SITE Research Center online event from January 16 to 20, 2022 on

Long Time Behavior and Singularity Formation in PDEs — Part IV

(the talk times are given in Gulf Standard Time)

Sunday, January 16

1.15 pm: Opening remarks by **Sehamuddin Galadari**, Senior Vice Provost of Research; Managing Director, Research Institute; Professor of Biology, New York University, Abu Dhabi

1.20 - 2.20 pm: **Zhifei Zhang**

- Title: Global regularity and asymptotic behavior for the steady Prandtl equation
- Abstract: In this talk, I will introduce some results about the global C^∞ regularity and asymptotic behavior for the steady Prandtl equation under favaroble pressure gradient.

2.30 – 3.30 pm: **Zineb Hassainia**

- Title: Time quasi-periodic vortex patches
- Abstract: We shall discuss the emergence of quasi periodic solutions for the 2D-Euler equations close to Kirchhoff elliptical vortices. We prove the existence of such solutions when the ellipse aspect ratio belongs to a suitable Cantor set. The proof is based on Nash-Moser implicit function theorem and KAM theory.

8.00 – 9.00 pm: **Tai-peng Tsai**

- Title: Remarks on local regularity of axisymmetric solutions to the 3D Navier-Stokes equations
- Abstract: A new local regularity criteria for the axisymmetric solutions to the 3D Navier-Stokes equations is investigated. It is slightly supercritical and implies an upper bound for the oscillation of $\Gamma=r u^{\theta}$: for any $0< \tau<1$, there exists a constant $c>0$, $$ |\Gamma(r,x_{3},t)|\leq N e^{-c\, |\ln r|^{\tau}},\ 0<r\leq \frac{1}{4}.$$
  This talk is based on a joint work with Hui Chen and Ting Zhang, arXiv:2201.01766

Monday, January 17

1.15 – 2.15 pm: **Edriss S. Titi**

- Title: Rigorous Justification of the Viscous Primitive Equations and Their Global Regularity Coupled to Moisture Dynamics
- Abstract: In this talk I will consider the vanishing small parameter limit behavior for two geophysical models involving small parameters. The first problem is concerning the rigorous

justification of the derivation of the Primitive Equations of planetary scale oceanic dynamics from the three-dimensional Navier-Stokes equations as the vanishing limit of the small aspect ratio of the depth to horizontal width. Specifically, I will show that the Navier-Stokes equations, after being scaled appropriately by the small aspect ratio parameter of the physical domain, converge strongly to the primitive equations, globally and uniformly in time, and the convergence rate is of the same order as the aspect ratio parameter. The second problem is concerning a tropical atmospheric model with moisture, that involves a convective adjustment relaxation time parameter $\varepsilon > 0$. In particular, I will present results concerning the limit behavior as $\varepsilon \to 0$.
In addition, I will also show the global well-posedeness of the coupled three-dimensional viscous Primitive Equations with a micro-physics phase change moisture model for cloud formation.

2.30 – 3.30 pm: **Thomas Alazard**

- Title: Entropies of free surface flows in fluid dynamics
- Abstract: I will discuss recent works with Didier Bresch, Nicolas Meunier and Didier Smets on the dynamics of a free surface carried by an incompressible flow obeying Darcy's law. This talk focuses on monotonicity properties of different kinds: maximum principles, Lyapunov functions and entropies. The analysis is based on exact identities which in turn allow us to study the Cauchy problem.

8.00 – 9.00 pm: **Yannick Sire**

- Title: Blow-up via parabolic gluing
- Abstract: We will present some recent results on the construction of blow-up solutions for critical parabolic problems of geometric flavor. Initiated in the recent years, the inner/outer parabolic gluing is a very versatile parabolic version of the well-known Lyapunov-Schmidt reduction in elliptic PDE theory. The method allows to prove rigorously some formal matching asymptotics (if any available) for several PDEs arising in porous media, geometric flows, etc….I will give an overview of the strategy and will present several applications to (variations of) the harmonic map flow, Yamabe flow and Yang-Mills flow. I will also present some open questions.

<u>Tuesday, January 18</u>

1.15 – 2.15 pm: **Tristan Buckmaster**

- Title: Smooth Imploding Solutions for 3D Compressible Fluids
- Abstract: Building upon the pioneering work of Merle-Rodnianski-Szeftel, we construct exact, smooth self-similar imploding solutions to the 3D isentropic compressible Euler equations for ideal gases for all adiabatic exponents. For the particular exponent 7/5, corresponding to air and akin to the result of Merle-Raphael-Rodnianski-Szeftel, we show the existence of a sequence of smooth, self-similar imploding solutions. In addition, we provide simplified proofs of linear stability and non-linear stability which allows us to construct asymptotically self-similar

imploding solutions to the compressible Navier-Stokes equations with density independent viscosity for the adiabatic exponent 7/5 and density bounded from below.

2.30 – 3.30 pm: **Charles Collot**

- Title: Soliton resolution for the radial quadratic wave equation in six space dimensions
- Abstract: This talk will present the soliton resolution for the radial quadratic semilinear wave equation in six dimensions. In a joint work with T. Duyckaerts, C. Kenig, and F. Merle, we prove that any spherically symmetric solution, that remains bounded in the energy norm, evolves asymptotically to a sum of decoupled stationary states, plus a radiation term. As a by-product of the approach we prove the non-existence of multisoliton solutions that do not emit any radiation. The proof follows the method initiated for large odd dimensions by Duyckaerts, Kenig and Merle, reducing the problem to ruling out the existence of such non-radiative multisolitons, by deriving a contradiction from a finite dimensional system of ordinary differential equations governing their modulation parameters. In comparison, the difficulty in six dimensions is the failure of certain channel of energy estimates and the related existence of a linear resonance. The main novelties are the obtention of new channel of energy estimates, as well as the classification of non-radiative solutions with small energy.

8.00 – 9.00 pm: **Dallas Albritton**

- Title: Non-uniqueness of Leray solutions of the forced Navier-Stokes equations
- Abstract: In this talk, we will explain our recent work with Elia Brué and Maria Colombo on non-uniqueness of Leray solutions of the forced Navier-Stokes equations. Our method is inspired by the work of Vishik (2018) on unstable vortices and the non-uniqueness scenarios of Jia and Šverák (2015), which we will also explain.

Wednesday, January 19

1.15 – 2.15 pm: **Mohamed Majdoub**

- Title: Source-type solutions for the stable thin-film equation
- Abstract: We investigate the existence and the boundary regularity of source-type self-similar solutions to the stable thin-film equation. We show that the higher order corrections are analytic with respect to three variables: the first one is just the spatial variable, whereas the second and third (except for mobility n = 2) are irrational powers of it.

2.30 – 3.30 pm: **Rupert Frank**

- Title: Time-dependent Bogoliubov-de-Gennes and Ginzburg-Landau equations
- Abstract: We study the time-dependent Bogoliubov--de-Gennes equations for generic translation-invariant fermionic many-body systems. For initial states that are close to thermal equilibrium states at temperatures near the critical temperature, we show that the magnitude of the order parameter stays approximately constant in time and, in particular, does not follow a

time-dependent Ginzburg--Landau equation, which is often employed as a phenomenological description and predicts a decay of the order parameter in time.

8.00 – 9.00 pm: **Fabio Pusateri**

- Title: Internal modes for quadratic Klein-Gordon in 3d
- Abstract: We consider quadratic Klein-Gordon equations with an external potential V in $3+1$ space-time dimensions. We assume that V is generic and decaying, and that the operator $-\Delta + V + m^2$ has an eigenvalue $\lambda^2 < m^2$. This is a so-called + 'internal mode' and gives rise to time-periodic localized solutions of the linear flow. Our main result shows that small nonlinear solutions slowly decay as the energy is transferred from the internal mode to the continuous spectrum, provided a natural Fermi golden rule holds. We obtain very precise asymptotic information including sharp rates of decay and the growth of weighted norms. Our results extend the seminal work of Soffer-Weinstein for cubic nonlinearities. This is joint work with Tristan Léger (Princeton University).

Thursday, January 20

1.15 – 2.15 pm: **Eliot Pacherie**

- Title: On the stability of the Ginzburg-Landau vortex
- Abstract: We investigate the orbital stability of the Ginzburg-Landau vortex of degree one for the Gross-Pitaevskii equation in dimension two. Our main result is a nonlinear coercivity estimate on the renormalized energy, from which we can deduce the stability. This is a joint work with Philippe Gravejat and Didier Smets.

6.15 – 7.15 pm: **Andrew Lawrie**

- Title: The soliton resolution conjecture for equivariant wave maps
- Abstract: I will present joint work with Jacek Jendrej (CRNS, Sorbonne Paris Nord) on equivariant wave maps with values in the two-sphere. We prove that every finite energy equivariant wave map resolves, as time passes, into a superposition of decoupled harmonic maps and radiation, settling the soliton resolution conjecture for this equation. As a byproduct of our analysis we also prove that there are no elastic collisions between pure multi-solitons. We will discuss some of the motivations behind soliton resolution and some of the techniques involved in the proof.

8.00 – 9.00 pm: **Benoit Pausader**

- Title: Global existence for the Euler-Coriolis system
- Abstract: For the 3D incompressible Euler equation, a distinguished solution is the flow leading to rigid rotation about an axis u(x,y,z)=(-y,x,0). We show that this solution is stable under small axisymmetric perturbations, and that the perturbations exist globally and scatter linearly back to the equilibrium through a dispersive effect induced by the linearized Euler-Coriolis equations.