On parameter identification in macroscopic models for pedestrian crowds

Part I: G. Jankowiak, A. Iuorio, P. Szmolyan
Part II: S. Gomes and A. Stuart

SITE seminar
NYU Abu Dhabi

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Microscopic modelling

Force based models: position of an individual is determined by forces acting on it.

- *Newton's laws of motion*

$$
\begin{aligned}
\mathrm{d}X_i &= V_i\,\mathrm{d}t\\
\mathrm{d}V_i &= F_i(X_1,\dots,X_N,V_1,\dots,V_N)\,\mathrm{d}t + \sigma_i\,\mathrm{d}B_i^t.
\end{aligned}
$$

$F_i \dots$ repulsion
follow the leader
...

  Here $X_i = X_i(t)$ is the location of the i-th individual, $V_i = V_i(t)$ its velocity, F_i the forces acting on it, and dB_i some additive noise.

- *Overdamped Langevin dynamics:*

$$\mathrm{d}X_i = F(X_1,\dots,X_N)\,\mathrm{d}t + \sigma_i\,\mathrm{d}B_i^t.$$

V_i

X_i

Stochastic optimal control: Each individual wants to minimise a cost functional

kinetic energy

$$J_i(X_1,\dots,X_N,V_i) = \mathbb{E}\Big(\int_0^T L_i(X_i,V_i) + F(X_1,\dots,X_N)\,\mathrm{d}t\Big)$$

under the constraint that $\mathrm{d}X_i = V_i\mathrm{d}t + \sigma_i\mathrm{d}B_i^t$, where L and F denote the running cost.

Microscopic modelling

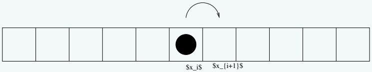
Lattice based models:

- *Consider a domain divided into a cells.*
- *Each cell centre represents a possible position of an individual.*
- *Individuals can jump from one cell to another with a certain transition rate.*

Probability p_i to find an individual at a discrete lattice site X_i:

$$\dot{p}_i = \mathcal{T}_{i+1}^{-} p_{i+1} + \mathcal{T}_{i-1}^{+} p_{i-1} - (\mathcal{T}_i^{+} + \mathcal{T}_i^{-}) p_i \quad \Leftarrow$$

cellular automata



Challenges on the microscopic level:

- *Highly nonlinear coupling*
- *Curse of dimensionality $\Rightarrow$ development of reduced models*
- *Computational complexity*

Macroscopic description

Nonlinear conservation law

Pedestrian density $\rho = \rho(x,t)$ satisfies

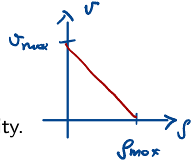
$$\partial_t \rho = \text{div}\left(D(\rho)\nabla(E'(\rho) - V + W * \rho\right).$$

where $V = V(x)$ is a given external potential, $D = D(\rho)$ a nonlinear diffusivity, $E = E(\rho)$ the internal energy and $W = W(x)$ an interaction energy.

Traffic flow model: Lighthill-Whitham-Richards (LWR) model

$$\partial_t \rho = \nabla \cdot (\rho v \nabla \phi).$$

where $\phi = \phi(x)$ is a given normalized potential and the velocity is computed via the *fundamental diagram:*



$$v(\rho) = v_{max}\left(1 - \frac{\rho}{\rho_{\text{max}}}\right)$$

where v_{max} denotes the maximum velocity and $\rho = \rho_{\text{max}}$ the maximum density.

Hughes model for pedestrian flow

Hughes model for pedestrian flow

$\Delta \varepsilon_1 \Delta \rho$

$$\partial_t \rho - \operatorname{div}(\rho \mathfrak{f}^2(\rho) \nabla \phi) = 0,$$

$$\varepsilon_2 \Delta \phi + |\nabla \phi| = \frac{1}{\mathfrak{f}(\rho) + \varepsilon_3}$$

Eikonal equation

shortest distance exit

Possible models for $\mathfrak{f}$: $\mathfrak{f}(\rho) = \rho_{max} - \rho$ *or* $\mathfrak{f}(\rho) = (\rho_{max} - \rho)^2$.

Analytical issues:

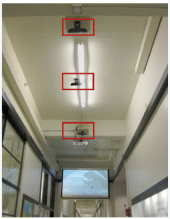
- nonlinear hyperbolic conversation law
- density dependent stationary Hamilton-Jacobi equation (eikonal type) $\rightarrow \phi \in \mathcal{C}^{0,1}$ only
- fully coupled system

Available analytic results (existence and uniqueness) only for a regularised version of the Hughes model in 1D. [1]

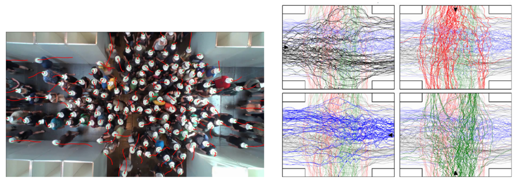
[1] M. DiFrancesco, P.A. Markowich, J.F. Pietschmann, MTW, *On Hughes model for pedestrian flow: the one-dimensional case*, JDE, 2011

Available data

- Video surveillance data: e.g. kinect cameras giving height profiles
- Experimental data: e.g. pedestrian data base of the Forschungszentrum in Jülich.



(a) Kinect sensors mounted on the ceiling.

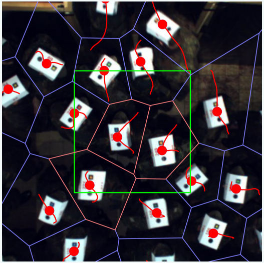


(b) Experiments in Juelich

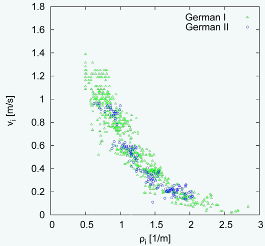
Left: Seer et al., *Validating social force based models with comprehensive real world motion data*, Transportation Research Procedia, 2014. **Right:** Courtesy of Armin Seyfried (Forschungszentrum Jülich), BaSiGo experiments (5 days, 31 experiments, 200 runs, 28 industrial cameras, 2200 participants in total)

Traffic flow $v(\rho) = v_{max} \left(1 - \frac{\rho}{\rho_{max}}\right)$

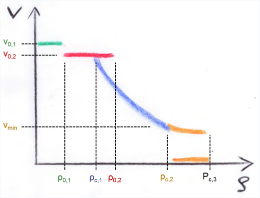
Fundamental diagram



(a) Voronoi diagram.



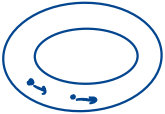
(b) Fundamental diagram

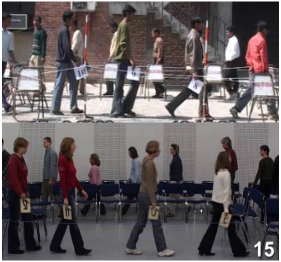


(c) Overview FD

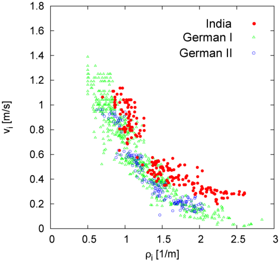
Steffen, B. and Seyfried, A. *Methods for measuring pedestrian density, flow, speed and direction with minimal scatter* Physica A, 2010

Cultural differences





(a) Experiments.



(b) Fundamental diagram.

U. Chattaraj, A. Seyfried, *Comparison of pedestrian fundamental diagram across cultures*, Advances in Complex Systems 12(3), 2009

Part I: Stationary profiles and asymptotic profiles of a PDE model for unidirectional pedestrian flows

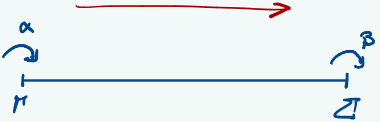
Goal: Understand the impact of inflow and outflow rates at entrances and exits as well as the geometry on pedestrian density profiles in a simple PDE model for unidirectional flows.

Collaborators: Gaspard Jankowiak, Annalisa Iuorio, and Peter Szmolyan

A macroscopic model for unidirectional pedestrian flows

Conservation law for the pedestrian density ρ:

$$\partial_t \rho + \nabla \cdot \mathbf{J} = 0\,,$$
$$\mathbf{J} = -\varepsilon \nabla \rho + \rho\,(1-\rho)\,\underset{\llcorner}{\mathbf{u}}\,, \quad \begin{pmatrix}1\\0\end{pmatrix}$$

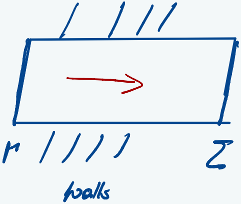


$f(\rho) = \rho_{max} - \rho \qquad \rho_{max} \equiv 1$

where ε denotes the diffusion coefficient and $\mathbf{u}$ is a given normalised vector field.

At the exit Σ and the entrance Γ we impose nonlinear boundary conditions and no flux along walls:

$$\begin{aligned} \mathbf{J}\cdot\mathbf{n} &= 0 && \text{on } \partial\Omega \setminus (\Gamma \cup \Sigma)\,,\\ -\mathbf{J}\cdot\mathbf{n} &= \alpha(1-\rho) && \text{on } \Gamma\,,\\ \mathbf{J}\cdot\mathbf{n} &= \beta\rho && \text{on } \Sigma\,. \end{aligned}$$



M. Burger and J.-F. Pietschmann *Flow characteristics in a crowded transport model* Nonlinearity, 29(11), 2016.

Stationary profiles for a straight corridors

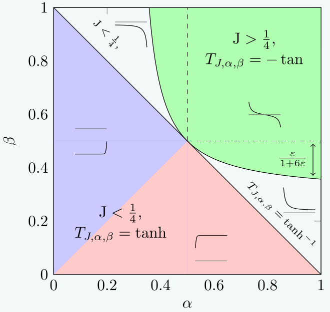
Density profiles in 1D:

$$\rho = \frac{1}{2} + \sqrt{|\mathrm{J} - \frac{1}{4}|}\; T_{\mathrm{J},\alpha,\beta}\left(\varepsilon^{-1}\sqrt{|\mathrm{J} - \frac{1}{4}|}(x-\xi)\right),$$

where $\mathrm{J} = \mathrm{J}(\alpha,\beta)$ and $\xi = \xi(\alpha,\beta) \in \mathbb{R}$, ξ is the value of x for which ρ takes the value $\frac{1}{2}$.

The profile shape is given by

$$T_{\mathrm{J},\alpha,\beta} = \begin{cases} -\tan & \text{if } \mathrm{J} > \frac{1}{4}, \\ \tanh & \text{if } \mathrm{J} < \frac{1}{4} \text{ and } \alpha+\beta < 1, \\ \tanh^{-1} & \text{if } \mathrm{J} < \frac{1}{4} \text{ and } \alpha+\beta > 1. \end{cases}$$



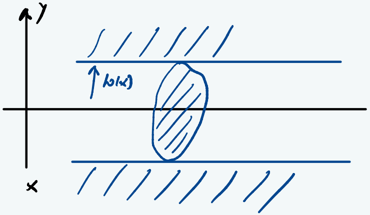
Stat. profiles for different in- and outflow rates.

want equation average density

The area averaged model

Consider radially symmetric domains $\Omega \in \mathbb{R}^2$

$$\Omega := \left\{(x, y)\ :\ x \in [0, L],\, y \in \frac{1}{2}[-w(x), w(x)]\right\}$$



where $w\colon [0, L] \to (0, \infty)$ is the width in the y-direction and vector fields

$$\mathbf{u}_x(x, -y) = \mathbf{u}_x(x, y) \text{ and } \mathbf{u}_y(x, -y) = -\mathbf{u}_y(x, y)\,,$$

$$|\mathbf{u}| = 1\,,$$

$$\mathbf{u} \cdot n = \begin{cases} -1 & \text{on } \Gamma, \\ 1 & \text{on } \Sigma. \end{cases}$$

Then the density ρ and the flux $\mathbf{J}$ are symmetric w.r.t the x-axis.

Integrating the conservation law w.r.t. y gives

$$\frac{\partial}{\partial t} \int_{-w(x)/2}^{w(x)/2} \rho \, dy + \int_{-w(x)/2}^{w(x)/2} \nabla \cdot \mathbf{J} \, dy = 0\,.$$

The area averaged model

Then we can derive an area averaged 1D model for the re-scaled density ρ:

$$\partial_t \rho + \partial_x \left(k(x)\left(-\varepsilon \partial_x \rho + \rho\,(1-\rho)\right)\right) = 0\,.$$

with boundary conditions

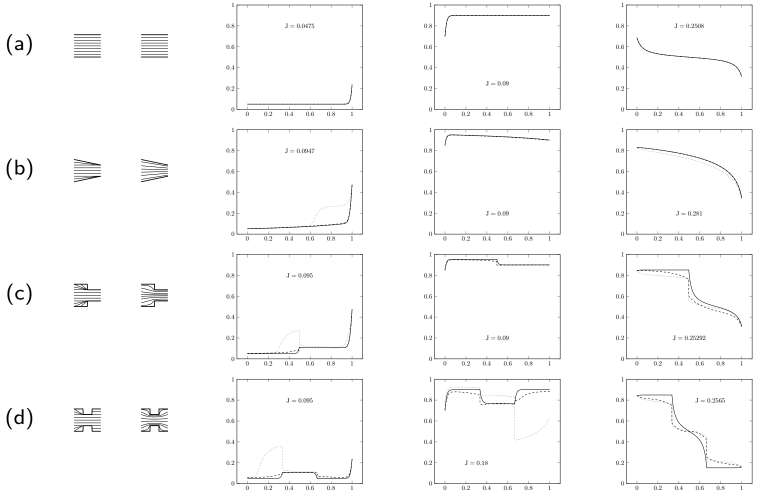
$$\begin{aligned} j &= \alpha\,(1-\rho) && \text{at } x = 0\,, \\ j &= \beta \rho && \text{at } x = L\,. \end{aligned}$$

Here $\underbrace{k(x) = \tilde{w}\langle \tilde{\mathbf{u}}_x \rangle}_{\text{'rescaled flow' over cross section}}$ where $\langle \cdot \rangle = w^{-1}\int \cdot\, dy$ and $x \to \tilde{x} = \int_0^x \langle \mathbf{u}_x \rangle (s) ds$.

In the following we wish to analyze the impact of α, β and $k(x)$ on the structure of the stationary profiles to this approximate model using

- Computational experiments.
- Geometric singular perturbation analysis (GSPT).

Geometry		**Regime**		
Streamlines		Low density	High density	High flux
$\mathbf{u}_e$	$\mathbf{u}_L$	$\alpha = 0.05,\ \beta = 0.2$	$\alpha = 0.3,\ \beta = 0.1$	$\alpha = 0.8,\ \beta = 0.8$



GSPT for smooth closing domain

We consider C^1 positive functions k, which satisfy

$$g(x) := \frac{1}{k(x)} \frac{dk(x)}{dx} < 0.$$

We introduce the new variable $\xi = x$ and include the equation $\dot{\xi} = 1$ to obtain:

$$\begin{aligned} \dot{j} &= -g(\xi) j, \\ \dot{\xi} &= 1, \\ \varepsilon \dot{\rho} &= \rho(1-\rho) - j. \end{aligned}$$

The respective boundary conditions are given by

$$\begin{aligned} j &= \alpha\,(1-\rho) && \text{at } \xi = 0, \\ j &= \beta\rho && \text{at } \xi = 1. \end{aligned}$$

GSPT

Scaling: Introduce $\chi = \frac{x}{\varepsilon}$ and obtain

$$
\begin{aligned}
j' &= -\varepsilon g(\xi) j, \\
\xi' &= \varepsilon, \\
\rho' &= \rho(1-\rho) - j,
\end{aligned}
$$

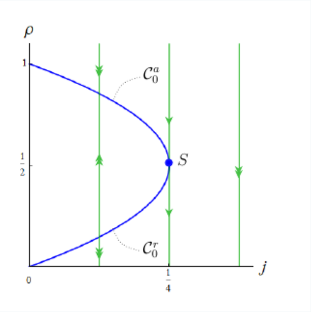
GSPT: analyze the singular limit ε on the slow and the fast scale separately and then glue the results together.

Layer problem (setting $\varepsilon = 0$ in the scaled system):

$$
\begin{aligned}
j' &= 0, \\
\xi' &= 0, \\
\rho' &= \rho(1-\rho) - j,
\end{aligned}
$$

Manifold of equilibria:

$$
\mathcal{C}_0 := \{(j, \xi, \rho) \ : \ j = \rho(1-\rho)\}.
$$



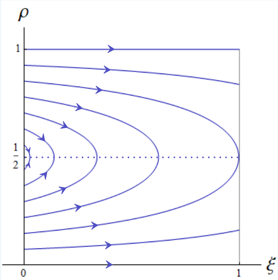
GSPT

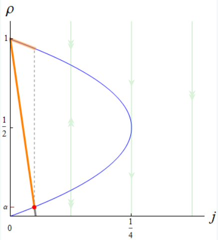
Reduced problem (setting $\varepsilon = 0$ in the original system)

$$\dot{j} = -g(\xi) j,$$
$$\dot{\xi} = 1.$$

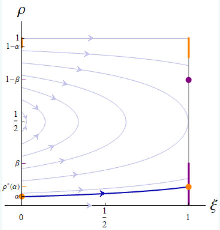
More advantageous to rewrite it in terms of ρ and ξ:

$$\dot{\rho} = -g(\xi) \frac{\rho(1-\rho)}{1-2\rho},$$
$$\dot{\xi} = 1.$$

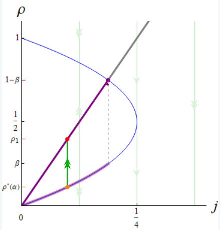




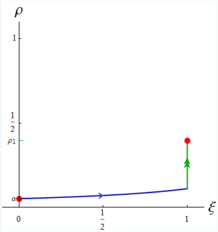
(a) $\xi = 0$



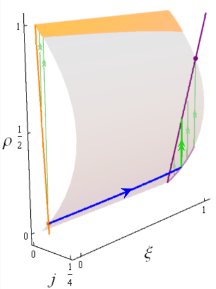
(b) $\xi \in [0, 1]$



(c) $\xi = 1$

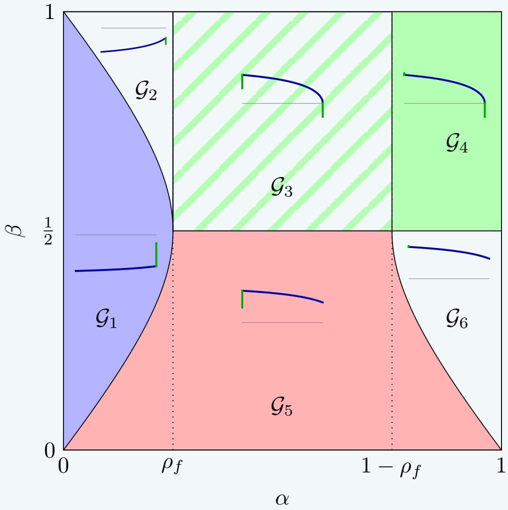


(d)

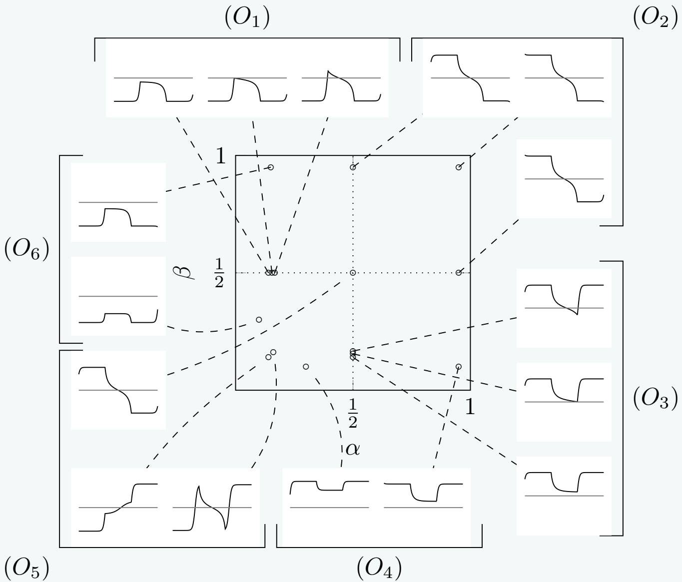


(e)

Stationary profiles for closing channel



Stationary profiles for a bottleneck



Part II: Parameter identification in pedestrian dynamics models

Goal: Use the Bayesian framework to estimate parameters in the fundamental diagram using individual trajectories.

Collaborators: Susana Gomes and Andrew Stuart

Macroscopic model for uni-directional flows

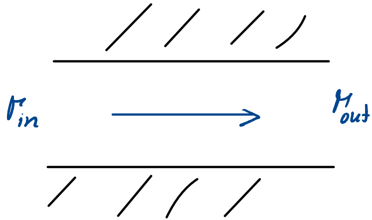
Evolution of the pedestrian density $\rho = \rho(x,t)$ can be described by a Fokker-Planck equation

$$\partial_t \rho(x,t) = \operatorname{div}\left(\Sigma \nabla \rho(x,t) - \rho(x,t) F(\rho)\right),$$

where $\Sigma = \operatorname{diag}(\sigma_1^2, \sigma_2^2)$ and

$$F(\rho) = f(\rho) \underbrace{e_1}_{\binom{1}{0}} \text{ and } f(\rho) = v_{\max}\left(1 - \frac{\rho}{\rho_{\max}}\right).$$

$\Leftarrow$ not scaled version of equ. in part I



Boundary conditions: for the flux $j = -\Sigma \nabla \rho + \rho F(\rho)$

$$\begin{aligned}
j \cdot n &= -a\big(\rho_{max} - \rho\big), && \text{for all } (x_1, x_2) \in \Gamma_{in},\\
j \cdot n &= b\rho, && \text{for all } (x_1, x_2) \in \Gamma_{out},\\
j \cdot n &= 0, && \text{for all } (x_1, x_2) \in \Gamma_{N},
\end{aligned}$$

Scaling

Let $\tilde{\rho} = \frac{\rho}{\rho_{max}}$, then the FPE equation can be rescaled as

$$\partial_t \tilde{\rho} = \nabla \cdot \left(\Sigma \nabla \tilde{\rho} - \tilde{\rho} \tilde{F}(\tilde{\rho}) \right)$$

where $\tilde{F}(\tilde{\rho}) = \tilde{f}(\tilde{\rho}) e_1$, with $\tilde{f}(\tilde{\rho}) = v_{\text{max}} \left(1 - \tilde{\rho}\right)$.

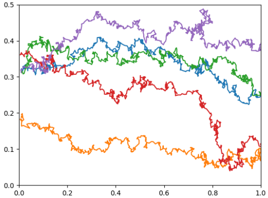
Boundary conditions:

$$\begin{aligned}
\tilde{j} \cdot n &= -a\big(1 - \tilde{\rho}\big), && \text{for all } (x_1, x_2) \in \Gamma_{in}, \\
\tilde{j} \cdot n &= b\tilde{\rho}, && \text{for all } (x_1, x_2) \in \Gamma_{out}, \\
\tilde{j} \cdot n &= 0, && \text{for all } (x_1, x_2) \in \Gamma_N.
\end{aligned}$$

where $\tilde{j} = -\Sigma \nabla \tilde{\rho} + v_{max}(1 - \tilde{\rho})\tilde{\rho} e_1$.

The maximum density ρ_{max} is not present in the scaled formulation.

← real data



5 realizations of the McKean-Vlasov process.

Microscopic dynamics:

Assumption: Individual trajectories are realizations of a McKean-Vlasov process

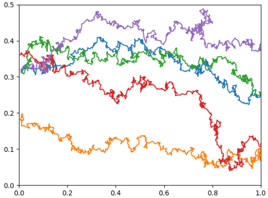
$$\mathrm{d}X(t) = F(\rho(X(t), t))\,\mathrm{d}t + \sqrt{2\Sigma}\mathrm{d}B(t)$$

where $B(\cdot)$ is a standard Brownian motion and ρ solves the FPE with $\rho_{\mathrm{max}} = 1$.

Consistent coupling: pdf of the McKean-Vlasov process satsfies the FPE.

Inverse problem:

Estimate v_{max} and ρ_{max} in the fundamental diagram using individual trajectories.



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Consistent coupling: pdf of the McKean-Vlasov process satsfies the FPE.

Inverse problem:

Estimate v_{max} and ρ_{max} in the fundamental diagram using individual trajectories.

The parameter ρ_{max} does not influence the SDE for trajectories.
$\Rightarrow$ the parameter ρ_{max} cannot be learned from the data available to us.

Parameter estimation using the Bayesian framework

Data-misfit function

$$\Phi(v;X) = \frac{1}{4}\int_0^T |\dot{X} - F\big(\rho(X(t),t);v\big)|_\Sigma^2,$$

However, the function $\Phi(v;\cdot)$ is almost surely infinite.
Hence, we perform a parametric estimation of $v_{\max}$ by minimizing

$$\mathcal{J}(v;X) := \Psi(v;X) + \underbrace{\frac{1}{2c}|v-m|^2}_{\text{prior: } v_{\max}\in\mathcal{N}(m,c)},$$

where

$$\Psi(v;X) = \frac{1}{4}\int_0^T \Big(|F\big(\rho(X(t),t);v\big)|_\Sigma^2\,\mathrm{d}t - 2\langle F\big(\rho(X(t),t);v\big),\,\mathrm{d}X(t)\rangle_\Sigma\Big).$$

- Functionals can be generalized for multiple trajectories
- We either sample from the posterior distribution

$$\mathbb{P}(v|X) \propto \exp(-\mathcal{J})\mathbf{1}(v>0)$$

  or minimize the negative log-likelihood of the functional $\mathcal{J}$.

Sampling from the posterior

Algorithm 1 The pCN algorithm

1: Set $k = 0$ and pick $v^{(0)}$.
2: **for** $k = 1, ..., N$, where N is the number of iterations, **do**
3: Propose $y^{(k)} = m + \sqrt{(1-\beta^2)}(v^{(k)} - m) + \beta\xi^{(k)}, \quad \xi^{(k)} \sim N(0, c)$.
4: Set $v^{(k+1)} = y^{(k)}$ with probability $\alpha_k := \alpha(v^{(k)}, y^{(k)})$
5: Set $v^{(k+1)} = v^{(k)}$ otherwise.
6: $k \to k + 1$.
7: **end for**

Sampling from the posterior

Algorithm 2 The pCN algorithm

Set $k = 0$ and pick $v^{(0)}$.
for $k = 1, ..., N$, where N is the number of iterations, **do**
  Propose $y^{(k)} = m + \sqrt{(1-\beta^2)}(v^{(k)} - m) + \beta\xi^{(k)}, \quad \xi^{(k)} \sim N(0, c)$.
  Set $v^{(k+1)} = y^{(k)}$ with probability $\alpha_k := \alpha(v^{(k)}, y^{(k)})$
  Set $v^{(k+1)} = v^{(k)}$ otherwise.
  $k \to k+1$.
end for

The crux of sampling: how to solve a nonlinear parabolic PDE more than a million times efficiently ?

Solving 2D parabolic PDEs quickly

- Solve 2D nonlinear Fokker-Planck equation of the form

$$\partial_t \rho(x,t) = \mathrm{div}(\Sigma \nabla \rho - \rho(1-\rho)\nabla \phi)$$

for a given $\Sigma = \mathrm{diag}(\sigma_1^2, \sigma_2^2)$ and potential $\phi = \phi(x_1, x_2)$.

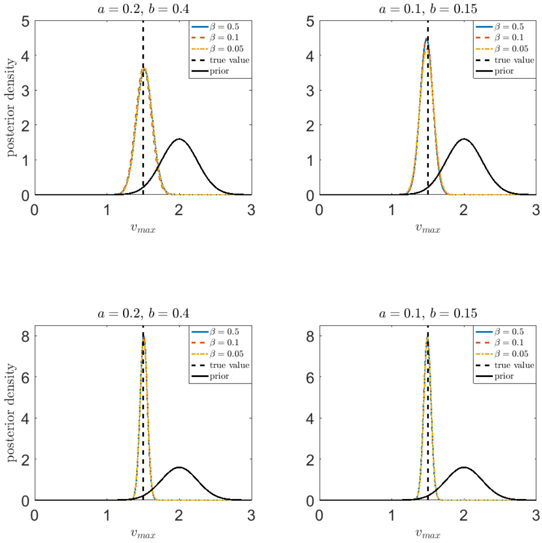
Discretization:

  - Space: HDG for the linear diffusion and DG for the non-linear convective part (with upwinding).
  - Time: 4-stage third order RK IMEX scheme in time.

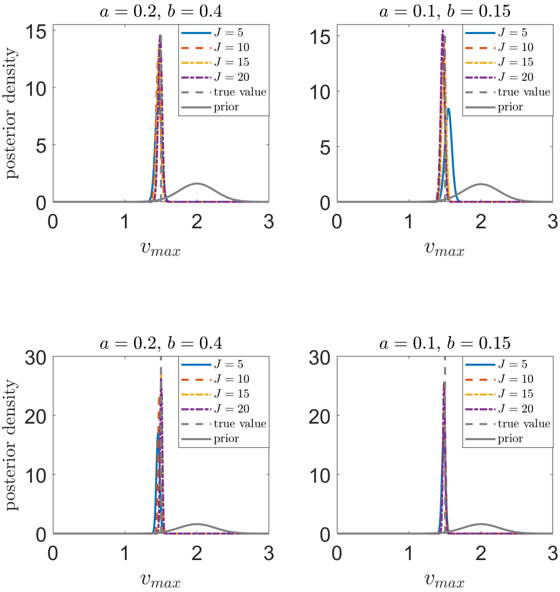
Allowed us to make larger time steps and use a much coarser grid.

- Unidirectional corridor: solve 1D stationary problem.

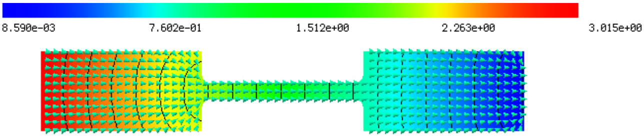
Varying β - TD vs. stat.



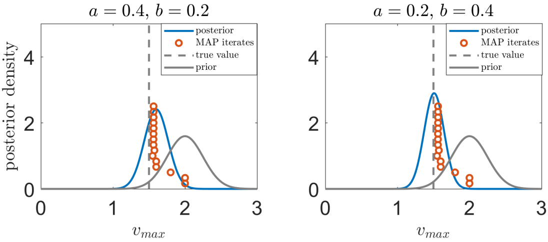
Varying # trajectories - TD vs. stat.



Bottleneck

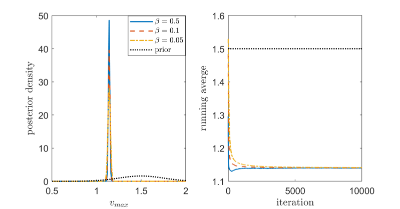


8.590e-03
7.602e-01
1.512e+00
2.263e+00
3.015e+00



$a = 0.4,\ b = 0.2$
$a = 0.2,\ b = 0.4$
posterior
MAP iterates
true value
prior
posterior density
v_{max}

Posterior of v_{max} using real trajectory data

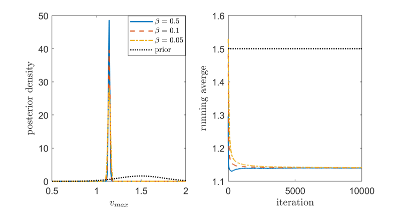


Inverse problem: Given ρ find v_{max}

Given $X_1 \dots X_N$ find v_{max}

Given $X_1 \dots X_N$ find v_{max} in SDE

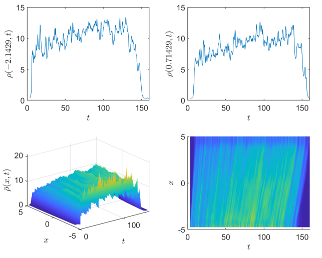
Posterior of v_{max} using real trajectory data



v_{max} α, β, σ

ρ_{max} ?

Are our assumptions on the data correct?



Average pedestrian density calculated from real trajectory.

Ongoing & future work:

- Use GSPT in case of more realistic geometries; such as corridors with piecewise constant width.
- Estimation of $v_{\max}$, Σ, as well as the inflow and outflow rate a and b using real data.
- Non-parametric estimation of the fundamental diagram.

Some more info:

- M. Burger and J.-F. Pietschmann *Flow characteristics in a crowded transport model* Nonlinearity, 29(11):3528–3550, 2016.
- A. Iuorio, G. Jankowiak, P. Szmolyan and MTW, *A PDE model for unidirectional flows: stationary profiles and asymptotic behavior*, J Math Annal Appl 510(2), 2020
- A. Iuorio, G. Jankowiak, P. Szmolyan, MTW, *Canards in a bottleneck*, accepted for publication at Physica D 2023
- S. Gomes, A. Stuart and MTW, *Parameter estimation for macroscopic pedestrian dynamics models from microscopic data*, SIAM Appl. Math. 79(4) 1475-1500, 2019

- Pedestrian Dynamics Data Archive: thanks to Armin Seyfried, Maik Boltes and all their co-workers !

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Thank you very much for your attention.