Final Exam

Closed book, 2 sheet of notes (double sided) allowed. No peeking into neighbors or unauthorized notes. No calculator or any electronics devices allowed. Cheating will result in getting an F on the course. Make sure you write your name and ID on the cover of the blue book. Write your answer in the blue book (or on the problem sheet when space is provided on the problem sheet).

Your Name ______________________________ **ID** __________________________

1. (10pt) For the image in the left figure (which only takes integer values of 0 to 3): a) determine its histogram, b) determine a transformation function that will attempt to equalize the histogram; c) Show the equalized image using your transformation function in the right figure; d) Show the histogram of the equalized image.

0	0	1	3
1	1	2	3
1	2	1	1
2	1	1	1

2. (10 pt) For the 2D filter H given below, where the center position corresponds to m=n=0: a) Is this filter separable? If yes, what is the horizontal and vertical filter? b) Based on the filter coefficients, can you tell what is the function of this filter overall, and what its function in the horizontal and vertical directions? c) Determine the discrete space Fourier Transform *H(u,v)* of the filter H, and sketch the one dimensional profiles *H(u,0), H(u,1/2), H(0,v), and H(1/2,v). You should use the property of separable functions for computing the DSFT when possible.* (d) Based on the frequency response you derived, can you tell what is the function of the filter? Is this observation consistent with what you get in part (b)?

$$H = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ -1 & -1 & -1 \end{bmatrix}$$

3. (10pt) For the image given below, determine its 3-level Gaussian Pyramid and Laplacian Pyramid. Use averaging of 2x2 pixels for downsampling, use nearest neighbor replication for interpolation. Also, demonstrate the reconstruction of the original image from the Laplacian pyramid.

6	5	4	3
5	4	3	2
4	3	2	1
3	2	1	1

4. (10pt) Suppose you are given a gray scale image with some text overlaid over it. You would like to remove the text by filling in appropriate gray values in those text pixels. Suppose you know where the text pixels are and furthermore you know that the underlying image has a sparse representation under a given dictionary. Describe how would you like to accomplish this task though solving some optimization problems. You may use the following notations: **Y** is a *Nx1* vector consisting of the observed gray values of all *N* pixels in the given image ordered into a 1D array, **D** is a *NxM* matrix consisting of *M (M>=N)* dictionary atoms, **X** is a vector consisting of *M* coefficients of the recovered image, **M** is a LxN masking matrix, indicating which pixels are non-text with *L (L<N)* denoting the total number of non-text pixels, so that **Y'=MY** is a vector containing all non-text pixels. Each row of **M** should have only one non-zero entry, and element $M_{l,n}=1$ if the *l*-th pixel in Y' corresponds to pixel *n* in the image. a) Set up a constrained optimization problem for solving **X** using the above notation, with the goal to minimize the number of non-zeros in coefficient vector **X**, while requiring the recovered non-text pixel values to be within a certain small distance to observed values (i.e. $\|\mathbf{Y}'-\mathbf{MDX}\|_2<\varepsilon$). b) Describe a non-constrained formulation where you combine the minimizing term and the constraint term of (a) using a weighted sum. c) Show a relaxed convex problem

formulation for the non-constrained problem. d) How do you obtain the completed image (with text pixels filled) after solving the optimization problem? Note: You do not need to describe how to solve the stated optimization problems.

5. (10 pt) We would like to use the histogram of oriented gradient (HoG) to describe an image patch. a) (8pt) Given the image patch below, generate its gradient images Ix and Iy and the gradient orientation image Io and finally the HoG. Write the resulting images in the figure below. Assume all possible orientations are quantized to only 8 directions (0, 45, 90, 135, 180, 225, 270, 315). Please use the simple difference operator to determine the image gradient: Ix(m,n)=I(m,n)-I(m,n-1); Iy(m,n)=I(m,n)-I(m-1,n). You can assume pixels outside the patch have values of 0. (b) (2pt) Is the HoG (after normalization) invariant to image contrast (i.e. the dynamic range of the gray values)? Is it invariant to image rotation?

0	0	0	0
0	1	1	1
1	1	1	1
1	1	1	1

Image I

Gradient Ix

Gradient Iy

Gradient Orientation Io

6. (10pt) You are given two images taken under different view angles. Assume that the two images are related by a homography mapping described below. Suppose you have detected *N* corresponding pairs of features in these two images, denoted by (u_n, v_n) in image 1 and (x_n, y_n) in image 2, *n=1,2,…,N*. Assume N>8. Describe how would you determine the homography parameters using the least squares method. You should set up the equations, express it in a matrix form, and show the solution of the equation.

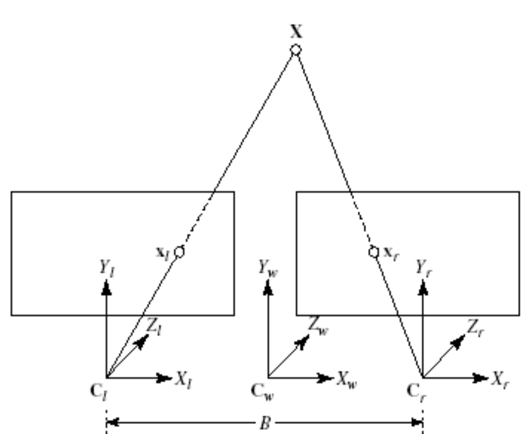
$$x(u,v) = \frac{a_0 + a_1 u + a_2 v}{1 + c_1 u + c_2 v}, \qquad y(u,v) = \frac{b_0 + b_1 u + b_2 v}{1 + c_1 u + c_2 v}$$

7. (10pt) Given two images of the same scene taken from two different view angles, we would like to stitch them into a single image based on feature correspondences. Describe the steps (in words) that you would take to accomplish this task. Be sure to include all the steps necessary, but you do not need to elaborate on how to do each step in detail. Hint: you should have at least 5 steps.

8. (10pt) Given two frames of a video *F(x,y)* and *G(x,y),* we would like to estimate the camera motion between the two frames modeled by an affine motion. That is, for every pixel *(x,y)* in F, we assume its motion vector can be described below. One way to solve the motion parameters is by using optical flow equation at every pixel. (a) Write down the optical flow equation at every pixel *(x,y)* of F based on the constant brightness assumption $F(x + d_x, y + d_y) = G(x, y)$. (b) Set up a set of equations for solving the global motion parameters based on the optical flow equations. Express the equation in matrix form and its least squares solution. (c) How would you detect moving objects which moves differently from the camera motion?

$$d_x(x,y) = a_0 + a_1 x + a_2 y, \qquad d_y(x,y) = b_0 + b_1 x + b_2 y$$

9. (10pt) Consider a stereo imaging system using two parallel cameras with a baseline distance of B, each camera with a focal length of F. The relationships between the image coordinates and 3D world coordinates are shown below. (a) (5pt) For each pixel (x_l, y) in the left image I_l, how do you estimate its 3D coordinate? (b) (5pt) Now assume each camera takes a video. Suppose that you have a stereo pair captured at time t, $I_{l,t}$ *and* $I_{r,t}$, and a stereo pair captured at time t-1, $I_{l,t-1}$ *and* $I_{r,t-1}$. For each pixel $(x_{l,t}, y_t)$ in the image $I_{l,t}$, how do you estimate its 3D movement between time t and t-1?



(a)

10. (10pt) The figure below shows 4 samples over a 2D plane (think of them as 4 pixels where each pixel is described by a feature vector of dimension 2). We would like to cluster them into two groups using the K-means method. Starting with the initial centroids illustrated in the top-left figure, show the results of several iterations of K-means in the figures provided until the iteration converges. You can use a big circle to include all samples in the same cluster in each iteration, and use triangles to indicate the cluster centroids. (Note that the lines in the figure are there to help you gauge the relative positions of the samples and intial centroids.