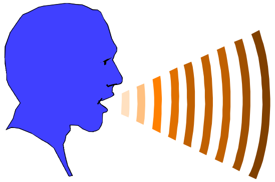

Image and Video Processing

Convolutional Networks for Image Processing (Part II)

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Many contents from Sundeep Rangan:
https://github.com/sdrangan/introml/blob/master/sequence.md

Outline (Part I)

- Supervised learning: General concepts
- Neural network architecture
  - From single perceptron to multi-layer perceptrons
- Convolutional network architecture
  - Why using convolution and many layers
  - Multichannel convolution
  - Pooling
- Deep networks
- Model training
  - Loss functions
  - Stochastic gradient descent: general concept
  - Data Preprocessing and Regularization
- Training, validation and testing and cross validation
- Demo: training a ConvNet classifier

Outline (Part II)

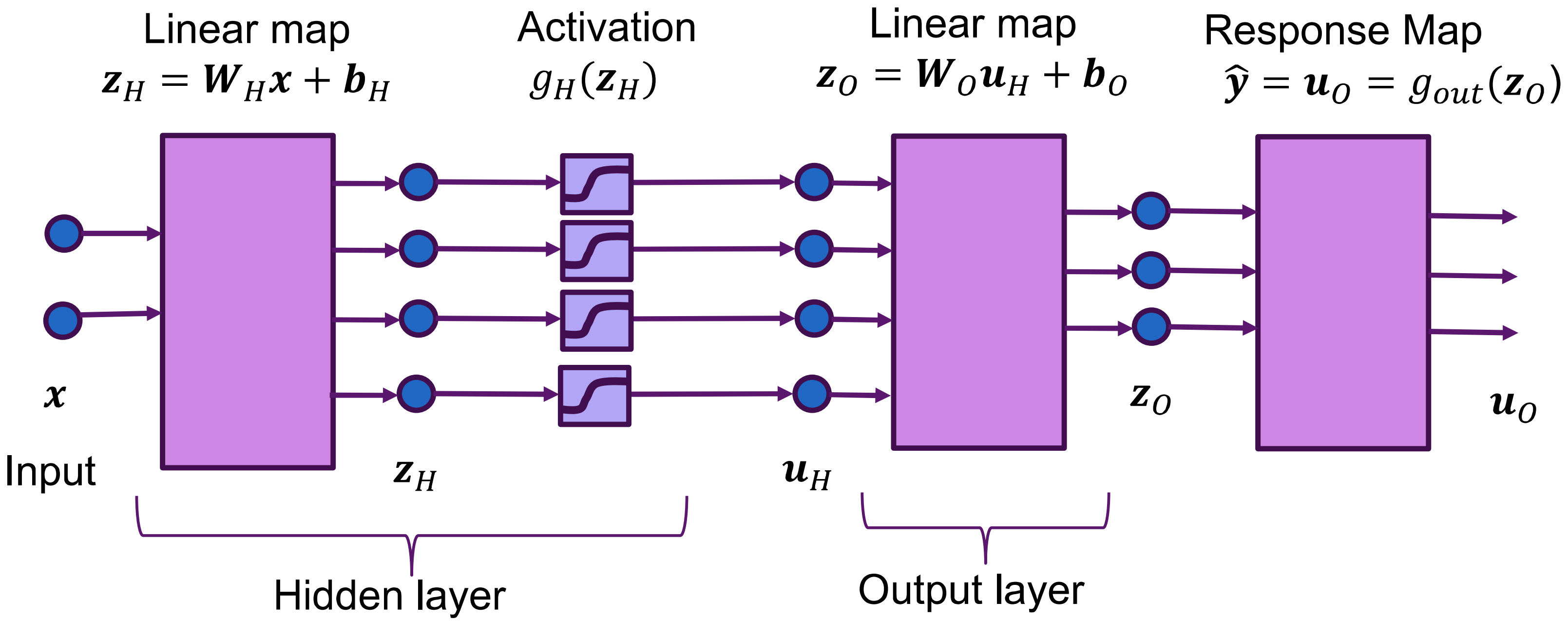
- Neural Nets and Conv Nets and Model Training (Review)
- Gradient calculation
- Some important extensions of conv. layers
- Popular classification models and transfer learning

Outline (Part III)

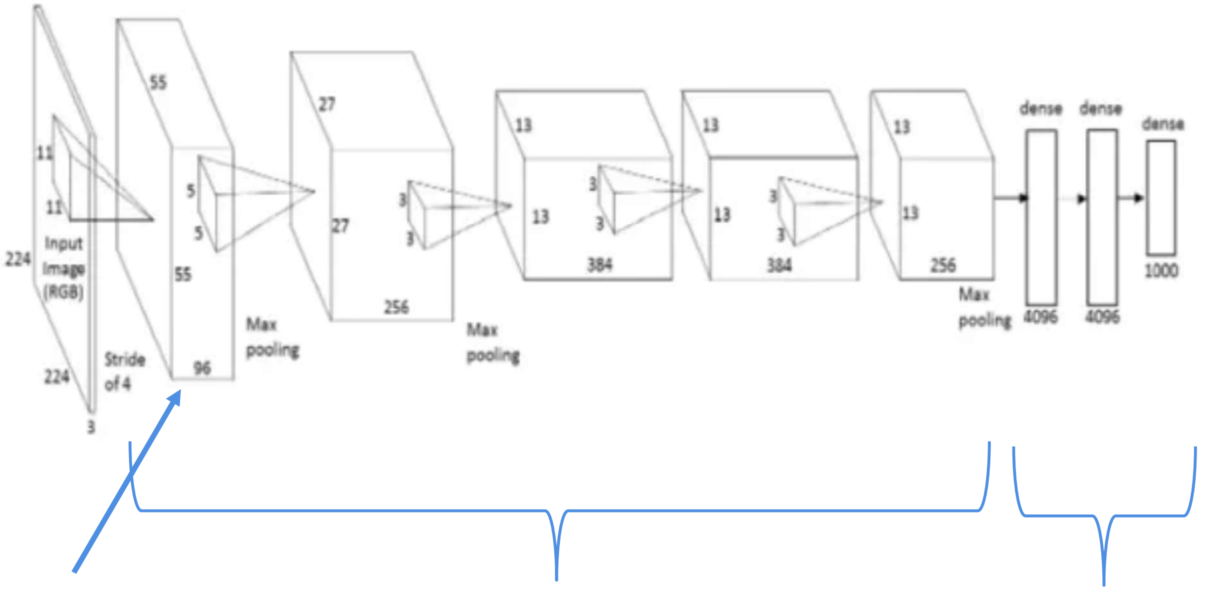
- Image to image autoencoder
- Semantic Segmentation using Multiresolution Autoencoder
- Object detection and classification
- Instance segmentation

Two-Layer Neural Net for Multiple Outputs

- Hidden layer: $\boldsymbol{z}_H = \boldsymbol{W}_H\boldsymbol{x} + \boldsymbol{b}_H, \quad \boldsymbol{u}_H = g_{act}(\boldsymbol{z}_H)$
- Output layer: $\boldsymbol{z}_O = \boldsymbol{W}_O\boldsymbol{u}_H + \boldsymbol{b}_O$
- Response map: $\hat{y} = u_O = g_{out}(\boldsymbol{z}_O)$


Linear map
$\boldsymbol{z}_H = \boldsymbol{W}_H\boldsymbol{x} + \boldsymbol{b}_H$
Activation
$g_H(\boldsymbol{z}_H)$
Linear map
$\boldsymbol{z}_O = \boldsymbol{W}_O\boldsymbol{u}_H + \boldsymbol{b}_O$
Response Map
$\hat{\boldsymbol{y}} = \boldsymbol{u}_O = g_{out}(\boldsymbol{z}_O)$
$\boldsymbol{x}$
Input
$\boldsymbol{z}_H$
$\boldsymbol{u}_H$
$\boldsymbol{z}_O$
$\boldsymbol{u}_O$
Hidden layer
Output layer

Example Conv. Network



96 feature maps of size 55x55 each

Convolutional layers
For feature extraction

2D convolution with Activation and pooling / sub-sampling

Fully connected layers
For Classification task

Matrix multiplication & activation

- Alex Net
- Each convolutional layer has:
  - 2D convolution
  - Activation (eg. ReLU)
  - Pooling or sub-sampling

Krizhevsky, Alex, Ilya Sutskever, and Geoffrey E. Hinton. "Imagenet classification with deep convolutional neural networks." *Advances in neural information processing systems*. 2012.

Training with Gradient Descent

- Given training data: $(\boldsymbol{x}_i, y_i), i = 1, \ldots, N$
- Learn parameters: $\theta = (W_H, b_H, W_o, b_o)$
  - Weights and biases for hidden and output layers
  - W_H are filter kernels in conv. layer
- Neural network training (like all training): Minimize loss function

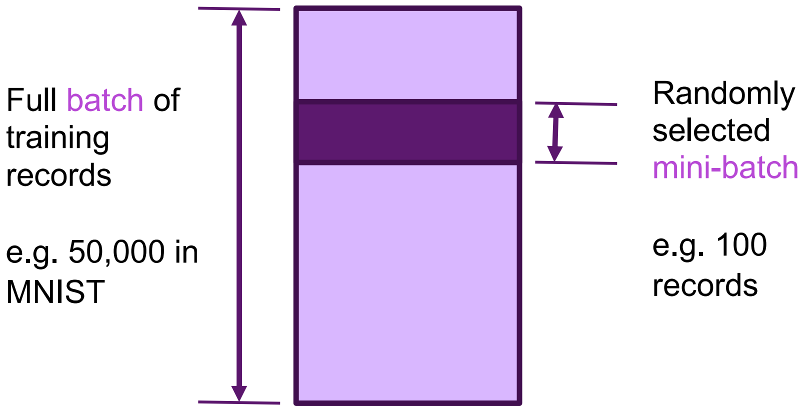
$$\hat{\theta} = \arg\min_{\theta} L(\theta), \qquad L(\theta) = \sum_{i=1}^{N} L_i(\theta, \boldsymbol{x}_i, y_i)$$

  - $L_i(\theta, \boldsymbol{x}_i, y_i)$ = loss on sample i for parameter θ
- Standard gradient descent:

$$\theta^{k+1} = \theta^k - \alpha \nabla L(\theta^k) = \theta^k - \alpha \sum_{i=1}^{N} \nabla L_i(\theta^k, \boldsymbol{x}_i, y_i)$$

  - Each iteration requires computing N loss functions and gradients
  - But, gradient computation is expensive when data size N large

Stochastic Gradient Descent



- In each step:
  - Select random small "mini-batch"
  - Evaluate gradient on mini-batch

- For $t = 1$ to N_{steps}
  - Select random mini-batch $I \subset \{1, \dots, N\}$
  - Compute gradient approximation: $g^t = \frac{1}{|I|} \sum_{i \in I} \nabla L(x_i, y_i, \theta)$
  - Update parameters: $\theta^{t+1} = \theta^t - \alpha^t g^t$

Loss Function: Regression

- Regression case:
  - y_i = target variable for sample i
  - Typically continuous valued
- Output layer:
  - $\hat{y}_i = z_{Oi}$ = estimate of y_i
- Loss function: Use L2 loss

$$L(\theta) = \sum_{i=1}^{N} (y_i - \hat{y}_i)^2$$

- For vector $\boldsymbol{y}_i = (y_{i1}, \dots, y_{iK})$, use vector L2 loss

$$L(\theta) = \sum_{i=1}^{N} \sum_{j=1}^{K} \left(y_{ik} - \hat{y}_{i,k}\right)^2$$

Loss Function: Binary Classification

- Binary classification:
  - Sample: x_i with label $y_i = \{0,1\}$ = class label,
  - Predicted output: $\hat{y}_i = P(y_i = 1|x_i, \theta)$; $1 - \hat{y}_i = P(y_i = 0|x_i, \theta)$
  - Output given by sigmoid on $z_{O,i}$: $\hat{y}_i = \frac{1}{1+e^{-z_{O,i}}}$

- Objective: maximize the likelihood (probability of y_i given x_i for all samples, assuming independence among samples)
  - $P(\boldsymbol{y}|\boldsymbol{X}, \boldsymbol{\theta}) = \prod_{i=1}^{N} P(y_i|x_i, \theta)$

- Maximizing the likelihood = minimizing negative log likelihood:

$$\begin{aligned} L(\theta) &= -\sum_{i=1}^{N} \ln P(y_i|x_i, \theta) \\ &= -\sum_{i=1}^{N} \underset{\substack{\uparrow \\ \text{activate when } y_i\text{=1}}}{y_i \ln \hat{y}_i} + \underset{\substack{\uparrow \\ \text{activate when } y_i\text{=0}}}{(1 - y_i) \ln (1 - \hat{y}_i)} \end{aligned}$$

  - Called the binary cross-entropy

Loss Function: Multi-Class Classification

- Use one-hot-encoding to describe the label y_i

$$y_i = (y_{i1}, \dots, y_{iK}), \quad y_{ik} = \begin{cases} 1 & y_i = k \\ 0 & y_i \neq k \end{cases} \quad k = 1, \dots, K$$

- Output: $\hat{y}_i = \left(\hat{y}_{i,1}, \dots, \hat{y}_{i,K} \right)$; $\hat{y}_{i,k} = P(y_i = k | x_i, \theta)$
  - Output given by softmax on $z_{O,i}$: $\hat{y}_{i,k} = \frac{e^{z_{O,ik}}}{\sum_{\ell} e^{z_{O,il}}}$

- Negative log-likelihood given by:

$$L(\theta) = -\sum_i \ln P(y_i = k | x_i, \theta) = -\sum_i \sum_{k=1}^{K} y_{ik} \ln \hat{y}_{i,k}$$

  - Called the categorical cross-entropy

How to compute gradients?

- For two-layer neural net: $\theta = (W_H, b_H, W_o, b_o)$
- Gradient is computed with respect to each parameter in each batch of M samples:

$$L(\theta) = \sum_{i=1}^{M} L_i(\theta, \boldsymbol{x}_i, y_i) \qquad \nabla L(\theta) = \sum_{i=1}^{M} \nabla L_i(\theta, \boldsymbol{x}_i, y_i)$$

$$\nabla L_i(\theta) = [\nabla_{W_H} L_i(\theta), \nabla_{b_H} L_i(\theta), \nabla_{W_O} L_i(\theta), \nabla_{b_O} L_i(\theta)]$$

- Gradient descent is performed on each parameter:

$$W_H \leftarrow W_H - \alpha \nabla_{W_H} L(\theta),$$
$$b_H \leftarrow b_H - \alpha \nabla_{b_H} L(\theta),$$
$$\ldots.$$

- How to compute $\nabla_{W_H} L_i(\theta), \nabla_{b_H} L_i(\theta)$, etc.?
- W_H, b_H, etc. are vectors and more generally tensors!
- Variables $\boldsymbol{x}_i$, $\boldsymbol{z}_i$, $\boldsymbol{u}_i$, $\hat{\boldsymbol{y}}_i$ are also tensors!

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- Neural Nets and Conv Nets and Model Training (Review)
- Gradient calculation
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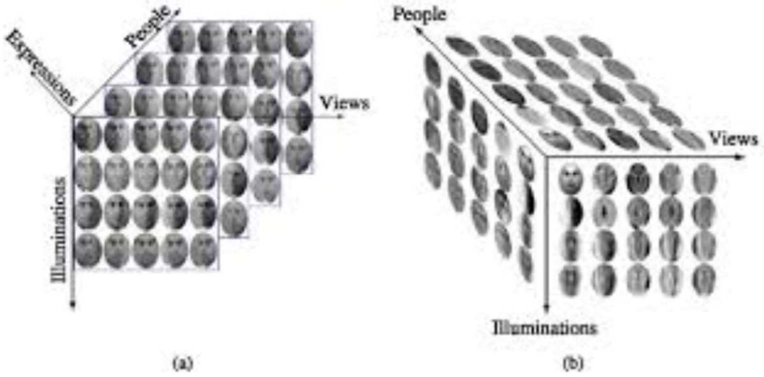
Gradient Calculation

- Tensor definition
- Tensor gradients
- Tensor gradient chain rule
- Backpropagation
- Forward and backward pass

What is a Tensor?

- A multi-dimensional array
- Examples:
  - 2D: A grayscale image [height x width]
  - 3D: A color image [height x width x rgb]
  - 4D: A collection of images [height x width x rgb x image number]
- Like numpy ndarray
- Basic unit in tensorflow
- Rank or order = Number of dimensions
  - Note: Rank has different meaning in linear algebra



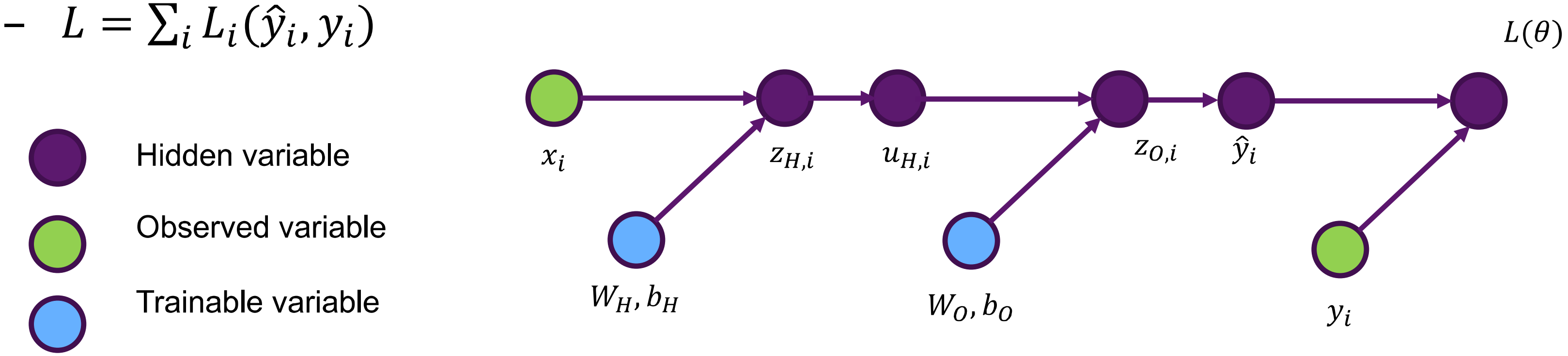
Tensor Gradient Examples

- Example 1: $f(w) = (w_1 w_2, w_1^2 + w_3^3) = \left(f_1(w), f_2(w)\right)$
  - 2 outputs, 3 inputs.
  - Gradient tensor is 2×3
  - $$\frac{\partial f(w)}{\partial w} = \begin{bmatrix} \frac{\partial f_1(w)}{\partial w_2} & \frac{\partial f_1(w)}{\partial w_2} & \frac{\partial f_1(w)}{\partial w_3} \\ \frac{\partial f_2(w)}{\partial w_1} & \frac{\partial f_2(w)}{\partial w_2} & \frac{\partial f_2(w)}{\partial w_3} \end{bmatrix} = \begin{bmatrix} w_2 & w_1 & 0 \\ 2w_1 & 0 & 3w_3^2 \end{bmatrix}$$

- Example 2: $z = f(w) = Aw,$ A is $M\times N, w\ is\ N\times 1$
  - M outputs, N inputs: $z_i = \sum_{j=1}^{N} A_{ij} w_j$
  - Gradient components: $\frac{\partial z_i}{\partial w_j} = A_{ij}$
  - $\frac{\partial f(w)}{\partial w} =?$
  - $$\frac{\partial f(w)}{\partial w} = \begin{bmatrix} \frac{\partial f_1(w)}{\partial w_1} & \cdots & \frac{\partial f_1(w)}{\partial w_N} \\ \cdots & \cdots & \cdots \\ \frac{\partial f_M(w)}{\partial w_1} & \cdots & \frac{\partial f_M(w)}{\partial w_N} \end{bmatrix} = \begin{bmatrix} A_{11} & \cdots & A_{1N} \\ \cdots & \cdots & \cdots \\ A_{M1} & \cdots & A_{MN} \end{bmatrix} = \mathrm{A}$$

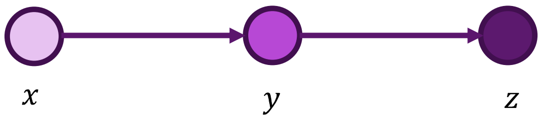
Computation Graph & Forward Pass

- Neural network loss function can be computed via a computation graph
- Sequence of operations starting from input data and network parameters to output
- Loss function computed via a forward pass in the computation graph
  - $z_{H,i} = W_H x_i + b_H$
  - $u_{H,i} = g_{act}(z_{H,i})$
  - $z_{O,i} = W_O u_{H,i} + b_O$
  - $\hat{y}_i = g_{out}(z_{O,i})$
  - $L = \sum_i L_i(\hat{y}_i, y_i)$



Chain Rule

- How do we compute gradient?
- Consider a three node computation graph:
  - $y = h(x),\ z = g(y)$
  - So $z = f(x) = g(h(x))$
  - What is $\frac{\partial z}{\partial x}$?



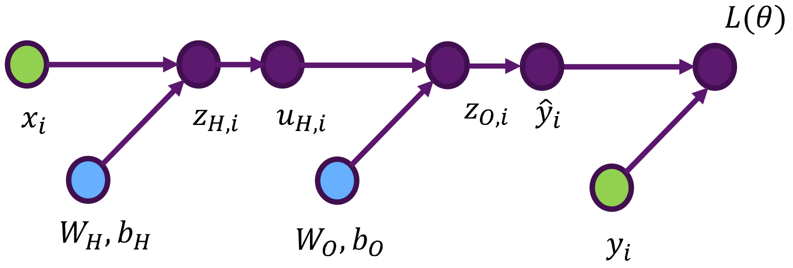
- If variables were scalars, we could compute gradients via chain rule:

$$\frac{\partial z}{\partial x} = \frac{\partial f(x)}{\partial x} = \frac{\partial g(y)}{\partial y}\frac{\partial h(x)}{\partial x}$$

- What happens for tensors?
  - Each output variable has to take gradient with respect to each parameter through chain rule
  - More formal definition in tensor notation in Appendix

Gradients on a Computation Graph

- Backpropagation: Compute gradients backwards
  - Use tensor dot products and chain rule
- First compute all derivatives of all the variables
  - $\partial L/\partial z_O = \langle \partial L/\partial \hat{y}\ , \partial \hat{y}\ /\partial z_O \rangle$
  - $\partial L/\partial u_H = \langle \partial L/\partial z_O\ , \partial z_O/\partial u_H \rangle$
  - $\partial L/\partial z_H = \langle \partial L/\partial u_H\ , \partial u_H/\partial z_H \rangle$
  - ($\partial \hat{y}\ /\partial z_O$ and $\partial u_H/\partial z_H$ is element wise)
- Then compute gradient of parameters:
  - $\partial L/\partial W_O = \langle \partial L/\partial z_O\ , \partial z_O/\partial W_O \rangle$
  - $\partial L/\partial b_O = \langle \partial L/\partial z_O\ , \partial z_O/\partial b_O \rangle$
  - $\partial L/\partial W_H = \langle \partial L/\partial z_H\ , \partial z_H/\partial W_H \rangle$
  - $\partial L/\partial b_H = \langle \partial L/\partial z_H\ , \partial z_H/\partial b_H \rangle$
  -

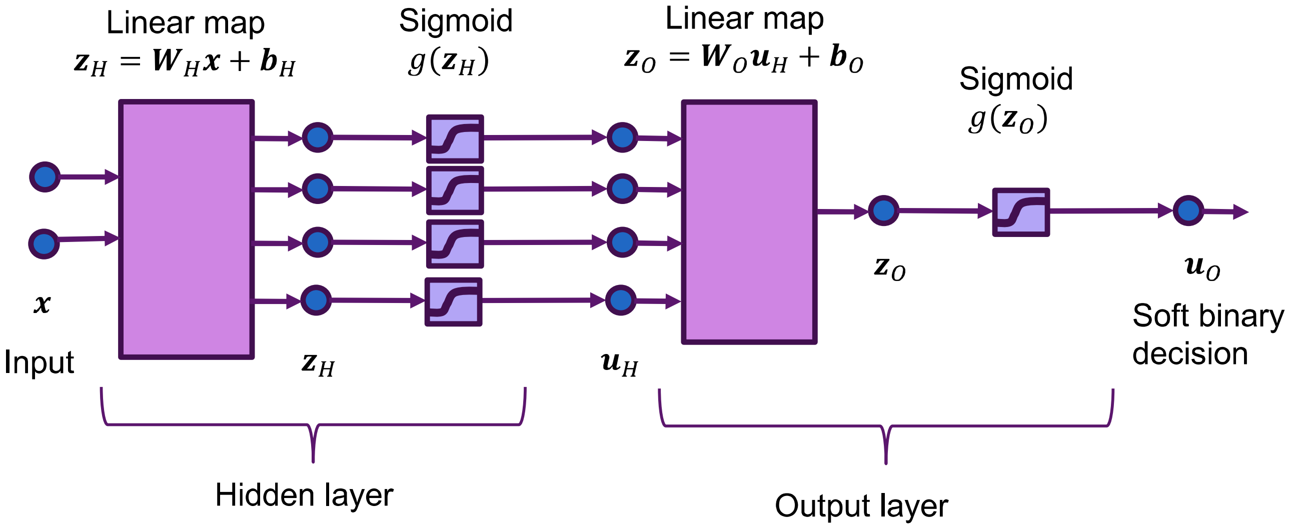


Example: a two layer binary classifier

- How to compute $\partial L/\partial W_O, \partial L/\partial b_O, \partial L/\partial W_H, \partial L/\partial b_H$?

$$L(\theta) = -\sum_{i=1}^{N} y_i \ln \hat{y}_i + (1-y_i)\ln(1-\hat{y}_i)$$

$$\hat{y}_i = \frac{1}{1+e^{-z_{O,i}}}; \quad \boldsymbol{z}_O = \boldsymbol{W}_O\boldsymbol{u}_H + \boldsymbol{b}_O; \boldsymbol{z}_H = \boldsymbol{W}_H\boldsymbol{u}_H + \boldsymbol{b}_H$$



Gradient for Output layer Parameters

$$L(\theta) = \sum L_i(\theta) \quad , \quad L_i(\theta) = -\sum y_i \ln \hat{y}_i + (1-y_i)\ln(1-\hat{y}_i)$$

$$\hat{y}_i = \frac{1}{1+e^{-Z_{o,i}}} \quad ; \quad \underset{(1\times1)}{Z_{o,i}} = \underset{(1\times4)}{W_o}\, \underset{(4\times1)}{u_{H,i}} + \underset{(1\times1)}{b_o} \; ;$$

$$u_{H,i} = \frac{1}{1+e^{-Z_{H,i}}} \qquad \underset{(4\times1)}{Z_{H,i}} = \underset{(4\times2)}{W_H}\, \underset{(2\times1)}{x_i} + \underset{(4\times1)}{b_H}$$

$$\frac{\partial L}{\partial \theta} = \sum \frac{\partial L_i}{\partial \theta} \qquad \theta = \{W_o, b_o, W_H, b_H\}$$

$$\underset{(1\times4)}{\frac{\partial L_i}{\partial W_o}} = \underset{(1\times1)}{\frac{\partial L_i}{\partial \hat{y}_i}} \cdot \underset{(1\times1)}{\frac{\partial \hat{y}_i}{\partial Z_{o,i}}} \cdot \underset{(1\times4)}{\frac{\partial Z_{o,i}}{\partial W_o}}$$

$$\frac{\partial Z_{o,i}}{\partial W_o} \Rightarrow \underset{(1\times4)}{u_{H,i}^T}$$

$$\frac{\partial \hat{y}_i}{\partial Z_{o,i}} \Rightarrow \frac{e^{-Z_{o,i}}}{(1+e^{-Z_{o,i}})^2}$$

$$\frac{\partial L_i}{\partial \hat{y}_i} \Downarrow \; -y_i\left(\frac{1}{\hat{y}_i}\right) - (1-y_i)\left(\frac{-1}{1-\hat{y}_i}\right)$$

$$\underset{(1\times1)}{\frac{\partial L_i}{\partial b_o}} = \frac{\partial L_i}{\partial \hat{y}_i} \cdot \frac{\partial \hat{y}_i}{\partial Z_{o,i}} \cdot \frac{\partial Z_{o,i}}{\partial b_o}$$

$$\frac{\partial Z_{o,i}}{\partial b_o} \Rightarrow 1$$

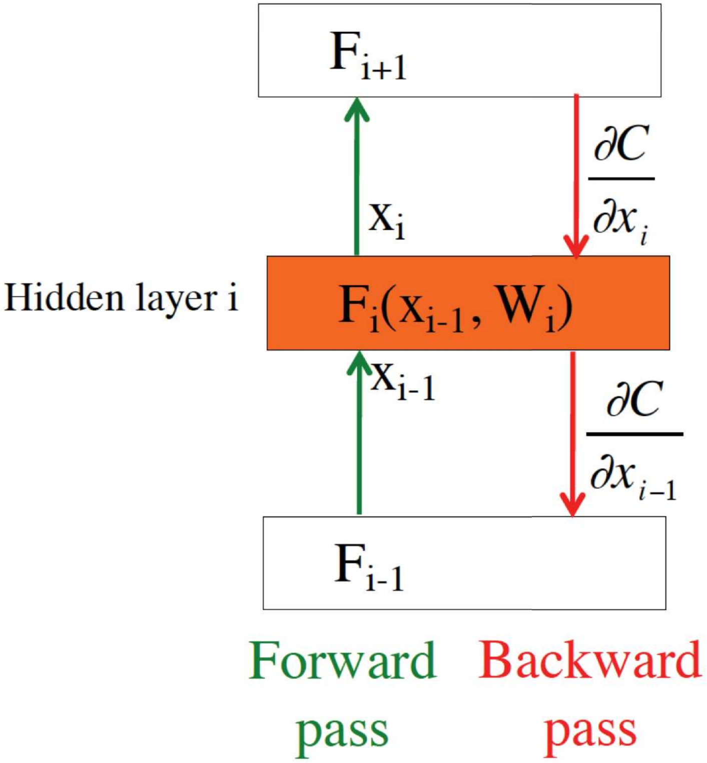
Gradient for Hideen layer Parameters

$$\underset{(4\times 2)}{\frac{\partial L_i}{\partial W_H}} = \begin{bmatrix} \frac{\partial L_i}{\partial w_{H,1}} \\ \frac{\partial L_i}{\partial w_{H,2}} \\ \frac{\partial L_i}{\partial w_{H,3}} \\ \frac{\partial L_i}{\partial w_{H,4}} \end{bmatrix} \qquad W_H = \begin{bmatrix} \underset{(1\times 2)}{w_{H,1}} \\ w_{H,2} \\ w_{H,3} \\ w_{H,4} \end{bmatrix}_{4\times 2} \qquad u_{H,i} = \begin{bmatrix} u_{H,1,i} \\ u_{H,2,i} \\ u_{H,3,i} \\ u_{H,4,i} \end{bmatrix}$$

$$\underset{(1\times 2)}{\frac{\partial L_i}{\partial w_{H,j}}} = \underbrace{\frac{\partial L_i}{\partial \hat{y}_i} \cdot \frac{\partial \hat{y}_i}{Z_{o,i}}}_{\text{See before}} \cdot \underbrace{\frac{\partial Z_{o,i}}{\partial u_{H,j,i}}}_{= \, w_{o,j}} \cdot \underbrace{\frac{\partial u_{H,j,i}}{\partial Z_{H,j,i}}}_{= \, \frac{e^{-Z_{H,j,i}}}{(1+e^{-Z_{H,j,i}})^2}} \cdot \underbrace{\frac{\partial Z_{H,j,i}}{\partial w_{H,j}}}_{= \, x_i^T \, (1\times 2)}$$

$$\frac{\partial L_i}{\partial b_{H,j}} = \overbrace{\frac{\partial L_i}{\partial \hat{y}_i} \frac{\partial \hat{y}_i}{\partial Z_{o,i}}} \frac{\partial Z_{o,i}}{\partial u_{H,j,i}} \frac{\partial u_{H,j,i}}{\partial Z_{H,j,i}} \underbrace{\frac{\partial Z_{H,j,i}}{\partial b_{H,j}}}_{= \, 1}$$

Backpropagation: layer i



- Layer i has two inputs (during training)

$$x_{i-1} \qquad \frac{\partial C}{\partial x_i}$$

- For layer i, we need the derivatives:

$$\frac{\partial F_i(x_{i-1}, w_i)}{\partial x_{i-1}} \qquad \frac{\partial F_i(x_{i-1}, w_i)}{\partial w_i}$$

- We compute the outputs

$$x_i = F_i(x_{i-1}, w_i)$$

$$\frac{\partial C}{\partial x_{i-1}} = \frac{\partial C}{\partial x_i} \cdot \frac{\partial F_i(x_{i-1}, w_i)}{\partial x_{i-1}}$$

- The weight update equation is:

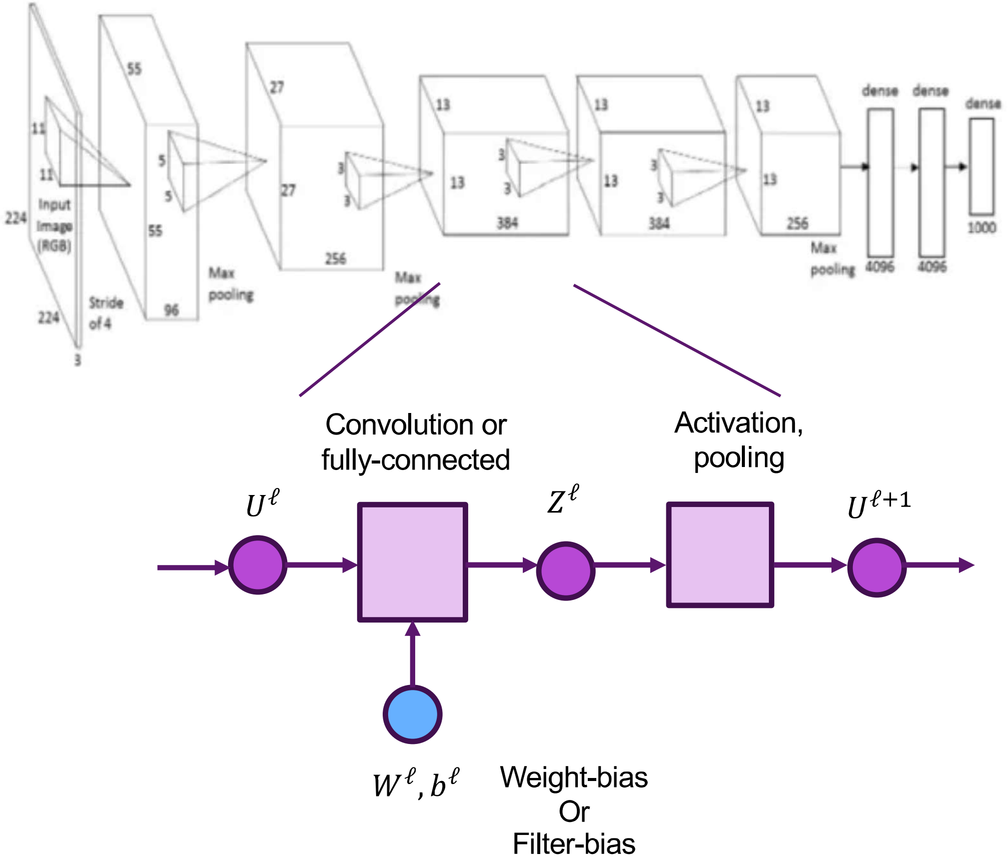
$$\frac{\partial C}{\partial w_i} = \frac{\partial C}{\partial x_i} \cdot \frac{\partial F_i(x_{i-1}, w_i)}{\partial w_i}$$

$$w_i^{k+1} \leftarrow w_i^k + \eta_t \frac{\partial E}{\partial w_i}$$

(sum over all training examples to get E)

From Fergus: https://cs.nyu.edu/~fergus/teaching/vision/2_neural_nets.pdf

Convolutional Networks



Convolution or fully-connected

Activation, pooling

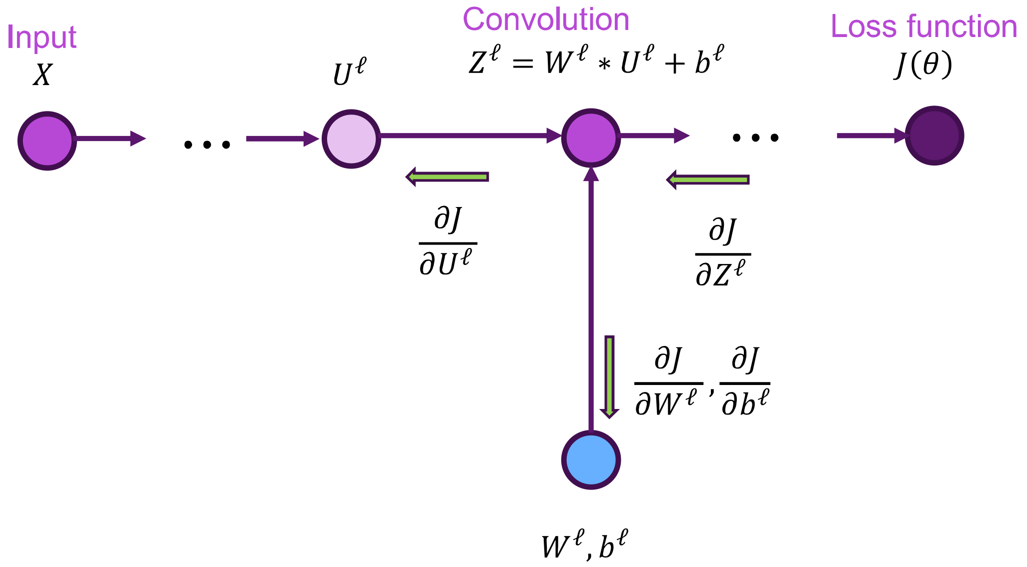
Weight-bias Or Filter-bias

$$Z[i_1, i_2, m] = \sum_{k_1=0}^{K_1-1} \sum_{k_2=0}^{K_2-1} \sum_{n=0}^{N_{in}-1} W[k_1, k_2, n, m] U[i_1 + k_1, i_2 + k_2, n] + b[m]$$

Back-Propagation in Convolutional Layers

- Convolutional layer in forward path
  $$Z^{\ell} = W^{\ell} * U^{\ell} + b^{\ell}$$
- During back-propagation:
  - Obtain gradient tensor from upstream layers $\frac{\partial J}{\partial Z^{\ell}}$
  - Need to compute downstream gradients:

$$\frac{\partial J}{\partial W^{\ell}}, \qquad \frac{\partial J}{\partial b^{\ell}}, \qquad \frac{\partial J}{\partial U^{\ell}}$$


Input
X
U^{ℓ}
Convolution
$Z^{\ell} = W^{\ell} * U^{\ell} + b^{\ell}$
Loss function
$J(\theta)$
…
…
$\frac{\partial J}{\partial U^{\ell}}$
$\frac{\partial J}{\partial Z^{\ell}}$
$\frac{\partial J}{\partial W^{\ell}}, \frac{\partial J}{\partial b^{\ell}}$
W^{ℓ}, b^{ℓ}

Gradient With Respect to Filter Weights

- Write convolution as:

$$Z[i_1, i_2, m] = \sum_{k_1=0}^{K_1-1} \sum_{k_2=0}^{K_2-1} \sum_{n=0}^{N_{in}-1} W[k_1, k_2, n, m] U[i_1 + k_1, i_2 + k_2, n] + b[m]$$

  - Drop layer index ℓ and sample index i
- Gradient wrt filter weights: $\frac{\partial Z[i_1,i_2,m]}{\partial W[k_1,k_2,n,m]} = U[i_1 + k_1, i_2 + k_2, n]$
  - *Note that the same filter is used for all pixels, need to sum gradients* $\frac{\partial Z[i_1,i_2,m]}{\partial W[k_1,k_2,n,m]}$ *for all* i_1, i_2:

$$\frac{\partial J}{\partial W[k_1,k_2,n,m]} = \sum_{i_1=1}^{N_1} \sum_{i_2=1}^{N_2} \frac{\partial Z[i_1,i_2,m]}{\partial W[k_1,k_2,n,m]} \frac{\partial J}{\partial Z[i_1,i_2,m]} = \sum_{i_1=1}^{N_1} \sum_{i_2=1}^{N_2} U[i_1 + k_1, i_2 + k_2, n] \frac{\partial J}{\partial Z[i_1,i_2,m]}$$

- Gradient wrt weights can be computed via convolution
  - Convolve input U with gradient tensor $\frac{\partial J}{\partial Z[i_1,i_2,m]}$
- Similar computations for gradients with respect to $\frac{\partial J}{\partial b}$
  - Homework! (for practice)

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- Gradient calculation
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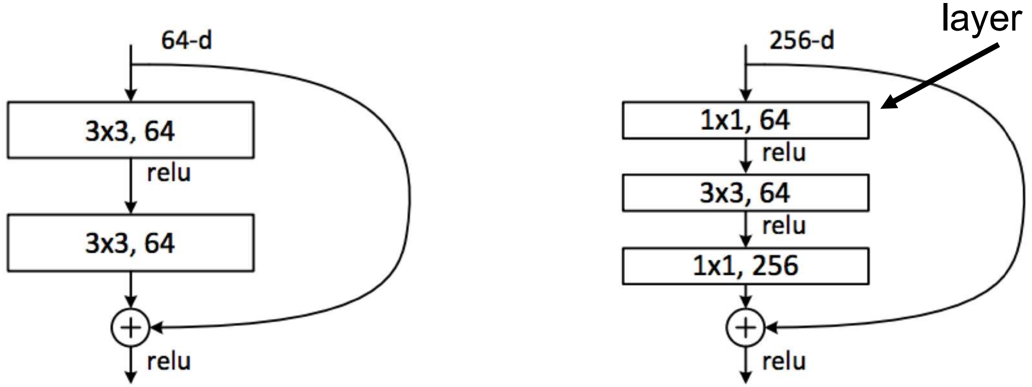
- Popular classification models and transfer learning

Some Important Extensions

- Residual connections
- Dense connections
- Dilated convolution

Residual Connections (ResNET)

- Really, really deep convnets don't train well
  - Gradient of final loss does not propagate back to earlier layers
- Key idea: introduce "pass through" into each layer for back propagation

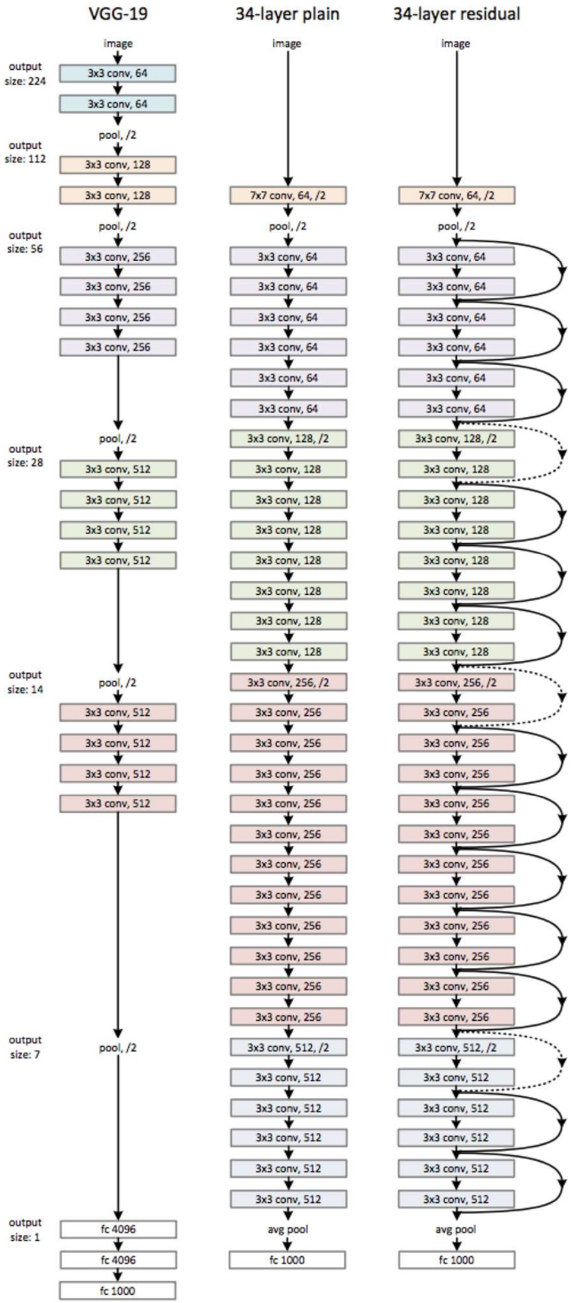


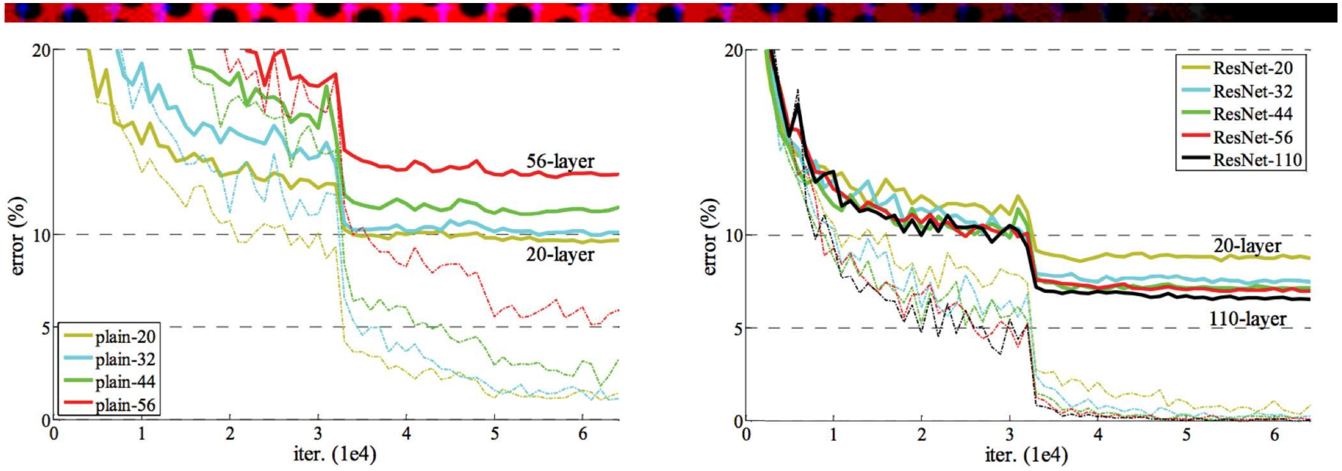
Bottleneck layer

Figure 5. A deeper residual function $\mathcal{F}$ for ImageNet. Left: a building block (on 56×56 feature maps) as in Fig. 3 for ResNet-34. Right: a "bottleneck" building block for ResNet-50/101/152.

He, Kaiming, Xiangyu Zhang, Shaoqing Ren, and Jian Sun. "Deep residual learning for image recognition." In *Proceedings of the IEEE conference on computer vision and pattern recognition*, pp. 770-778. 2016. http://openaccess.thecvf.com/content_cvpr_2016/papers/He_Deep_Residual_Learning_CVPR_2016_paper.pdf

Benefit of residual connection



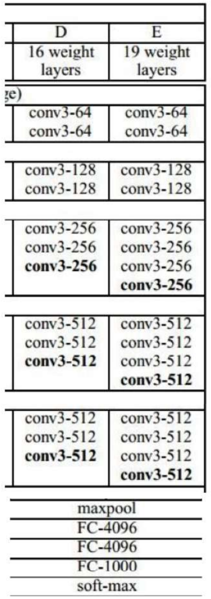


W/o residual layer: deeper networks perform worse even for the training data.
W/ residual layer: deeper networks perform better!

Using shortcut 2 is theoretically optimal
Demystifying ResNet
https://arxiv.org/abs/1611.01186

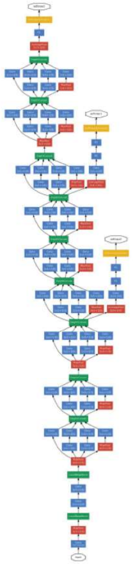
Revolution of Depth

Case Studies



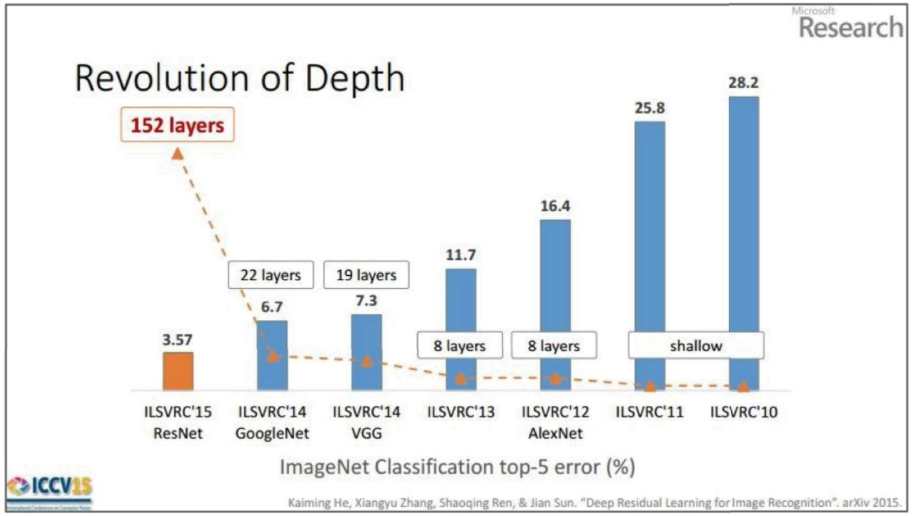
D	E
16 weight layers	19 weight layers
ge)	
conv3-64 conv3-64	conv3-64 conv3-64
conv3-128 conv3-128	conv3-128 conv3-128
conv3-256 conv3-256 **conv3-256**	conv3-256 conv3-256 conv3-256 **conv3-256**
conv3-512 conv3-512 **conv3-512**	conv3-512 conv3-512 conv3-512 **conv3-512**
conv3-512 conv3-512 **conv3-512**	conv3-512 conv3-512 conv3-512 **conv3-512**
maxpool	
FC-4096	
FC-4096	
FC-1000	
soft-max	

VGG
(2014)



GoogLeNet
(2014)

ResNet
(2015)



Kaiming He, Xiangyu Zhang, Shaoqing Ren, & Jian Sun. "Deep Residual Learning for Image Recognition". arXiv 2015.

From: http://cs231n.stanford.edu/slides/2016/winter1516_lecture8.pdf

A variation of residual connection: Concatenation (DenseNet)

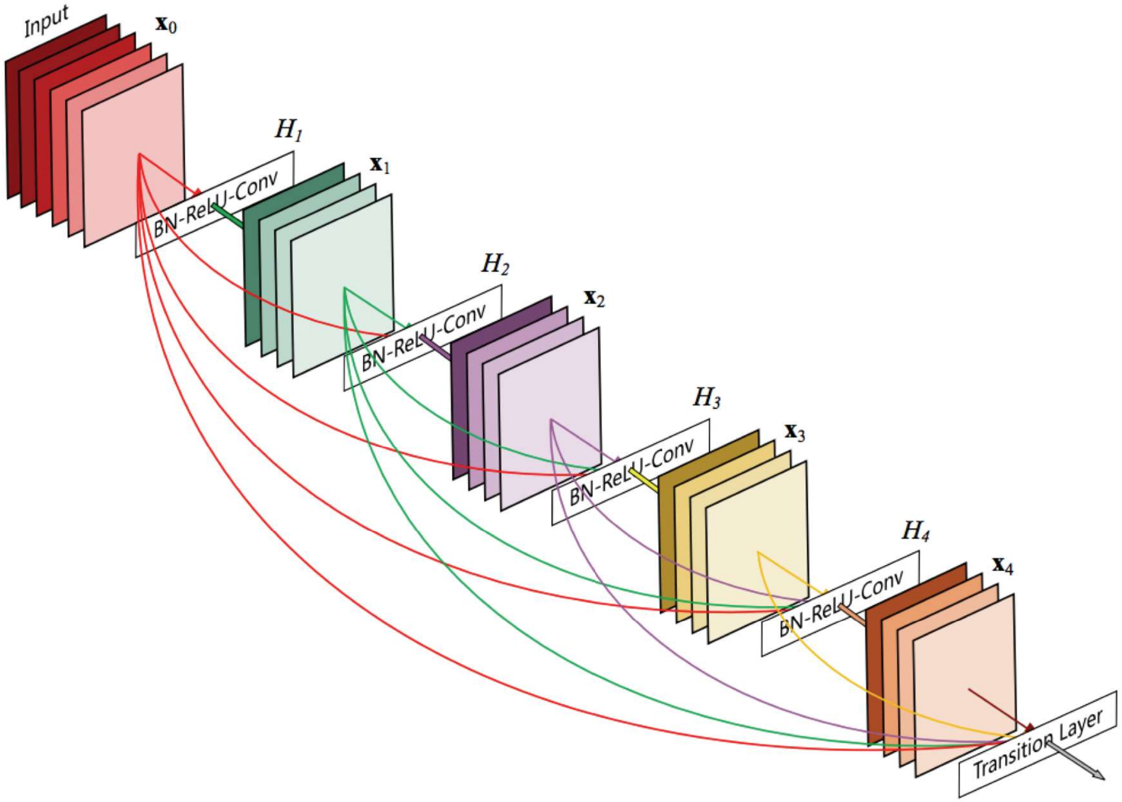


Figure 1: A 5-layer dense block with a growth rate of $k = 4$. Each layer takes all preceding feature-maps as input.

- Feature maps of all preceding layers are concatenated and used as input for the current layer.
- Facilitate gradient back propagation, as with residual connection
- Strengthen feature forward propagation and reuse

From: Huang, Gao, Zhuang Liu, Laurens Van Der Maaten, and Kilian Q. Weinberger. "Densely connected convolutional networks." In *Proceedings of the IEEE conference on computer vision and pattern recognition*, pp. 4700-4708. 2017.

Stacking Dense Blocks

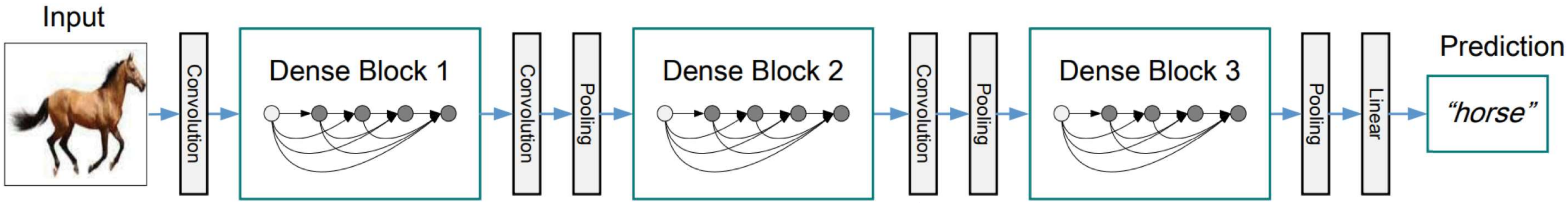
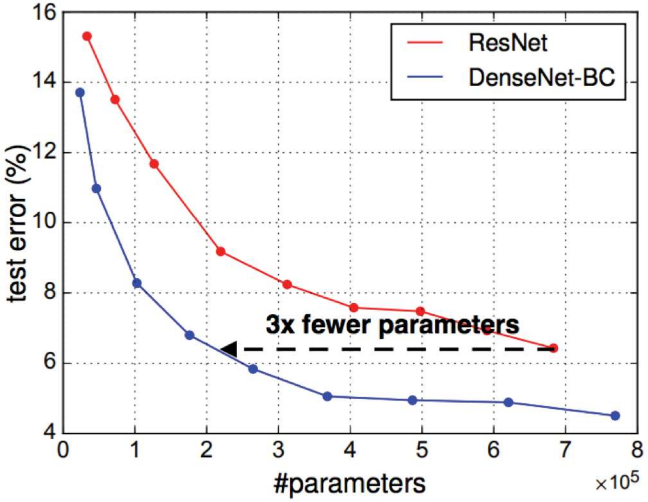


Figure 2: A deep DenseNet with three dense blocks. The layers between two adjacent blocks are referred to as transition layers and change feature-map sizes via convolution and pooling.

Use bottleneck layer (1x1 conv) to reduce the number of feature maps between blocks

- Can use fewer layers to achieve same performance as ResNET



From: Huang, Gao, Zhuang Liu, Laurens Van Der Maaten, and Kilian Q. Weinberger. "Densely connected convolutional networks." In *Proceedings of the IEEE conference on computer vision and pattern recognition*, pp. 4700-4708. 2017.

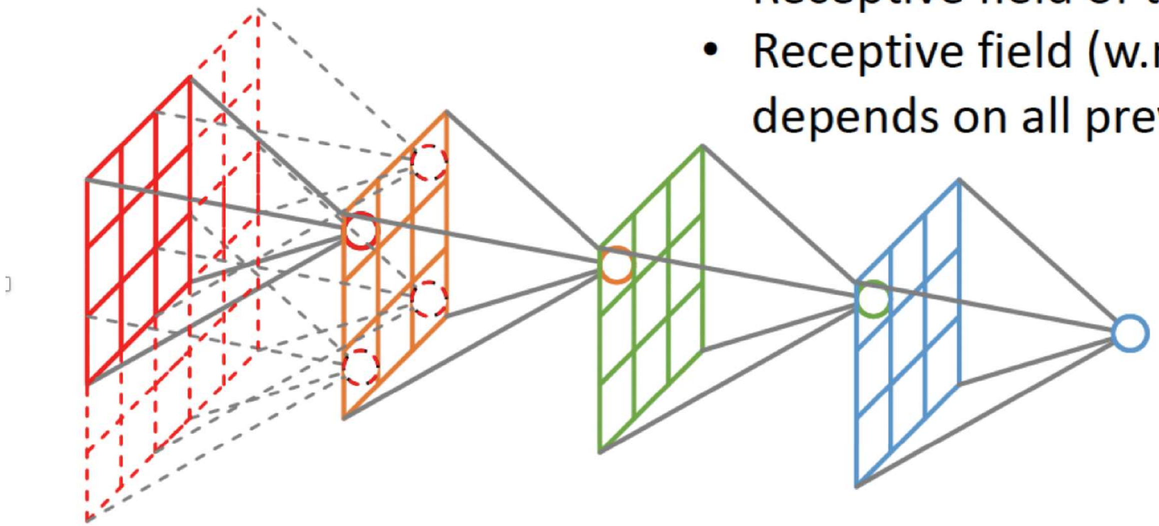
Dilated Convolution

- Large perceptive field is important to incorporate global information
- How to increase the perceptive field
  - Larger filter
  - More layers of small filters
  - Dilated conv.

Figure from Fergus: https://cs.nyu.edu/~fergus/teaching/vision/3_convnets.pdf

Non-Dialated Convoltuon

- Receptive field of the first layer is the filter size
- Receptive field (w.r.t. input image) of a deeper layer depends on all previous layers' filter size and strides



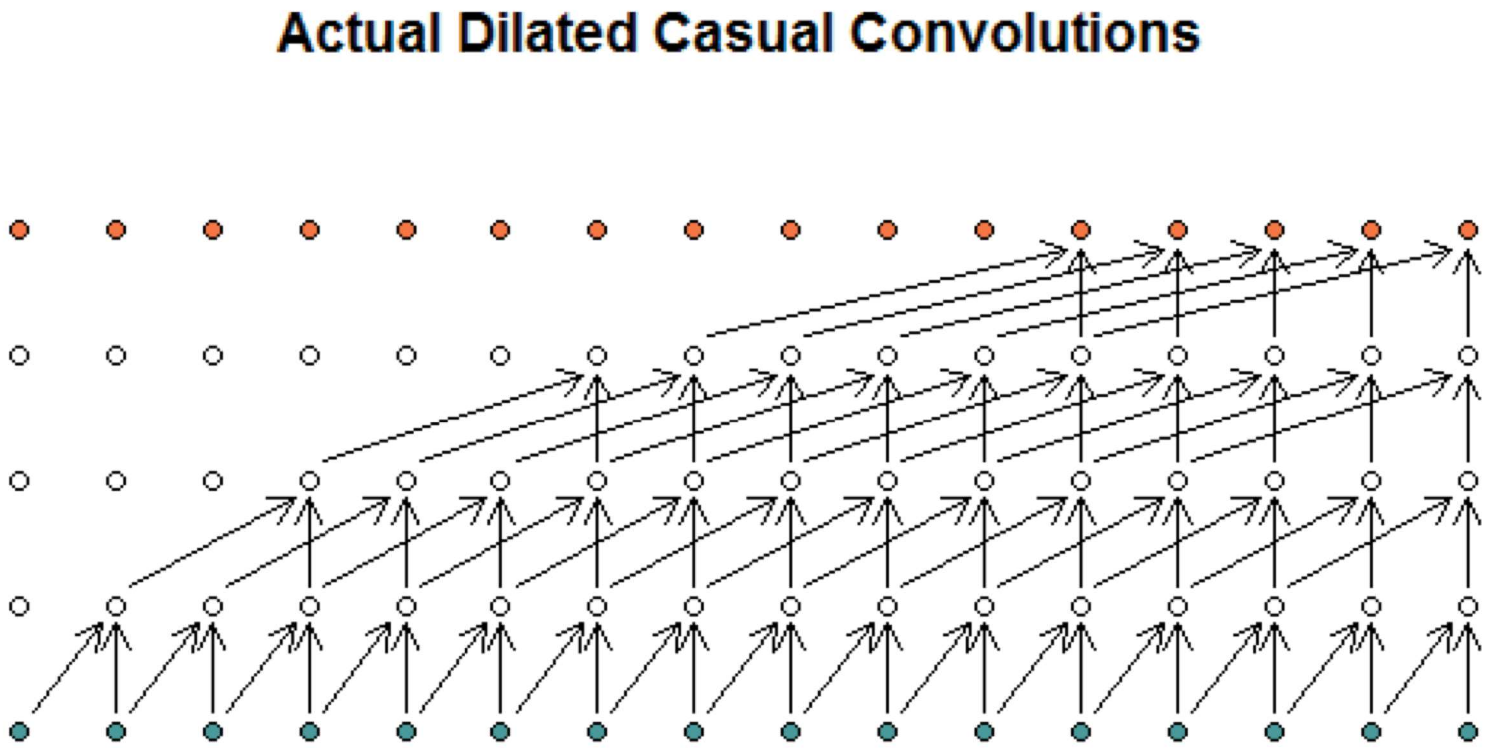
3x3

5x5

7x7

Filter size KxK, L-th layer: K+ 2L [K/2]
Grows linearly with # of layers

Dilated Conv in 1D



Dilation rate

4

3

2

1

Figure from https://i.stack.imgur.com/RmJSu.png

Multiscale processing while maintaining original resolution!
Used for speech waveform generation.

Dilated Conv. In 2D

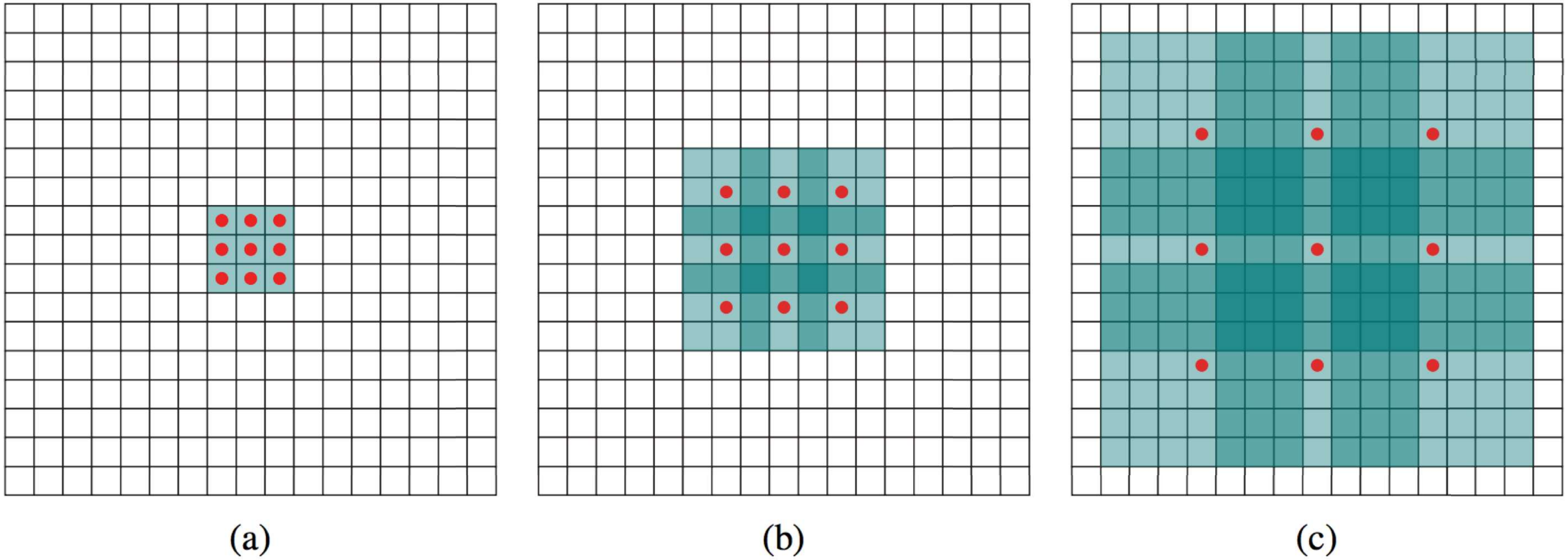


Figure 1: Systematic dilation supports exponential expansion of the receptive field without loss of resolution or coverage. (a) F_1 is produced from F_0 by a 1-dilated convolution; each element in F_1 has a receptive field of 3×3. (b) F_2 is produced from F_1 by a 2-dilated convolution; each element in F_2 has a receptive field of 7×7. (c) F_3 is produced from F_2 by a 4-dilated convolution; each element in F_3 has a receptive field of 15×15. The number of parameters associated with each layer is identical. The receptive field grows exponentially while the number of parameters grows linearly.

Yu, Fisher, and Vladlen Koltun. "Multi-scale context aggregation by dilated convolutions." *arXiv preprint arXiv:1511.07122* (2015).

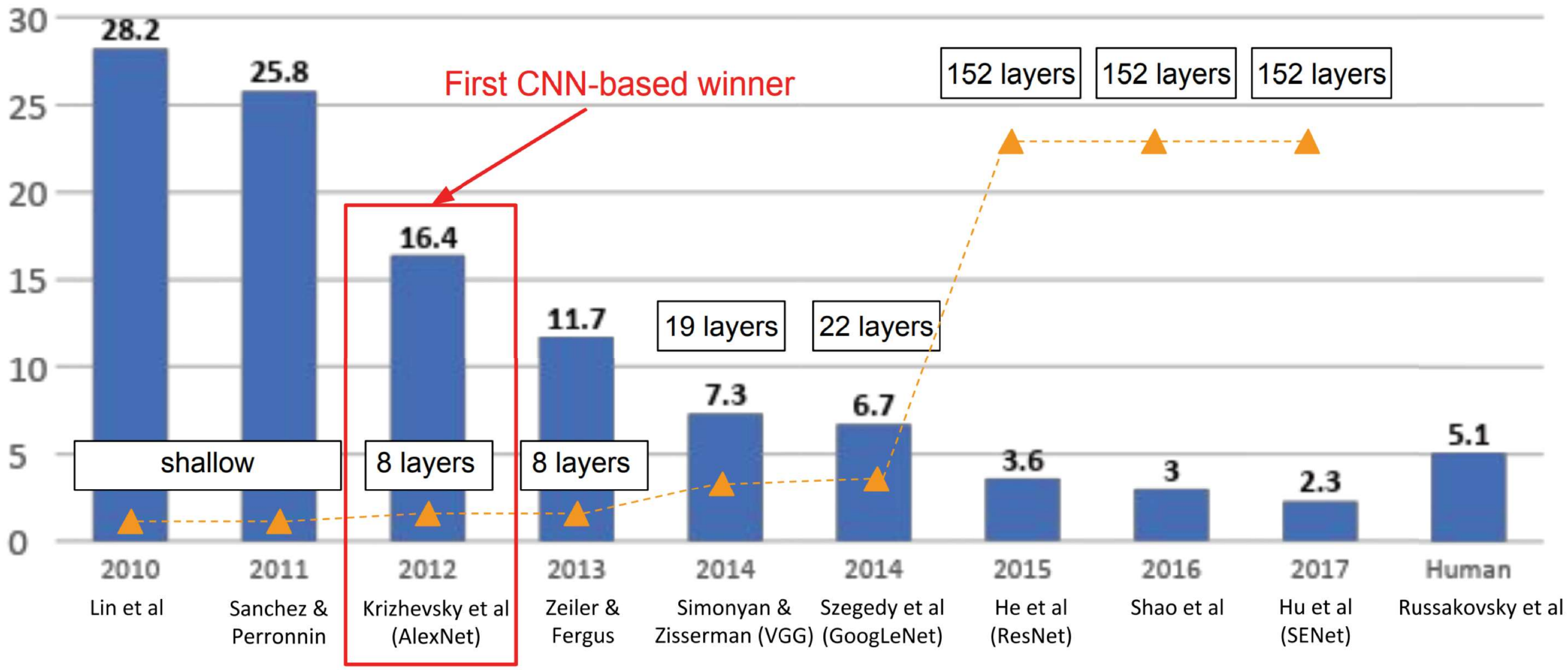
Multiscale processing while maintaining original resolution!
Good for dense prediction: image to image

Outline (Part II)

- Neural Nets and Conv Nets and Model Training (Review)
- Gradient calculation
- Some important extensions of conv. layers
- ➡ Popular classification models and transfer learning

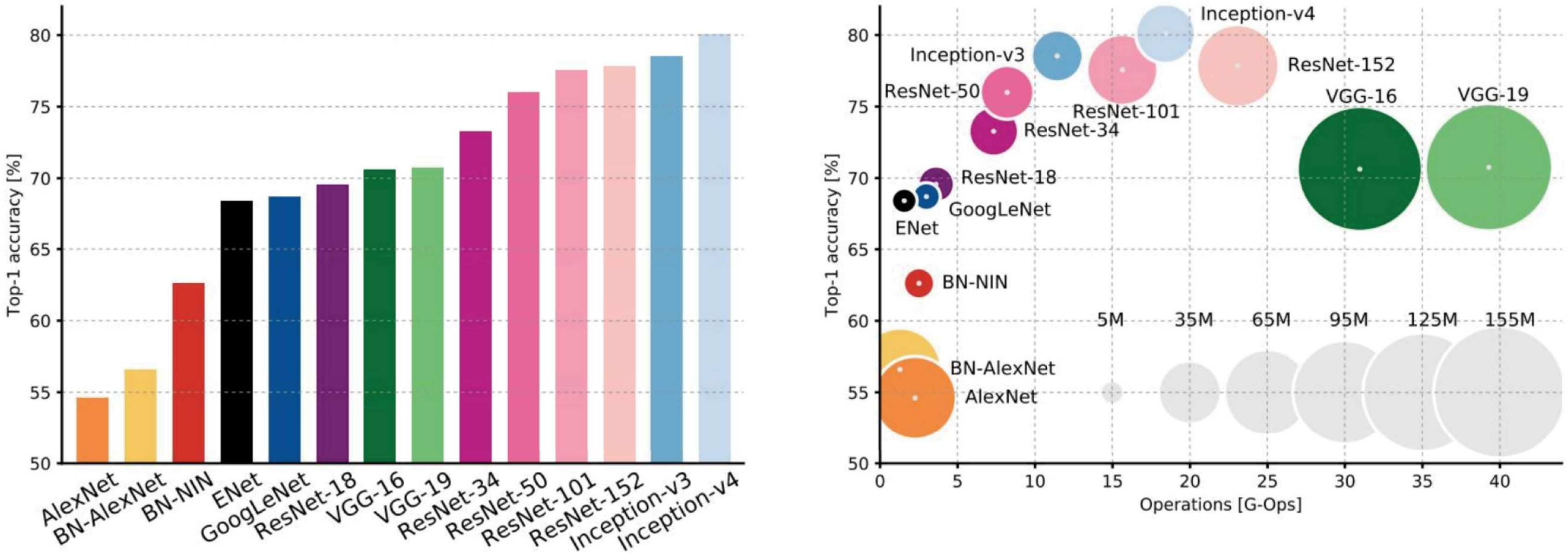
Well-Known Models

ImageNet Large Scale Visual Recognition Challenge (ILSVRC) winners



http://cs231n.stanford.edu/slides/2018/cs231n_2018_lecture09.pdf

Performance vs. Complexity



An Analysis of Deep Neural Network Models for Practical Applications, 2017.

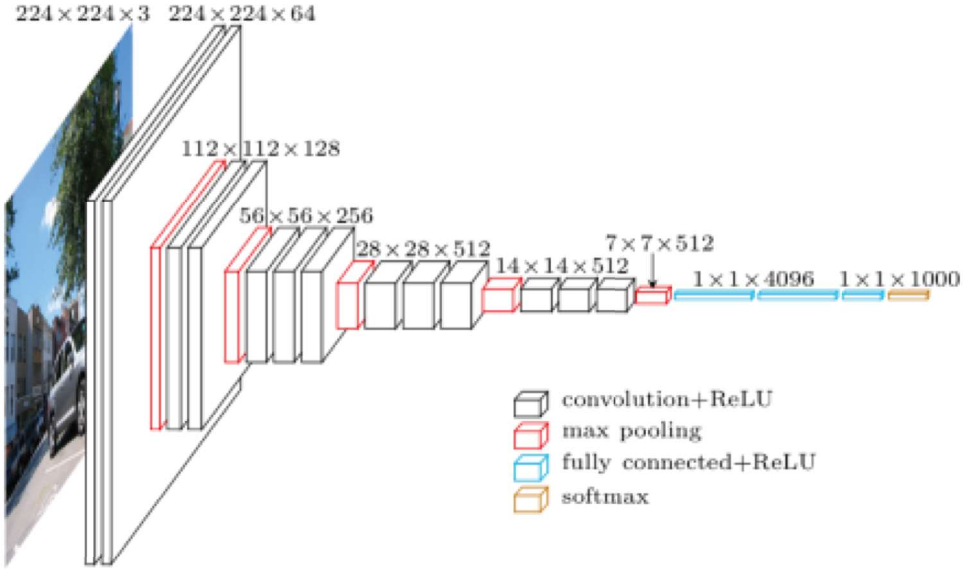
http://cs231n.stanford.edu/slides/2018/cs231n_2018_lecture09.pdf

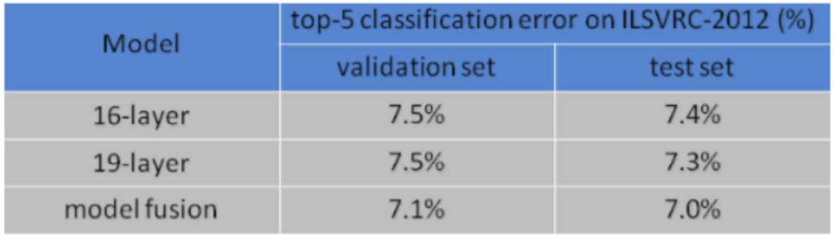
Transfer Learning

- For image classification or other applications, training from scratch takes tremendous resources
- Instead, can refine the VGG or other well trained networks
- Can use VGG convolutional layers, and retrain only the fully connected layers (possibly some later convolutional layers) for different problems.
- Or can use VGG conv layers as the “initial model” and further refine.
- Computer Assignment (optional): load VGG model, and fix all conv. layers, retrain additional fully connected layers for different image classification tasks, try and compare different training tricks
  - Using Flickr API (courtesy of Sundeep Rangan) for downloading images for a given keyword

VGG16

- From the Visual Geometry Group
  - Oxford, UK
- Won ImageNet ILSVRC-2014
- Remains a very good network
- Lower layers are often used as feature extraction layers for other tasks





Model	top-5 classification error on ILSVRC-2012 (%)	
	validation set	test set
16-layer	7.5%	7.4%
19-layer	7.5%	7.3%
model fusion	7.1%	7.0%

http://www.robots.ox.ac.uk/~vgg/research/very_deep/

K. Simonyan, A. Zisserman
Very Deep Convolutional Networks for Large-Scale Image Recognition
arXiv technical report, 2014

Transfer Learning

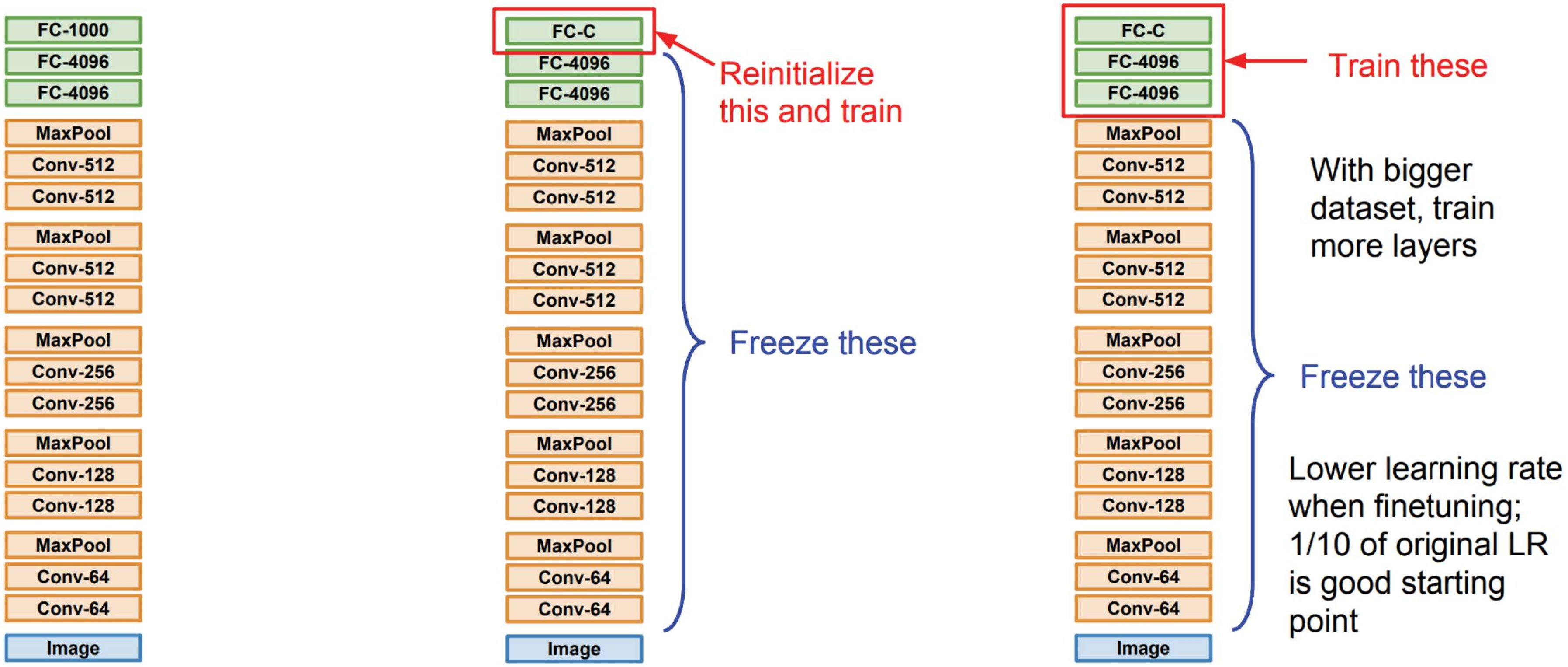
Transfer Learning with CNNs

Donahue et al, “DeCAF: A Deep Convolutional Activation Feature for Generic Visual Recognition”, ICML 2014
Razavian et al, “CNN Features Off-the-Shelf: An Astounding Baseline for Recognition”, CVPR Workshops 2014

1. Train on Imagenet

2. Small Dataset (C classes)

3. Bigger dataset



Reinitialize this and train

Freeze these

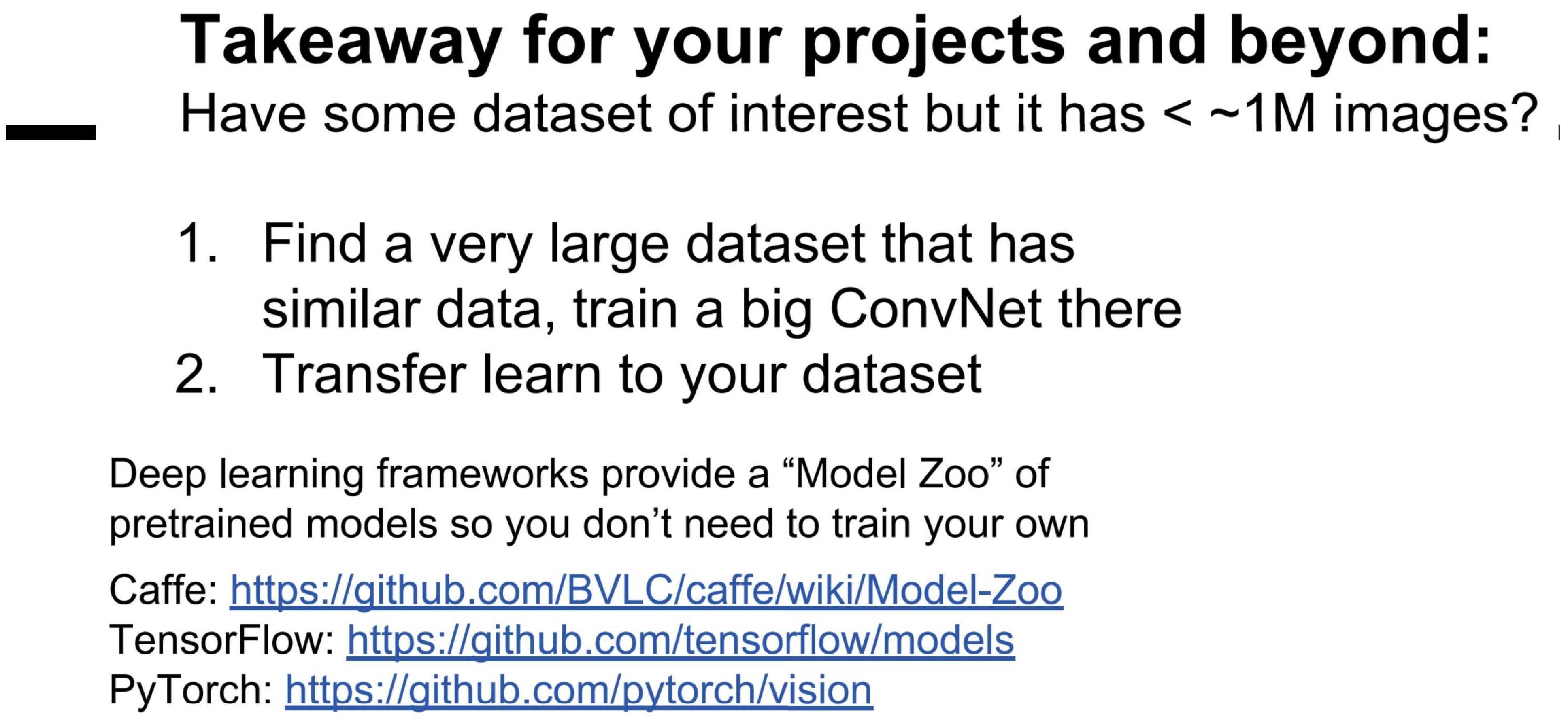
Train these

With bigger dataset, train more layers

Freeze these

Lower learning rate when finetuning; 1/10 of original LR is good starting point

From http://cs231n.stanford.edu/slides/2018/cs231n_2018_lecture07.pdf



Takeaway for your projects and beyond:

Have some dataset of interest but it has < ~1M images?

1. Find a very large dataset that has similar data, train a big ConvNet there
2. Transfer learn to your dataset

Deep learning frameworks provide a “Model Zoo” of pretrained models so you don’t need to train your own

Caffe: https://github.com/BVLC/caffe/wiki/Model-Zoo
TensorFlow: https://github.com/tensorflow/models
PyTorch: https://github.com/pytorch/vision

From http://cs231n.stanford.edu/slides/2018/cs231n_2018_lecture07.pdf

Summary

- Gradient Calculation through backpropagation
  - Tensor gradient, Tensor chain rule
- Residual and dense connections to ease gradient back propagation
- Dilated convolution for increasing perceptive field
- Transfer learning

Recommended Readings

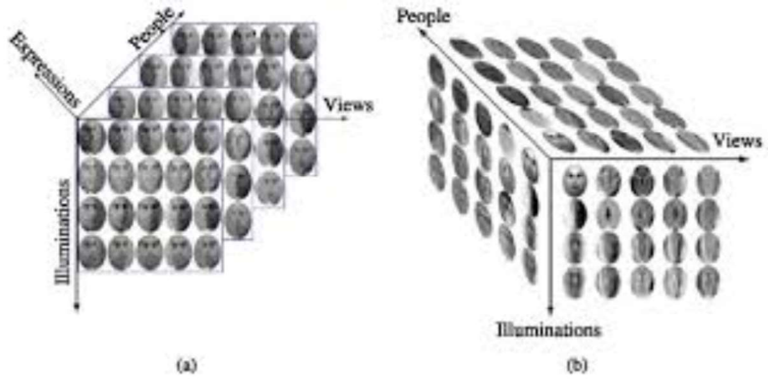
- For tensor gradient calculation and backpropagation:
  - Lecture material of Sundeep Rangan (see Appendix)
  - https://github.com/sdrangan/introml/blob/master/sequence.md
  - Unit on neural net and convolution networks
- For vision applications:
  - Stanford course by Feifei Li, et al: CS231n: Convolutional Neural Networks for Visual Recognition, Spring 2018: http://cs231n.stanford.edu/
  - Popular network case studies: http://cs231n.stanford.edu/slides/2018/cs231n_2018_lecture09.pdf
  - Learning GPU and PyTorch and TensorFlow: http://cs231n.stanford.edu/slides/2018/cs231n_2018_lecture08.pdf
  - Video available for previous offerings:
    - https://www.youtube.com/playlist?list=PL3FW7Lu3i5JvHM8ljYj-zLfQRF3EO8sYvhttps://www.youtube.com/playlist?list=PL3FW7Lu3i5JvHM8ljYj-zLfQRF3EO8sYv

Appendix: Tensor Gradient Calculation (Optional)

- Tensor definition
- Tensor gradients
- Tensor gradient chain rule
- Backpropagation
- Forward and backward pass

What is a Tensor?

- A multi-dimensional array
- Examples:
  - 2D: A grayscale image [height x width]
  - 3D: A color image [height x width x rgb]
  - 4D: A collection of images [height x width x rgb x image number]
- Like numpy ndarray
- Basic unit in tensorflow
- Rank or order = Number of dimensions
  - Note: Rank has different meaning in linear algebra



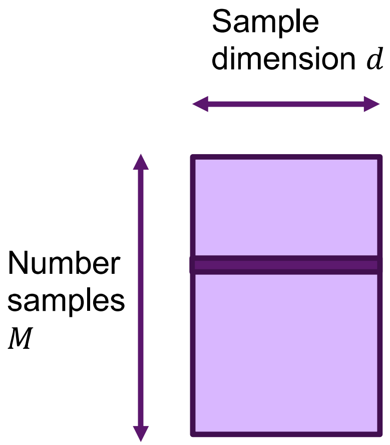
Indexing Tensors

- Suppose $\boldsymbol{X}$ is a tensor of order N
- Index with a multi-index $X[i_1, \dots, i_N]$
  - May also use subscript: $\mathrm{X}_{i_1,\dots,i_N}$

- Example: Suppose $\boldsymbol{X}$ = collection of images [height x width x rgb x image number]
  - What is the order of $\boldsymbol{X}$?
  - $\boldsymbol{X}$ is an order 4 tensor
  - $X[100,150,1,30] =$ pixel (100,150) for color channel 1 (green) on image 30
- If $i_1 \in \{0, \dots, d_1 - 1\}, i_2 \in \{0, \dots, d_2 - 1\}$, ... then total number of elements = $d_1 d_2 \dots d_N$

Tensors and Neural Networks

- Need to be consistent with indexing
- For a single input x:
  - Input x: vector of dimension d
  - Hidden layer: z_H, u_H: vectors of dimension N_H
  - Outputs: z_O: dimension K
- A batch of inputs with M samples:
  - Input x: Matrix of dimension $M \times d$
  - Hidden layer: z_H, u_H: vectors of dimension $M \times N_H$
  - Outputs: z_O: dimension $M \times K$
- Can generalize to other shapes of input



Gradient for Tensor Inputs & Outputs

- How do we consider general tensor inputs and outputs?
- General setting: $\boldsymbol{y} = f(\boldsymbol{x})$
  - $\boldsymbol{x}$ is a tensor of order N, $\boldsymbol{y}$ is a tensor of order M
- Gradient tensor: A tensor of order $N + M$

$$\left[\frac{\partial f(x)}{\partial x}\right]_{i_1,\ldots i_M, j_1,\ldots,j_N} = \frac{\partial f_{i_1,\ldots,i_M}(x)}{\partial x_{j_1,\ldots,j_N}}$$

  - Tensor has the derivative of every output with respect to every input.
- Ex: $\boldsymbol{x}$ has shape (50,30), $\boldsymbol{y}$ has shape (10,20,40)
  - $\frac{\partial f(x)}{\partial x}$ has shape (10,20,40,50,30)
  - 10(20)(40)(50)(30) = 1.2(10)^7 elements

Gradient Examples 1 and 2

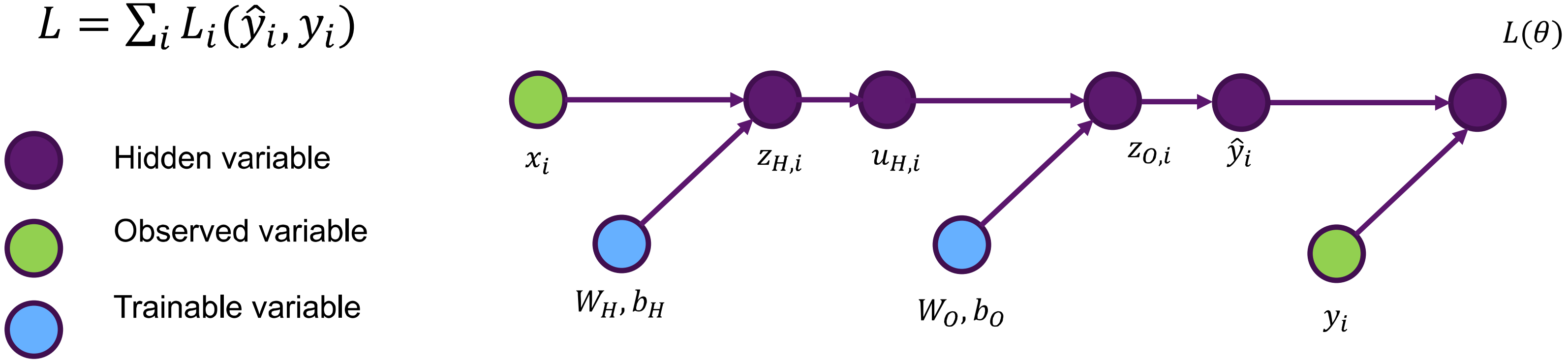
- Example 1: $f(w) = (w_1 w_2, w_1^2 + w_3^3) = \left(f_1(w), f_2(w)\right)$
  - 2 outputs, 3 inputs.
  - Gradient tensor is 2×3

$$\frac{\partial f(w)}{\partial w} = \begin{bmatrix} \frac{\partial f_1(w)}{\partial w} \\ \frac{\partial f_1(w)}{\partial w} \end{bmatrix} = \begin{bmatrix} \frac{\partial f_1(w)}{\partial w_1} & \frac{\partial f_1(w)}{\partial w_2} & \frac{\partial f_1(w)}{\partial w_3} \\ \frac{\partial f_2(w)}{\partial w_1} & \frac{\partial f_2(w)}{\partial w_2} & \frac{\partial f_2(w)}{\partial w_3} \end{bmatrix} = \begin{bmatrix} w_2 & w_1 & 0 \\ 2w_1 & 0 & 3w_3^2 \end{bmatrix}$$

- Example 2: $z = f(w) = Aw$, A is $M\times N, w\ is\ N\times1$
  - M outputs, N inputs: $z_i = \sum_{j=1}^{N} A_{ij} w_j$
  - Gradient components: $\frac{\partial z_i}{\partial w_j} = A_{ij}$
  - $\frac{\partial f(w)}{\partial w} =?$
  - $\frac{\partial f(w)}{\partial w} = \mathrm{A}$

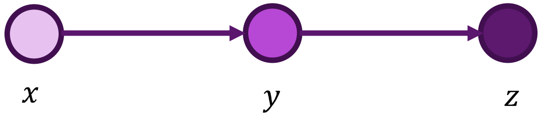
Computation Graph & Forward Pass

- Neural network loss function can be computed via a computation graph
- Sequence of operations starting from measured data and parameters
- Loss function computed via a forward pass in the computation graph
  - $z_{H,i} = W_H x_i + b_H$
  - $u_{H,i} = g_{act}(z_{H,i})$
  - $z_{O,i} = W_O u_{H,i} + b_O$
  - $\hat{y}_i = g_{out}(z_{O,i})$
  - $L = \sum_i L_i(\hat{y}_i, y_i)$



Chain Rule

- How do we compute gradient?
- Consider a three node computation graph:
  - $y = h(x), \ z = g(y)$
  - So $z = f(x) = g(h(x))$
  - What is $\frac{\partial z}{\partial x}$?



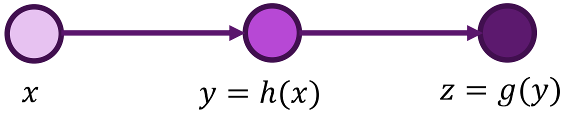
- If variables were scalars, we could compute gradients via chain rule:

$$\frac{\partial z}{\partial x} = \frac{\partial f(x)}{\partial x} = \frac{\partial g(y)}{\partial y} \frac{\partial h(x)}{\partial x}$$

- What happens for tensors?

Tensor Chain Rule

- Consider Tensor case:
  - x has shape $(n_1, \ldots, n_N)$,
  - y has shape $(m_1, \ldots, m_M)$
  - z has shape $(r_1, \ldots, r_R)$



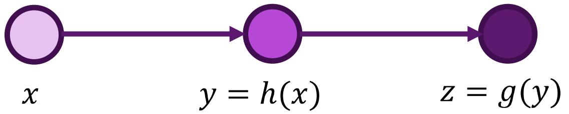
- First, compute gradient tensors between input and output of each node:
  - $\frac{\partial g(y)}{\partial y}$ has shape $(r_1, \ldots, r_R, m_1, \ldots, m_M)$
  - $\frac{\partial h(x)}{\partial x}$ has shape $(m_1, \ldots, m_M, n_1, \ldots, n_N)$
- Next, apply tensor chain rule:

  $\frac{\partial z}{\partial x}$ has shape $(r_1, \ldots, r_R, n_1, \ldots, n_N)$: How to compute this?

$$\frac{\partial z_{i_1,\ldots,i_R}}{\partial x_{j_1,\ldots,j_N}} = \frac{\partial f_{i_1,\ldots,i_R}(x)}{\partial x_{j_1,\ldots,j_N}} = \underbrace{\sum_{k_1=1}^{m_1} \cdots \sum_{k_M=1}^{m_M}}_{\text{Sum over indices of } \boldsymbol{y}} \frac{\partial g_{i_1,\ldots,i_R}(y)}{\partial y_{k_1,\ldots k_M}} \frac{\partial h_{k_1,\ldots k_M}(x)}{\partial x_{j_1,\ldots j_N}}$$

$$\frac{\partial z}{\partial x} = \frac{\partial f(x)}{\partial x} = \left\langle \frac{\partial g(y)}{\partial y}, \frac{\partial h(x)}{\partial x} \right\rangle$$

Tensor Chain Rule Summary



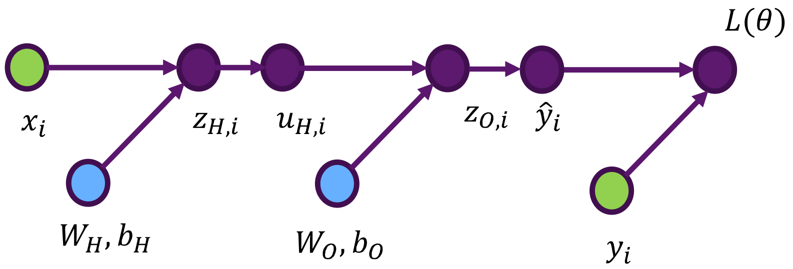
- It is all about keeping track of indices!
- Step 1. Decide on some indexing
  - $x_{j_1,\dots j_N}$, $y_{k_1,\dots k_M}$, $z_{i_1,\dots,i_R}$
- Step 2. Compute all partial derivatives $\frac{\partial g_{i_1,\dots,i_R}(y)}{\partial y_{k_1,\dots k_M}}$ and $\frac{\partial h_{k_1,\dots k_M}(x)}{\partial x_{j_1,\dots j_N}}$
- Step 3. Use tensor chain rule

$$\frac{\partial z_{i_1,\dots,i_R}}{\partial x_{j_1,\dots,j_N}} = \frac{\partial f_{i_1,\dots,i_R}(x)}{\partial x_{j_1,\dots j_N}} = \sum_{k_1=1}^{m_1} \cdots \sum_{k_M=1}^{m_M} \frac{\partial g_{i_1,\dots,i_R}(y)}{\partial y_{k_1,\dots k_M}} \frac{\partial h_{k_1,\dots k_M}(x)}{\partial x_{j_1,\dots j_N}}$$

- Sometimes write with tensor inner product

$$\frac{\partial z}{\partial x} = \frac{\partial f(x)}{\partial x} = \left\langle \frac{\partial g(y)}{\partial y}, \frac{\partial h(x)}{\partial x} \right\rangle$$

Gradients on a Computation Graph



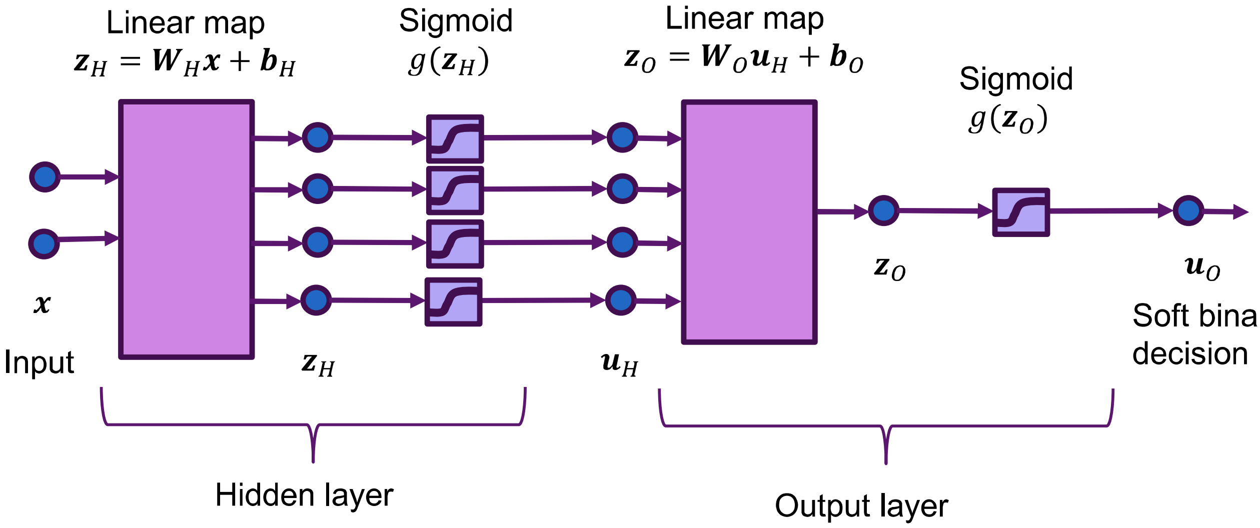
- Backpropagation: Compute gradients backwards
  - Use tensor dot products and chain rule
- First compute all derivatives of all the variables
  - $\partial L/\partial z_O = \langle \partial L/\partial \hat{y} \,, \partial \hat{y} \,/ \partial z_O \rangle$
  - $\partial L/\partial u_H = \langle \partial L/\partial z_O \,, \partial z_O/\partial u_H \rangle$
  - $\partial L/\partial z_H = \langle \partial L/\partial u_H \,, \partial u_H/\partial z_H \rangle$
  - ($\partial \hat{y} \,/ \partial z_O$ and $\partial u_H/\partial z_H$ is element wise)
- Then compute gradient of parameters:
  - $\partial L/\partial W_O = \langle \partial L/\partial z_O \,, \partial z_O/\partial W_O \rangle$
  - $\partial L/\partial b_O = \langle \partial L/\partial z_O \,, \partial z_O/\partial b_O \rangle$
  - $\partial L/\partial W_H = \langle \partial L/\partial z_H \,, \partial z_H/\partial W_H \rangle$
  - $\partial L/\partial b_H = \langle \partial L/\partial z_H \,, \partial z_H/\partial b_H \rangle$
  -

Example: Last layer of a Binary Classifier

- How to compute $\partial L/\partial W_O, \partial L/\partial b_O$?

$$L(\theta) = -\sum_{i=1}^{N} y_i \ln \hat{y}_i + (1-y_i)\ln(1-\hat{y}_i)$$

$$\hat{y}_i = \frac{1}{1+e^{-z_{O,i}}}; \quad \boldsymbol{z}_O = \boldsymbol{W}_O \boldsymbol{u}_H + \boldsymbol{b}_O$$



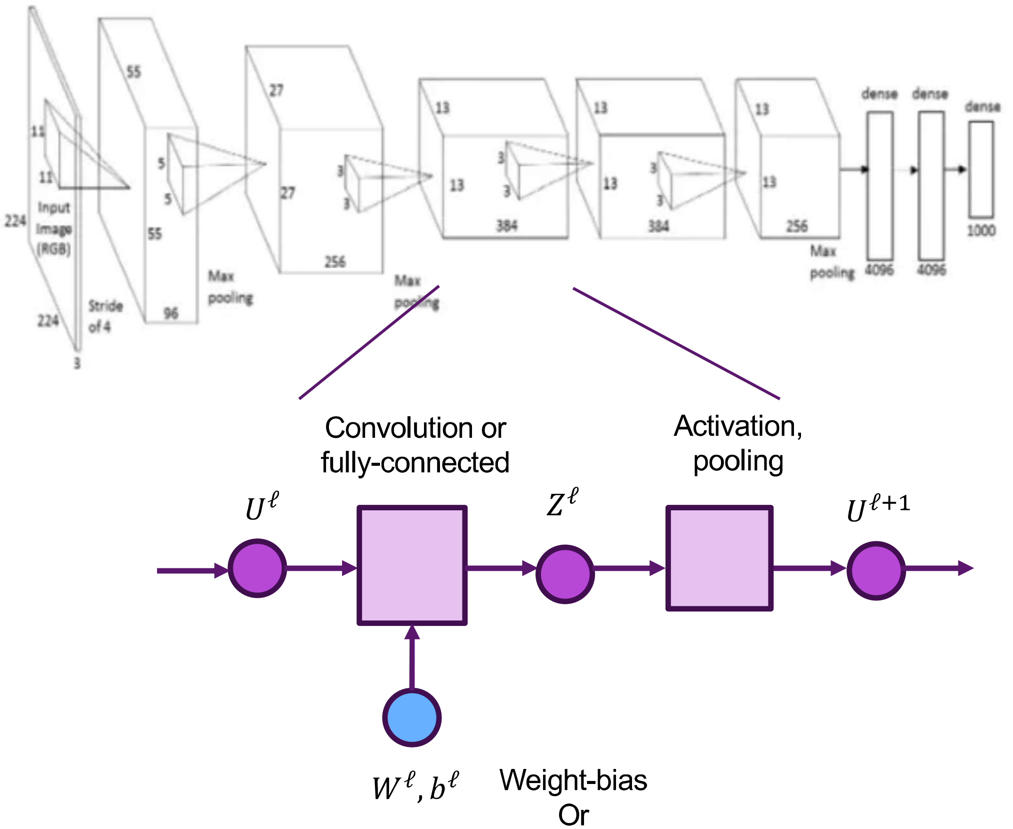
This part could be convolutional layers

Example: Last layer of a Binary Classifier

- Go through on the board

Indexing Multi-Layer Networks

- Similar to two-layer NNs
  - But must keep track of layers
- Consider batch of image inputs:
  - $X[i,j,k,n]$, (sample,row,col,channel)
- Input tensor at layer ℓ:
  - $U^{\ell}[i,j,k,n]$ for convolutional layer
  - $U^{\ell}[i,n]$ for fully connected layer
- Output tensor from linear transform:
  - $Z^{\ell}[i,j,k,n]$ or $Z^{\ell}[i,n]$
- Output tensor after activation / pooling:
  - $U^{\ell+1}[i,j,k,n]$ or $U^{\ell+1}[i,n]$



Convolution or fully-connected

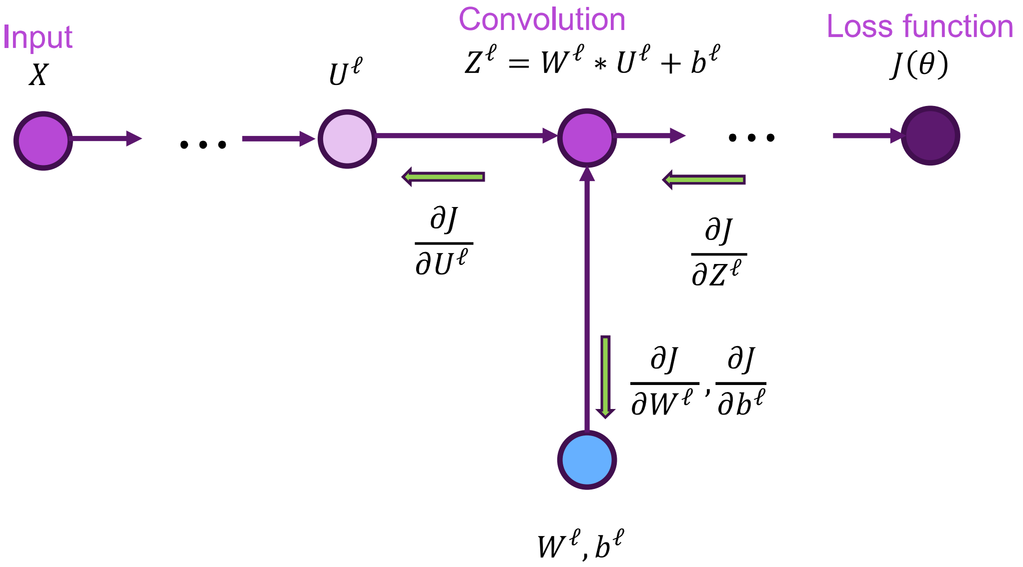
Activation, pooling

Weight-bias Or Filter-bias

Back-Propagation in Convolutional Layers

- Convolutional layer in forward path
  $$Z^{\ell} = W^{\ell} * U^{\ell} + b^{\ell}$$
- During back-propagation:
  - Obtain gradient tensor from upstream layers $\frac{\partial J}{\partial Z^{\ell}}$
  - Need to compute downstream gradients:

$$\frac{\partial J}{\partial W^{\ell}}, \qquad \frac{\partial J}{\partial b^{\ell}}, \qquad \frac{\partial J}{\partial U^{\ell}}$$


Input
X
U^{ℓ}
Convolution
$Z^{\ell} = W^{\ell} * U^{\ell} + b^{\ell}$
Loss function
$J(\theta)$
$\frac{\partial J}{\partial U^{\ell}}$
$\frac{\partial J}{\partial Z^{\ell}}$
$\frac{\partial J}{\partial W^{\ell}}, \frac{\partial J}{\partial b^{\ell}}$
W^{ℓ}, b^{ℓ}

Gradient With Respect to Filter Weights

- Write convolution as:

$$Z[i_1, i_2, m] = \sum_{k_1=0}^{K_1-1} \sum_{k_2=0}^{K_2-1} \sum_{n=0}^{N_{in}-1} W[k_1, k_2, n, m] U[i_1 + k_1, i_2 + k_2, n] + b[m]$$

  - Drop layer index ℓ and sample index i
- Gradient wrt filter weights: $\frac{\partial Z[i_1,i_2,m]}{\partial W[k_1,k_2,n,m]} = U[i_1 + k_1, i_2 + k_2, n]$
  - *Note that the same filter is used for all pixels, need to sum gradients* $\frac{\partial Z[i_1,i_2,m]}{\partial W[k_1,k_2,n,m]}$ *for all* i_1, i_2:

$$\frac{\partial J}{\partial W[k_1,k_2,n,m]} = \sum_{i_1=1}^{N_1} \sum_{i_2=1}^{N_2} \frac{\partial Z[i_1,i_2,m]}{\partial W[k_1,k_2,n,m]} \frac{\partial J}{\partial Z[i_1,i_2,m]} = \sum_{i_1=1}^{N_1} \sum_{i_2=1}^{N_2} U[i_1 + k_1, i_2 + k_2, n] \frac{\partial J}{\partial Z[i_1,i_2,m]}$$

- Gradient wrt weights can be computed via convolution
  - Convolve input U with gradient tensor $\frac{\partial J}{\partial Z[i_1,i_2,m]}$
- Similar computations for gradients with respect to $\frac{\partial J}{\partial b}$
  - Homework!