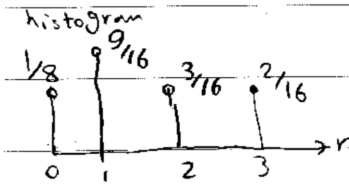
Problem 1:

part (a):



$N = 16 \quad N_0 = 2 \quad N_1 = 9 \quad N_2 = 3 \quad N_3 = 2$

Part (b):

Values	Prob	sum of prop	round{3x Σ Prob}
0	2/16	2/16	round{6/16} = 0
1	9/16	11/16	round{33/16} = 2
2	3/16	14/16	round{42/16} = 3
3	2/16	16/16	round{3} = 3

mapping

0 → 0

1 → 2

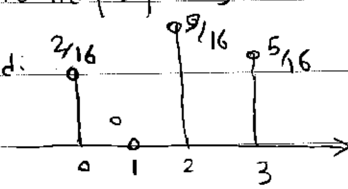
2 → 3

3 → 3

part (c):

0	0	2	3
2	2	3	3
2	3	2	2
3	2	2	2

part d:



Problem 2:

Part (a) $H = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ -1 & -1 & -1 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \end{bmatrix}$

$\Rightarrow h_x = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \qquad h_y = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \Rightarrow$ it is seprable.

Part (b) filter $\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$ is a difference filter $\Rightarrow$ high pass filter

filter $\begin{bmatrix} 1 & 1 & 1 \end{bmatrix}$ is an averaging filter $\Rightarrow$ low pass filter

the over all filter is taking derivative in $\downarrow$ direction and average in $\rightarrow$ direction $\Rightarrow$ detects horizental edges.

Part (c)

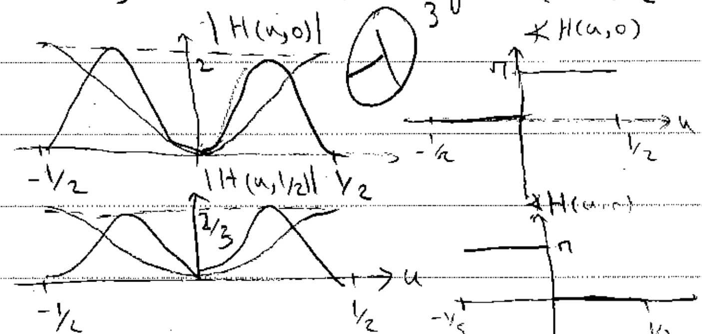
$$\mathcal{F}\{h_x\} = \frac{1}{\sqrt{3}}\left[-e^{+j2\pi u} + e^{-j2\pi u}\right] = \frac{-2j}{\sqrt{3}}\sin(2\pi u)$$

$$\mathcal{F}\{h_y\} = \frac{1}{\sqrt{3}}\left[1 + 2\cos(2\pi v)\right] \Rightarrow H(u,v) = -\frac{2j}{3}\sin(2\pi u)\left[1+2\cos(2\pi v)\right]$$

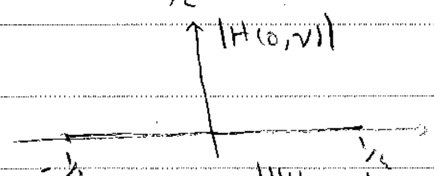
Bandpass $\rightarrow$

$H(u,0) = -2j\sin(2\pi u)$

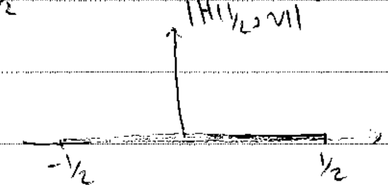
$H(u,\tfrac{1}{2}) = \frac{2j}{3}\sin(2\pi u)$



$H(0,v) = 0$



$H(\tfrac{1}{2},v) = 0$



part(d): filter acts as a horizontal edge detector.

Problem 3:

$$F_1 = \begin{bmatrix} 6 & 5 & 4 & 3 \\ 5 & 4 & 3 & 2 \\ 4 & 3 & 2 & 1 \\ 3 & 2 & 1 & 1 \end{bmatrix} \Rightarrow \text{after averaging} \quad G_1 = \begin{bmatrix} 5 & 3 \\ 3 & 1 \end{bmatrix}$$

$$\Rightarrow \text{interpolating} \quad I_1 = \begin{bmatrix} 5 & 5 & 3 & 3 \\ 5 & 5 & 3 & 3 \\ 3 & 3 & 1 & 1 \\ 3 & 3 & 1 & 1 \end{bmatrix} \xRightarrow{\text{subtract}} L_1 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & -1 & 0 & -1 \\ 1 & 0 & 1 & 0 \\ 0 & -1 & 0 & -1 \end{bmatrix}$$

next level

$$\text{averaging on } G_1 \Rightarrow G_2 = [3] \Rightarrow I_2 = \begin{bmatrix} 3 & 3 \\ 3 & 3 \end{bmatrix} \Rightarrow L_2 = \begin{bmatrix} 2 & 0 \\ 0 & -2 \end{bmatrix}$$

for re construction we have:

$$G_2 = [3] \xrightarrow{\text{inter polate}} I_2 = \begin{bmatrix} 3 & 3 \\ 3 & 3 \end{bmatrix} \xrightarrow{\text{add to } L_2} \Rightarrow G_1 = \begin{bmatrix} 5 & 3 \\ 3 & +1 \end{bmatrix}$$

$$G_1 = \begin{bmatrix} 5 & 3 \\ 3 & 1 \end{bmatrix} \xrightarrow{\text{inter polate}} I_1 = \begin{bmatrix} 5 & 5 & 3 & 3 \\ 5 & 5 & 3 & 3 \\ 3 & 3 & 1 & 1 \\ 3 & 3 & 1 & 1 \end{bmatrix} \xrightarrow[L_1]{\text{add to}} F = \begin{bmatrix} 6 & 5 & 4 & 3 \\ 5 & 4 & 3 & 2 \\ 4 & 3 & 2 & 1 \\ 3 & 2 & 1 & 1 \end{bmatrix}$$

Problem 4: $Y_{N\times 1}$ ⇝ Observed gray values of N pixels.

part (a): $\min_{X} \|X\|_0$

s.t. $\|Y' - MDX\|_2 < \varepsilon$

this problem is not convex since L0-norm is not a convex norm and it is NP-Hard to solve.

Part (b): using techniques in convex an equivalant problem is

$\min_{X} \|Y' - MDX\|_2^2$

s.t. $\|X\|_0 < S$

which also is equal to $\min_{X} \|Y' - MDX\|_2^2 + \mu \|X\|_0$

Part (c): one relaxation to get convexity is changing L0-norm to L_1-norm ⇒ $\min_{X} \|Y' - MDX\|_2^2 + \lambda \|X\|_1$

where L_1-norm is sum of absolute values.

Part (d) when we calculate X^{opt} by solving the problem We can calculate the missing pixels by $(I-M) D X^{opt}$ and we can replace them by text in image. assuming the observation is noisy we can have $D X^{opt}$ as output to reduce the effect of noise too.

Wrong notation

Problem 5:

Part (a):

$I_x(m,n) = I(m,n) - I(m,n-1)$ assuming out side is 0

0	0	0	0
0	1	0	0
1	0	0	0
1	0	0	0

I_x

0	0	0	0
0	1	1	1
1	0	0	0
0	0	0	0

I_y

Grad magnitude CE

Not edge.

2

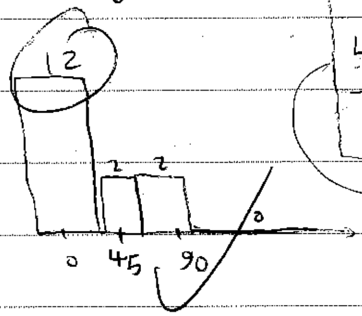
$\Rightarrow \theta = \tan^{-1}\left(\frac{I_x}{I_y}\right)$

0	0	0	0
0	45	90	90
45	0	0	0
0	0	0	0

Should be given special symbol.

this is created due to the fact that we padded zero to the left.

$\Rightarrow$ HoG



Part (b): if we normalize (and Threshold in the sift algorithm) then the HoG is invariant to light change and intensity change.

this histogram is not rotation invariant since we did not circularly shift the histogram to get the main dominant direction in the first bin. but in the actual SIFT discriptor we actuly calculate the dominant direction of the patch and shift the histograms of sub-patches in the first bin to get rotation invariant result.

Problem 6:

$$x = \frac{a_0 + a_1 u + a_2 v}{1 + c_1 u + c_2 v} \Rightarrow x + c_1 x u + c_2 x v = a_0 + a_1 u + a_2 v$$

$$\Rightarrow \begin{cases} x = a_0 + a_1 u + a_2 v - c_1 x u - c_2 x v \\ y = b_0 + b_1 u + b_2 v - c_1 y u - c_2 y v \end{cases}$$

$$\Rightarrow \underbrace{\begin{bmatrix} \vdots \\ 1 & u_n & v_n & 0 & 0 & 0 & -x_n u_n & -x_n v_n \\ 0 & 0 & 0 & 1 & u_n & v_n & -y_n u_n & -y_n v_n \\ \vdots \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ b_0 \\ b_1 \\ b_2 \\ c_1 \\ c_2 \end{bmatrix}}_{\bar{a}} = \underbrace{\begin{bmatrix} \vdots \\ x_n \\ y_n \\ \vdots \end{bmatrix}}_{x}$$

We want to have $Aa = x$ if $N = 8 \Rightarrow \bar{a} = A^{-1} x$

if $N > 8$ which is usually the case we solve least squares problem:

$N > 8 \Rightarrow$ over determinde $\Rightarrow \min_a \|Aa - x\|_2^2$

$$\Rightarrow \frac{\partial}{\partial a} \|x - Aa\|_2^2 = -2A^T x + 2A^T A a \Rightarrow a^{opt} = (A^T A)^{-1} A^T x$$

one other method is DLT:

$$\begin{bmatrix} \vdots \\ 1 & u_n & v_n & 0 & 0 & 0 & -x_n & -u_n x_n & -v_n x_n \\ 0 & 0 & 0 & 1 & u_n & v_n & -y_n & -y_n u_n & -y_n v_n \\ \vdots \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ b_0 \\ b_1 \\ b_2 \\ c_0 \\ c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} \vdots \\ 0 \\ \vdots \end{bmatrix}$$

$\Rightarrow Aa = 0$ least squares $\Rightarrow \min \|Aa\|_2^2 = a^T A^T A a$

Solution a^{opt} = eigen vector of $A^T A$ with the minimal evalue.

Problem 7:

for this problem we choose to use harris corners in laplacian as Feature points and sift as descriptor.

step 1: find the Laplacian scale images for both Im1 and Im2 this is done using filters $\sigma_n = \sigma_0 k^n$ and down sampling and then subtracting adjacent images.

step 2: extract Harris feature points in multiple scales and for each detect the charactristic scale.

step 3: for each feature point in Images create the SIFT descriptor and save all feature points and their descriptor.

step 4: try to find corresponding points between Image 1 and Image 2. this is achived by comparing the descriptors and taking the one that has the nearest distanse. since there might be ambigaity we have:

$d_1 < d_2$ if $d_1 < T_d$, $d_1/d_2 < T_r \Rightarrow$ keep d_1 as match else delet both.

step 5: after finding the matching points use Least squares or RANSAC to find the homography or affine mapping that best describes the mapping between two images.

step 6: use the mapping to warp one Image to other cordinate (this is done with inverse mapping and interpolation)

step 7: stich two images to get the result. this can be done using pyramid Blending. some smoothing might be required.

Problem 8:

Part (a):

$$F(x+dx, y+dy) - G(x,y) = 0 \Rightarrow \frac{\partial F}{\partial x} dx + \frac{\partial F}{\partial y} dy + F(x,y) - G(x,y) = 0$$

$$\Rightarrow \text{optical flow eq: } \frac{\partial F}{\partial x} dx + \frac{\partial F}{\partial y} dy + F(x,y) - G(x,y) = 0$$

$$\begin{bmatrix} d(x,a) \\ d(y,a) \end{bmatrix} = \begin{bmatrix} a_0 + a_1 x + a_2 y \\ b_0 + b_1 x + b_2 y \end{bmatrix} = \begin{bmatrix} 1 & x_n & y_n & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & x_n & y_n \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ b_0 \\ b_1 \\ b_2 \end{bmatrix}$$

simple case Affine:

$$dx = \begin{bmatrix} 1 & x_n & y_n \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} \Rightarrow dx = Aa, \; dy = Ab \Rightarrow \frac{\partial F}{\partial x} A(x,y)\, a + \frac{\partial F}{\partial y} A(x,y)\, b + F(x,y) - G(x,y) = 0$$

(b)

Least squares problem: $E_{OF} = \frac{1}{2} \sum_{x \in B} \left| \frac{\partial F}{\partial x} A(x,y)\, a + \frac{\partial F}{\partial y} A(x,y)\, b + F(x,y) - G(x,y) \right|^2$

we try to minimize $E_{OF} \Rightarrow \frac{\partial E_{OF}}{\partial a} = 0 \quad \frac{\partial E_{OF}}{\partial b} = 0$

and we get the following equation.

$$\begin{bmatrix} \sum_{x \in B} \frac{\partial^2 F}{\partial x^2} A^T A & \sum_{x \in B} \frac{\partial F}{\partial x} \frac{\partial F}{\partial y} A^T A \\ \sum_{x \in B} \frac{\partial F}{\partial x} \frac{\partial F}{\partial y} A^T A & \sum_{x \in B} \frac{\partial^2 F}{\partial y^2} A^T A \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} \sum_x (G(x,y) - F(x,y)) \frac{\partial F}{\partial x} A^T \\ \sum_{x \in B} (G(x,y) - F(x,y)) \frac{\partial F}{\partial y} A^T \end{bmatrix}$$

$$\Rightarrow Sa = t \Rightarrow a = S^{-1} t$$

in order to find the moving object we can compensate for camera motion based on the mapping we found and use other motion estimation methods like EBMA to find the moving object based on warped image and image 1.

-1 ?

You just need to detect pixels with large error after global motion compensation.

Problem 9:

part (a):

when we have corresponding point in right image we can find disparity and then we have:

$$X_\ell = X + B/2 \quad X_r = X - B/2 \Rightarrow x_\ell = F\frac{X+B/2}{Z} \quad x_r = F\frac{X-B/2}{Z}$$

$$\Rightarrow d = x_\ell - x_r = \frac{FB}{Z} \Rightarrow \frac{F}{Z} = \frac{d}{B} \Rightarrow \frac{Z}{F} = \frac{B}{d} \Rightarrow Z = \frac{FB}{d}$$

$$Y = y\frac{Z}{F} = y\frac{B}{d} \qquad X = \frac{x_\ell + x_r}{2}\frac{Z}{F} = \frac{x_\ell + x_r}{2}\frac{B}{d}$$

Part (b):

first we find the mapping between $I_{\ell,t}$ and $I_{r,t}$ to find the corresponding points using these corresponding point and result of part (a) we can find Z^t, X^t, Y^t position 3D at time t using the same method we can find the corresponding points on $I_{\ell,t-1}$, $I_{r,t-1}$ and recover $Z^{t-1}, X^{t-1}, Y^{t-1}$

$$\Rightarrow \Delta Z = Z^t - Z^{t-1}$$

$$\Delta X = X^t - X^{t-1}$$

$$\Delta Y = Y^t - Y^{t-1}$$

-2 ④ Need to do motion estimation to ~~track~~ find the corresponding pixel in Frame t (in say $I_r(t)$) for a point in Frame t-1, say $I_{r,t-1}$.

10. (10pt) The figure below shows 4 samples over a 2D plane (think of them as 4 pixels where each pixel is described by a feature vector of dimension 2). We would like to cluster them into two groups using the K-means method. Starting with the initial centroids illustrated in the top-left figure, show the results of several iterations of K-means in the figures provided until the iteration converges. You can use a big circle to include all samples in the same cluster in each iteration, and use triangles to indicate the cluster centroids. (Note that the lines in the figure are there to help you gauge the relative positions of the samples and intial centroids.