1 Supplemental Appendix for "National Conflict in a Federal System"

Derivation of $\Psi^*(\cdot)$

From the text,

$$\begin{aligned}\Psi^*(F) &= \int_{\underline{\alpha}}^{F} Fp(\alpha)d\alpha + \int_{F}^{\overline{\alpha}} \alpha p(\alpha)d\alpha \\ &= FP(F) + \int_{F}^{\overline{\alpha}} \alpha p(\alpha)d\alpha \end{aligned} \quad \text{(A.1)}$$

Integrating the second expression in the second line of (A.1) by parts,

$$\begin{aligned}\int_{F}^{\overline{\alpha}} \alpha p(\alpha)d\alpha &= \left[\alpha P(\alpha) - \int P(\alpha)d\alpha\right]_{F}^{\overline{\alpha}} \\ &= \overline{\alpha}P(\overline{\alpha}) - \hat{P}(\overline{\alpha}) - FP(F) + \hat{P}(F),\end{aligned}$$

where $\hat{P}(x) = \int_{\underline{\alpha}}^{x} P(\alpha)d\alpha$, i.e., the integral of the cdf. Substituting into (A.1) and noting that $P(\overline{\alpha}) = 1$, we have

$$\Psi^*(F) = \overline{\alpha} - \hat{P}(\overline{\alpha}) + \hat{P}(F). \quad \text{(A.2)}$$

By definition, $E[\alpha] = \int_{\underline{\alpha}}^{\overline{\alpha}} \alpha p(\alpha)d\alpha$. Integrating by parts as above gives

$$\int_{\underline{\alpha}}^{\overline{\alpha}} \alpha p(\alpha)d\alpha = \overline{\alpha}P(\overline{\alpha}) - \hat{P}(\overline{\alpha}) - \underline{\alpha}P(\underline{\alpha}) + \hat{P}(\underline{\alpha}).$$

Noting that $P(\overline{\alpha}) = 1$, and $P(\underline{\alpha}) = \hat{P}(\underline{\alpha}) = 0$, we have $E[\alpha] = \overline{\alpha} - \hat{P}(\overline{\alpha})$. Substituting into (A.2), $\Psi^*(F) = E[\alpha] + \hat{P}(F)$.

Lemma 1 *Let $\tilde{\alpha}$ be the modal value of α. In a federal system, a state's preferences are single-peaked on $F \in [0, \overline{Z}]$ if and only if one of the following four conditions holds:*

(a) $\tilde{\alpha} \geq \alpha$ and $p(F) \leq \frac{1}{\alpha\beta}$ for all $F > \alpha$;

(b) $\tilde{\alpha} \geq \alpha$ and $p(\alpha) \geq \frac{1}{\alpha\beta}$;

(c) $\tilde{\alpha} < \alpha$; *or*

(d) Conditions (a) through (c) are violated, but $\overline{Z} < \check{F}_\alpha$, *where* $\check{F}_\alpha$ *is the value of* F *corresponding to a local minimum in* $u(F)$ *for* $F > \alpha$.

Otherwise, state i *has double-peaked preferences.*

Proof. First, note that $\forall F < \alpha$, $u(F)$ is strictly increasing and convex. Second, note that for $F > \alpha$, $\frac{\partial u}{\partial F} = \alpha + \alpha\beta P(F) - F$. At $F = \alpha$, this quantity is equal to $\alpha\beta P(\alpha)$, which is strictly positive. For sufficiently large $F > \alpha$, this quantity is strictly negative. Next, observe that $\frac{\partial^2 u}{\partial F^2} = \alpha\beta p(F) - 1$. Rearranging, this quantity is negative if and only if

$$p(F) < \frac{1}{\alpha\beta}. \tag{A.3}$$

(Necessity). Violation of (a), (b), and (c) imply $\tilde{\alpha} \geq \alpha$, $p(\alpha) < \frac{1}{\alpha\beta}$, and $p(\tilde{\alpha}) \geq \frac{1}{\alpha\beta}$; in this case, $u(F)$ is first concave, then convex, then concave in F for $F > \alpha$. Given $\frac{\partial u}{\partial F}|_{F=\alpha} > 0$ and $\lim_{F\to\infty} \frac{\partial u}{\partial F} < 0$, this implies double-peakedness if F is not constrained as it is in (d).

(Sufficiency).

(a) If inequality (A.3) holds for all $F > \alpha$, then $u(F)$ is strictly concave in that range. Given $\frac{\partial u}{\partial F}|_{F=\alpha} > 0$ and $\lim_{F\to\infty} \frac{\partial u}{\partial F} < 0$, this implies single-peakedness.

(b) Log-concavity of $p(\cdot)$ implies unimodality. This condition therefore implies that $u(F)$ is first convex, and then concave in F for $F > \alpha$. Given $\frac{\partial u}{\partial F}|_{F=\alpha} > 0$ and $\lim_{F\to\infty} \frac{\partial u}{\partial F} < 0$, this implies single-peakedness.

(c) If $\tilde{\alpha} < \alpha$, then unimodality implies $p(F)$ is either first convex and then concave in F for $F > \alpha$, or concave for all $F > \alpha$. In either case, given $\frac{\partial u}{\partial F}|_{F=\alpha} > 0$ and $\lim_{F\to\infty} \frac{\partial u}{\partial F} < 0$, this implies single-peakedness.

(d) As noted above, when conditions (a) through (c) do not hold but (d) does, single-peakedness is established by construction.

■

Proof of Proposition 1

From equation (2),

$$\frac{\partial u(F;\alpha,\beta)}{\partial F} = \begin{cases} \alpha\beta P(F) \text{ if } F < \alpha \\ \alpha - F + \alpha\beta P(F) \text{ otherwise.} \end{cases} \tag{A.4}$$

The statement in the proposition requires that

$$-\left.\frac{\partial u}{\partial F}\right|_{F=\hat{F}_\alpha-\Delta} > \left.\frac{\partial u}{\partial F}\right|_{F=\hat{F}_\alpha+\Delta} \tag{A.5}$$

for Δ sufficiently large. For sufficiently large Δ, $\frac{\partial u}{\partial F}$ evaluated at $\hat{F}_\alpha - \Delta$ is given by the first line, and $\frac{\partial u}{\partial F}$ evaluated at $\hat{F}_\alpha + \Delta$ by the second line, of (A.4). Substituting, (A.5) is equivalent to

$$-\alpha\beta P(\hat{F}_\alpha - \Delta) > \alpha - (\hat{F}_\alpha + \Delta) + \alpha\beta P(\hat{F}_\alpha + \Delta).$$

From the first order condition for an interior optimum, $\hat{F}_\alpha = \alpha + \alpha\beta P(\hat{F}_\alpha)$. Substituting and rearranging yields

$$\Delta > \alpha\beta\left(P(\hat{F}_\alpha - \Delta) + P(\hat{F}_\alpha + \Delta) - P(\hat{F}_\alpha)\right).$$

The right side of this inequality is bounded between 0 and $\alpha\beta$. Thus for sufficiently large Δ the inequality holds. ■

Proof of Proposition 2

Under unitary government, a state's first order condition is given by $\hat{F}_\alpha^{uni} = \alpha(1+\beta)$, while under federalism it is given by $\hat{F}_\alpha^{fed} = \alpha(1+\beta P(\hat{F}_\alpha^{fed}))$. It is immediate that the first value is weakly higher than the second, and strictly if $P(\hat{F}_\alpha^{fed}) < 1$ (i.e., $\hat{F}_\alpha^{fed} < \overline{\alpha}$). ■

Proof of Proposition 3

There are two cases to consider. First, suppose $\hat{F}^{fed}_{\alpha_L}$ is effectively federal, and α_H is sufficiently high that $\hat{F}^{fed}_{\alpha_H}$ is effectively centralized. Then $\hat{F}^{fed}_{\alpha_H} = \hat{F}^{uni}_{\alpha_H}$ and, by Proposition 2, $\hat{F}^{fed}_{\alpha_L} < \hat{F}^{uni}_{\alpha_L}$. Therefore $\hat{F}^{fed}_{\alpha_H} - \hat{F}^{fed}_{\alpha_L} > \hat{F}^{uni}_{\alpha_H} - \hat{F}^{uni}_{\alpha_L}$.

Second, suppose both $\hat{F}^{fed}_{\alpha_H}$ and $\hat{F}^{fed}_{\alpha_L}$ are effectively federal. Then from the first order conditions for $\hat{F}^{fed}_{\alpha}$ and $\hat{F}^{uni}_{\alpha}$, polarization under federalism is equal to $\alpha_H(1+\beta P(\hat{F}^{fed}_{\alpha_H})) - \alpha_L(1+\beta P(\hat{F}^{fed}_{\alpha_L}))$ and under unitary government is $(\alpha_H - \alpha_L)(1+\beta)$. Comparing these expressions yields the necessary and sufficient condition given in the Proposition. ■

Proof of Proposition 4

Let $A(F;\beta,P(\cdot)) \equiv \frac{F}{1+\beta P(F)}$ denote the value of α that would yield F as an ideal point. From the expressions for states' marginal utilities in (A.4),

$$\begin{aligned} mRP^-(F;\beta,P(\cdot)) &\equiv \int_{\underline{\alpha}}^{A(F;\cdot)} (F - \alpha(1+\beta P(F)))p(\alpha)d\alpha \qquad \text{and} \\ mRP^+(F;\beta,P(\cdot)) &\equiv \int_{A(F;\cdot)}^{F} (\alpha(1+\beta P(F)) - F)p(\alpha)d\alpha + \beta P(F)\int_F^{\overline{\alpha}} \alpha p(\alpha)d\alpha \\ &. \end{aligned} \tag{A.6}$$

Substituting for $A(F)$, integrating by parts (see derivation of $\Psi^*(\cdot)$ above for details) and rearranging yields

$$\begin{aligned} mRP^-(F;\beta,P(\cdot)) &= (1+\beta P(F))\hat{P}(A(F;\cdot)) \qquad \text{and} \\ mRP^+(F;\beta,P(\cdot)) &= (1+\beta P(F))\hat{P}(A(F;\cdot)) + \beta P(F)E[\alpha] - \hat{P}(F). \end{aligned} \tag{A.7}$$

1. $\frac{\partial mRP^-}{\partial F} = (1+\beta P(F))P(A(F))\frac{\partial A(F)}{\partial F} + \beta p(F)\hat{P}(A(F)) > 0$ for all $F > \underline{\alpha}$ (noting that

$\frac{\partial A(F)}{\partial F} > 0$), and

$$\frac{\partial mRP^{+}}{\partial F} = \frac{\partial mRP^{-}}{\partial F} + \beta p(F)E[\alpha] - P(F).$$

Having established $\frac{\partial mRP^{-}}{\partial F} > 0$, it is sufficient to demonstrate that $\beta p(F)E[\alpha] - P(F) > 0$ for sufficiently small values of F. Rearranging, the sufficient condition is $\frac{p(F)}{P(F)} > (\beta E[\alpha])^{-1}$. From the definition of log-concavity, $\frac{p(F)}{P(F)}$ is strictly decreasing. Further, $\lim_{F\to\underline{\alpha}\downarrow} \frac{p(F)}{P(F)} = \infty$. Therefore the condition holds for sufficiently small values of F.

2. We proceed by showing that there exists an $\underline{F}$ such that the result holds for $\overline{F} = F^*$. Comparing the expressions from (A.7), $mRP^- > mRP^+$ if and only if $\delta(F) > \beta E[\alpha]$ (where, as above, $\delta(F) = \frac{\hat{P}(F)}{P(F)}$). From the proof of Proposition **??**, at equality this statement defines F^* implicitly. Via monotonicity of $\delta(\cdot)$, therefore, $mRP^- > mRP^+$ if and only if $F > F^*$. Suppose $F > F^*$, so $mRP = mRP^-$. As mRP^- is strictly increasing, it is minimized at $mRP^-(F^*) > 0$. Suppose $F < F^*$, so $mRP = mRP^+$. From part (1), mRP^+ is increasing for sufficiently small values of F. From the second line of (A.7), $RP^+(\underline{\alpha}) = 0$. As mRP^+ is therefore increasing from zero, there must be some $\underline{F}$ such that for all $F < \underline{F}$, $mRP^+(F) < mRP^-(F^*)$.

■

Proof of Proposition 5

Let $\tilde{A}(F;\beta) \equiv \frac{F}{1+\beta}$ denote the value of α that would yield F as an ideal point in a unitary system. Then

$$mRP^{-}_{uni}(F;\beta,P(\cdot)) = \int_{\underline{\alpha}}^{\tilde{A}(F;\cdot)} (F - \alpha(1+\beta))p(\alpha)d\alpha. \tag{A.8}$$

Comparing this expression to the first line of (A.6), it is immediate that both the upper bound of integration, and the integrand, of the expression for $mRP^{-}_{uni}(F)$ are smaller than for their counterparts under effective federalism. Therefore, mRP^- is strictly higher under federalism than in a unitary system for any F that is effectively federal under the former.

The corresponding expression for mRP^+ in a unitary system is

$$mRP^+_{uni}(F;\beta,P(\cdot)) = \int_{\tilde{A}(F;\cdot)}^{\overline{\alpha}} (\alpha(1+\beta)-F)p(\alpha)d\alpha.$$

Noting that $\tilde{A}(F;\cdot) < A(F;\cdot)$ under effective federalism, mRP^+_{uni} can be expressed as

$$\int_{\tilde{A}(F;\cdot)}^{A(F;\cdot)} (\alpha(1+\beta)-F)p(\alpha)d\alpha + \int_{A(F;\cdot)}^{F} (\alpha(1+\beta)-F)p(\alpha)d\alpha + \int_{F}^{\overline{\alpha}} (\alpha(1+\beta)-F)p(\alpha)d\alpha.$$

Comparing this expression to the second line of (A.6), mRP^+ is strictly greater under the unitary than federal institutions if and only if

$$\int_{\tilde{A}(F;\cdot)}^{A(F;\cdot)} (\alpha(1+\beta)-F)p(\alpha)d\alpha + \int_{A(F;\cdot)}^{F} \alpha\beta(1-P(F))p(\alpha)d\alpha + \int_{F}^{\overline{\alpha}} (\alpha(1+\beta(1-P(F)))-F)p(\alpha)d\alpha > 0.$$

The first integral is strictly positive. Rearranging terms, the second and third integrals may be expressed as

$$\int_{A(F;\cdot)}^{\overline{\alpha}} \alpha\beta(1-P(F))p(\alpha)d\alpha + \int_{F}^{\overline{\alpha}} (\alpha-F)p(\alpha)d\alpha.$$

Each of these terms is strictly positive under effective federalism. Therefore the inequality holds. ■

Proof of Remark 1

We proceed by conjecturing that the equilibrium national policy F^* is effectively federal, and then establishing the condition in which this is consistent with equilibrium play. From the text, $S^*_L = 0$ and $S^*_H = \alpha_H - F^*$. Substituting into the states' utility functions and differentiating with respect to c^{fed}_H and c^{fed}_L respectively yields the first-order conditions:

$$\begin{aligned} \beta\alpha_H - \nu c^{fed}_H &= 0 \\ F^\circ - \alpha_L + c^{fed}_H - (1+\nu)c^{fed}_L &= 0 \end{aligned}$$

Solving this system of equations yields

$$c_H^{*fed} = \frac{\beta\alpha_H}{\nu}; \quad c_L^{*fed} = \frac{\beta\alpha_H + \nu(F^\circ - \alpha_L)}{\nu(\nu+1)}. \tag{A.9}$$

(Second-order conditions establish trivially that (c_H^{*fed}, c_L^{*fed}) is a global maximum.) c_H^{*fed} is independent of F°, while c_L^{*fed} is strictly increasing in F°. Therefore total deadweight loss, $\frac{\nu}{2}((c_H^{*fed})^2 + (c_L^{*fed})^2)$, is strictly increasing in F°.

If $F^* < \alpha_H$, it is is effectively federal. In equilibrium, $F^* = F^\circ + c_H^{*fed} - c_L^{*fed}$. Substituting from (A.9) and simplifying, this condition holds for all effectively federal status quo policies ($F^\circ < \alpha_H$) if and only if

$$\beta < \frac{\alpha_H - \alpha_L}{\alpha_H}, \tag{A.10}$$

establishing the initial conjecture.

■

Proof of Remark 2

First, note that L's ideal policy under unitary governance would be effectively federal under federalism if and only if $\alpha_L(1+\beta) < \alpha_H$, or $\beta < \frac{\alpha_H - \alpha_L}{\alpha_L}$, a condition that is always satisfied if (A.10) is met.

Substituting into the states' utility functions under unitary governance and differentiating with respect to c_H^{uni} and c_L^{uni} respectively yields the first-order conditions:

$$\begin{aligned} \alpha_H(\beta+1) - F^\circ - (1+\nu)c_H^{uni} + c_L^{uni} &= 0 \\ -\alpha_L(\beta+1) + F^\circ + c_H^{uni} - (1+\nu)c_L^{uni} &= 0. \end{aligned}$$

Solving this system of equations yields

$$c_H^{*uni} = \frac{(\beta+1)((\nu+1)\alpha_H - \alpha_L) - \nu F^\circ}{\nu(\nu+2)}; \quad c_L^{*uni} = \frac{(\beta+1)(\alpha_H - (\nu+1)\alpha_L) + \nu F^\circ}{\nu(\nu+2)} \tag{A.11}$$

(Second-order conditions establish trivially that (c_H^{*uni}, c_L^{*uni}) is a global maximum.)

For a given status quo policy F°, total deadweight loss from conflict under federalism exceeds that under unitary governance if and only if

$$(c_L^{*fed})^2 + (c_H^{*fed})^2 > (c_L^{*uni})^2 + (c_H^{*uni})^2, \tag{A.12}$$

which is equivalent to

$$(c_L^{*fed} - c_L^{*uni})(c_L^{*fed} + c_L^{*uni}) > (c_H^{*uni} - c_H^{*fed})(c_H^{*fed} + c_H^{*uni}) \tag{A.13}$$

For this inequality to hold, it is sufficient that both $c_L^{*fed} > c_L^{*uni}$ and $c_H^{*fed} > c_H^{*uni}$. Evaluated at $F^\circ = \alpha_H$, these conditions are equivalent to

$$\begin{aligned}(\beta\nu^2 + 2\beta\nu + \beta + 1)\alpha_L + (\beta - 1)\alpha_H &> 0 \text{ and} \\ (\beta + 1)\alpha_L + (\beta - 1)\alpha_H &> 0.\end{aligned}$$

Note that the first condition is implied by the second, which holds if and only if

$$\beta > \frac{\alpha_H - \alpha_L}{\alpha_H + \alpha_L} \tag{A.14}$$

By continuity, this condition will be met in an open ball around $F^\circ = \alpha_H$. The right side of (A.14) is strictly less than the right side of (A.10). Thus $\beta \in (\frac{\alpha_H - \alpha_L}{\alpha_H + \alpha_L}, \frac{\alpha_H - \alpha_L}{\alpha_H})$ is sufficient for the Proposition to hold.

2 Additional Analysis Not Presented in the Main Text

Lemma 2 (Majority Voting Equilibrium) *States' induced preferences over federal policies are single-crossing; thus, under simple majority rule, $\hat{F}_{\alpha_m}$, the most preferred national policy of the state with the median preference parameter, $\alpha = \alpha_m$, is an equilibrium.*

Proof of Lemma 2

Differentiating (2) with respect to F and again with respect to α yields

$$\frac{\partial^2 u(F;\alpha,\cdot)}{\partial F \partial \alpha} = \begin{cases} \beta P(F) \text{ if } F < \alpha \\ 1 + \beta P(F) \text{ otherwise.} \end{cases} \tag{A.15}$$

Both the first and second lines of (A.15) are strictly positive, implying increasing differences, which are sufficient for single-crossing. Given single-crossing preferences, a majority rule voting equilibrium exists, and the median state will be decisive (Gans and Smart 1996).

■

Proposition 6 (Equilibrium Versus Efficient National Policymaking) *Suppose $p(\cdot)$ is symmetric and $\hat{F}_{\alpha_m}$ is effectively federal. Then:*

1. *The national policy arrived at under simple majority rule is strictly higher than the aggregate welfare-maximizing national policy; and*

2. *If the bargaining protocol B is supermajoritarian, then the aggregate welfare-maximizing national policy is either below or within B's associated gridlock interval.*

Proof of Proposition 6

Integrating (2) over $p(\alpha)$, aggregate welfare is given by

$$W \equiv \int_{\underline{\alpha}}^{\overline{\alpha}} \alpha\beta(E[\alpha] + \hat{P}(F))p(\alpha)d\alpha + \int_{\underline{\alpha}}^{F} \left(\alpha F - \frac{F^2}{2}\right) p(\alpha)d\alpha + \int_{F}^{\overline{\alpha}} \frac{\alpha^2}{2} p(\alpha)d\alpha. \tag{A.16}$$

Via the Leibniz integral rule, marginal aggregate welfare is

$$\frac{\partial W}{\partial F} = (\beta E[\alpha] - \delta(F))P(F), \tag{A.17}$$

where where $\delta(F) \equiv \frac{\hat{P}(F)}{P(F)} = F - E[\alpha|\alpha < F]$ is the *mean advantage over inferiors* function from reliability theory. Any $F < \underline{\alpha}$ is Pareto dominated. Lemma 1 of Bagnoli and Bergstrom

(2005) shows for log-concave $p(\cdot)$ that $\delta(F)$ is strictly increasing in F (from zero at $F = \underline{\alpha}$). Therefore F^* is unique and defined implicitly by the first order condition $\delta(F^*) = \beta E[\alpha]$ (or by the corner $\overline{Z}$ when $\delta(\overline{Z}) < \beta E[\alpha]$).

1. Under symmetry, $E[\alpha] = \alpha_m$. Therefore, from above, $\delta(F^*) = \beta\alpha_m$. Since $\delta(\cdot)$ is monotone increasing for log-concave densities, its inverse exists and is also monotone increasing. Therefore $F^* = \delta^{-1}(\beta\alpha_m)$, and $F^* < \hat{F}_{\alpha_m}$ if and only if

$$\delta(\hat{F}_{\alpha_m}) > \beta\alpha_m. \tag{A.18}$$

Recalling that $\delta(F) = \frac{\hat{P}(F)}{P(F)}$, substituting into (A.18) and rearranging yields

$$\hat{P}(\hat{F}_{\alpha_m}) > \alpha_m\beta P(\hat{F}_{\alpha_m}). \tag{A.19}$$

$\hat{F}_{\alpha_m}$ is defined implicitly by the first order condition $\alpha_m\beta P(\hat{F}_{\alpha_m}) = \hat{F}_{\alpha_m} - \alpha_m$. Substituting into (A.19) yields the condition

$$\hat{P}(\hat{F}_{\alpha_m}) > \hat{F}_{\alpha_m} - \alpha_m. \tag{A.20}$$

Note that at $\beta = 0$, $\hat{F}_{\alpha_m} = \alpha_m$ and (A.20) holds trivially. Recall from the derivation of $\Psi^*(F)$ above (and given symmetry) that $\alpha_m = \overline{\alpha} - \hat{P}(\overline{\alpha})$, or $\hat{P}(\overline{\alpha}) = \overline{\alpha} - \alpha_m$. Also note that for all $\hat{F}_{\alpha_m} > \overline{\alpha}$, $\frac{\partial \hat{P}(\hat{F}_{\alpha_m})}{\partial \hat{F}_{\alpha_m}} = P(\hat{F}_{\alpha_m}) = 1$. Therefore for all $\hat{F}_{\alpha_m} \geq \overline{\alpha}$, (A.20) holds at equality, which in turn implies $\hat{F}_{\alpha_m} = F^*$.

Next, assume $\hat{F}_{\alpha_m} < \overline{\alpha}$. Then the derivative of the left side of (A.20), $P(\hat{F}_{\alpha_m})$, is strictly less than one, while the derivative of the right side is equal to one. Suppose there exists some $\hat{F}'_{\alpha_m} < \overline{\alpha}$ such that (A.20) does not hold. Given convexity of $\hat{P}(\cdot)$, $\hat{P}(\alpha_m) > 0$, $\hat{F}_{\alpha_m} \geq \alpha_m$, and $\hat{P}(\overline{\alpha}) = \overline{\alpha} - \alpha_m$, it must then be the case that there exists some $\hat{F}''_{\alpha_m} \in (\hat{F}'_{\alpha_m}, \overline{\alpha}]$ such that $\left.\frac{\partial \hat{P}}{\partial \hat{F}_{\alpha_m}}\right|_{\hat{F}_{\alpha_m} = \hat{F}''_{\alpha_m}} > 1$, a contradiction. Therefore (A.20) holds for all $\hat{F}_{\alpha_m} < \overline{\alpha}$.

2. Follows immediately from part 1 and the assumption that $\hat{F}_{\alpha_m}$ lies within the gridlock interval.

■

References

Edlin, Aaron S., and Chris Shannon. 1998. "Strict Monotonicity in Comparative Statics." *Journal of Economic Theory* 81: 201-219.

Gans, Joshua S., and Michael Smart. 1996. "Majority Voting with Single-Crossing Preferences." *Journal of Public Economics* 59: 219-237.