Tradable Carbon Permits Auctions Under Regulation and Competition

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Bruno Kamdem[1] and Moustapha Pemy[2]

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[1] NYU Tandon School of Engineering, Department of Finance & Risk Engineering, bgk8384@nyu.edu and Johns Hopkins University, Department of Applied Mathematics & Statistics, bkamdem1@jhu.edu
[2] Towson University, Department of Applied Mathematics, mpemy@towson.edu

Outline

Climate Policies

- Since climate change is a pressing existential threat, governments around the world are going to need a mosaic of integrated energy solutions
- How can we reduce our consumption of fossil fuels at lowest cost? To achieve this goal, there are two main climate policies:
  1. Carbon tax
  2. Permit market ((1)Grandfathering (giving companies permits based on historical output or emissions) (2) Auction)
- A key reason for which economists and investors are attracted to auctions is that the latter give the commodity market the option to self-determine

Carbon Emission Rate

- In this section of our paper, we track the carbon emission rate using a mean reverting process. This approach is not new (similar to interest rate)
- Our work and technique in this section follow the likes of Hull-White (which derives a two-factor interest model using the Vasicek model and the extended CIR model

Oldrich A Vasicek. An equilibrium characterization of the term structure. **Journal of Financial Economics**, 5(2):177–188, 1977

John Hull and Alan White. Pricing interest-rate derivative securities. **The Review of Financial Studies**, 3(4):573–592, 1990

A Two-Factor Model for Pricing Carbon Derivative

$$\begin{cases} dZ_t &= Z_t\left[(\mu + \lambda u_t)dt + \sigma_1 dW_{1,t}\right] \\ du_t &= (m - bu_t)dt + \sigma_2 dW_{2,t} \\ Z_s &= z, \quad u_s = u, \quad 0 \leq s \leq t < \infty, \quad b > 0 \end{cases} \tag{1}$$

- Z_t: carbon permit price (geometric Brownian process)
- u_t: emission rate (Ornstein-Uhlenbeck process)
- $W_{1,t}$, $W_{2,t}$: correlated Wiener processes
  $\rho \in (-1, 1)$, $dW_{1,t} dW_{2,t} = \rho dt$

Model Overview

u_t

↓

Itô Lemma
$Y_t = \ln(Z_t) + \delta u_t$

↓

Itô isometry
$\int_s^t e^{-b(t-\xi)} dW_{2,\xi}$

↓

Carbon Permit
$Z_s e^{k(s,t,u_s)+W_{3,\sigma_3(s,t)^2}}$

↓

Distribution Function
$F(y; s, t, Z_s, u_s)$

↓

Density Function
$f(x; s, t, Z_s, u_s)$

↓

Carbon Call Option:
$C(s, t, z, u, K)$

↓

Carbon Put Option:
$P(s, t, z, u, K)$

Itô Isometry

1.
$$u_t = e^{-b(t-s)}u_s + \frac{m}{b}(1 - e^{-b(t-s)}) + \sigma_2 \int_s^t e^{-b(t-\xi)} dW_{2,\xi}. \tag{2}$$

2.
$$E\left[\left(\int_s^t e^{-b(t-\xi)} dW_{2,\xi}\right)^2\right] = \int_s^t e^{-2b(t-\xi)} ds = \frac{1}{2b}\left(1 - e^{-2b(t-s)}\right) \tag{3}$$

3.
$$Z_t = Z_s e^{k(s,t,u_s)+W_{3,\sigma_3(s,t)^2}} \tag{4}$$

Bernt Øksendal. **Stochastic Differential Equations: An Introduction with Applications**. Springer, 2010

The Carbon Call Option

$$C(s,t,z,u,K) = ze^{-\mu(t-s)+k(s,t,u)+\frac{1}{2}\sigma_3^2(s,t)}\left[N(d_1) - Ke^{-\mu(t-s)}N(d_2)\right] \quad (5)$$

- $d_1 := \dfrac{\ln(\frac{z}{K}) + k(s,t,u)}{\sigma_3(s,t)} + \sigma_3(s,t)$
- $d_2 := \dfrac{\ln(\frac{z}{K}) + k(s,t,u)}{\sigma_3(s,t)}$
- $k(s,t,u_s) := \dfrac{\lambda}{b}u_s + \left(\mu - \dfrac{1}{2}\sigma_1^2 + \dfrac{\lambda}{b}m\right)(t-s) - \dfrac{\lambda}{b}\left\{e^{-b(t-s)}u_s + \dfrac{m}{b}\left[1 - e^{-b(t-s)}\right]\right\}$
- $\sigma_3(s,t)^2 = \sigma_1^2 t + \dfrac{\lambda^2\sigma_2^2}{b^2}\left[\sqrt{t} - \sqrt{h(s,t)}\right]^2 + 2\rho\dfrac{\sigma_1\sigma_2\lambda\sqrt{t}|\sqrt{t} - \sqrt{h(s,t)}|}{b}$

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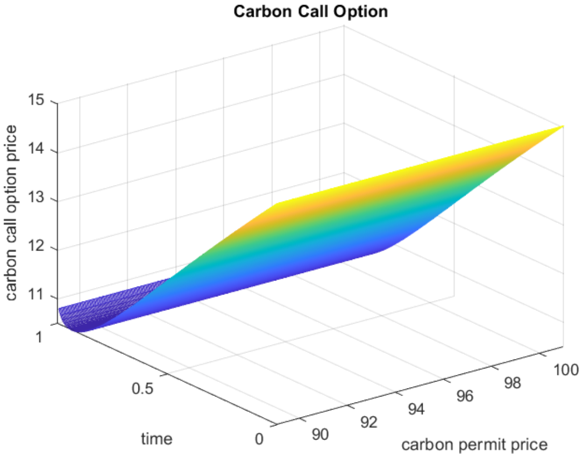
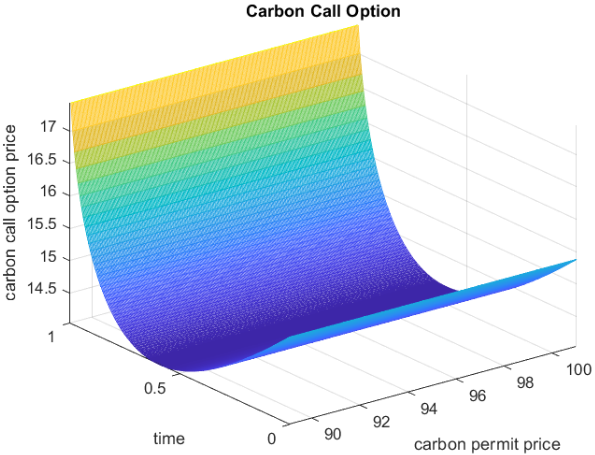
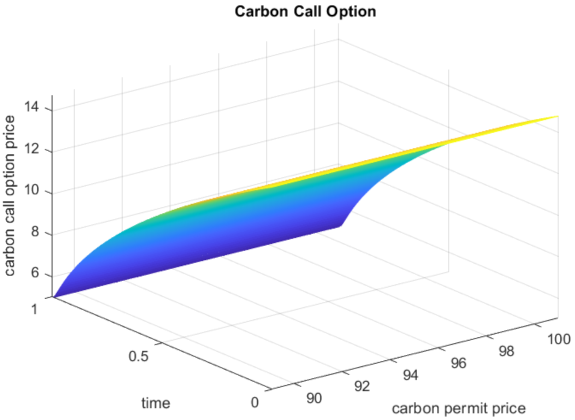
$\lambda \approx 4\%$



$\lambda \approx 10\%$



$\lambda = 0$


Carbon Call Option
carbon call option price
14
12
10
8
6
1
0.5
0
time
90
92
94
96
98
100
carbon permit price

Decreasing Emission Rate as Decarbonization Rate

- We successfully solve the dynamics of the carbon auction trading system and derive the carbon call option and the carbon put option in closed form
  - The carbon option permit starts as a GBM and is driven by a mean reverting rate
  - The carbon call option is analogous to the classical BS model, but is not the classical BS because the drift is not linear but is exponential
  - As t goes to infinity, our carbon model may behaves as BS because the mean-reverting part is destroyed
  - If t is not very large, we still have a mean reverting contribution.
- Term Structure of Carbon Emission Rate
  - We can think of decreasing emission rate as some sort of decarbonization rate. Achieving decarbonization involves pricing carbon through regulation
  - Trajectory needed to meet the Paris Agreement goal of 1.5° and avoid catastrophic climate change requires a proper calibration of the decarbonisation rate.
  - A suitable decarbonization (emission) rate is required to halve global emissions by 2030 and to reach net zero by mid-by mid-century
- Implications
  1. Implications for the Commodity Market (structure carbon emission rate to tame extracting and drilling activities)
  2. Implications for Regenerative Agriculture (structure the carbon emission rate to stimulate carbon sequestration)

Peter Carr (a Stellar Intellectual, a Rare Humanist)

"ne mourra jamais pour la deuxième fois"