Exam

Closed book, 2 sheet of notes (double sided) allowed. No peeking into neighbors or unauthorized notes. No calculator or any electronics devices allowed. Cheating will result in getting an F on the course. Make sure you write your name and ID on the cover of the blue book. Write your answer in the blue book (or on the problem sheet when space is provided on the problem sheet).

Your Name ______________________________ **ID** _________________________

1. (10pt) The histograms of three images are illustrated in Figs. (a) to (c). Choose one of the three transformations given in Figs. (d) to (f) such that the transformed image has a nearly flat histogram.

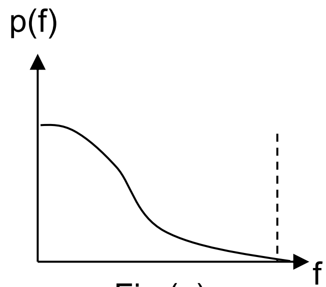

Fig.(a)

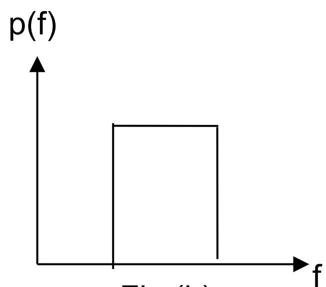

Fig.(b)

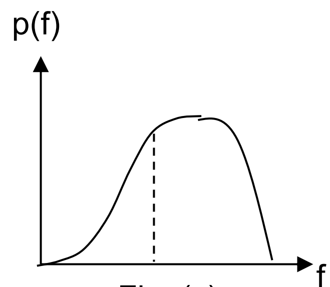

Fig. (c)

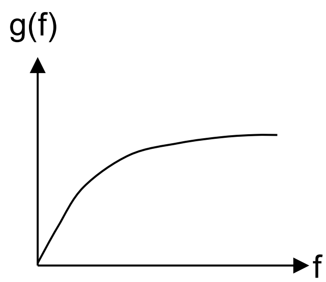

Fig.(d)

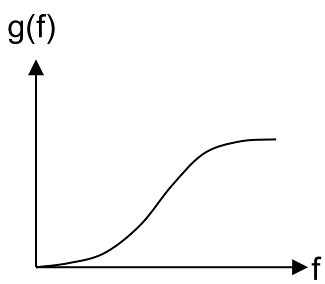

Fig.(e)

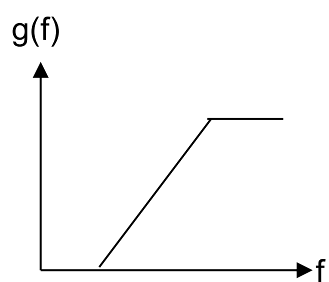

Fig.(f)

2. (10 pt) A 2D filter H is given below, where the center position corresponds to m=n=0. (a, 6pt) The filter H can be decomposed into the sum of two filters as shown above. Is each filter (*H, H_1 and H_2*) separable? If so, give the one dimensional filters in horizontal (*y*) and vertical (*x*) directions. Also, based on the filter coefficients, can you tell what is the function of each filter (*H, H_1 and H_2*). (4) Determine the DSFT *H(u,v)* of the filter H by computing the DSFT of the subfilters *H_1 and H_2*.

$$H = \frac{1}{8}\begin{bmatrix} -1 & 2 & -1 \\ 2 & 4 & 2 \\ -1 & 2 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \frac{1}{8}\begin{bmatrix} -1 & 2 & -1 \\ 2 & -4 & 2 \\ -1 & 2 & -1 \end{bmatrix} = H_1 + H_2$$

3. (10pt) For the image given below, determine its 1-level wavelet decomposition using Haar wavelets. You should use separable processing. First generate two subbands corresponding to row wise decomposition (generating images L and H), and then apply wavelet decomposition column wise to generate 4 subimages (LL, LH, HL, HH). For simplicity, you can assume the analysis stage simply uses a filter of [1 1] and [-1, 1]. That is, given two samples A and B, the low band signal is A+B, the high band signal is A-B. Draw the resulting images in the figures given below.

1	1	1	1
1	1	2	2
1	2	1	2
1	2	2	1

L H

Row wise decomposition

LL HL

LH HH

Column wise decomposition

4. (10 pt) Magnetic Resonance Imaging (MRI) works by (with some approximations) measuring the 2D Discrete Fourier Transform of each slice of an organ. If all samples in the Fourier space (known as k-space) are measured, the image slice can be easily reconstructed by using Inverse DFT. However, imaging the complete k-space takes long time. To reduce the scan time, it is desirable to measure only a subset of points in the k-space (the total number of samples is significantly less than the total number of pixels). Your task is to recover the image slice $\boldsymbol{x}$ from the measured subset of DTFT coefficients, $\boldsymbol{y}$. Obviously, this is an underdetermined problem. One prior knowledge you can use is that the absolute difference between horizontally adjacent pixels and that between two vertically adjacent pixels are typically small (large only when there are strong edges). Describe how would you formulate this as a quadratic optimization problem and derive a closed form solution. Please define all the notations you use.
Hint: You could think of $\boldsymbol{x}$ as a 1D vector reshaped from a 2D image row by row, and similarly $\boldsymbol{y}$ a 1D vector consisting of the measured DFT coefficients. You can write that $\boldsymbol{y} = \boldsymbol{F}\boldsymbol{x}$, but you have to stay clearly how to define the matrix $\boldsymbol{F}$ (you just need to say what is the physical meaning of each row of $\boldsymbol{F}$). Also, a vector consisting of the differences between every two horizontally adjacent pixels can be written as $\boldsymbol{H}\boldsymbol{x}$. Similarly, the vector corresponding to vertical differences can be written as $\boldsymbol{V}\boldsymbol{x}$. You should define clearly matrices $\boldsymbol{H}$ and $\boldsymbol{V}$.

5. (10 pt) We would like to use the histogram of oriented gradient (HoG) to describe an image patch. a) (8pt) Given the image patch below, generate its gradient images Ix and Iy and the gradient orientation image Io and finally the HoG. For pixels where the gradient orientation is undefined, use a notation NA. Write the resulting images in the figure below. Assume all possible orientations are quantized to only 8 directions (0, 45, 90, 135, 180, 225, 270, 315). Please use the simple difference operator to determine the image gradient: Ix(m,n)=I(m,n)-I(m,n-1); Iy(m,n)=I(m,n)-I(m-1,n). You can assume pixels outside the patch have values of 0. (b) (2pt) Is the HoG (after normalization) invariant to image contrast (i.e. the dynamic range of the gray values)? Is it invariant to image rotation?

0	1	0	0
0	1	1	0
0	0	1	1
0	0	0	0

Image I

Gradient Ix

Gradient Iy

Gradient Orientation Io

6. (10pt) Suppose you would like to segment an image into K regions so that pixels with similar colors are put into the same region. Assuming pixel n is described by a feature vector $\mathbf{f}_n$. a) Describe the K-means algorithm that you could use to perform the segmentation. Describe it in a ``pseudo code''. b) Compared to K-means, what are the advantages of the GMM algorithm? List two advantages.

7. (15pt) You are given two images taken under different view angles, and you want to align them. Assume that there are quite significant variation in the ambient illumination when the two pictures are taken. Furthermore, assume that the underlying scene is quite far and can be approximated by a plane. a) (2pt) What would be good model that you could use to describe the geometric mapping function between the two images? b) (3pt) Would you use a feature-based or intensity-based approach to determine the mapping function? why? c) (10pt) Describe the steps that you would use to find the mapping parameters.

8. (15pt) Consider block wise motion estimation between two frames. a) (5pt) Let us first consider the exhaustive block matching algorithm (EBMA) algorithm. Suppose the image size is WxH. Block size is BxB. You perform integer-pel accuracy search with a search range of R. What is the total number of operations? (For simplicity, assume the number of operations for calculating the error between two blocks is B^2). b)(5pt) Now consider the hierarchical block matching algorithm (HBMA). Suppose you use two resolution level. At the top level, you use integer accuracy search and the search range at the top level is R/2. You get the initial motion vector at the bottom level by interpolating the motion vector obtained at the top level and you use half-pel accuracy search at the bottom level with a search range of 1. What is the total number of operations? You can ignore the computation needed to generate the low resolution images and for interpolation between image levels and for halfpel search. c) (5pt)s Between EBMA and HBMA, which method has less computation? Which one is expected to yield better motion estimation?

9. (10pt) Consider a parallel stereo imaging system with baseline distance *B* and focus length *F* (see below). Suppose that for an object point at world coordinate *(X,Y,Z)*, its image position in the left and right view are (x_l, y) and (x_r, y), respectively.

a. (2 pt) Describe how to estimate the 3D position *X,Y,Z* from x_l, x_r, y.

b. (8 pt) Suppose we want to generate an intermediate view, whose camera center has a distance of *B*/4 away from the world coordinate origin, as shown below. Describe an algorithm that you will use to generate the intermediate view. For this problem, you could ignore the difficulty that the possible position in the image to be generated for a given pixel in the left or right image may not be an integer.