Behavioral Crowd Dynamics

Recent Results and Safety

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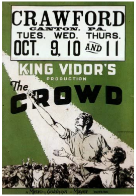
1.1. Behavioral Crowd Dynamics



The study of human crowds can contribute to the well-being of our society. The study generates challenging analytic and computational problems. The dynamics is influenced by social interactions and collective learning. The modeling requires a multiscale vision and accounts for the quality and geometry of the venue where the dynamics occur.

1. **Behavioral Crowds: Complexity and Key Problems**
2. **Mathematical Tools and Social Dynamics**
3. **Pattern Dynamics and Safety Problems**

1.2. Behavioral Crowds: Complexity and Key Problems

• The greatest part of known models are based on the assumption of rational, almost optimal, behaviors of individuals. However, real conditions can show irrational behaviors that can generate events, where crisis substitute safety.

• When irrational behaviors appear, small deviations in the input can create large deviations in the output. Some of these events are not easily predictable, however once they appeared, a rational interpretation can often explain them. The term "black swan" is a metaphoric expression used by Nassam Taleb to denote not predictable events.

• In some cases it might happen that antagonist groups fight in a crowd. The consequence is that unsafe situations can involve not aware citizens and generate safety-security problems.

• Recent studies have been focused on the modeling of heterogeneous awareness of the crowd to contagion problems.

1.3. Behavioral Crowds: Complexity and Key Problems

• The recent literature on crowd modeling has enlightened the need of a modeling approach, where the behavioral features of crowds, to be viewed as a living, hence complex system, are taken into account.

• The most important feature is the ability to express a strategy which is heterogeneously distributed among walkers and it depends on their state and on that of those in their surrounding environment.

• Heterogeneity can include a possible presence of different groups. For instance, attracting the crowd to their own strategy towards the selection of optimal routes among the various available ones.

• Stress conditions which, in some cases are simply induced by overcrowding, can have an important influence on the dynamics crowds and affect safety.

1.4. Behavioral Crowds: Complexity and Key Problems

Modeling Strategy

1. Assessment of the complexity features of crowds viewed as living systems;
2. Selection of the social phenomena to be inserted in the model;
3. Subdivision into different groups related both to social and mechanical features which have to be precisely referred to the type of dynamics which is object of modeling;
4. Selection of the modeling scale and derivation of a mathematical structure consistent with the requirements in the first three items;
5. Derivation of models by inserting, into the said structure, the mathematical description of interactions for both social and mechanical dynamics including their reciprocal interplay.

1.5. Behavioral Crowds: Complexity and Key Problems

- **A Multiscale Vision:** Modeling of individual based interactions are used firstly to derive models at the microscopic scale and, subsequently, kinetic type models, namely at the mesoscopic scale. Asymptotic methods lead to hydrodynamical models.

- **The Role of Emotional State and Pattern Formation:** The presence of stress can induce significant modification in the overall self-organization, and hence on the collective dynamics. A deep understanding of possible social dynamics can contribute to account for the interactions of different even antagonist groups.

- **Safety-Security Problems:** Modeling and simulations can support crisis management by means of platforms where the process of selecting the most appropriate actions towards safety can be developed by predictive engines which refer the real dynamics to a database, where a huge number of simulations are stored.

1.6. Behavioral Crowds: Complexity and Key Problems

Rarefied, high density, contagion and heterogeneous flows

A Personal Quest on Kinetic Theory of Crowd Dynamics

• N. Bellomo and A. Bellouquid, and D. Knopoff, From the micro-scale to collective crowd dynamics, *SIAM Multiscale Model. Simul.*, **11**, (2013), 943–963.

• N. Bellomo and L. Gibelli, Toward a mathematical theory of behavioral-social dynamics for pedestrian crowds, *Math. Models Methods Appl. Sci.*, **25**, (2015), 2417–2437.

• N. Bellomo, L. Gibelli, and N. Outada, On the Interplay between Behavioral Dynamics and Social Interactions in Human Crowds, *Kinetic Related Models*, **12**, (2019), 397–409.

• N. Bellomo, L. Gibelli, A. Quaini, and A. Reali, Towards a mathematical theory of behavioral human crowds, *Math. Models Methods Appl. Sci.*, **32(2)**, (2022), 321–358. "Open access".

• N. Bellomo, J. Liao, A. Quaini, L. Russo, and C. Siettos, Human behavioral crowds review, critical analysis, and research perspectives, *Math. Models Methods Appl. Sci.*, **33**, (2023), https://doi.org/10.1142/S0218202523500379. "Open access".

Recent articles on behavioral crowd dynamics

• N.L. Kontorovsky, C.G. Ferrari, J. P. Pinasco, and N. Saintier, Kinetic modeling of coupled epidemic and behavior dynamics: The social impact of public policies, *Math. Models Methods Appl. Sci.*, **32**, 2037–2076, (2022).

• D. Kim and A. Quaini, Coupling kinetic theory approaches for pedestrian dynamics and disease contagion in a confined environment, *Math. Models Methods Appl. Sci.*, **30(9)**, (2020).

• J. Liao and L. Zhou, A kinetic modeling of crowd evacuation with several groups in complex venues, *Math. Models Methods Appl. Sci.*, **32(10)**, 1785–1805, (2022).

• Y. Bi, D. Li, and Y. Luo, Combining keyframes and image classification for violent behavior recognition, *Applied Sciences*, 12, 8014, (2022).

• J. Ma, M. Wang, and L. Li, Research on crowd dynamic risk management based on the psychological stress perception function, *Journal Statistical Mechanics*, 123405, (2022).

1.9. Behavioral Crowds: Complexity and Key Problems

• M. Grave, A. Viguerie, G.F. Barros, A. Reali, R.F.S. Andrade, and A.L.G.A. Coutinho, Modeling nonlocal behavior in epidemics via a reaction–diffusion system incorporating population movement along a network, *Computer Methods in Applied Mechanics and Engineering*, 401, 115541, (2022).

• Z. Sabeur and B. A. Zavar, Crowd behaviour understanding using computer vision, **Crowd Dynamics, Volume 3**, *Series: Modelling Simulations Science Engineering Technology*, 49–72, (2022).

• N. Bakhdil, A. El Mousaoui, A. Hakim, A kinetic theory approach to model pedestrian social groups in bounded domains, *Kinetic Related Models*, **24**, (2023), doi:10.3934/krm.2023017.

• P. Agnelli, B. Buffa, D.A. Knopoff, and G. Torres, A spatial kinetic model of crowd evacuation dynamics with infectious disease contagion, *Bulletin Math. Biol.*, **85(4)**, Article 23, (2023).

1.10. Behavioral Crowds: Complexity and Key Problems

Motivation and EU Projects

• **Safety in evacuation dynamics:** *Modeling and simulations can contribute to decision making of crisis managers in charge to guide evacuation dynamics.*

• **Contagion awareness:** *Modeling and simulations can contribute to improve the awareness of contagion risk in citizens.*

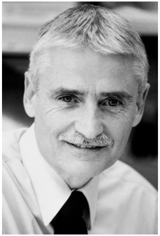
• **Pollution problems in cities:** *Modeling and simulations of traffic flows, vehicles and crowds, can contribute to mitigate the pollution risk.*

• **eVACUATE:** *A holistic, scenario independent, situation-awareness and guidance system for sustaining the Active Evacuation Route for large crowds.* **Seventh Framework Programme**.

• **Safeciti:** *Simulation Platform for the Analysis of Crowd Turmoil in Urban Environments with Training and Predictive Capabilities.* **Seventh Framework Programme**.

1.11. Behavioral Crowds: Complexity and Key Problems

The quest towards mathematical tools to describe behavioral crowd dynamics should not forget about the ambitious goal of developing a mathematical theory of living systems.

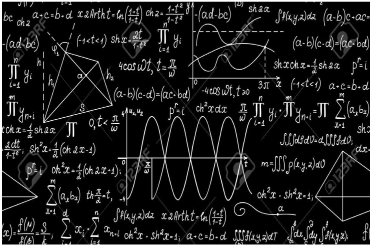


- N. Bellomo, A. Bellouquid, L. Gibelli, and N. Outada, **A Quest Towards a Mathematical Theory of Living Systems**, Birkhäuser, New York, (2017).

End of Part I

2.1. Mathematical Tools and Social Dynamics

Mathematical tools towards social dynamics in crowds



This part of the lecture focuses on the search for mathematical tools to model crowd dynamics accounting for collective social dynamics. We do start for the study of mathematical structures at each scale and, out of a critical analysis, we select the kinetic theory approach.

A key issue of the search for mathematical tools is the the approach rely on recent results on the mathematical theory of living systems.

2.2. Mathematical Tools and Social Dynamics

The dynamics depends on the geometry and quality of the venue

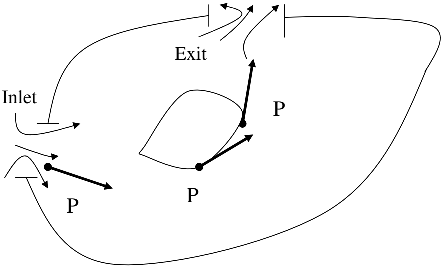


Figure – Domain with exit and internal obstacles.

The set of all walls, including that of obstacles and of entrance and exit doors, is denoted by Σ.
The physical quality of the venue is modeled by a parameter $\alpha \in [0, 1]$, where $\alpha = 0$ and $\alpha = 1$ denote the worst and best quality, respectively.

2.3. Mathematical Tools and Social Dynamics

Micro-scale structures Consider a human crowd with N pedestrians each labeled by the subscript $i \in \{1, \dots, N\}$, their state is defined by: position: $\mathbf{x}_i = \mathbf{x}_i(t) = (x_i(t), y_i(t))$; velocity: $\mathbf{v}_i = \mathbf{v}_i(t) = (v_{ix}(t), v_{iy}(t))$; and activity (social variable): $\mathbf{u}_i = \mathbf{u}_i(t)$.

The framework corresponds to a pseudo-Newtonian mechanics:

$$\begin{cases} \dfrac{d\mathbf{u}_i}{dt} = \mathbf{z}_i, \\ \dfrac{d\mathbf{z}_i}{dt} = \displaystyle\sum_{j \in \Omega_i} \boldsymbol{\psi}_i(\mathbf{x}_i, \mathbf{v}_i, \mathbf{u}_i, \mathbf{x}_j, \mathbf{v}_j, \mathbf{u}_j; \alpha, \Sigma), \\ \dfrac{d\mathbf{x}_i}{dt} = \mathbf{v}_i, \\ \dfrac{d\mathbf{v}_i}{dt} = \displaystyle\sum_{j \in \Omega_i} \boldsymbol{\varphi}_i(\mathbf{x}_i, \mathbf{v}_i, \mathbf{u}_i, \mathbf{x}_j, \mathbf{v}_j, \mathbf{u}_j; \alpha, \Sigma), \end{cases}$$

where Ω_i be the interaction domain of the i-pedestrian, the notation $j \in \Omega_i$ indicates summation of all j-particles in Ω_i.

Macro-scale structures, where $(\rho, \xi, \mathbf{u})$ define the state of the system:

- $\rho = \rho(t, \mathbf{x})$ is the local *density* of the crowd at the point $\mathbf{x}$ and time t, normalized with respect to the maximum packing density ρ_M.
- $\xi = \xi(t, \mathbf{x}) = \xi = \xi(t, \mathbf{x})\, \omega(t, \mathbf{x})$ is the mean *velocity* at the point $\mathbf{x}$ and time t, where the speed ξ is normalized with respect to the maximum limit speed ξ_M. ξ is the dimensionless mean speed and ω is the unit vector giving the direction of the local mean velocity;
- $\mathbf{u} = \mathbf{u}(t, \mathbf{x})$ is the dimensionless local mean *activity* representing the specific social-emotional state considered in each case study, with $\mathbf{u} \in D_{\mathbf{u}}$ for a specific parameter domain.

The interaction domain $\Omega = \Omega(t, \mathbf{x}; \omega(t, \mathbf{x}))$ is analogous to that at the lower scales. The pedestrians at $\mathbf{x}$ perceive the action of all pedestrians in Ω)non'local interactions).

2.5. Mathematical Tools and Social Dynamics

Macro-scale structures

At the macroscopic scale, the human crowd is described by a second order differential system for $\rho(t, \mathbf{x})$, $\xi(t, \mathbf{x})$ and $\mathbf{u}(t, \mathbf{x})$:

$$\begin{cases} \dfrac{\partial \rho}{\partial t} + \nabla_{\mathbf{x}} \cdot (\rho\, \xi) = 0, \\ \dfrac{\partial \xi}{\partial t} + \xi \cdot \nabla_{\mathbf{x}} \xi = \mathbf{A}[\rho, \xi, \mathbf{u}], \\ \dfrac{\partial \mathbf{u}}{\partial t} + \nabla_{\mathbf{x}} \cdot (\mathbf{u}\, \xi) = \mathbf{S}[\rho, \xi, \mathbf{u}], \end{cases} \tag{1}$$

where $\mathbf{A}$ is a pseudo-mechanical acceleration acting on pedestrians in the infinitesimal volume $d\mathbf{x}$ and $\mathbf{S}$ is a source term that implements locally the emotional state generated by the interaction with the surrounding pedestrians. Square brackets denotes that functional dependence with respect to its arguments.

• *Mesoscopic (kinetic):* The micro-state of the interacting entities is identified by the position and velocity, while their representation is delivered by a suitable probability distribution function over the micro-state:

$$f_i = f_i(t, \mathbf{x}, v, \theta, u), \quad \mathbf{x} \in \Sigma \subset \mathbb{R}^3, \quad v \in [0,1], \quad \theta \in [0, 2\pi) \quad u \in [0,1],$$

where the overall system is subdivided into functional subsystems labeled by the subscript i.

Macroscopic observable quantities can be obtained by weighted moments. The local *density* and *mean velocity*;

$$\rho_i(t,\mathbf{x}) = \int_0^1 \int_0^{2\pi} \int_0^1 f_i(t, \mathbf{x}, v, \theta, u)\, v dv\, d\theta\, du,$$

$$\boldsymbol{\xi}_i(t,\mathbf{x}) = \frac{1}{\rho_i(t,\mathbf{x})} \int_0^1 \int_0^{2\pi} \int_0^1 \mathbf{v}\, f_i(t, \mathbf{x}, v, \theta, u)\, v dv\, d\theta\, du.$$

Common features in the modeling of interactions

1. All a-particles have a visibility angle related to their velocity direction and a visibility radius depending on the quality and shape of the venue. Within this visibility area, they can have a sensitivity domain depending on the local density.
2. All a-particles are subject to different stimuli, namely trend towards a direction corresponding to a meeting point, a walking direction, attraction by the motion of the other a-particles which, however, is contrasted by a desire to avoid overcrowded areas.
3. The choice of the velocity direction corresponds to a weighted selection of the stimuli related to the quality of the venue, the emotional state and the local density.
4. Subsequently, a-particles adapt their speed to local density accounting for the quality of the venue.
5. The emotional states is not a constant quantity, but it acts as a microscopic variable to be inserted in the interactions at each scale.

- **Rationale of the kinetic theory approach**
  - The overall system is subdivided into functional subsystems constituted by entities, called active particles, whose individual state is called activity.
  - The state of each functional subsystem is defined by a probability distribution over the micro-scale state: position, velocity, and activity.
  - The dynamics of the probability distribution is obtained by a balance of number of particles within elementary volume of the space of the microscopic states, where the dynamics of inflow and outflow of particles is related to interactions a modeled by stochastic games.

Representation and interactions

• The state of functional subsystems is defined by: $f_i = f_i(t, \mathbf{x}, v, \theta, u)$.

• Interactions involve, at each time t and for each FS, three types of a-particles *which play the game*. The *test particle*, the *field particle*, and the *candidate particle*. Their distribution functions are, respectively $f_i(t, \mathbf{x}, \mathbf{v}, u)$, $f_k(t, \mathbf{x}, \mathbf{v}^*, u^*)$, and $f_h(t, \mathbf{x}, \mathbf{v}_*, u_*)$. The *test particle*, is representative, for each FS, of the whole system. It loses its state by interaction with the *field particles*, while the *candidate particle* can acquire, in probability, the micro-state of the test particle.

• Interactions, which are nonlocal and nonlinear, can be modeled by the following quantities: *Interaction domain* Ω_s, *interaction rate* η, *transition probability density* $\mathcal{A}$ which can depend on the micro-state and on the distribution function of the interacting particles.

Hierarchy of the decision process

Interactions correspond to a *decision process* by which each walker develops a strategy obtained by the following sequence of decisions:

1. Contagion of the emotional state u;
2. Selection of the walking direction;
3. Selection of the walking speed.

Decisions are supposed to be sequentially dependent

$$\mathcal{A}(\mathbf{v}_* \to \mathbf{v}, u_* \to u) = \mathcal{A}^u(u_* \to u) \times \mathcal{A}^\theta(\theta_* \to \theta) \times \mathcal{A}^v(v_* \to v).$$

which depend also on the quality of the venue-environment.

2.11. Mathematical Tools and Social Dynamics

Contagion of the emotional state The dynamics by which the stress initially in Σ_s diffuse among all walkers is driven by the highest value:

$$u^* > u_* \, : \, \mathcal{A}^u(u_* \to u|u_*, u^*) = \delta\big(u - u_* - \varepsilon(u^* - u_*)(1 - u_*)\big),$$

$$u^* \leq u_* \, : \, \mathcal{A}^u(u_* \to u|u_*, u^*) = \delta\big(u - u_*\big).$$

Dynamics of the velocity direction: At high density, walkers try to drift apart from the more congested area moving in the direction of the less congested areas, while at low density, walkers head for the target unless their level of anxiety is high and induces a trend to follow the mean stream.

Dynamics of the speed: Once the direction of motion has been selected, walkers adjust their speed to the local density and mean speed conditions. If the walker's speed is lower (higher) than the mean speed, then the trend of the walkers increase (decreases) the speed.

2.12. Mathematical Tools and Social Dynamics

Role of the perceived density : Walkers moving along a direction perceive a density higher (lower) than the real one in the presence of positive (negative) gradients. The attraction towards the target as it increases by decreasing density.

Role of the emotional state: Increasing the emotional state increases the attraction to the stream.

Role of the walls and obstacles: The presence of walls, which is modified by the distance from the wall, acts in a way that the search of less congested areas decreases with decreasing distance.

Boundary conditions: Let f^r and f^i denote, respectively, the distribution function after and before the interaction with the wall with directions θ_i and θ_r. Then a reflection model states the boundary conditions

$$f^r(t,\mathbf{x},v,\theta_r,u) = \frac{|\mathbf{v}_i \cdot \mathbf{n}|}{|\mathbf{v}_r \cdot \mathbf{n}|} \int R(\theta_i \to \theta_r)\, f^i(t,\mathbf{x},v,\theta_i,u)\, d\theta_i'.$$

2.13. Mathematical Tools and Social Dynamics

A general structure can be derived for a multi-component crowd with FSs crossing:

$$
\begin{aligned}
& (\partial_t + \mathbf{v}\cdot\partial_{\mathbf{x}})\; f_i(t,\mathbf{x},\mathbf{v},u) = Q_i[\mathbf{f}](t,\mathbf{x},\mathbf{v},u) \\
& \quad = \sum_{h=1}^{n}\sum_{k=1}^{n}\int_{D\times D} \eta_{hk}[\mathbf{f}](\mathbf{x},\mathbf{v}_*,\mathbf{v}^*,u_*,u^*;\alpha) \\
& \qquad \times \mathcal{A}_{hk}^{i}[\mathbf{f}](\mathbf{v}_*\to\mathbf{v},u_*\to u|\mathbf{v}_*,\mathbf{v}^*,u_*,u^*;\alpha) \\
& \qquad \times f_h(t,\mathbf{x},\mathbf{v}_*,u_*) f_k(t,\mathbf{x},\mathbf{v}^*,u^*)\,d\mathbf{v}_*\,d\mathbf{v}^*\,du_*\,du^* \\
& \quad - f_i(t,\mathbf{x},\mathbf{v},u)\sum_{k=1}^{n}\int_{D}\eta_{ik}[\mathbf{f}](\mathbf{x},\mathbf{v},\mathbf{v}^*,u,u^*;\alpha) f_k(t,\mathbf{x},\mathbf{v}^*,u^*)\,d\mathbf{v}^*\,du^*.
\end{aligned}
$$

Models are obtained by implementing this structure by models of micro-scale interactions.

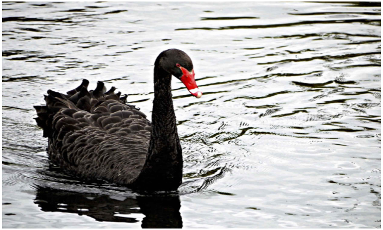
Technical details are available in N. Bellomo, L. Gibelli, A. Quaini, and A. Reali, Towards a mathematical theory of behavioral human crowds, *Math. Models Methods Appl. Sci.*, **32(2)**, (2022), 321–358. "Open access".

The derivation of macroscopic equations from the underlying description at the microscopic scale, within the kinetic theory framework, corresponds to the Sixth Hilbert Problem.

• D. Hilbert, Mathematical problems, *Bull. Amer. Math. Soc.*, **8(10)**, 437-479, (1902).

End Part II

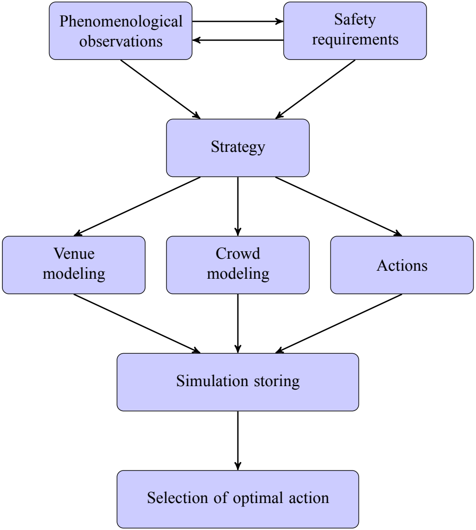
3.1. Pattern Dynamics and Safety Problems



This Part 3 presents a number of simulations developed by Monte Carlo Particle simulations based on the mathematical model presented in Part 2.
We have in mind simulations that can provide useful indications to crisis managers.
Finally some research perspectives are reported.

3.2. Pattern Dynamics and Safety Problems

Simulations can be developed to support crisis managers.



Case study I: The specific features of the dynamics are simply described. A movie will show how the dynamics evolves in time.

The crowd is constituted by two groups moving to opposite directions in a rectangular venue of $20\,m \times 5\,m$.

The group on the left is composed of 40 people uniformly distributed in a rectangular area $4\,m \times 4\,m$ with the initial emotional state set to $u \simeq 0.4$ while the group on the right is composed of 20 people uniformly distributed in a rectangular area of $4\,m \times 2\,m$ with an higher level of stressful condition, namely $u \simeq 0.8$.

The speed ξ, at initial time, is also homogeneously distributed over all walkers at a value $\xi_0 \cong u_0$.

3.4. Pattern Dynamics and Safety Problems

Case study II: The crowd is waiting on a platform of an underground and it is uniformly distributed. Then the crowd receives an order to evacuate. Simulations compare the exit of a crowd with and without contagion of emotional state.

Case study III: The crowd of 30ML person is confined in a square where some of the natural exits have been closed. The dynamics shows the onset of dangerous aggregations during forced evacuations.

3.6. Pattern Dynamics and Safety Problems

A forward look to research perspectives. Examples of social dynamics to be included in crowds modeling

1. **Awareness to contagion dynamics:** Awareness modifies the walkers' trajectories in order to preserve social distances. Actually the behavior differs to the case of strss by perception of danger.
2. **Leaders dynamics:** A few leaders can drive the crowd along optimal paths by attracting them to their walking strategy.
3. **Competition between antagonist groups:** Extreme situations, where antagonist groups contrast each other in a crowd.
4. **Learning dynamics from vocal or visual signalling:** Vocal or visual signals are used to control the dynamics of the crowd in risk situations.

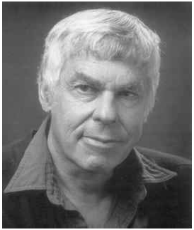
3.7. Pattern Dynamics and Safety Problems

On the Sixth Hilbert Problem: Derivation of hydrodynamic models from the underlying description at the microscopic scale.

> *... To treat in the same manner, by means of axioms, those physical sciences in which already today mathematics plays an important part.*
>
> *... As to the axioms of the theory of probabilities, it seems to me desirable that their logical investigation should be accompanied by a rigorous and satisfactory development of the method of mean values in mathematical physics, and in particular in the kinetic theory of gases. ... Boltzmann's work....*

The micro-macro derivation for models of crowds dynamics in unbounded domains, has been treated in N. B. and A. Bellouquid, *Comm. Math. Sci.*, **13** (2015). *Proving how the structure of macroscopic models is modified by social behaviors and by the presence of walls or obstacles, is a highly challenging open problem*.

Can a unified approach can be designed for crowds and swarms?



*The answer is definitely YES as the two collective systems present common features. However, one has to go beyond the classical theory of swarms and look for a mathematical theory of **behavioral** swarms.*

- N. Bellomo, S-Y. Ha, and N. Outada, Towards a mathematical theory of behavioral swarms, *ESAIM: Control, Optimisation and Calculus of Variations*, **26** (2020), 125.

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The END - Thank You!