SITE Research Center online event from January 10 to 14, 2021 on

Long Time Behavior and Singularity Formation in PDEs — Part II

(the talk times are given in Gulf Standard Time)

Sunday, January 10

4.20 – 4.30pm: **Arlie Petters**, Provost NYU Abu Dhabi

4.30 - 5.30pm: **Jean-Marc Delort**

- Title: Long time dispersive estimates for perturbations of a kink solution of one dimensional cubic wave equations.
- Abstract:

Title: Long time dispersive estimates for perturbations of a kink solution of one dimensional cubic wave equations.

Abstract: A kink is a stationary solution to a cubic one dimensional wave equation $(\partial_t^2 - \partial_x^2)\phi = \phi - \phi^3$ that has different limits when x goes to $-\infty$ and $+\infty$, like $H(x) = \tanh(x/\sqrt{2})$.

The main result in this talk gives, for small odd perturbations of size ϵ of the kink that are smooth enough and have some space decay, *explicit* rates of time decay for the solution up to time ϵ^{-4+0}.

This is joint work with Nader Masmoudi.

8.00 - 9.00pm: **Sasha Kiselev**

- Title: The flow of polynomial roots under differentiation
- Abstract: The question of how roots of polynomials move under differentiation is classical. Contributions to this subject have been made by Gauss, Lucas, Marcel Riesz, Polya and many others. In 2018, Stefan Steinerberger derived formally a PDE that should describe the dynamics of roots under differentiation in certain situations. Interestingly, Dimitri Shlyakhtenko and Terry Tao have later formally obtained the same PDE as the evolution equation for free fractional convolution of a measure, an object appearing in free probability. The PDE in question is of hydrodynamic type and bears a striking resemblance to the models used in mathematical biology to describe collective behavior and flocking of various species - such as fish, birds or ants. The equation is critical, but due to strongly nonlinear form of its coefficients, proving global regularity for its solutions is harder than for equations such as Burgers, SQG or Euler alignment model. I will discuss joint work with Changhui Tan in which we establish global regularity of Steinerberger's equation and make a rigorous connection between its solutions and evolution of roots under differentiation for a class of trigonometric polynomials.

9.00 - 10.00pm: **Andrea Nahmod**

- Title: Random tensors and propagation of randomness under the NLS flow

- Abstract: In this talk we focus on the time dynamics of solutions of periodic nonlinear Schrödinger with random initial data and the key underlying difficulty of understanding how randomness propagates under the nonlinear flow. We discuss the theory of random tensors, a powerful new framework that we developed with Yu Deng and Haitian Yue.

Monday, January 11

4.30 - 5.30pm: **Belkacem Said-HouariHouari**

- Title: Global Well–Posedness Of The Cauchy Problem For The 3D Jordan–Moore–Gibson–Thompson Equation
- Abstract: We consider the Cauchy problem of a third-order in time nonlinear equation known as the Jordan—Moore-Gibson-Thompson equation arising in acoustics as an alternative model to the well-known Kuznetsov equation. We show a local existence result in appropriate function spaces, and using the energy method together with a bootstrap argument, we prove a global existence result for small data, without using the linear decay. Finally, we obtain polynomial decay rates in time for a norm related to the solution.

8.00 - 9.00pm: **Luis Vega**

- Title: Riemann's Non-Differentiable Function And The Binormal Curvature Flow (Joint work with Valeria Banica)
- Abstract: We make a connection between a famous analytical object introduced in the 1860s by Riemann, as well as some variants of it, and a nonlinear geometric PDE, the binormal curvature flow. As a consequence this analytical object has a non-obvious non- linear geometric interpretation. We recall that the binormal flow is a standard model for the evolution of vortex filaments. We prove the existence of solutions of the binormal flow with smooth trajectories that are as close as desired to curves with a multifractal behavior. Finally, we show that this behavior falls within the multifractal formalism of Frisch and Parisi, which is conjectured to govern turbulent fluids.

9.00 - 10.00pm: **Emmanuel Grenier**

- Title: Instability of viscous shear layers
- Abstract: In this talk we review recent results on the instability of shear layers for the incompressible Navier Stokes equations as the viscosity goes to 0.

Tuesday, January 12

4.30 - 5.30pm: **Boualem Djehiche**

- Title: On a problem of Mass Transport of Particles Towards a Target
- Abstract: Boualem will review some recent results on a problem of Mass Transport of Particles Towards a Target in a Brownian environment.

8.00 - 9.00pm: **Vlad Vicol**

- Title: Shock formation and vorticity creation for compressible Euler
- Abstract: We discuss the formation of singularities (shocks) for the compressible Euler equations with the ideal gas law. We provide a constructive proof of stable shock formation from smooth initial datum, of finite energy, and with no vacuum regions. Additionally, for the non-isentropic problem, we show that sounds waves interact with entropy waves to produce vorticity at the shock. This talk is based on joint work with Tristan Buckmaster and Steve Shkoller.

9.00 - 10.00pm: **Edriss Titi**

- Title: The Inviscid Primitive Equations and the Effect of Rotation
- Abstract: Large scale dynamics of the oceans and the atmosphere is governed by the primitive equations (PEs). It is well-known that the three-dimensional viscous primitive equations are globally well-posed in Sobolev spaces. In this talk, I will discuss the ill-posedness in Sobolev spaces, the local well-posedness in the space of analytic functions, and the finite-time blowup of solutions to the three-dimensional inviscid PEs with rotation (Coriolis force). Eventually, I will also show, in the case of “well-prepared" analytic initial data, the regularizing effect of the Coriolis force by providing a lower bound for the life-span of the solutions which grows toward infinity with the rotation rate. The latter is achieved by a delicate analysis of a simple limit resonant system whose solution approximates the corresponding solution of the 3D inviscid PEs with the same initial data.

Wednesday, January 13

4.30 - 5.30pm: **Nejla Nouaili**

- Title: Construction of blowup solutions for the Complex Ginzburg-Landau equation with critical parameters
- Abstract: In this talk Nejla will present a construction of a solution for the Complex Ginzburg-Landau (CGL) equation in a general critical case, which blows up in finite time T only at one blow-up point. She will also give a sharp description of its profile. These is joint work with G.K.Duong and H.Zaag.

8.00 - 9.00pm: **Massimiliano Berti**

- Title: Quasi-periodic solutions in fluid equations
- Abstract: In this talk Massimiliano will present some new results about bifurcation of time quasi-periodic solutions for fluid equations

9.00 - 10.00pm: **Hajer Bahouri**

- Title: Global well-posedness for the derivative nonlinear Schrödinger equation
- Abstract: In this work, in collaboration with Galina Perelman, we prove that the derivative nonlinear Schrödinger equation is globally well-posed for general Cauchy data in $H^{\frac 1

2}(\R)$. The proof of our result is achieved by combining the profile decomposition techniques with the integrability structure of the equation.

Thursday, January 14

4.30 - 5.30pm: **Anne-Laure Dalibard**

- Title: Stability of propagation fronts in congestion models
- Abstract: The purpose of this talk is to present two 1d congestion models: a « soft » congestion model in which the congestion effects are described by a singular pressure law, and a « hard » congestion model in which the dynamics are different in congested zones (incompressible dynamics) and in the non-congested ones (compressible dynamics). For each model, we exhibit traveling fronts and investigate their stability. This is a joint work with Charlotte Perrin.

8.00 - 9.00pm: **Jose Carrillo**

- Title: Nonlinear Aggregation-Diffusion Equations: Gradient Flows, Free Energies and Phase Transitions
- Abstract: The main goal of this talk is to discuss the state-of-the-art in understanding the phenomena of phase transitions for a range of nonlinear Fokker-Planck equations with linear and nonlinear diffusion. They appear as natural macroscopic PDE descriptions of the collective behavior of particles such as Cucker-Smale models for consensus, the Keller Segel model for chemotaxis, and the Kuramoto model for synchronization. We will show the existence of phase transitions in a variety of these models using the natural free energy of the system and their interpretation as natural gradient flow structure with respect to the Wasserstein distance in probability measures. We will discuss both theoretical aspects as well as numerical schemes and simulations keeping those properties at the discrete level.

9.00 - 10.00pm: **Charles Collot**

- Title: On the stability of equilibria for infinitely many particles
- Abstract:

We study the evolution of a system of particles. Instead of the usual Hartree equation for density matrices, we consider the following equivalent model, proposed by de Suzzoni, of a Hartree type equation but for a random field:

$$iX_t = -\Delta X + (w * \mathbb{E}(|X|^2))X.$$

Above, $X : [0, T] \times \mathbb{R}^d \times \Omega$ is a time-dependent random field, w a pair interaction potential, $*$ the convolution product and $\mathbb{E}$ the expectation. This equation admits equilibria which are random Gaussian fields whose laws are invariant by time and space translations. They are hence not localised and represent an infinite number of particles. We give a stability result under certain hypotheses, by showing that small perturbations scatter as $t \to \pm\infty$ to linear waves. This is joint work with de Suzzoni.

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