Mathematical modeling of the Covid-19 in the UAE.

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Motivation of the work

- There is a increase fatalities among people with comorbidity have required governments to take different measures that have restricted human mobility and put many economies on hold
- Overwhelmed the health care capacities of many countries, which requires an immediate increase of acute and critical beds
- Providing the healthcare personnel (HCP) with enough and proper personal protective equipment (PPE).
- As the pandemic is still taking many lives everyday, and in the absence of specific treatment or vaccine, it is very important to understand if the non-pharmaceutical interventions (NPI) are effective in reducing the burden on the public health.

Introduction of the model

$$\begin{cases} \dfrac{\partial S(t,a)}{\partial t} + \dfrac{\partial S(t,a)}{\partial a} & = & -\beta S(t,a)\displaystyle\int_0^{+\infty} I(t,\theta)d\theta - \alpha\beta C(t)S(t,a), \\ \dfrac{\partial I(t,a)}{\partial t} + \dfrac{\partial I(t,a)}{\partial a} & = & \beta S(t,a)\displaystyle\int_0^{+\infty} I(t,\theta)d\theta + \alpha\beta C(t)S(t,a) \\ & & -(\gamma + \delta(a))I(t,a), \\ \dfrac{dC(t)}{dt} & = & \displaystyle\int_0^{+\infty} \delta(a)I(t,a)da - \mu C(t), \\ S(t,0) & = & 0, \\ I(t,0) & = & 0, \\ S(0,a) & = & S_0(a) \in L^1(0,+\infty), \\ I(0,a) & = & I_0(a) \in L^1(0,+\infty), \\ C(0) & = & C_0 \in \mathbb{R}^+, \end{cases} \tag{1}$$

Bentout, Tridane, Djilali, Touaoula, Age-structured Modeling of COVID-19 Epidemic in the USA, UAE and Algeria, Alexandria Engineering Journal, 2020

We make the assumption that the infectioness of the COVID-19 depending on the age of the patient:

$$\delta(a) = \begin{cases} \delta^* & \text{if} \quad a \geq \tau, \\ 0 & \text{if} \quad a < \tau, \end{cases} \tag{2}$$

We define the basic reproduction number $\mathscr{R}_0$, which represents the number of the new infectious cases by one infected individual in the infection period during a given time of spread of the COVID-19

$$\mathscr{R}_0 = \beta S_0 \int_0^\infty e^{-\gamma a - \int_0^a \delta(\sigma) d\sigma} \left(1 + \frac{\alpha \delta(a)}{\mu}\right) da.$$

Data

Country	USA	UAE	Algeria
Population size (million)	328,2	9,631	42,23

Table: The population sizes of in USA, UAE, Algeria

Country	USA	UAE	Algeria
over 60	29.71%	9.8%	13.58%

Table: The percentage of the aged individuals in USA, UAE, Algeria

well-posedness of the systems

We define the following space

$$\mathbb{H} = L^1(0,+\infty) \times L^1(0,+\infty) \times \mathbb{R}^+,$$

with $L^1(0,+\infty)$ is the space of functions that are positive and Lebesgue integrable, which is equipped with the norm

$$||(\phi,\psi,\chi)||_{\mathbb{H}} = \int_0^{+\infty} |\phi(\theta)|d\theta + \int_0^{+\infty} |\psi(\theta)|d\theta + |\chi|.$$

We suppose that the initial conditions $(S_0, I_0, C_0) \in \mathbb{H}$. We can prove that the model has a unique positive solution for initial conditions that belongs to the space $\mathbb{H}$.

Fractions equations

We calculate the total fraction of the infected individuals and the susceptible ones, we obtain

$$\begin{aligned} \frac{ds(t)}{dt} &= -\beta s(t)(i(t)+\alpha C(t)), \\ \frac{di(t)}{dt} &= (\beta s(t)-\gamma)i(t)+\alpha\beta s(t)C(t)-\int_{\tau}^{+\infty}\delta(a)I(t,a)da, \\ \frac{dC(t)}{dt} &= \int_{\tau}^{+\infty}\delta(a)I(t,a)-\mu C(t), \end{aligned} \tag{3}$$

where

$$i(t)=\int_0^{+\infty}I(t,a)da \quad \text{and} \quad s(t)=\int_0^{+\infty}s(t,a)da.$$

We also have

$$0\leq\int_{\tau}^{+\infty}\delta(a)I(t,a)da\leq\delta^* i(t). \tag{4}$$

We get the following inequality:

$$0<C(t)<C(0)\mathrm{e}^{-\mu t}+\delta^*\mathrm{e}^{-\mu t}\int_0^t i(s)ds. \tag{5}$$

Exponential increase of the pandemic

We assume that the total fraction of the infected individuals $i(t)$ has the following special form:

$$i(t) = x_1 e^{x_2 t}, \tag{6}$$

x_1 represents the proportion of the declared infection cases. To obtaining the best value of x_2 , we use the squares method for approximation . The obtained results are highlighted in Table below. We get

$$\beta = \frac{i'(t) + \int_\tau^{+\infty} \delta(a) I(t,a) da,}{(s(t) - \gamma) i(t) + \alpha s(t) C(t)}. \tag{7}$$

we have the following inequality:

$$\widetilde{\beta} \leq \beta \leq \widehat{\beta},$$

with

$$\widetilde{\beta} = \frac{(x_2 + \gamma)\chi_1}{s(0)x_1 + \alpha s(0) C(0)} \text{ and } \widehat{\beta} = \frac{x_2 + \gamma + \delta^*}{s(0)}.$$

By a variation of the value of β in the interval $[\widetilde{\beta}, \widehat{\beta}]$, we retrieve the

Parameters estimations

Country	x_1	x_2	$\widetilde{\beta}$	β	$\widehat{\beta}$	$\mathscr{R}_0$
USA	4.9725×10^{-4}	0.091	0.1966	0.2364	0.2526	2.2540
UAE	6.7139×10^{-5}	0.15	0.2528	0.2679	0.3033	2.6491
Algeria	1.44×10^{-4}	0.09	0.1920	0.2172	0.2425	2.1490

Table: Estimation of x_1,x_2, the transmission rate and the basic reproduction number.

Data fitting

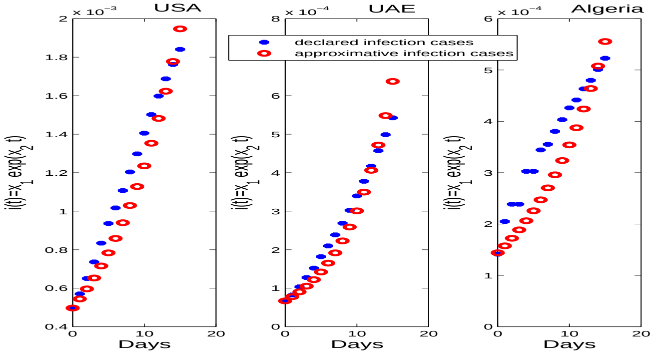


Figure: The approximation x_1 and x_2 using the WHO data where the values used in Table 3 are used for each sample and $t = 0$ represents the date 01/04/2020.

For the USA

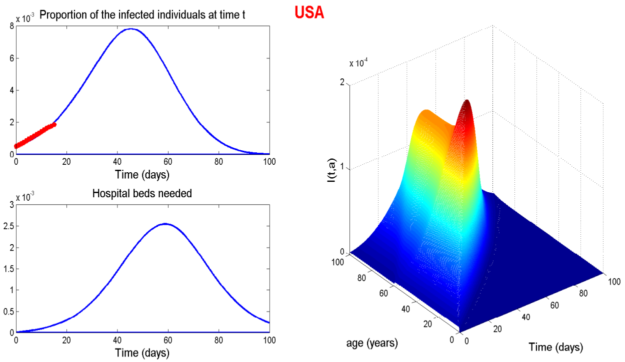


Figure: Numerical simulation of the spread of COVID-19 epidemic in USA with data fitting, where the red color curve is the declared infection cases by WHO and the blue curves is the approximations offered by the model.

For the UAE

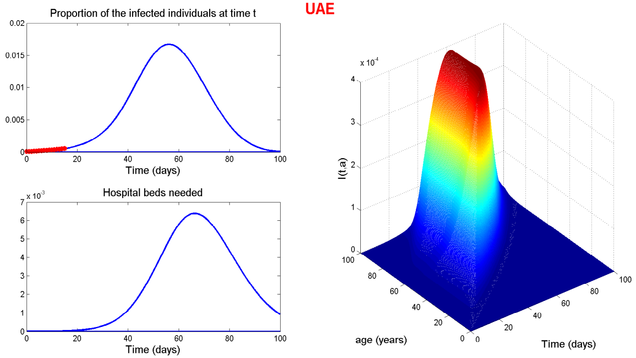


Figure: Predicting the spread of COVID-19 epidemic in UAE, where the red color curve is the declared infection cases by WHO and the blue curves is the approximations offered by the model

For Algeria

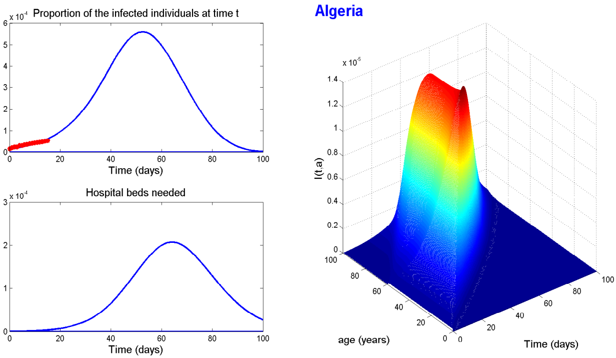


Figure: Predicting the spread of COVID-19 epidemic in Algeria , where the red color curve is the declared infection cases by WHO and the blue curves is the approximations offered by the model

By using the same method as the previous section, we obtain the values of x_1 and x_2 on the following figure

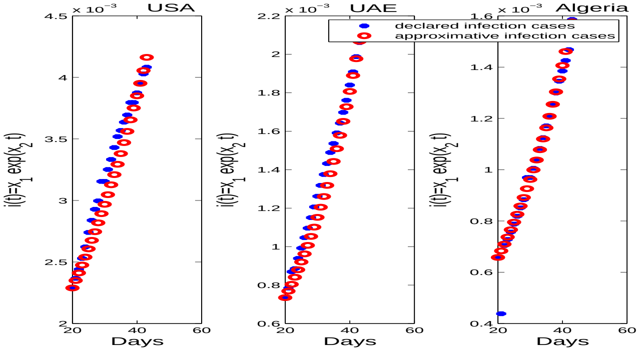


Figure: The method of approximating x_1 and x_2 using the WHO data, where the values used in Table below are used for each sample and $t = 20$ represents the

Parameters estimation

Country	x_1	x_2	$\widetilde{\beta}$	β	$\widehat{\beta}$	$\mathscr{R}_0$
USA	0.0023	0.0260	0.1368	0.2074	0.2155	2.0822
UAE	7.3458×10^{-4}	0.045	0.1466	0.1769	0.1972	1.7768
Algeria	6.5779×10^{-5}	0.038	0.1433	0.1571	0.19	1.5777

Table: Estimation of x_1, x_2, the transmission rate and the basic reproduction number on the period between April 21^{st} to May 15^{th}, 2020.

The peak estimations

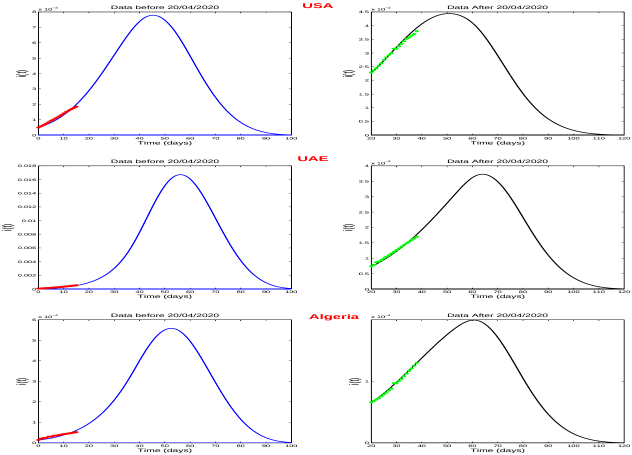


Figure: The influence of the measures taken after 21/04/2020 on the value of the pandemic peak of COVID-19.

The peak estimations

Country	USA	UAE	Algeria
The number of infected a the peak without NPI	2 555 400	165 360	23 052
The number of infected a the peak with NPI	1 968 358	45 560	12355

Table: The effect of the NPI taken after 21/04/2020 on the number of infected cases at the peak of pandemic in each country.

The impact of the full lockdown

Since not all the countries were able to stop the community transmission of the COVID-19 by the NPI, we would like to know how a full lockdown We consider the transmission rate as a decreasing function of time.
The transmission rate as a constant before applying the lockdown and it exponentially decreases after applying the lockdown.
We suggest the following form of $\beta(t)$, if it was applied, would have affected the progress of the COVID-19.

$$\beta(t) = \begin{cases} \beta_0 \text{ if } t < T \\ \beta_0 \mathrm{e}^{-\mu(t-N)} \text{ if } t > T. \end{cases} \tag{8}$$

The impact of the full lockdown

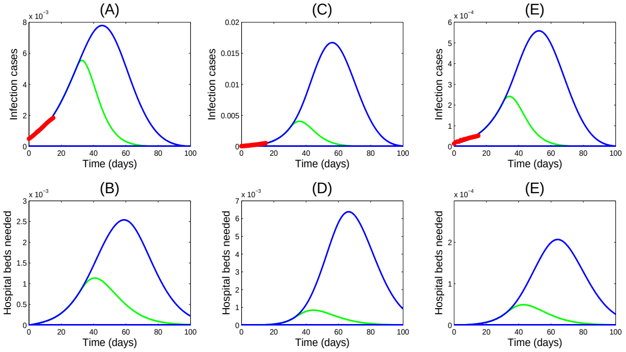


Figure: The effect of the full lockdown on the spread of COVID-19 in the USA ((A) and (B)), the UAE ((C) and (D)) and Algeria ((E) and (F)). The blue figures represent the case of the absence of restriction and the green curves represent the case of the full lockdown.

In the absence of the restriction

	ASU	UAE	Algeria
End of pandemic	$t = 90$	$t = 100$	$t = 94$
Time of the peak of the number of IC	$t = 45$	$t = 56$	$t = 51$
Number of IC at this peak	2 555 400	165 360	23 052
Time of the peak of the number of HBN	$t = 58$	$t = 65$	$t = 63$
Peak of the of the number of HBN	833 690	63 138	8 545

Table: The main characteristic of the pandemic in the absence of the restriction of human movement strategy. The time $t = 0$ refers to the date 01/04/2020.

The case of full lockdown

	ASU	UAE	Algeria
End of pandemic	$t = 63$	$t = 63$	$t = 63$
Time of the peak of the number of IC	$t = 30$	$t = 35$	$t = 30$
Number of IC at this peak	1 846 800	39 829	9 995
Time of the peak of the number of HBN	$t = 40$	$t = 42$	$t = 42$
Peak of the of the number of HBN	372 840	8 275	2 039

Table: The main characteristic of the pandemic in the case of full lockdown. The time $t = 0$ refers to the date 01/04/2020.

The effect the PPE on the spread of COVID-19

The Goal is to investigate the efficacy of the availability of the PPE on the spread of the COVID-19 among the HCP.
The parameter that represents this efficacy is α. In reality, as the number of the infected cases increases the HCP are obligated to increase their work load, which makes them more susceptible to infection, particularly if there is a shortage or an improper use of the PPE. Therefore, we choose the following special form:

$$\alpha(i(t)) = 1 - e^{-\rho N \int_0^{+\infty} I(t,a) da}.$$

N is the total size of the population and ρ is the percentage of individuals that uses the PPE.

The effect the PPE on the spread of COVID-19

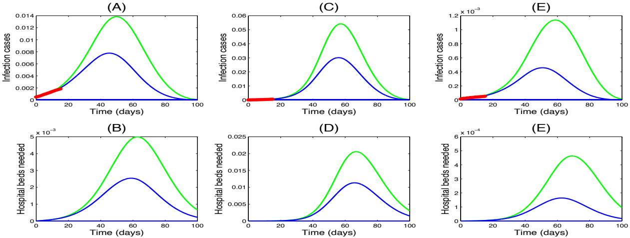


Figure: The effect the PPE on the spread of COVID-19 in USA (A and B), UAE (C and D) and Algeria (E and F), where the blue figures represent the case where α is constant and the green curves represent the case where the PPE depends on the number of infected.

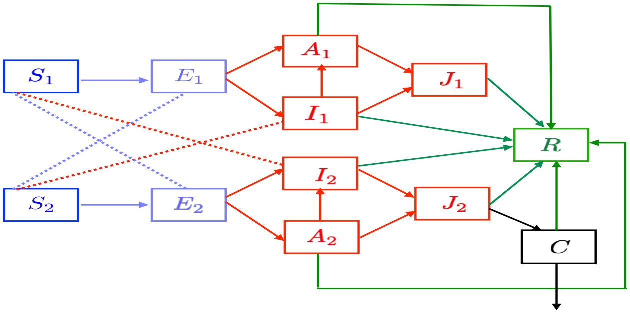


Figure: Schema of the Comorbidity Model

AlBlooshi, Tridane, Djilali, Al Jassmi, The impact of the comorbidity on the progress of the Covid-19 in the United Arab Emirates, Results in Physics, 2021

Mathematical Model

$$
\left\{
\begin{array}{rcl}
\dot{S_1} &=& -(\beta_{I_1}^1 I_1 + \beta_{A_1}^1 A_1 + \beta_{I_2}^1 I_2 + \beta_{A_2}^1 A_2) S_1,\\
\dot{S_2} &=& -(\beta_{I_1}^2 I_1 + \beta_{A_1}^2 A_1 + \beta_{I_2}^2 I_2 + \beta_{A_2}^2 A_2) S_2,\\
\dot{E_1} &=& (\beta_{I_1}^1 I_1 + \beta_{A_1}^1 A_1 + \beta_{I_2}^1 I_2 + \beta_{A_2}^1 A_2) S_1 - (\gamma_1 + \nu_1) E_1,\\
\dot{E_2} &=& (\beta_{I_1}^2 I_1 + \beta_{A_1}^2 A_1 + \beta_{I_2}^2 I_2 + \beta_{A_2}^2 A_2) S_2 - (\gamma_2 + \nu_2) E_2,\\
\dot{A_1} &=& \gamma_1 E_1 - (\kappa_1 + \varphi_1 + \alpha_{A_1}) A_1,\\
\dot{A_2} &=& \gamma_2 E_2 - (\kappa_2 + \varphi_2 + \alpha_{A_2}) A_2,\\
\dot{I_1} &=& \kappa_1 A_1 + \nu_1 E_1 - (\xi_1 + \alpha_{I_1}) I_1,\\
\dot{I_2} &=& \kappa_2 A_2 + \nu_2 E_2 - (\xi_2 + \alpha_{I_2}) I_2,\\
\dot{J_1} &=& \varphi_1 A_1 + \xi_1 I_1 - \alpha_{J_1} J_1,\\
\dot{J_2} &=& \varphi_2 A_2 + \xi_2 I_2 - (\eta + \alpha_{J_2}) J_2,\\
\dot{R} &=& \alpha_{A_1} A_1 + \alpha_{A_2} A_2 + \alpha_{I_1} I_1 + \alpha_{I_2} I_2 + \alpha_{J_1} J_1 + \alpha_{J_2} J_2 + \alpha_C C,\\
\dot{C} &=& \eta J_2 - (\mu + \alpha_{\mathbf{C}}) C,\\
\dot{D} &=& \mu C,
\end{array}
\right. \tag{9}
$$

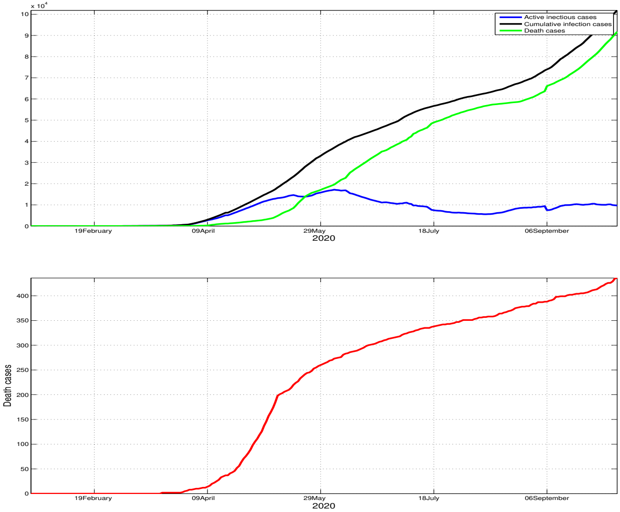


Figure: Evolution of the COVID19 in the UAE

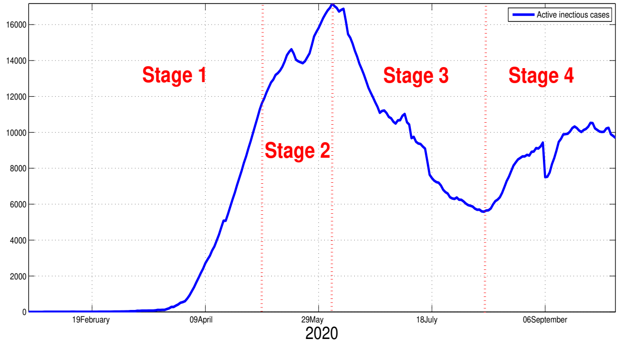


Figure: The stages of the COVID-19 in the UAE

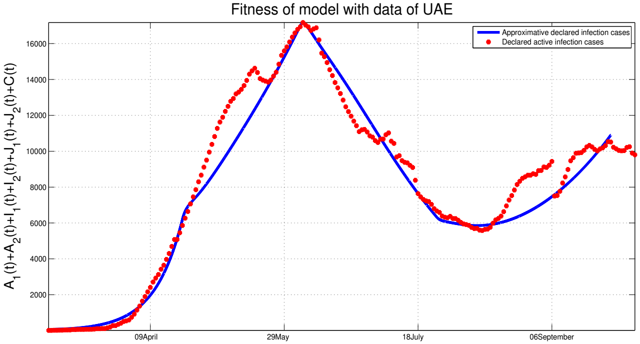


Figure: Fitting solution of our model to the active cases

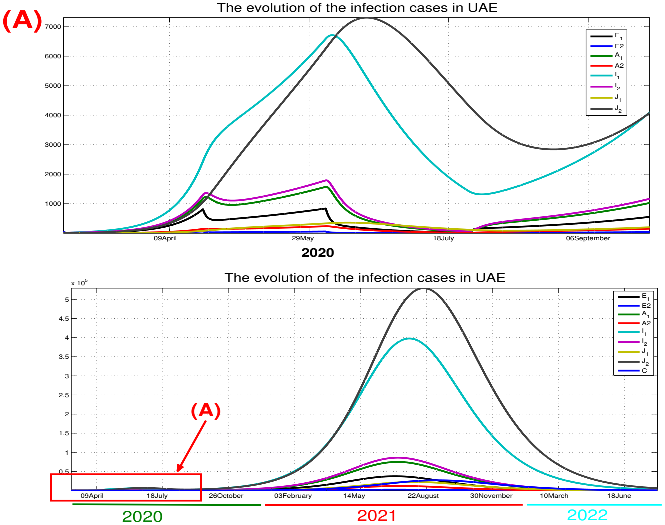


Figure: The evolution of the infection cases in UAE.

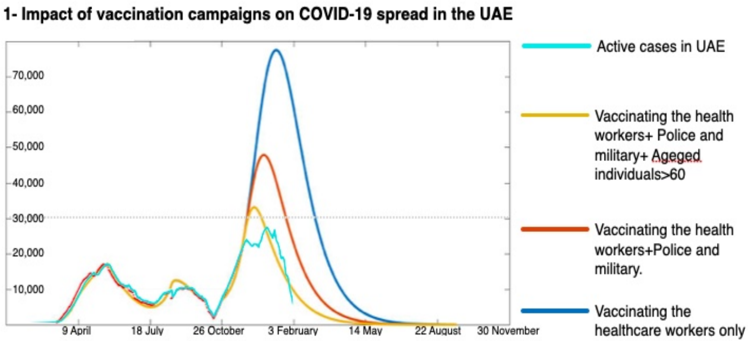


Figure: Impact of vaccination campaigns on COVID-19 spread in the UAE

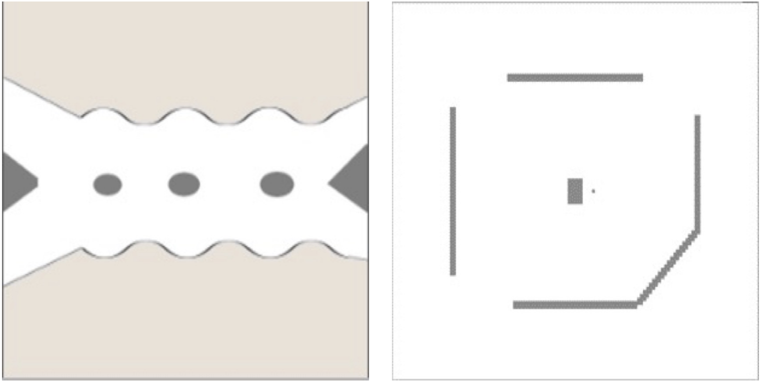


Figure: Environment model of the Tawaf and Ramy al-Jamarat rituals

Al-Shaery, Hejase, Tridane, Farooqi, El Jassmi, Agent-based modeling of the Hajj rituals with a possible spread of COVID-19, Sustainability, 2021

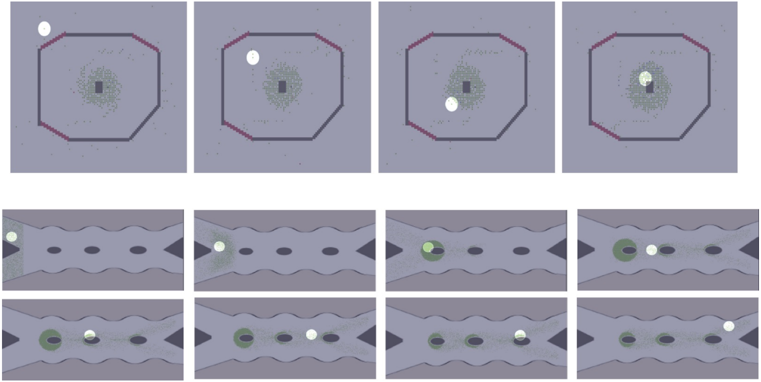


Figure: Crowd mobility during the rituals

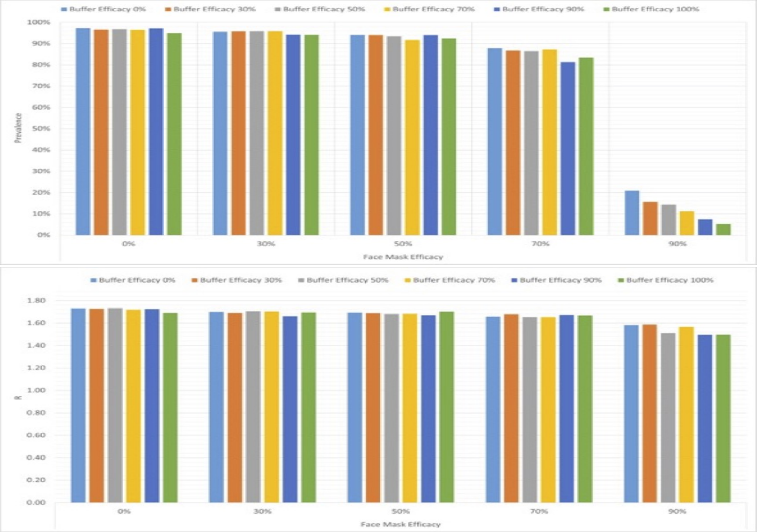


Figure: Effect of control measures on the prevalence of COVID-19 in Tawaf

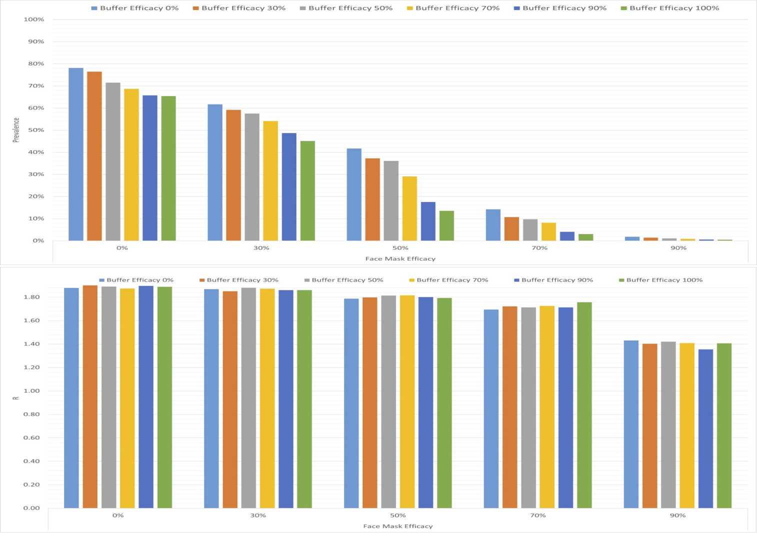


Figure: Effect of control measures on the prevalence of COVID-19 in Ramy al-Jamarat

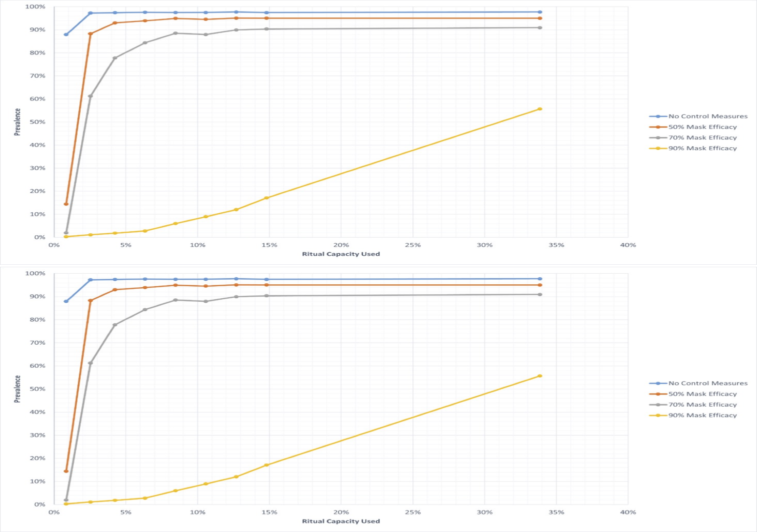


Figure: Crowd control in this context limits the capacity in Tawaf

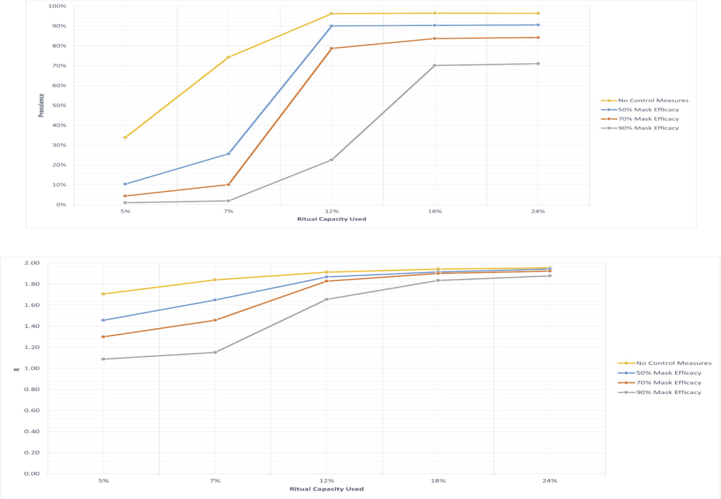


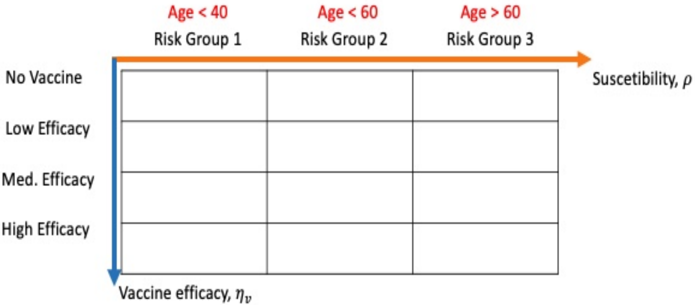
Figure: Crowd control in this context limits the capacity in Ramy al-Jamarat

Figure: Effect of vaccinations efficacies on big gatherings and crowds movement using Agent-based modeling

Hajj Vaccine Simulation

Using the same NetLogo-based implementation:

- Agents are divided by vaccine type and risk group.
- Each vaccine type reflects a different efficacy, while risk group reflect susceptibility to the virus (for example, effect of age).



Genetic Algorithm Implementation

Fitness evaluation

All candidate solutions are simulated using the Hajj simulator in NetLogo and the disease prevalence is collected. The fitness is defined as

$$\boldsymbol{f(c) = K_1\eta + K_2D + K_3(1 - R)}$$

- **Disease prevalence.**

$$\eta \in [0, 1]$$

Where $\eta = 1$ represents the entire population is infected.

- **Population diversity (Simpson's Index).**

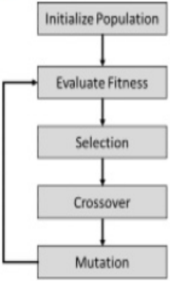
$$D = \sum_{i=1}^{R} \frac{n_i(n_i - 1)}{N(N - 1)} \in [0, 1]$$

Where $D = 0$ represents infinite diversity, $D = 1$ represents no diversity [1].

- **Ritual capacity.**

$$R = \frac{p_6}{N_{MAX}} \in [0, 1]$$

Where N_{MAX} is a hyperparameter representing the maximum capacity.



[1] Simpson, Edward H. "Measurement of diversity." *nature* 163.4148 (1949): 688-688.

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