Midterm 2: QM 1
Shabani, Fall 2021

Q1:

Consider the one-dimensional motion of a particle of mass μ in the potential

$$V(x) = V_0\,(x/a)^{2n}\,,$$

where n is a positive integer and $V_0 > 0$. Discuss qualitatively the distribution of energy eigenvalues and the parities, if any, of the corresponding eigenfunctions. Use the uncertainty principle to get an order-of-magnitude estimate for the lowest energy eigenvalue. Specialize this estimate to the cases $n = 1$ and $n \to \infty$. State what $V(x)$ becomes in these cases and compare the estimates with your previous experience.

Q2:

Consider a particle of mass m subject to a one-dimensional potential of the following form,

$$V(x) = \begin{cases} \frac{1}{2}m\omega^2x^2\,, & \text{for } x > 0, \\ \infty\,, & \text{for } x \leq 0. \end{cases}$$

(a) Determine the possible the bound state energy values of the particle.

(b) What is the normalized ground state wave function in the coordinate representation?

Q3:

The Dirac delta function can be thought of as the limiting case of a rectangle of area 1, as the height goes to infinity and the width goes to zero. Show that the delta-function well is a "weak" potential (even though it is infinitely deep), in the sense that $z_0 \to 0$ (check your notes). Determine the bound state energy for the delta-function potential, by treating it as the limit of a finite square well. Check that your answer with delta function bound state solution.

Q4:

An approximate model for the problem of an atom near a wall is to consider a particle moving under the influence of the one-dimensional potential given by:

$$x > -d : V(x) = -V_0\delta(x)$$

$$x < -d : V(x) = \infty$$

Basically, an infinite wall at minus d (where d is a positive number) and delta function at zero.

(a) Find the modification of the bound-state energy caused by the wall when it is far away. Explain also how far is far away.

(b) What is the exact condition on V_0 and d for the existence of at least one bound state?

Q5:

A rigid body with moment of inertia of I_z, rotates freely in the x-y plane. Let ϕ be the angle between the x-axis and the rotator axis.

(a) Find the energy eigenvalues and the corresponding eigenfunctions.

(b) At time t = 0 the rotator is described by a wave packet $\psi(0) = A\ Sin^2\phi$ Find $\psi(t)$ for t > 0.