SITE RESEARCH CENTER ON STABILITY, INSTABILITY, AND TURBULENCE

SITE Research Center Talk Series September 2022 to August 23

SEPTEMBER 1

Qingtang Su

- Title: **The nonlinear modulational instability of the Stokes waves in 2d water waves**
- Abstract: The Stokes waves are periodic symmetric steady water waves travelling at a constant speed, which plays a fundamental role in the study of water waves. It was observed by Benjamin and Feir in 1967 that the Stokes waves are subject to the modulational instability. However, the rigorous mathematical proof was missing for a long time. In this talk, we will discuss how to prove the nonlinear modulational instability of the Stokes waves in the context of 2d full water waves. This is joint work with Gong Chen.

SEPTEMBER 15

Anna Mazzucato

- Title: **Irregular transport and loss of regularity for transport equations**
- Abstract: She will present recent results concerning examples of loss of regularity for solutions to linear transport equations with advecting field in Sobolev spaces below the Lipscitz class. She will discuss how this loss is generic and can be made instantaneous and total (that is, there exists smooth initial data for which the solution leaves instantaneously any Sobolev space of positive order). This is joint work with Giovanni Alberti, Gianluca Crippa, Gautam Iyer, and Tarek Elgindi.

SEPTEMBER 28

Khaled Saleh

- Title: **Asymptotic preserving numerical schemes for low Mach number flows**
- Abstract: Since seminal papers published in the middle of the sixties, low-order staggered schemes for incompressible flow computations have received a considerable attention. The staggered discretization is a space structured or unstructured discretization where the scalar unknowns (the pressure) are located at the cell centers while the vector unknowns (the velocity) are located at the cells faces. This discretization is essentially motivated by the fact that it combines a low computational cost with the so-called inf-sup or LBB stability condition, which prevents from the odd-even decoupling of the pressure. For several years, an important effort has been dedicated to the extension of staggered numerical schemes for the approximation of compressible flows. In this talk, I will focus on the barotropic compressible Navier-Stokes equations. I will prove that, as the Mach number tends to zero, the solution of the implicit staggered scheme for these equations converges towards the solution of the standard staggered scheme for the incompressible Navier-Stokes equations. In particular, the numerical density tends towards a constant as the Mach number tends to zero. Such a result follows from a similar analysis to that of Lions and Masmoudi (1998) at the continuous level for weak solutions of the barotropic compressible Navier-Stokes equations. It extends to other time discretizations such as the so-called pressure-correction scheme (adapted to compressible models). These numerical schemes are used in practice in industrial codes such as CALIF3S, a code developed by the IRSN (French Institut for Radioprotection and Nuclear Safety) for the simulation of deflagrations.

SEPTEMBER 29

[Julián David Gutiérrez Pineda](#)

- Title: **Machine learning architectures for mean-field games models of price formation**
- Abstract: In this talk, we approach the solution of mean-field game systems arising in price formation models employing machine learning. We use a min-max characterization of the optimal control and price variables. We guarantee the convergence of the training algorithm using first-order conditions of the underlying optimal control problem. Numerical results for linear-quadratic models illustrate our results.

OCTOBER 12

[Baoping Liu](#)

- Title: **Wellposedness for the KdV hierarchy**
- Abstract: The KdV hierarchy is a hierarchy of integrable equations generalizing the KdV equation. Using the modified Muria transform, we first relate it to the Gardner hierarchy, and by exploiting the idea of approximate flow initiated by Killip-Visan, we show that the whole hierarchy is wellposed for initial data in H^{-1}(R). This is based on joint work with H.Koch and F. Klaus.

OCTOBER 26

[Jussi Behrndt](#)

- Title: **Differential operators with singular potentials**
- Abstract: Schrödinger and Dirac operators with singular interactions supported on curves and surfaces represent a mathematically interesting class of differential operators in which one is able to derive numerous relations between spectral and geometric properties. At the same time such Hamiltonians are physically useful, for instance in quantum mechanics, where they describe various nanostructures as, e.g., leaky quantum graphs or graphene, as well as in the theory of photonic crystals. This is one of the main reasons why such operators with delta-interactions supported on manifolds have attracted a lot of attention by mathematicians and physicists in the recent past. The aim of this lecture is to give an introduction to the topic and to discuss some qualitative spectral properties of self-adjoint Schrödinger and Dirac operators with singular potentials. We first briefly review some classical results for regular potentials from the literature and turn to more recent developments afterwards. One of our main objectives is to investigate Dirac operators with delta-potentials supported on curves or hyperplanes, where it is necessary to distinguish the so-called non-critical and critical cases for the strength of the singular perturbation. In particular, it turns out that Dirac operators with singular potentials in the critical case have some unexpected spectral properties.

NOVEMBER 2

[EL Houcine Bergou](#)

- Title: **Stochastic zero order methods for unconstrained minimization**
- Abstract: In this work we consider the unconstrained minimization problem of a smooth function in a setting where only function evaluations are possible or approximation of the function values. We propose novel randomized derivative-free algorithms and analyze their iteration complexity. At each iteration, our method generates a random search direction according to a certain fixed probability law. Our assumptions on this law are very mild: roughly speaking, all laws which do not concentrate all measure on any halfspace passing through the origin will work. For instance, we allow for the uniform

distribution on the sphere and also distributions that concentrate all measure on a positive spanning set. The complexity bounds of our methods depend on the probability law via a simple characteristic closelyrelated to the cosine measure which is used in the analysis of deterministic direct search kind of methods. Unlike in the case of deterministic direct search methods, where the complexity depends quadratically on the dimension. For our methods we have only a linear dependence.

NOVEMBER 9

Changzhen Sun

- Title: **Uniform regularity in the low Mach number and the inviscid limits for the non-isentropic system in domains with boundaries**
- Abstract: In this talk, we are concerned with the propagation of the regularity for the non-isentropic Navier-Stokes system in a domain with boundaries, uniformly in the Mach number and the Reynolds number, in a general setting of ill-prepared data. This is an essential step towards the study of the low Mach number and the inviscid limit for strong solutions. The main obstacle to estabilish such uniform regularity estimates lie in the existence of fast oscillating acoustic waves (due to the ill-prepared assumption) and the interactions of two kinds of boundary layers (due to the large time temperature variation and the non-vanishing thermal conductivity). To start, I will first explain the ideas to propagate the high regularity for the isentropic Navier-Stokes system uniformly only in Mach number, which was a joint work with Professor N. Masmoudi and F. Rousset.

NOVEMBER 23

Manuel Gnann

- Title: **Non-negative Martingale Solutions to the Stochastic Thin-Film Equation with Nonlinear Gradient Noise**
- Abstract: We prove the existence of non-negative martingale solutions to a class of stochastic degenerate-parabolic fourth-order PDEs arising in surface-tension driven thin-film flow influenced by thermal noise. The construction applies to a range of mobilites including the cubic one which occurs under the assumption of a no-slip condition at the liquid-solid interface. Since their introduction more than 15 years ago, by Davidovitch, Moro, and Stone and by Grün, Mecke, and Rauscher, the existence of solutions to stochastic thin-film equations for cubic mobilities has been an open problem, even in the case of sufficiently regular noise. Our proof of global-in-time solutions relies on a careful combination of entropy and energy estimates in conjunction with a tailor-made approximation procedure to control the formation of shocks caused by the nonlinear stochastic scalar conservation law structure of the noise. The talk is based on joint work with Konstantinos Dareiotis (University of Leeds), Benjamin Gess (Bielefeld University/MPI Leipzig), and Günther Grün (University of Erlangen-Nuremberg)

NOVEMBER 29

Juntao Huang

- Title: **Structure-preserving machine learning moment closures for the radiative transfer equation**
- Abstract: In this talk, we are concerned with the propagation of the regularity for the non-isentropic Navier-Stokes system in a domain with boundaries, uniformly in the Mach number and the Reynolds number, in a general setting of ill-prepared data. This is an essential step towards the study of the low Mach number and the inviscid limit for strong solutions. The main obstacle to estabilish such uniform

regularity estimates lie in the existence of fast oscillating acoustic waves (due to the ill-prepared assumption) and the interactions of two kinds of boundary layers (due to the large time temperature variation and the non-vanishing thermal conductivity). To start, I will first explain the ideas to propagate the high regularity for the isentropic Navier-Stokes system uniformly only in Mach number, which was a joint work with Professor N. Masmoudi and F. Rousset.

DECEMBER 14

Erol Gelenbe

- Title: **The Random Neural Network and its Generalizations (G-Networks)**

JANUARY 11

Boualem Djehiche

- Title: **On a class of time-inconsistent optimal stopping problems**
- Abstract: I will review some recent results on optimal stopping problems for time-inconsistent models including utility functions with hyperbolic discounting and recursive utility functions of mean-field type.

JANUARY 25

Satyvir Singh

- Title: **High fidelity simulations on the development of shock-induced hydrodynamic instabilities**
- Abstract: The hydrodynamic instability research in fluid mechanics establishes whether a flow is stable or unstable, and if so, how these instabilities produce turbulent mixing. The Richtmyer–Meshkov (RM) instability is a shock-driven hydrodynamic instability that occurs in a combination with the Kelvin–Helmholtz instability when an initially perturbed surface separating by distinct fluid properties is driven by an incident shock wave. The RM instability can be considered as the impulsive limit of Rayleigh–Taylor instability where primary perturbations expand across the surface and ultimately emerge into a turbulent fluid mixing as the uniform gravitational acceleration increases. The studies on development of shock-induced instability are essential for the investigation of difficult issues related to shock propagation through arbitrarily inhomogeneous materials because of its wide range of applications, such as inertial confinement fusion, supersonic combustion, and supernova explosions. In this talk, high-fidelity simulations on the development of shock-induced hydrodynamic instabilities for light/heavy bubbles of various shapes are presented. The focus is placed on presenting more intuitive details of the flow-fields visualizations, wave patterns, bubble deformation, vorticity production, and enstrophy evolution. For these simulations, two-dimensional compressible Euler/Navier-Fourier equations are simulated with a high-order mixed-type modal discontinuous Galerkin method. Additionally, a thorough investigation is made into the impact of shock strength, Atwood number, aspect ratios, and bulk viscosity in diatomic and polyatomic gases on the flow morphologies of shock-induced hydrodynamic instabilities.

FEBRUARY 15

Riccardo Montalto

- Title: **Quasi-periodic solutions and inviscid limit for Euler and Navier Stokes equations via KAM methods**
- Abstract: In this talk I will discuss some recent results on Euler and Navier Stokes equations concerning the construction of quasi-periodic solutions and the problem of the invscid limit for the Navier Stokes equation. I will discuss the following two results:
  1) Construction of quasi-periodic solutions for the Euler equation with a time quasi-periodic external force, bifurcating from a constant, diophantine velocity field

2) I shall discuss the inviscid limit problem from the perspective of KAM theory, namely I shall prove the existence of quasi-periodic solutions of the Navier Stokes equation converging to the one of the Euler equation constructed in 1) .
The main difficulty is that this is a singular limit problem. We overcome this difficulty by implementing a normal form methods which allow to prove sharp estimates (global in time) w.r. to the viscosity parameter.

FEBRUARY 15

Diogo Gomes

- Title: **Hessian Riemannian flows in mean-field games**
- Abstract: Hessian Riemannian flows are a powerful tool for the construction of numerical schemes for monotone mean-field games that have their origin in constrained optimization problems. In this talk, we discuss the general construction of these flows for monotone mean-field games, their existence and regularity properties, and their asymptotic convergence.

FEBRUARY 20

Jonas Sauer

- Title: **Time-Periodic Weighted Lp-Estimates**
- Abstract: In between elliptic PDEs, which do not depend on time (think of the steady-state Stokes equations), and honest parabolic PDEs, which do depend on time and are started at a given initial value (think of the in stationary Stokes equations), there are time-periodic parabolic PDEs: On the one hand, time-independent solutions to the elliptic PDE are also trivially time-periodic, which gives periodic problems an elliptic touch, on the other hand solutions to the initial value problem which are not constant in time might very well be periodic.
I want to advocate for time-periodic problems not being the little sister of either elliptic or parabolic problems, but being a connector between the two and a class of its own right. This is highlighted by a direct method for showing a priori Lp estimates for time-periodic, linear, partial differential equations. The method is generic and can be applied to a wide range of problems, for example the Stokes equations and boundary value problems of Agmon-Douglas-Nirenberg type. In the talk, I will present these ideas and show how they can be extended to the setting of weighted Lp estimates, which is advantageous for extrapolation techniques and rougher boundary data.
Parts of the talk are based on joint works with Yasunori Maekawa and Mads Kyed.

FEBRUARY 22

Alexander Migdal

- Title: **Vortex lines, anomalous dissipation, and intermittency**
- Abstract: We develop a new theory of circulation statistics in strong turbulence ($\nu \rightarrow 0$ in the Navier-Stokes equation), treated as a degenerate fixed point of a Hopf equation. We use spherical Clebsch variables to parametrize vorticity in the stationary singular Euler flow. This flow has a tangent velocity gap due to the phase gap in the angular Clebsch variable across a discontinuity surface bounded by a stationary loop C in space. We find a circular vortex with a singular core on this loop, regularized as a limit of the Burgers vortex. We compute anomalous contributions to the Euler Hamiltonian, helicity, and the energy flow, staying finite in the vanishing viscosity limit. The normalization constant in the spherical Clebsch variables is determined from the energy balance between incoming flow and anomalous dissipation. The randomness (spontaneous stochastization) comes from the Gaussian fluctuations of a background velocity due to random locations of remote vortex structures. Assuming weak fluctuations of the background velocity field, we compute the

probability distribution of velocity circulation Γ, which decays exponentially with pre-exponential factor $1/\sqrt{\Gamma}$ in perfect match with numerical simulations of conventional forced Navier-Stokes equations on periodic lattice $8K^3$. We also compute effective multifractal indexes for the tails of velocity circulation probability density as a function of conditional probability below that tail. The anomalous dimensions are independent of this probability and decrease as inverse powers of the logarithm of the size of the loop.

MARCH 8

Mohamed Majdoub

- Title: **The Fujita exponent for a Hardy-Henon equation with a spatial-temporal forcing term**
- Abstract: The purpose of this work is to analyze the well-posedness and the blow-up of solutions of the higher-order parabolic semilinear equation
$$
u_t+(-\Delta)^{d}u=|x|^{\alpha}|u|^{p}+\zeta(t){\mathbf w}(x) \ \quad\mbox{for } (x,t)\in\mathbb{R}^{N}\times(0,\infty),
$$
where $d\in (0,1)\cup \mathbb{N}$, $p>1$, $-\alpha\in(0,\min(2d,N))$ or $\alpha\geq 0$ and ζ as well as ${\mathbf w}$ are suitable given functions. The novelty here regards to previous works lies in considering a forcing term $\zeta(t){\mathbf w}(x)$ depending on both time and space variables, a fractional or high-order Laplace operator $(-\Delta)^{d}$, and a weighted (possibly singular) nonlinearity $|x|^{\alpha}|u|^{p}$. We prove small Lebesgue data global existence for p larger than a critical value depending on the behavior of ζ. Furthermore, the global existence fails to hold for p less than the critical value under the additional condition $\int_{\mathbb{R}^N}\textbf{w}(x)\,dx>0$. As a consequence, we derive the explicit value of the Fujita exponent $p_{F}(\sigma)$ in the case $\zeta(t)=t^\sigma,\, \sigma>-1$.

MARCH 16

Claudia Garcia

- Title: **Time periodic solutions for the 3D quasi-geostrophic model**
- Abstract: In this talk we will discuss on time periodic solutions for the inviscid 3D quasi-geostrophic model. We will show the existence of non-trivial rotating patches obtained by suitable perturbations of stationary solutions. These stationary solutions are obtained by generic revolutions shapes around the vertical axis. The construction of these special solutions is done using the bifurcation theory. More precisely, through the Crandall-Rabinowitz Theorem. We will obtain rotating patches for the simply connected case and for the doubly connected case. It is a joint work with Claudia García and Taoufik Hmidi.

MARCH 16

Joan Mateu

- Title: **Self-similar spirals for the generalized surface quasi-geostrophic equations**
- Abstract: In this talk, we will construct a large class of non-trivial (non radial) self-similar solutions of the generalized surface quasi-geostrophic equation. To the best of our knowledge, this is the first rigorous construction of any self-similar solution for these equations. Moreover, the solutions are of spiral type, locally integrable and may have a change of sign. This is a joint work with Javier Gómez-Serrano.

APRIL 4

David Ambrose

- Title: **Some existence results for mean field games**
- Abstract: When considering N-player differential games, making the approximation that there are instead infinitely many agents leads to the mean field games system of PDEs. This system has two unknowns, the probability distribution of the players, and the value function being optimized by a representative agent. One of these satisfies a forward parabolic equation and the other satisfies a backward parabolic equation. The forward parabolic equation comes with initial data while terminal data (at a fixed time T>0) is specified for the backward parabolic equation. We will describe some existence results for this coupled forward-backward system, without assuming that the nonlinearity (the Hamiltonian) has any special structures such as convexity or monotonicity. Results presented will including treating a specific system which has been given as a model of household savings and wealth.

APRIL 24

Alfio Borzi

- Title: **Some existence results for mean field games**
- Abstract: This talk is devoted to the formulation of ensemble optimal control problems governed by kinetic models. The starting point for this presentation is the Liouville equation, which is the fundamental building block of models that govern the evolution of probability and material density functions like the Fokker-Planck equation and the Boltzmann equation. In this framework, optimal control problems arise in a multitude of application fields ranging from plasma physics to pedestrians' motion, where it is required to design control mechanisms that are able to drive the underlying stochastic process or microscopic system in order to perform given tasks. It is shown that many of these tasks can be formulated in terms of ensemble (expected value) cost functionals. It is also illustrated how ensemble cost functionals allow to draw a connection between open-loop Fokker-Planck control problems and Hamilton-Jacobi-Bellman problems arising in the computation of closed-loop controls. The talk is concluded with a brief discussion on Fokker-Planck Nash games for modelling the avoidance problem in, e.g., pedestrian motion.

APRIL 24

Alpár R. Mészáros

- Title: **A variational approach to first order kinetic mean field games with local couplings**
- Abstract: First order kinetic mean field games formally describe the Nash equilibria of deterministic differential games where agents control their acceleration, asymptotically in the limit as the number of agents tends to infinity. The known results for the well-posedness theory of mean field games with control on the acceleration assume either that the running and final costs are regularizing functionals of the density variable, or the presence of noise, i.e. a second-order system. In this talk we describe how to construct global in time weak solutions to a first order mean field games system involving kinetic transport operators, where the costs are local (hence non-regularizing) functions of the density variable with polynomial growth. We show the uniqueness of these solutions on the support of the agent density. The heart of the analysis is to characterize solutions through two convex optimization problems in duality. We will introduce a notion of 'reachable set', built from the initial agent

distribution, that allows us to work with initial measures with or without compact support. In this way we are able to obtain crucial estimates on minimizing sequences for merely bounded and continuous initial measures. These are then carefully combined with L^1-type averaging lemmas from kinetic theory to obtain pre-compactness for the minimizing sequence. Finally, under stronger convexity and monotonicity assumptions on the data, we can prove higher order Sobolev estimates of the solutions. The talk will be based on a joint work with M. Griffin-Pickering (UCL).

APRIL 25

Carlo Berselli

- Title: **On rotational eddy viscosity models**
- Abstract: We consider the Baldwin-Lomax model, which is a rotational model proposed to describe turbulent flows at statistical equilibrium. This method is specifically designed to address the problem of a turbulent motion taking place in a bounded domain, with Dirichlet boundary conditions at solid boundaries. Possible extensions and applications to models for ocean currents are discussed.

APRIL 25

Marco Cirant

- Title: **Mean Field Games with aggregation: existence and nonexistence of equilibria**
- Abstract: I will discuss the issue of existence of solutions to viscous Mean Field Games systems in the so-called anti-monotone regime, that describe Nash equilibria in differential games involving a large population of identical players aiming at aggregating. The problem can be recast into the optimal control of a system whose state is driven by a Fokker-Planck equation. I will show the role of the aggregation strength in the existence of equilibria, which may correspond to global or local minima of a suitable functional, or their nonexistence. The stationary and the evolutive case, which correspond to long-time and fixed time horizon optimization respectively, will be discussed and compared.

MAY 1

Idriss Mazari-Fouquer

- Title: **Propagation fronts and Mean Field Games: an approach to the tragedy of the commons**
- Abstract: In this talk, we will present a work in collaboration with Z. Kobeissi and D. Ruiz-Balet where we analyse an optimal harvesting problem from the point of view of Mean Field Games. Our goal is to show that, one the one hand, a purely selfish harvesting strategies, whereby each single fisherman acts in his best interest, can drive the population to extinction while a coordinated plan of action, where fishermen would coordinate, would actually lead to a survival of the fishes' population, and to a higher harvested yield for every single fisherman. Mathematically, we will use a travelling wave approach, focusing on a bistable model for which, when no fisherman is present, travelling wave solutions are invading. We will model the behaviour of fisherman using the MFG formalism.

MAY 11

Nicola Bellomo

- Title: **Behavioral Crowd Dynamics: Recent Results and Safety**

- Abstract: The study of human crowds can contribute to the well-being of our society. The study generates challenging analytical and computational problems. Dynamics are influenced by social interactions and collective learning. Modeling requires a multiscale view and takes into account the quality and geometry of the place where the dynamics occur. This lecture aims to provide an answer that can be given to the following ve key questions: Why a crowd is a \social, hence complex," system? How mathematical sciences can contribute to understand the \behavioral dynamics of crowds"? How the crowd behaves in extreme situations such as panic and how models can depict them? The answer to the key question takes advantage of recent research activity documented in the titles of the following bibliography. The answer opens to challenging research perspectives.
    - N.B., A. Bellouquid, L. Gibelli, and N. Outada, A Quest Towards a Mathematical Theory of Living Systems, Birkh•auser, New York, (2017).
    - N. B., N. Bellouquid, L. Gibelli, and N. Outada, On the Interplay between Behavioral Dynamics and Social Interactions in Human Crowds, Kinetic Related Models, 12, (2019), 397{409, (2019).
    - N. B., L. Gibelli, A. Quaini, and A. Reali, Towards a mathematical theory of behavioral human crowds, Math. Models Methods Appl. Sci., 32(2), (2022), 321{358. \Open access".

JUNE 1

Marie-Therese Wolfram

- Title: On parameter identification in macroscopic models for pedestrian crowds
- Abstract: There has been a significant increase in the modelling, analysis and calibration of models for pedestrian crowds in the last years. In this talk I will present different mathematical models for crowds - such as the social force model or the Hughes model - and discuss their respective analytical and computational challenges. I will then focus on the problem of estimating parameters in macroscopic pedestrian models using trajectory data. I will use the Bayesian framework to perform the identification and analyse the performance of the developed methodologies for different experimental settings.

JUNE 22

Alessio Porretta

- Title: **Weak solutions in Mean-field game systems with applications to optimal transport and congestion models**
- Abstract: In mean-field game theory, Nash equilibria are described through solutions of PDE systems coupling Hamilton-Jacobi and Fokker-Planck equations. When the models involve local functions of the density in the cost functionals, this leads to study PDEs in non regular setting. In this context a good notion of weak solutions to MFG systems is crucial to characterize singular limits, asymptotic regimes etc..A typical example occurs for vanishing viscosity limits as well as for optimal transport problems with congestion effects.