\mathbf{a} + \frac{\partial\psi_2}{\partial x}\frac{\partial\psi_2}{\partial y}\mathbf{A}(x,y)^T\mathbf{A}(x,y)\mathbf{b} + \left(\psi_2(x,y) - \psi_1(x,y)\right)\frac{\partial\psi_2}{\partial x}\mathbf{A}(x,y)^T\right) = 0$$

$$\frac{\partial E}{\partial\mathbf{b}} = \sum_{\mathbf{x}\in B}\left(\frac{\partial\psi_2}{\partial x}\mathbf{A}(x,y)\mathbf{a} + \frac{\partial\psi_2}{\partial y}\mathbf{A}(x,y)\mathbf{b} + \psi_2(x,y) - \psi_1(x,y)\right)\frac{\partial\psi_2}{\partial y}\mathbf{A}(x,y)^T$$

$$= \sum_{\mathbf{x}\in B}\left(\frac{\partial\psi_2}{\partial x}\frac{\partial\psi_2}{\partial y}\mathbf{A}(x,y)^T\mathbf{A}(x,y)\mathbf{a} + \frac{\partial\psi_2}{\partial y}\frac{\partial\psi_2}{\partial y}\mathbf{A}(x,y)^T\mathbf{A}(x,y)\mathbf{b} + \left(\psi_2(x,y) - \psi_1(x,y)\right)\frac{\partial\psi_2}{\partial y}\mathbf{A}(x,y)^T\right) = 0$$

This leads to a linear equation

$$\begin{bmatrix} \sum_{\mathbf{x}\in B}\left(\frac{\partial\psi_2}{\partial x}\frac{\partial\psi_2}{\partial x}\mathbf{A}(x,y)^T\mathbf{A}(x,y)\right) & \sum_{\mathbf{x}\in B}\left(\frac{\partial\psi_2}{\partial x}\frac{\partial\psi_2}{\partial y}\mathbf{A}(x,y)^T\mathbf{A}(x,y)\right) \\ \sum_{\mathbf{x}\in B}\left(\frac{\partial\psi_2}{\partial x}\frac{\partial\psi_2}{\partial y}\mathbf{A}(x,y)^T\mathbf{A}(x,y)\right) & \sum_{\mathbf{x}\in B}\left(\frac{\partial\psi_2}{\partial y}\frac{\partial\psi_2}{\partial y}\mathbf{A}(x,y)^T\mathbf{A}(x,y)\right) \end{bmatrix}\begin{bmatrix}\mathbf{a}\\ \mathbf{b}\end{bmatrix} = \begin{bmatrix} \sum_{\mathbf{x}\in B}\left(\psi_1(x,y) - \psi_2(x,y)\right)\frac{\partial\psi_2}{\partial x}\mathbf{A}(x,y)^T \\ \sum_{\mathbf{x}\in B}\left(\psi_1(x,y) - \psi_2(x,y)\right)\frac{\partial\psi_2}{\partial y}\mathbf{A}(x,y)^T \end{bmatrix}$$

A Closer Look at the Equation …

$$\begin{bmatrix} \sum_{\mathbf{x}\in B}\left(\frac{\partial\psi_2}{\partial x}\frac{\partial\psi_2}{\partial x}\mathbf{A}(x,y)^T\mathbf{A}(x,y)\right) & \sum_{\mathbf{x}\in B}\left(\frac{\partial\psi_2}{\partial x}\frac{\partial\psi_2}{\partial y}\mathbf{A}(x,y)^T\mathbf{A}(x,y)\right) \\ \sum_{\mathbf{x}\in B}\left(\frac{\partial\psi_2}{\partial x}\frac{\partial\psi_2}{\partial y}\mathbf{A}(x,y)^T\mathbf{A}(x,y)\right) & \sum_{\mathbf{x}\in B}\left(\frac{\partial\psi_2}{\partial y}\frac{\partial\psi_2}{\partial y}\mathbf{A}(x,y)^T\mathbf{A}(x,y)\right) \end{bmatrix}\begin{bmatrix}\mathbf{a}\\ \mathbf{b}\end{bmatrix} = \begin{bmatrix} \sum_{\mathbf{x}\in B}\left(\psi_1(x,y)-\psi_2(x,y)\right)\frac{\partial\psi_2}{\partial x}\mathbf{A}(x,y)^T \\ \sum_{\mathbf{x}\in B}\left(\psi_1(x,y)-\psi_2(x,y)\right)\frac{\partial\psi_2}{\partial y}\mathbf{A}(x,y)^T \end{bmatrix} \Rightarrow \mathbf{S}\begin{bmatrix}\mathbf{a}\\ \mathbf{b}\end{bmatrix} = \mathbf{t}$$

$$\mathbf{B}(x,y) = \mathbf{A}(x,y)^T\mathbf{A}(x,y) = \begin{bmatrix}1\\ x\\ y\end{bmatrix}\begin{bmatrix}1 & x & y\end{bmatrix} = \begin{bmatrix}1 & x & y\\ x & x^2 & xy\\ y & xy & y^2\end{bmatrix}$$

$$\mathbf{S} = \begin{bmatrix} \sum\left(G_x(x,y)\right)^2\mathbf{B}(x,y) & \sum G_x(x,y)G_y(x,y)\mathbf{B}(x,y) \\ \sum G_x(x,y)G_y(x,y)\mathbf{B}(x,y) & \sum\left(G_y(x,y)\right)^2\mathbf{B}(x,y) \end{bmatrix}$$

$$\mathbf{t} = \begin{bmatrix} \sum e(x,y)G_x(x,y) \\ \sum e(x,y)G_x(x,y)x \\ \sum e(x,y)G_x(x,y)y \\ \sum e(x,y)G_y(x,y) \\ \sum e(x,y)G_y(x,y)x \\ \sum e(x,y)G_y(x,y)y \end{bmatrix}; \quad \begin{bmatrix} a_0\\ a_1\\ a_2\\ b_0\\ b_1\\ b_2 \end{bmatrix} = \mathbf{S}^{-1}\mathbf{t}$$

Iterations using the Optical Flow Approach

- The solution is accurate only if the true motion is small.
- When the true motion is not necessarily small, use an iterative approach

At the (l+1)-th iteration:

Update the predicted image, error image, and gradient images using

$$\psi_2^{(l)}=warp(\psi_2,T^{(l)}),e^{(l)}=\psi_1-\psi_2^{(l)},G_x^{(l)}=warp(G_x,T^{(l)}),G_y^{(l)}=warp(G_y,T^{(l)}),$$

Find the new affine mapping T=($\mathbf{a}$,$\mathbf{b}$) using the optical flow based approach, with ψ_2,e,G_x , G_y replaced by $\psi_2^{(l)},e^{(l)},G_x^{(l)}$, $G_y^{(l)}$.

Update the overall mapping function $T^{(l+1)}=T(\mathbf{a},\mathbf{b})\circ T^{(l)}$.

$$T^{(l+1)}: x\rightarrow x'=x+a_0+a_1x+a_2y, y\rightarrow y'=y+b_0+b_1x+b_2y$$

$$T^{(l)}:\quad x'\rightarrow x''=x'+a_0^{(l)}+a_1^{(l)}x'+a_2^{(l)}y'; y'\rightarrow y''=y'+b_0^{(l)}+b_1^{(l)}x'+b_2^{(l)}y';$$

Overall motion:

$$x''=x+a_0+a_1x+a_2y+a_0^{(l)}+a_1^{(l)}(x+a_0+a_1x+a_2y)+a_2^{(l)}(y+b_0+b_1x+b_2y)$$
$$=x+a_0^{(l+1)}+a_1^{(l+1)}x+a_2^{(l+1)}y$$

$$a_0^{(l+1)}=a_0+a_0^{(l)}+a_1^{(l)}a_0+a_2^{(l)}b_0, a_1^{(l+1)}=a_1+a_1^{(l)}(1+a_1)+a_2^{(l)}b_1, a_2^{(l+1)}=a_2+a_1^{(l)}a_2+a_2^{(l)}(1+b_2)$$

$$y''=y+b_0^{(l+1)}+b_1^{(l+1)}x+b_2^{(l+1)}y$$

$$b_0^{(l+1)}=b_0+b_0^{(l)}+b_1^{(l)}a_0+b_2^{(l)}b_0, b_1^{(l+1)}=b_1+b_1^{(l)}(1+a_1)+b_2^{(l)}b_1, b_2^{(l+1)}=b_2+b_1^{(l)}a_2+b_2^{(l)}(1+b_2)$$

Indirect Estimation of Global Motion

- First find the dense motion field using pixel-based or block-based approach (e.g. EBMA), or find motion vectors at selected feature points, resulting in a sequence of data pairs pairs

$$(x_n, y_n),(d_{x,n}, d_{y,n}), n = 1,2,\ldots,N$$

- Then finding the motion model parameters to satisfy the equations:

$$d_x(x_n, y_n; \mathbf{a}) = d_{x,n}, d_y(x_n, y_n; \mathbf{b}) = d_{y,n}, n = 1,2,\ldots,N$$

- The parameters for dx and dy can be solved separately by fitting the models for dx and dy separately.

Least Squares Fitting for Affine Model

Fitting error for dx: $E_{fit,x} = \sum w_n (d_x(\mathbf{x}_n;\mathbf{a}) - d_{x,n})^2.$

Affine motion: $d_x(\mathbf{x}_n;\mathbf{a}) = [\mathbf{A}_n]\mathbf{a},$

$$[\mathbf{A}_n] = \begin{bmatrix} 1 & x_n & y_n \end{bmatrix}, \mathbf{a} = \begin{bmatrix} a_0 & a_1 & a_2 \end{bmatrix}^T$$

$$\frac{\partial E_{fit}}{\partial \mathbf{a}} = \sum w_n [\mathbf{A}_n]^T ([\mathbf{A}_n]\mathbf{a} - d_{x,n}) = 0 \rightarrow \mathbf{Q}\mathbf{a} = \mathbf{r}_x \rightarrow \mathbf{a} = \mathbf{Q}^{-1}\mathbf{r}_x$$

$$[\mathbf{A}_n]^T[\mathbf{A}_n] = \begin{bmatrix} 1 & x_n & y_n \\ x_n & x_n^2 & x_n y_n \\ y_n & x_n y_n & y_n^2 \end{bmatrix}, [\mathbf{A}_n]^T d_{x,n} = \begin{bmatrix} d_{x,n} \\ x_n d_{x,n} \\ y_n d_{x,n} \end{bmatrix}$$

$$\mathbf{Q} = \sum w_n [\mathbf{A}_n]^T[\mathbf{A}_n] = \begin{bmatrix} \sum w_n & \sum w_n x_n & \sum w_n y_n \\ \sum w_n x_n & \sum w_n x_n^2 & \sum w_n x_n y_n \\ \sum w_n y_n & \sum w_n x_n y_n & \sum w_n y_n^2 \end{bmatrix}, \quad \mathbf{r}_x = \sum w_n [\mathbf{A}_n]^T d_{x,n} = \begin{bmatrix} \sum w_n d_{x,n} \\ \sum w_n x_n d_{x,n} \\ \sum w_n y_n d_{x,n} \end{bmatrix}$$

Similarly,

$$\mathbf{b} = \mathbf{Q}^{-1}\mathbf{r}_y, \quad \mathbf{r}_y = \sum w_n [\mathbf{A}_n]^T d_{y,n} = \begin{bmatrix} \sum w_n d_{y,n} \\ \sum w_n x_n d_{y,n} \\ \sum w_n y_n d_{y,n} \end{bmatrix}$$

- Weighting w_n coefficients depend on the accuracy of estimated motion at $\mathbf{x}_n$.

Problem with Least Squares Fitting

- Estimated motions at some pixels may be grossly wrong (outliers)
- Outliers can significantly impact the global motion estimation results when using square error, leading to errors in object detection as well
- Alternatives:
  - Instead of minimizing the square error in terms of DFD, OF or motion parameter fitting error, minimize the L0 norm (very hard)
  - Convex relaxation: Minimize L1 norm instead.
    - Computationally more demanding than minimizing L2 error, but solvable.
  - RANSAC method if based on feature correspondence

How to determine the parameters of Homography given a dense motion field?

- Previously described for the case using a set of corresponding feature points.
  - Feature correspondence
  - Least squares or RANSAC
- Same approach can be applied to all pixels
  - RANSAC may be not practical

Pop Quiz

- What is the difference between direct and indirect methods?
- What are the two ways to set up your intensity-based objective function?
- What is the pros and cons using the optical flow equation to set up your optimization criterion?
- How would you estimate the global translation based on the optical flow equation?
- How would you estimate the homography parameters between two frames using the indirect method?

Camera Motion Estimation: Applications

- To enable object detection under camera motion
- To enable registration of two frames under different camera views and stitching
- To form a complete background image from a video sequence captured by a moving camera

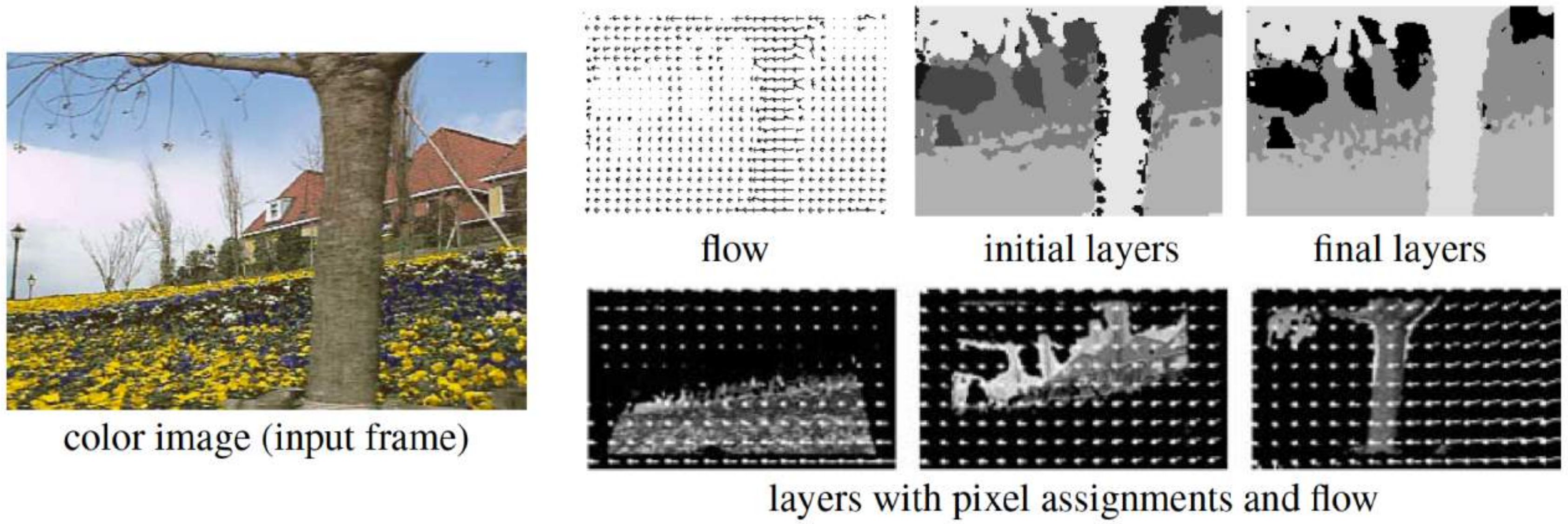
Moving Object Detection under Camera Motion

- Moving objects induce different motion than the camera motion
- Because moving objects are typically small in a frame, the pixels corresponding to these objects can be considered as “outliers”
- First determine camera motion by minimizing the error over all pixels (or feature points), then detect pixels with large error (in intensity or position)
- Moving objects are the outliers!

Region-Based Motion Estimation

- Assumption: the scene consists of multiple objects, with the region corresponding to each object (or sub-object) having a coherent motion
  - Physically more correct than block-based, mesh-based, global motion model
- Method:
  - Region First: Segment the frame into multiple regions based on texture/edges, then estimate motion in each region using the global motion estimation method
  - Motion First: Estimate a dense motion field, then segment the motion field so that motion in each region can be accurately modeled by a single set of parameters
    - This can be done by clustering: partition all all pixels into different groups based on their motion similarity, using e.g. K-means algorithm
  - Joint region-segmentation and motion estimation: iterate the two processes

Layered Motion Estimation


color image (input frame)
flow
initial layers
final layers
layers with pixel assignments and flow

'igure 8.15 Layered motion estimation results (Wang and Adelson 1994) © 1994 IEEE.

Wang, J. Y. A. and Adelson, E. H. (1994). Representing moving images with layers. IEEE Transactions on Image Processing , 3(5):625–638.

See [Szeliski2010] Sec. 8.5.

Video Shot Boundary Detection (or Scene Change Detection)

- A video often contains different shots, each has a coherent scene
- How to divide a video into separate shots or detect scene change?
  - An important first step for video analysis
- Simple approach:
  - Frame difference: if sum of DFD is large, there is a scene change
  - Sensitive to changes due to camera motion, object motion, illumination variation
  - Does not work well in gradual transitions
- More advanced approaches
  - Based on difference in color histogram, entropy of color distribution
  - Machine learning based approach

TRECVID Competition

- TRECVID (TREC Video Retrieval Evaluation) is sponsored by NIST to encourage research in digital video indexing and retrieval. It has focused on different video analysis tasks. Shot boundary detection was one
  - http://trecvid.nist.gov/

- Smeaton, Alan F., Paul Over, and Aiden R. Doherty. "Video shot boundary detection: Seven years of TRECVid activity." *Computer Vision and Image Understanding* 114.4 (2010): 411-418. http://doras.dcu.ie/4080/1/sbretro.pdf. Contain results up to 2005
- Liu, Z., Gibbon, D., Zavesky, E., Shahraray, B., & Haffner, P. (2007, November). AT&T research at trecvid 2006. In *Proc. TRECVID Workshop* (pp. 19-26). (best in TRECVid2006 SBD Competition) https://www.researchgate.net/profile/Behzad_Shahraray/publication/224718827_A_Fast_Comprehensive_Shot_Boundary_Determination_System/links/02e7e51a8b6cea0546000000.pdf

Video Stabilization

- A video may be unstable due to unwanted camera motion
- Especially prevalent in home video captured by hand-held cameras with a "shaky hand"
- Also prevalent in aerial surveillance video
- Goal: remove the motion due to unwanted camera motion, so that the video plays smoothly
- Demo
  - http://www.sri.com/newsroom/video/acadia-real-time-video-stabilization-demo

General Approach

- Estimate camera motion between every two adjacent frames
  - Assuming a global translation is often sufficient
  - More generally using affine to account for tilt or homography to account for out of plane rotation
  - Can use either feature-based on intensity-based approaches
- Smooth motion parameters in time (to remove shaking, but keep the smooth camera motion)
- Warping each frame so that it undergoes the smoothed motion between frames
  - Remove undesired global motion due to hand shaking
- Filling missing pixels (on the border) in each frame (image completion)

Global Motion Smoothing

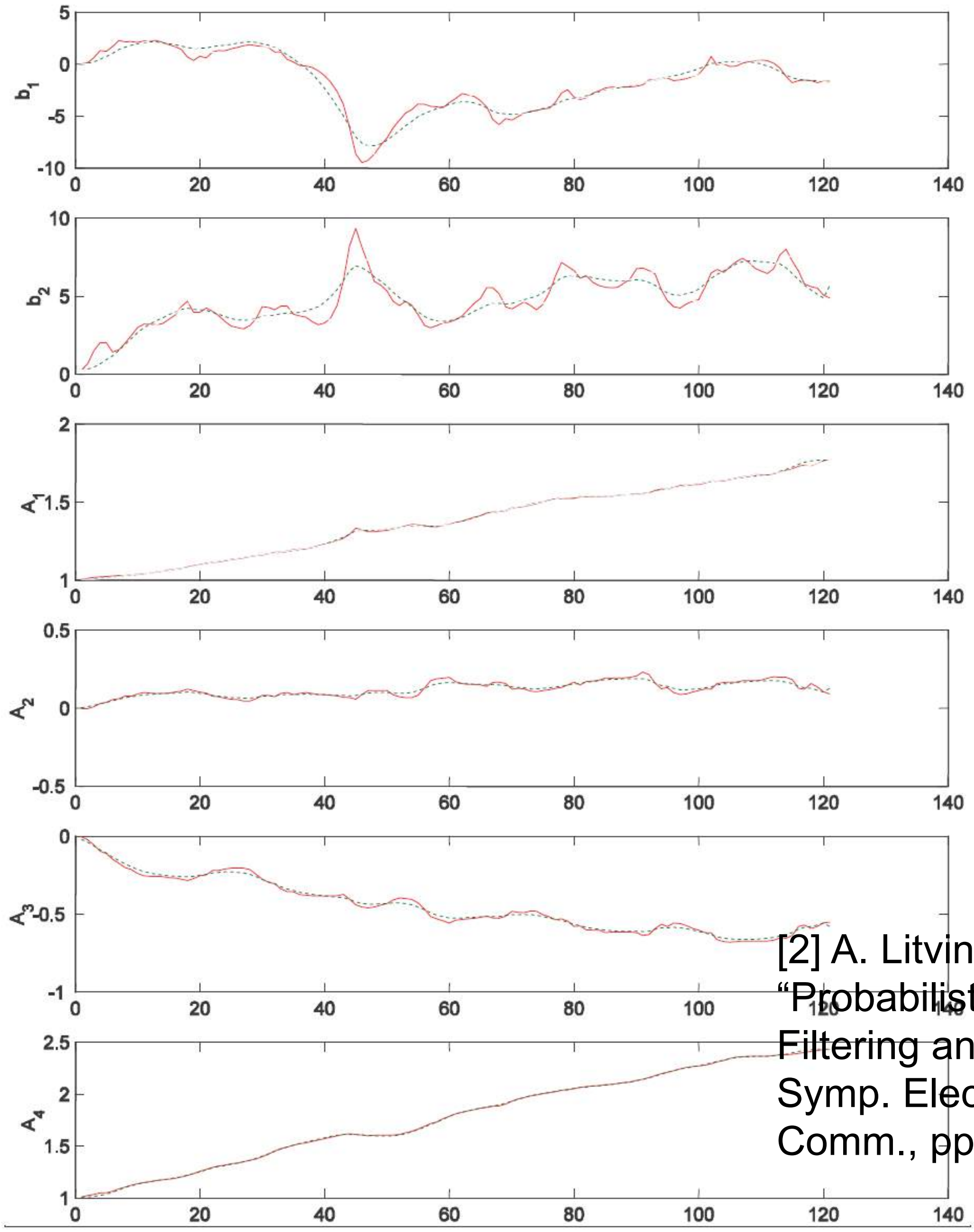


Figure 5: Red straight lines – Inter-frame motion parameters obtained for sequence A; Green dashed lines - intentional cumulative transform parameters estimated using smoothing Kalman filtering

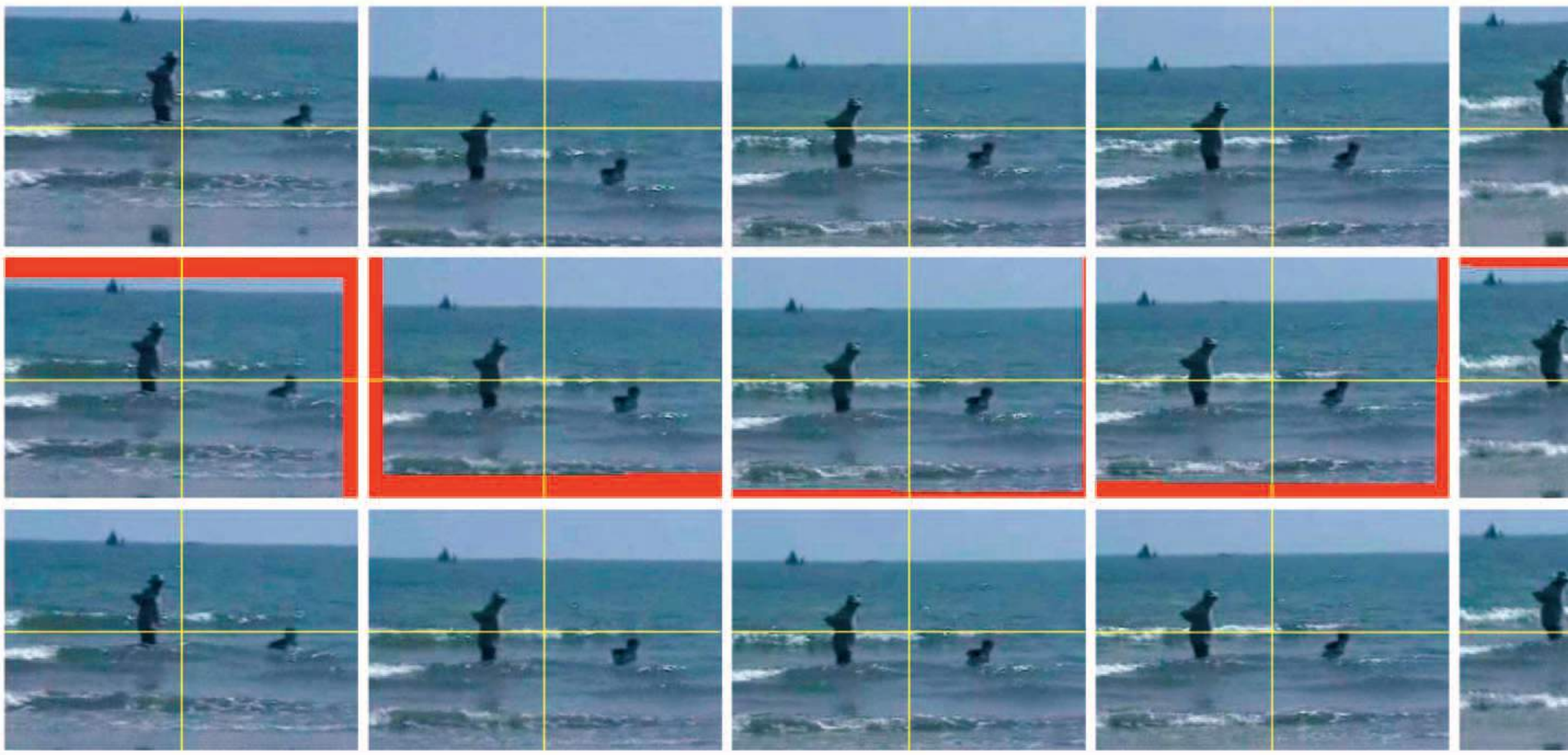
Kalman filtering approach: Separating observed motion parameters into intentional and unwanted motion [Ref 2]

[2] A. Litvin, J. Konrad, and W. Karl, "Probabilistic Video Stabilization Using Kalman Filtering and Mosaicking," Proc. IS&T/SPIE Symp. Electronic Imaging, Image, and Video Comm., pp. 663-674, 2003.

Motion Correction Needed

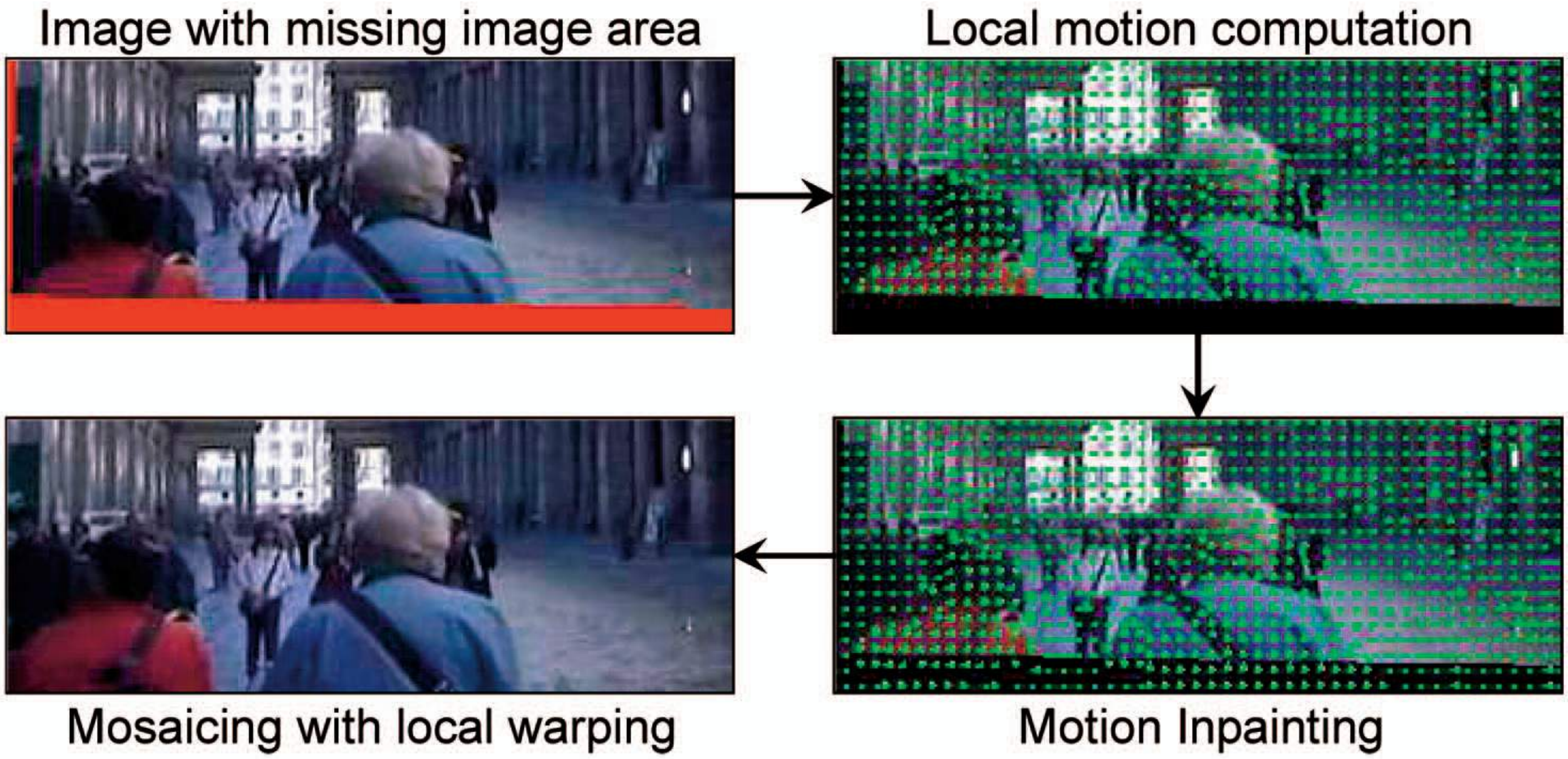
- Original global motion parameters between successive frames I_{n-1} and I_n: a_n
  - $x_n = x_{n-1} + h(x_{n-1}, a_n)$
- Smoothed parameters: b_n
- Correction needed for frame n
  - Observed pixel location $x_n = x_{n-1} + h(x_{n-1}, a_n)$
  - Desired pixel location $x'_n = x_{n-1} + h(x_{n-1}, b_n)$
  - Correction needed: $x'_n = x_n + h(x_{n-1}, b_n) - h(x_{n-1}, a_n) = x_n + d(x_{n-1}, b_{n,} a_n)$
  - Should warp x_n in I_x to x'_n in stabilized frame I'_n
  - Using inverse mapping $I'(x'_n) = I(x_n = x'_n - d(x_{n-1}, b_{n,} a_n))$
- Special case: Consider only a global translation
  - $d(x_{n-1}, b_{n,} a_n) = b_{n,} - a_n$: difference in smoothed translation and original translation

Sample Results: Considering Translation only



From: Y. Matsushita, E. Ofek, W. Ge, X. Tang, H.-Y. Shum, "Full-Frame Video Stabilization with Motion Inpainting," IEEE TRANSACTIONS ON PATTERN ANALYSIS AND MACHINE INTELLIGENCE, VOL. 28, NO. 7, JULY 2006

Video Completion by Motion Inpainting [Ref 1]



[1] Y. Matsushita, E. Ofek, W. Ge, X. Tang, H.-Y. Shum, “Full-Frame Video Stabilization with Motion Inpainting,” IEEE TRANSACTIONS ON PATTERN ANALYSIS AND MACHINE INTELLIGENCE, VOL. 28, NO. 7, JULY 2006

Pop Quiz

- What are the major steps in stabilization?

Pop Quiz (Answer)

- What are the major steps in stabilization?
- Global motion (mapping) estimation -> motion parameter smoothing -> warping based on the difference of the smoothed motion and the measured motion -> image completion

Summary (1)

- Simple approach for background modeling/ moving object detection: Using average or median of frames to form the background image. Pixels different from the background are considered to belong to moving objects.
- Not robust to changes due to illumination, shadow, background dynamics, and noise.
- Robust PCA solution:
  - smooth background forms a low rank matrix, moving object leads to sparse entries in the matrix, Solve Low rank + sparse decomposition problem
- When the camera is moving, one has to estimate the camera motion and finding regions with different motion as objects.
- Camera motion induced motion field models:
  - Homography is accurate when the imaged scene is faraway
  - Affine is accurate when the camera motion is in plane (rotation, zooming, shifting)

Summary (2)

- Camera motion parameters can be estimated by minimizing the DFD error over all pixels in a frame
  - No closed form solution, can be solved using gradient descent
  - Should know how to set up the energy function, the gradient, and the iterations
- Camera motion parameters can also be estimated by minimizing the optical flow equation error
  - Can lead to a linear equation with closed-form solutions
  - Should know how to set up the energy function, obtain the linear equation by setting the gradient to zero
  - Optical flow equation is only accurate if the motion at every pixel is small, which is usually not true for camera motion.
  - Can get around by iterative warping.
- Video shot boundary detection
  - Based on difference in histogram or entropy
- Video stabilization
  - Global motion estimation, motion parameter smoothing, warping, missing pixel interpolation

Tools for motion estimation, object detection and tracking

- Python tools for motion estimation and object detection
  - http://docs.opencv.org/2.4/modules/video/doc/motion_analysis_and_object_tracking.html
  - cv2.calcOpticalFlowPyrLK()
    - This function computes the flow at a set of feature points, using pyramid representation
  - cv2.calcOpticalFlowFarneback()
    - This function computes dense flow (at every pixel)
  - cv2.BackgroundSubtractorMOG2()
  - cv2.BackgroundSubtractorMOG()
- KLT tracker
  - https://github.com/TimSC/PyFeatureTrack (3rd party package)
- Tutorial on object detection and tracking using OpenCV
  - https://www.intorobotics.com/how-to-detect-and-track-object-with-opencv/

Useful Resources for Background Subtraction

- Background subtraction:
  - http://bmc.iut-auvergne.com/ (some datasets)
  - https://sites.google.com/site/backgroundsubtraction/ (algorithm, datasets, codes)
  - changedetection.net (large dataset)

Reading Assignments

- [Szeliski2010] Richard Szeliski, Computer Vision: Algorithms and Applications. 2010. Chap. 8. (Mainly 8.2 and 8.5 for this lecture)
- [Wang2002] Wang, et al, Digital video processing and communications. Sec. 6.7,6.8, Apx. A, B.
- T. Bouwmans, E. Zahzah, "Robust PCA via Principal Component Pursuit: A Review for a Comparative Evaluation in Video Surveillance", Special Issue on Background Models Challenge, Computer Vision and Image Understanding, CVIU 2014, Volume 122, pages 22–34, May 2014. [pdf]

- **Other optional references:**
- Candès, E. J., Li, X., Ma, Y., & Wright, J. (2011). Robust principal component analysis?. *Journal of the ACM (JACM)*, *58*(3), 11.
- T. Bouwmans, "Traditional and Recent Approaches in Background Modeling for Foreground Detection: An Overview", Computer Science Review, 2014.[pdf]
- Andrews Sobral and Antoine Vacavant. A comprehensive review of background subtraction algorithms evaluated with synthetic and real videos. *Computer Vision and Image Understanding*, 122:4-21, 2014
- Yilmaz, Alper, Omar Javed, and Mubarak Shah. "Object tracking: A survey." *Acm computing surveys (CSUR)* 38.4 (2006): 13. http://7xq232.com1.z0.glb.clouddn.com/talk/2013.12.20-Student.Workshop.pdf
- Wu, Yi, Jongwoo Lim, and Ming-Hsuan Yang. "Online object tracking: A benchmark." *Proceedings of the IEEE conference on computer vision and pattern recognition*. 2013. http://www.cv-foundation.org/openaccess/content_cvpr_2013/papers/Wu_Online_Object_Tracking_2013_CVPR_paper.pdf
- Stauffer, Chris, and W. Eric L. Grimson. "Adaptive background mixture models for real-time tracking." *Computer Vision and Pattern Recognition, 1999. IEEE Computer Society Conference on.*. Vol. 2. IEEE, 1999.

Written Assignment

- Answer all the quiz questions
- Suppose you model the global motion between two frames by a bilinear transform. You would like to determine the bilinear transform parameters by minimizing the sum of squared differences between corresponding pixels in the two frames. Formulate the minimization function and its gradient with respect to the motion parameters. From the gradient, can you find the closed-form solution? If yes, derive the solution. If not, describe a gradient descent algorithm for finding the solution.
- Instead of minimizing the sum of squared differences between corresponding pixels in the two frames, you can assume the motion is small and apply the optical flow constraint at each pixel. Formulate the minimization function and its gradient with respect to the motion parameters. From the gradient, can you find the closed-form solution? If yes, derive the solution. If not, describe a gradient descent algorithm for finding the solution.