Final Exam

Closed book, 1 sheet of notes (double sided) allowed. No peeking into neighbors or unauthorized notes. No calculator or any electronics devices allowed. Cheating will result in getting an F on the course. Make sure you write your name and ID on the cover of the blue book. Write your answer in the blue book (or on the problem sheet when space is provided on the problem sheet).

Your Name _____ **ID** _____

1. (10pt) For the image in the left figure (which only takes integer values of 0 to 3): a) (2pt) determine its histogram, b) (4pt) determine a transformation function that will attempt to equalize the histogram; c) (2pt) Show the equalized image using your transformation function in the right figure; d) (2pt) Show the histogram of the equalized image.

0	0	1	2
1	1	2	2
1	2	2	3
2	2	3	1

2. (10 pt) For the 2D filter H given below, where the center position corresponds to m=n=0: a) (2pt) Is this filter separable? If yes, what is the horizontal and vertical filter? b) (1pt) Based on the filter coefficients, can you tell what is the function of this filter overall, and what its function in the horizontal and vertical directions? c) (6pt) Determine the discrete space Fourier Transform *H(u,v)* of the filter H, and sketch the one dimensional profiles *H(u,0), H(u,1/2), H(0,v), and H(1/2,v). You should use the property of separable functions for computing the DSFT when possible. Note that u represents the vertical frequency, v the horizontal frequency.* (d) (1pt) Based on the frequency response you derived, can you tell what is the function of the filter? Is this observation consistent with what you get in part (b)?

$$H = \begin{bmatrix} -1 & 2 & -1 \\ -2 & 4 & -2 \\ -1 & 2 & -1 \end{bmatrix}$$

3. (10pt) For the image given below, determine its 3-level Gaussian Pyramid and Laplacian Pyramid. Use averaging of 2x2 pixels for downsampling, use nearest neighbor replication for interpolation. Also, demonstrate the reconstruction of the original image from the Laplacian pyramid.

1	2	3	4
2	3	4	5
3	4	5	6
4	6	6	6

4. (10pt) The image captured by a camera can generally be considered a blurred version of the original image X plus noise. Mathematically, suppose you reorder the image row-by-row into a vector, and denote the original image by a vector x, the captured image by a vector b, we can write b=Hx+n, where H depends on the underlying blurring filter.
a) (2pt) Suppose the blurring operation can be described as b(m,n)=(x(m-1,n)+x(m,n-1)+x(m,n)+x(m,n+1)+x(m+1,n))/5. Derive the corresponding blurring matrix H.
b) (2pt) One way to deblur the image is by minimizing the mean square error $J=|b-Hx|^2$. This problem has a closed-form solution. Derive the solution.
c) (6pt) The solution using above approach can be severely affected by pixels with large noise. To overcome this problem, we can instead minimize the L1 norm of the error J= |b-Hx|. Formulate this problem into a form that can be solved by ADMM (show the original constrained optimization problem and the augmented Lagrangian). Furthermore, show and the ADMM iterations to solve the problem.

5. (12 pt) Answer the following questions about feature detection.
   (a) Why a point along an edge is not a good feature point?
   (b) What does the Harris feature detector detect?
   (c) Is Harris feature detector intensity invariant? Why?
   (d) Is Harris feature detector rotation invariant? Why?
   (e) Is Harris feature detector scale invariant? Why?
   (f) How do you make the Harris feature detector scale invariant?

6. (15pt) You are given two images taken under different view angles. Assume that the two images are related by a homography mapping.
   (a) (5pt) Given 4 pairs of corresponding feature point positions: (u_n, v_n) in image 1 and (x_n, y_n) in image 2, *n=1,2,…,4*. How do you determine the homography mapping parameters?
   (b) (10pt) Suppose you have detected *N* (N>>4) corresponding pairs of features in these two images, but not all the correspondences are correct. Describe how would you determine the homography parameters using the RANSAC method. Write it as an algorithm flow chart

7. (15pt) Consider block wise motion estimation between two frames.
   (a) (10pt) Write a pseudo code for performing exhaustive block matching at *half-pel* accuracy. If you prefer to write in python or MATLAB, it is fine. Assume that you are given the anchor frame image I1 saved in a 2D array, and a target frame image I2 also saved in a 2D array, the frame size is HxW, the block size is NxN, the search range is RxR. You save the resulting block motion vectors in two separate arrays Mx and My of size H/N x W/N. (assuming H and W are both integer multiples of N). For simplicity, you can ignore the boundary issues by assuming motion vectors are zero for those blocks that are in the R-pixel-wide border along all four sides of the frame (assume R is an integer multiple of N), and performing motion estimation only for blocks that are within the border. You can call a imresize(Img, F) function to increase the resolution of Img by a factor of F.
   (b) (5pt) What is the total number of operations? (For simplicity, assume the number of operations for calculating the error between two blocks is B^2).

8. (18 pt) Consider the following network architecture used for binary image segmentation in your computer assignment.
   (a) (10pt) Determine the number of trainable parameters. Write down clearly the number of parameters in each module, so that you can get partial credits.
   (b) (3pt) What is the perceptive field sizes of the top left layer, the middle left layer, and the bottom left layer, respectively?
   (c) (3pt) What type of loss function would you use to train this network? Define it explicitly. Define your variables clearly.
   (d) (2pt) Why is the skip connection important?