HW3: Shabani – QM1 Due Oct 7th

1.

Consider a particle moving in one dimension with Hamiltonian H given by

$$H = \frac{p^2}{2m} + V(x)\,.$$

Show that the expectation values $\langle x \rangle$ and $\langle p \rangle$ are time-dependent functions that satisfy the following differential equations:

$$\begin{aligned} \frac{d}{dt}\langle x \rangle &= \frac{1}{m}\langle p \rangle\,, \\ \frac{d}{dt}\langle p \rangle &= -\left\langle \frac{\partial V}{\partial x} \right\rangle. \end{aligned}$$

2.

Consider the gaussian wavefunction

$$\psi(x) = N \exp\Big(-\frac{1}{2}\frac{x^2}{a^2}\Big)\,, \qquad (1)$$

where $N \in \mathbb{R}$ a is a real positive constant with units of length. The integrals

$$\begin{aligned} \int_{-\infty}^{\infty} dx e^{-\alpha x^2 + \beta x} &= \sqrt{\frac{\pi}{\alpha}} \exp\Big(\frac{\beta^2}{4\alpha}\Big), \quad \mathrm{Re}(\alpha) > 0\,, \\ \int_{-\infty}^{\infty} dx\, x^2 e^{-\alpha x^2} &= \frac{1}{2\alpha}\int_{-\infty}^{\infty} dx\, e^{-\alpha x^2} \end{aligned}$$

Use the position space wavefunction (1) to calculate the uncertainties Δx and Δp. Confirm that your answer saturates the Heisenberg uncertainty product

$$\Delta x \Delta p \geq \tfrac{\hbar}{2}\,.$$

3. If two operators commute, then show that they can have the same set of eigenfunctions. If two operators commute and consequently have the same set of eigenfunctions, then show that the corresponding physical quantities can be evaluated or measured exactly simultaneously with no limit on the uncertainty.