HW1: Shabani – QM1 Due Sept 23rd

1. Find the eigenvectors of Pauli matrix $\sigma_Y = \begin{pmatrix} 0 & -\mathrm{i} \\ \mathrm{i} & 0 \end{pmatrix}$

2. Assume the Hamiltonian of a quantum two-level system in form of $\widehat{H} = \begin{pmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{pmatrix}$.

The Hamiltonian is Hermitian with real H_{11} *and* H_{22} *such that* $H_{11} > H_{22}$.

a. Show that you can write $\widehat{H}$ in the form of $\widehat{H} = \widehat{H}_0 + \widehat{H}_1$ where $\widehat{H}_0$ is matrix with a global energy shift and $\widehat{H}_1$ is in form of $\widehat{H_1} = \begin{pmatrix} \epsilon & \Delta - i\widetilde{\Delta} \\ \Delta + i\widetilde{\Delta} & -\epsilon \end{pmatrix}$

b. Find eigenvalues of $\widehat{H}_0$ and $\widehat{H}_1$.

c. Express $\widehat{H}_1$ in terms of Pauli matrices.

3. A quantum system is said to possess a 'symmetry' if the Hamiltonian operator, H, is invariant under the associated transformation. For example, H acting on x could be the same as H acting on x+d, where d is a constant We can call H acting on x+d, H_B. In general, he symmetry could be translation, rotation etc. If the system is invariant under a symmetry, then one can show we can write $H_B = U^\dagger HU$ where U is unitary operator.

Show that if H_B and H are invariant then $[H,U] = 0$.