Mathematical models and methods for crowd dynamics control

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University of Verona

10th October 2023

SITE Research Center online seminar

Overview

Control problems for multi-agent systems

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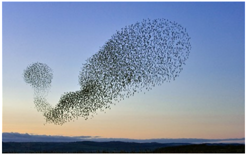 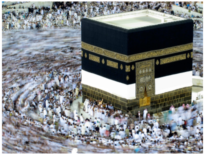 

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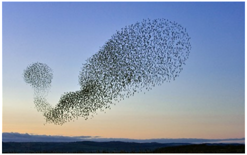
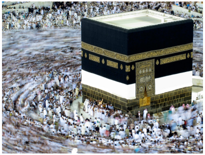


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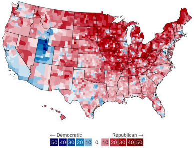

or reconstructing their interactions from their observations.

Self-organization via attraction & repulsion dynamics

Multi-scale framework

microscopic models

(Newton's equations,
Molecular dynamics, ...)

$\searrow$ $(N \to \infty)$

kinetic models

(Boltzmann, Enskong,
Vlasov-Fokker-Planck, ...)

$\searrow$ (*equilibrium closure*)

macroscopic models

(Euler, Navier-Stokes,
moment systems, ...)

Optimization across scales

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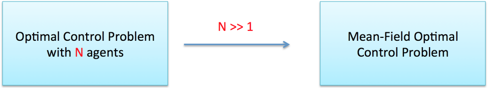
  Hence we want to reduce the problem complexity introducing a mean-field description.[1]
- In this direction large interest has been shown to the so called mean-field optimal control and mean-field games in several **mathematical fields** (game theory, stochastic processes, analysis of PDEs, optimal control. . .), and in many **applications** (consensus or milling enforcement, evacuation problems, optimal taxation, network formation, vaccination strategies . . .).[2]



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Control of a large crowd via few sparse agents

- We want to enforce a desired behavior via **sparse action** of the control.
- We consider the case of ‘few’ microscopic agents (leaders) controlling a continuos density of agents (followers) [3]

Dynamic of N agents + Optimal Control Problem with m agents	$\xrightarrow{N >> 1}$	Mean-Field equation + Optimal Control Problem with m agents

$$\min_u \; J^{N,m}(u)$$

$$\begin{cases} \dot{x}_i &= F_1(x_i, x) + F_2(x_i, y) \\ \dot{y}_k &= G_1(y_k, x) + G_2(y_k, y) + u_k \end{cases}$$

$$\min_u \; J^{m}(u)$$

$$\begin{cases} \partial_t f &= -\nabla \cdot ((\mathcal{F}_1[f] + \mathcal{F}_2[g^m])\, f) \\ \dot{y}_k &= G_1(y_k, x) + G_2(y_k, y) + u_k \end{cases}$$

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Pedestrian dynamics : optimized strategies in evacuation problems

Target

Safely drive a crowd outside an unknown environment Ω via minimal intervention.

$$\min \{t | t \geq 0, x_i(t) \notin \Omega, i = 1, \dots, N\}.$$

Pedestrian dynamics : optimized strategies in evacuation problems

- Our goal is to evacuate a crowd of individuals from an environment they don't know under limited visibility [4] .

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- We propose a mesoscopic description of this dynamics when the number of pedestrian is large.

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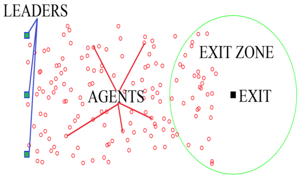

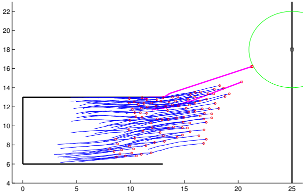

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  - an isotropic topological alignment force, i.e., given $\mathcal{N} \in \mathbb{N}$, the i-th agent aligns with those inside $\mathcal{B}_{\mathcal{N}}(x_i, t)$, the smallest ball at time t containing at least $\mathcal{N}$ agents.

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  - a random walk, in order to explore the unknown environment ;
- if the exit is visible
  - a sharp motion toward the exit.

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Metric vs. topological interaction : alignment is topological since empirical evidence suggests that only close neighbors play a role.

Microscopic model

For $i = 1, \dots, N^{\mathrm{F}}$ and $k = 1, \dots, N^{\mathrm{L}}$

$$\begin{cases} \dot{x}_i &= v_i, \\ \dot{v}_i &= A(x_i, v_i) + \sum_{j=1}^{N^{\mathrm{F}}} H(x_i, v_i, x_j, v_j) + \sum_{\ell=1}^{N^{\mathrm{L}}} H(x_i, v_i, y_\ell, w_\ell) \\ \dot{y}_k &= w_k = \sum_{j=1}^{N^{\mathrm{F}}} R_{\zeta,r}(y_k, x_j) + \sum_{\ell=1}^{N^{\mathrm{L}}} R_{\zeta,r}(y_k, y_\ell) + \color{red}{u_k}, \end{cases}$$

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- Let $\theta(x)$ represents the characteristic function of the target's visibility zone and

  $$A(x,v) := (1-\theta(x))C^{\mathrm{Z}}(z-v) + \theta(x)C^{\mathrm{D}}\left(\frac{x^{\mathrm{D}}-x}{|x^{\mathrm{D}}-x|} - v\right) + C^{\mathrm{V}}(\alpha^2 - |v|^2)v,$$

  where $z \sim \mathcal{N}(0, \sigma^2)$, α is the characteristic speed.

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$$R_{\gamma,r}(x,y) = \begin{cases} e^{-|y-x|^\gamma}\,\frac{y-x}{|y-x|} & \text{if } y \in B_r(x)\setminus\{x\}, \\ 0 & \text{otherwise}; \end{cases}$$

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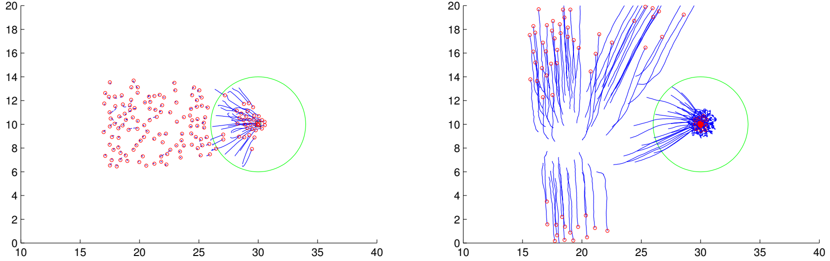
- In the dynamics of y_k, $\zeta \neq \gamma$.

Dynamics of followers

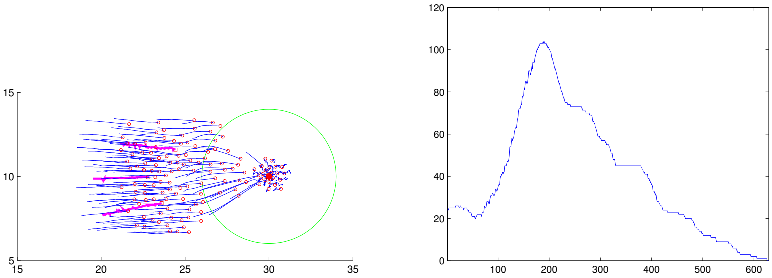


Followers dynamics without leaders

Control strategy : "dumb"

The control $u_k : [0, T] \to \mathbb{R}^d_k, \quad k = 1, \ldots, N^L$ as "dumb" strategy :

$$u_k(t) = \left(\frac{x^{\mathrm{D}} - y_k(t)}{|x^{\mathrm{D}} - y_k(t)|} - y_k(t) \right)$$



(Left) Dynamics with "dumb" strategy and (Right) occupancy of the exit's visibility zone

Control strategy : “dumb”

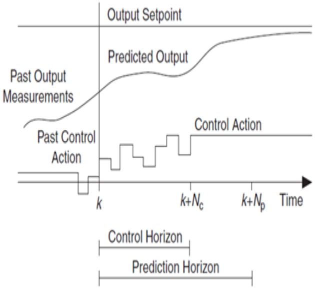
(Left) Dynamics with

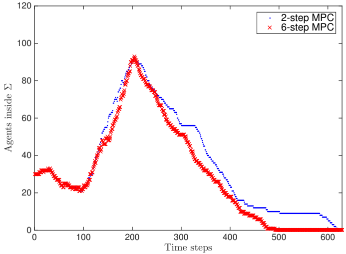
Control strategy : MPC

The control $u_k : [0,T] \to \mathbb{R}^d, \quad k = 1, \ldots, N^L$, with u_k minimizing the functional

$$J(u) = \int_t^{t+T_p} \left[C^{\mathrm{F}} \sum_{i=1}^{N^{\mathrm{F}}} \|x_i - x^{\mathrm{D}}\|^2 + C^{\mathrm{L}} \sum_{i=1}^{N^{\mathrm{F}}} \sum_{k=1}^{N^{\mathrm{L}}} \|x_i - y_k\|^2 + \nu \sum_{k=1}^{N^{\mathrm{L}}} \|u_k\|^2 \right] d\tau$$

applying *Model Predictive Control* (MPC) technique.





(Left) MPC scheme and (Right) occupancy of the exit's visibility zone

Smart strategy : Modified Compass Search

The control $u_k : [0,T] \to \mathbb{R}^d_k, \quad k = 1,\dots,N^L$, u_k minimizes the functional

$$J(u) = \min \left\{ t | t \geq 0, x_i(t) \notin \Omega, i = 1,\dots,N \right\}.$$

To minimize such functional we resort on a metaheursitic optimization method, named Modified Compass Search [5].

5. C. Audet, K.-C. Dang, and D. Orban 2014

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- *leaders' trajectories are piecewise constant ;*
- *starting from an initial guess, at each iteration we modify the current best strategy with small random variations ;*

5. C. Audet, K.-C. Dang, and D. Orban 2014

Smart strategy : Modified Compass Search

The control $u_k : [0,T] \to \mathbb{R}^d_k, \quad k = 1, \dots, N^L$, u_k minimizes the functional

$$J(u) = \min \{ t | t \geq 0, x_i(t) \notin \Omega, i = 1, \dots, N \} .$$

To minimize such functional we resort on a metaheursitic optimization method, named Modified Compass Search [5].

- *leaders' trajectories are piecewise constant ;*
- *starting from an initial guess, at each iteration we modify the current best strategy with small random variations ;*
- *we keep the variation if the evaluated cost is smaller than before ;*

5. C. Audet, K.-C. Dang, and D. Orban 2014

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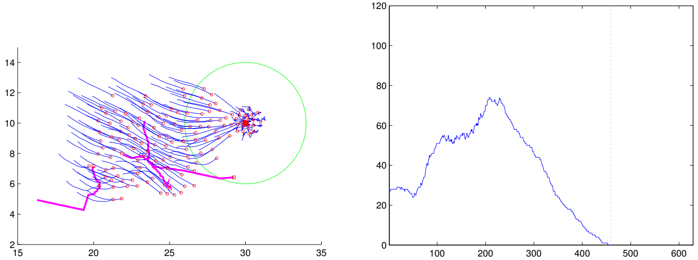
- *leaders' trajectories are piecewise constant ;*
- *starting from an initial guess, at each iteration we modify the current best strategy with small random variations ;*
- *we keep the variation if the evaluated cost is smaller than before ;*
- *the method generates a sequence converging to a local minimum.*

5. C. Audet, K.-C. Dang, and D. Orban 2014

Clog up effect around exit

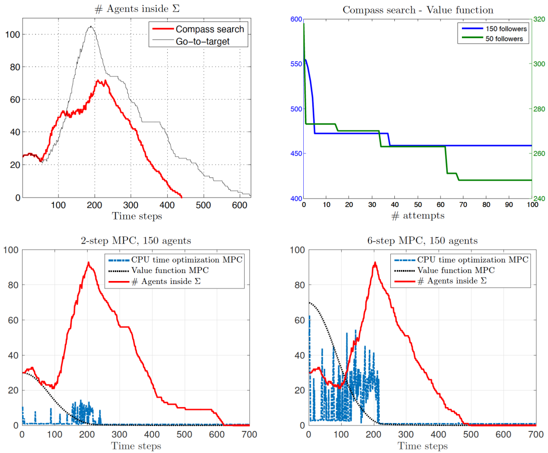


(Left) Dynamics with "smart" strategy and (Right) occupancy of the exit's visibility zone

Clog up effect around exit

Dynamics with "smart"

MPC vs Compass Search

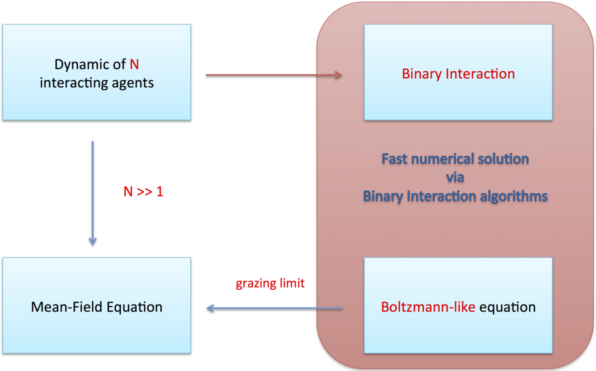


Kinetic approximation

- When the number of interacting agents N^{F} is very large $\Rightarrow$ huge system of ODEs and **control problem untreatable**
- The idea is to describe the followers-leaders dynamics considering a continuos density function for followers and maintaining leaders at the microscopic level. Thus recovering a mesoscopic+microscopic system [6].
- Here we follow a **kinetic approximation** of the crowd dynamics and to approach the **mean-field limit** [7].
- This approach allows to develop asymptotic *DSMC methods* for the efficient simulation of the dynamics [8].

6. J. A. Canizo, J. A. Carrillo, J. Rosado '10, J. A. Carrillo, Y. P. Choi, M. Hauray, S. Salem '15
7. B. Duering, P. Markowich, J.-F. Pietschmann, M.T. Wolfram '09, J. A. Carrillo, M. Fornasier, G. Toscani and F. Vecil, '10, L. Pareschi, G. Toscani '13, G.A., L. Pareschi, M. Zanella '14
8. A.V. Bobylev, K. Nanbu, '00 ; R.E. Caflisch, L.Pareschi, G. Dimarco '10.

Kinetic approximation



A binary interactions approach

- When a follower (x, v) interacts with another follower $(\hat{x}, \hat{v})$ or a leader $(\tilde{x}, \tilde{v})$, they update their state variables according to

$$\text{(FF)} \begin{cases} v^* & = v + \eta^F \Big[\underbrace{\theta(x) C^{\mathrm{z}} \xi + S(x, v)}_{A(x,v)} + \rho^F H(x, v, \hat{x}, \hat{v})\Big] \\ \hat{v}^* & = \hat{v} + \eta^F \left[\theta(\hat{x}) C^{\mathrm{z}} \xi + S(\hat{x}, \hat{v}) + \rho^F H(\hat{x}, \hat{v}, x, v)\right] \end{cases}$$

9. J. Haskovec '13, A. Blanchet, P. Degond '17-18 ; G.A. F. Ferrarese '23.

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  where η^F, η^L are the interaction strength and $\xi \sim \mathcal{N}(0,\varsigma^2)$.
- We consider the following densities for follower and leaders,

$$
f = f(t,x,v) \qquad g(t,x,v) = \sum_{k=1}^{N^{\mathrm{L}}} \delta_{y_k(t),w_k(t)}(x,v)
$$

- We assume that the total mass of the followers and leaders are such that

$$
\rho^F = \int f(x,v)dxdv = N^F \text{ and } \rho^L = \int g(x,v)dxdv = N^L.
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9. J. Haskovec '13, A. Blanchet, P. Degond '17-18 ; G.A. F. Ferrarese '23.

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$$

- To simplify the discussion we neglect the **topological interaction** in $H(\cdot)$.[9]

9. J. Haskovec '13, A. Blanchet, P. Degond '17-18 ; G.A. F. Ferrarese '23.

Boltzmann-Povzner dynamics

We assume that f, g satify Boltzmann-Povzner dynamics[10] + the ODEs for leaders

$$\begin{cases} \partial_t f + v \cdot \nabla_x f = \lambda^{\mathrm{F}} Q(f,f) + \lambda^{\mathrm{L}} Q(f,g), \\ \dot{y}_k = \displaystyle\int_{\mathbb{R}^{2d}} R_{\zeta,r}(y_k, x) f(x,v) dx\, dv + \sum_{\ell=1}^{N^{\mathrm{L}}} R_{\zeta,r}(y_k, y_\ell) + u_k, \end{cases} \tag{1}$$

where λ^{F} and λ^{L} are the interaction frequencies and

10. A.K. Povzner 1962

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$$
Q(f,f) = \mathbb{E}\left[\int_{\mathbb{R}^{2d}} \left(\frac{1}{J_{\mathrm{FF}}} f(x,v_*) f(\hat{x},\hat{v}_*) - f(x,v) f(\hat{x},\hat{v}) \right) d\hat{x}\, d\hat{v} \right],
$$

$$
Q(f,g) = \mathbb{E}\left[\int_{\mathbb{R}^{2d}} \left(\frac{1}{J_{\mathrm{FL}}} f(x,v_{**}) g(\tilde{x},\tilde{v}_*) - f(x,v) g(\tilde{x},\tilde{v}) \right) d\tilde{x}\, d\tilde{v} \right].
$$

with v_*, w_*, v_{**}, w_{**} are the pre-interaction velocities and
$\mathbb{E}[\cdot]$ is the expected value w.r.t. ξ.

10. A.K. Povzner 1962

Theorem : Grazing collision limit

Fix the control u. Let consider the following scaling

$$\eta^F = \eta^L = \varepsilon, \quad \lambda^{\mathrm{F}} = \frac{1}{\varepsilon N^{\mathrm{F}}}, \quad \lambda^{\mathrm{L}} = \frac{1}{\varepsilon N^{\mathrm{L}}}, \quad \varsigma^2 = \frac{\sigma^2}{\varepsilon}$$

and define $(f^\varepsilon, y^\varepsilon)$ be a solution of (1). Then, as $\varepsilon \to 0$, $(f^\varepsilon, y^\varepsilon)$ converges pointwise to a solution of the Fokker-Planck-type equation

$$\begin{cases} \partial_t f + v \cdot \nabla_x f = -\nabla_v \cdot ((S + \mathcal{H}[f] + \mathcal{H}[g])f) + \dfrac{1}{2}\sigma^2(\theta C^{\mathrm{z}})^2 \Delta_v f, \\ \dot{y}_k = \displaystyle\int_{\mathbb{R}^{2d}} R_{\zeta,r}(y_k, x) f(x,v)\, dx\, dv + \sum_{\ell=1}^{N^{\mathrm{L}}} R_{\zeta,r}(y_k, y_\ell) + u_k, \quad k = 1, \dots, N^{\mathrm{L}} \end{cases}$$

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which is the "mean-field limit" of the microscopic model. Where we recall that

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$$S(x,v) = -\theta(x) C^{\mathrm{Z}} v + (1-\theta(x)) C^{\mathrm{D}} \left(\frac{x^{\mathrm{D}} - x}{|x^{\mathrm{D}} - x|} - v \right) + C^{\mathrm{V}}(\alpha^2 - |v|^2) v.$$

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Idea of the proof

- We write a weak formulation of (1), therefore given $\mathcal{T}_\delta \supseteq \mathcal{C}_c^\infty(\mathbb{R}^d \times \mathbb{R}^d; \mathbb{R})$ a suitable space function, for any test function $\varphi \in \mathcal{T}_\delta$,

$$\lambda \langle Q(f,f), \varphi \rangle = \lambda \mathbb{E}\left(\int_{\mathbb{R}^{4d}} (\varphi(x, v^*) - \varphi(x,v))\, f(x,v) f(\hat{x}, \hat{v})\; dx dv d\hat{x} d\hat{v} \right)$$

- We derive the Taylor expansion around $v^* - v$ up to the second order of the interaction operator in the weak formulation the operator.

$$\lambda \langle Q(f,f), \varphi \rangle = \lambda \mathbb{E}\left(\int_{\mathbb{R}^{4d}} \nabla_v \varphi(x,v) \cdot (v^* - v) f(x,v) f(\hat{x}, \hat{v})\; dx dv d\hat{x} d\hat{v} \right) + \frac{\lambda}{2} \mathbb{E}(K^2[f,\varphi]) + \lambda R_\varphi,$$

- Using the grazing collision scaling, and taking the limit for $\varepsilon \to 0$ for any $\varphi \in \mathcal{T}_\delta$ we obtain the mean-field equation.

(Asymptotic) DSMC methods

We can simulate the **binary interaction dynamic** with small values of ε in order to approximate the mesoscopic model[11].

- We use a **splitting method**, for transport and collisional part of the scaled Boltzmann equation.
- We rewrite the collision step as

$$\partial_t f = \frac{1}{\varepsilon}\left[Q_\varepsilon^{F,+}(f,f) - \rho^F f\right] + \frac{1}{\varepsilon}\left[Q_\varepsilon^{L,+}(f,g) - \rho^L f\right]$$

  where Q_ε^+ is the **gain part** of the collision operator.
- We use a Monte-Carlo method to perform the evolution of the density f.

11. G.A., L. Pareschi, 2013

Nanbu-like algorithm

- Let us consider a time interval $[0,T]$ and discretize it in n_{tot} intervals of size Δt. Denote with $f^n(x,v)$ the approximation of $f(x,v,n\Delta t)$.
- The *forward Euler* scheme writes

$$f^{n+1} = \left(1 - \frac{(\rho^F + \rho^L)\Delta t}{\varepsilon}\right) f^n + \frac{\rho^F \Delta t}{\varepsilon} \frac{Q_\varepsilon^{+,F}(f^n, f^n)}{\rho^F} + \frac{\rho^L \Delta t}{\varepsilon} \frac{Q_\varepsilon^{+,L}(f^n, g^n)}{\rho^L}$$

  Under the restriction that $\Delta t \leq \varepsilon/(\rho^F + \rho^L)$ then f^{n+1} is a **convex combination** of densities.

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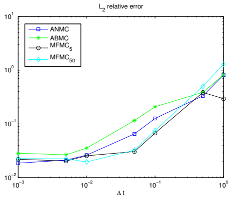
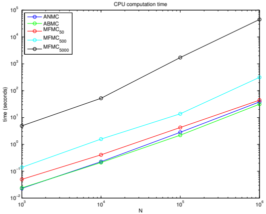
  Under the restriction that $\Delta t \leq \varepsilon/(\rho^F + \rho^L)$ then f^{n+1} is a **convex combination** of densities.
- The algorithms cost is linear, $O(N_s)$, w.r.t. to the number of sample particles N_s used to reconstruct the density f^n.

Convergence of the Binary Interaction Algorithms





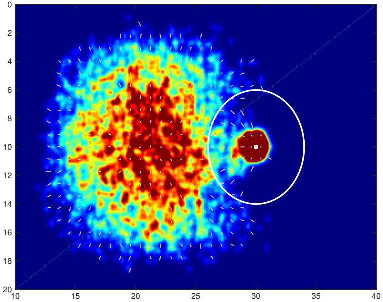
Left : Relative error, $\|f^{\varepsilon} - f^{0}\|_{L^2_r}$, after $\varepsilon = \sqrt{1/N_s}$ the error is not improving, due to the statistical fluctuations of the method.

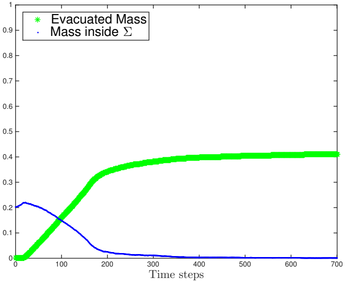
Right : CPU time for $\varepsilon = 0.01$ time step $\Delta t = \varepsilon$, $T = 1$.

In case of **topological interaction** one has to additionally consider the cost of approximate the setcontaining the closest agents. This step can be performed via construction of a grid, or relying on a k-NN search on a tree. In this case the computationl cost becomes $O(N_s \log(N_s))$ [12]

12. G.A. F. Ferrarese '23.

Uncontrolled case



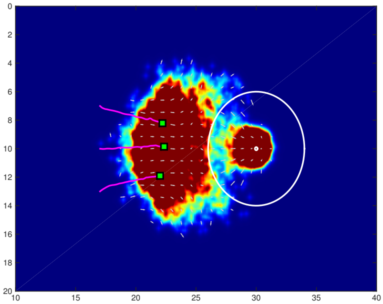


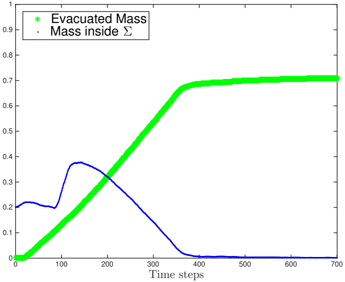
Uncontrolled evolution of followers' density.

Uncontrolled case

Dumb strategy

Control $u : [0, T] \to \mathbb{R}^d$ as a "dumb" strategy : $u_k(t) = \left(\frac{x^{\mathrm{D}} - y_k(t)}{|x^{\mathrm{D}} - y_k(t)|} - y_k(t) \right);$





Evolution of followers' density under microscopic leaders' fixed strategy.

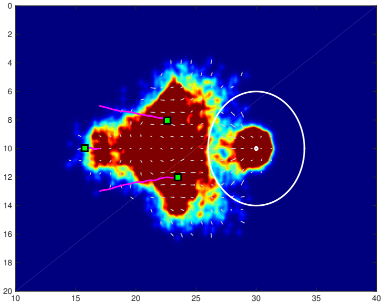
Dumb strategy

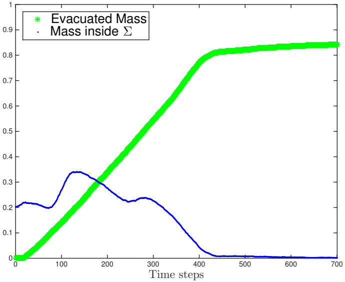
Smart strategy

Control $u : [0, T] \to \mathbb{R}^d$ as a "smart" strategy such that

$$\min_u J(u) = \int_0^T \left(P(t) + \nu \sum_{k=1}^{N^L} |u_k(t)|^2 \right) dt,$$

where $P(t)$ represents the number of followers outside exit at time t.



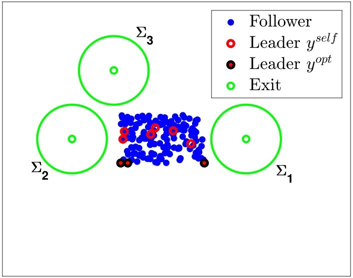


Evolution of followers' density under microscopic leaders' best strategy.

Dumb strategy

Evacuation with multiple exits

Denote by $\Omega \equiv \mathbb{R}^d$ the walking area, identify the different exits by $x_e^{\mathrm{D}} \in \Omega$ with $e = 1, \ldots, N_e$ and assume that the target is completely visible from any point belonging to Σ_e.



Evacuation with multiple exits

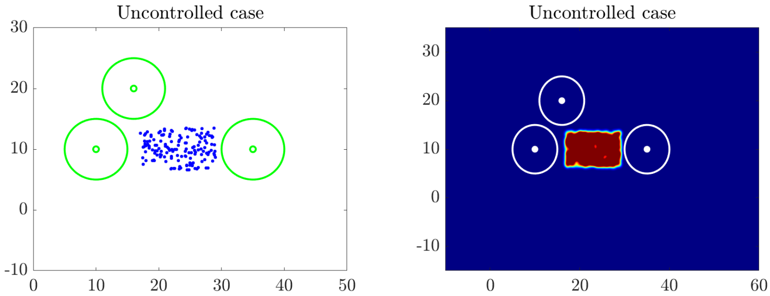


Figure – Uncontrolled case.

Evacuation with multiple exits : 'dumb' strategy

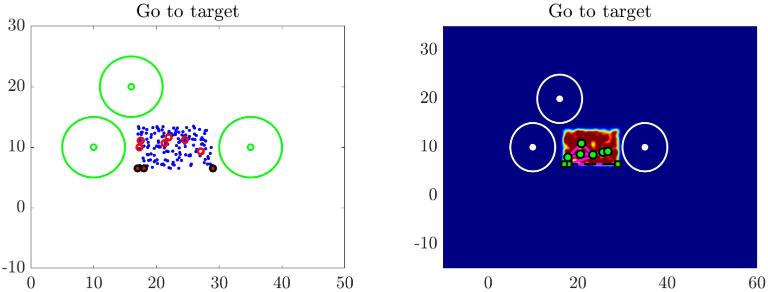


Figure – Go to target strategy.

$$u_k^{\text{self/opt}}(t) = \beta \frac{x_k^{\mathrm{D}} - y_k(t)}{\|x_k^{\mathrm{D}}(t) - y_k(t)\|} + (1-\beta)(m_F(t) - y_k(t)),$$

where x_k^{D} is the target position at time t, $m_F(t)$ is the followers centre of mass. We assume $\beta = 1$ for selfish leaders and $\beta \in [0,1]$ for optimized leaders.

Control framework

- Evacuation time

$$\mathcal{J}(\mathbf{x},\mathbf{y},\mathbf{u}^{\text{opt}}) = \{ t > 0 \mid (x_i(t), y_j(t)) \notin \Omega \} .$$

- Total mass with multiple exits

$$\mathcal{J}(\mathbf{x},\mathbf{y},\mathbf{u}^{\text{opt}}) = \int_{\mathbb{R}^d} \int_{\Omega \setminus \cup_e \Sigma_e} (f^{N^{\mathrm{F}}}(T,x,v) + g^{N^{\mathrm{L}}}(T,x,v)) dx dv.$$

- Optimal mass splitting over multiple exits

$$\mathcal{J}(\mathbf{x},\mathbf{y},\mathbf{u}^{\text{opt}}) = \sum_{e=1}^{N_e} \left| \mathcal{M}_e^F(T) - \mathcal{M}_e^{\text{des}} \right|^2 .$$

where $\mathcal{M}_e^{\text{des}}$ is the desired mass to be reached in the visibility area Σ_e and $\mathcal{M}_e^F(T)$ is the total mass of followers and leaders who reached exit x_e^{D} up to final time T.

Optimized CS strategy : minimum time evacuation

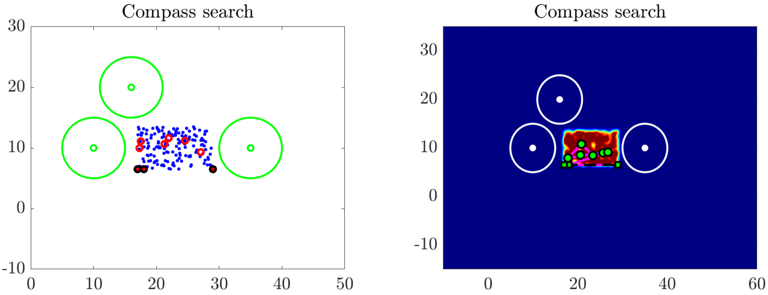


Figure – Optimized compass search strategy.

$$\mathbf{u}^{\text{opt}}(t) = \arg\min_{\mathbf{u}^{\text{opt}}} \mathcal{J}(\mathbf{x},\mathbf{y},\mathbf{u}^{\text{opt}}) = \{t > 0 \mid (x_i(t), y_j(t)) \notin \Omega \ \}\,.$$

Minimum time evacuation

	uncontrolled	go to target	CS (50 it)
Evacuation time	>1000	>1000	768
Total mass evacuated	46%	98%	100%

Table – Performance of leader strategies.

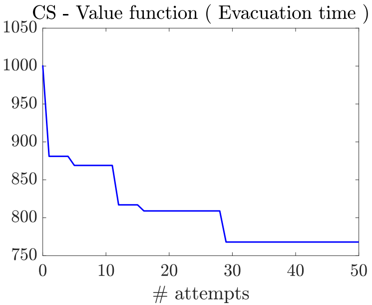
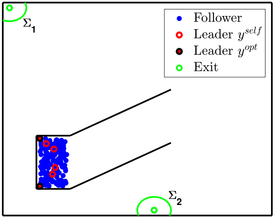


Figure – Decrease of the value functional as a function of the compass search iteration.

Mass maximization in presence of obstacles

- Consider two rooms, one inside the other.
- Assume the internal room to be bounded by three walls while the external room by four walls.



- Assume that walls can be perceived only by physical contact (evacuation in case of null visibility).
- At initial time leaders and followers are uniformly distributed in the inner room.

Mass maximization : microscopic case

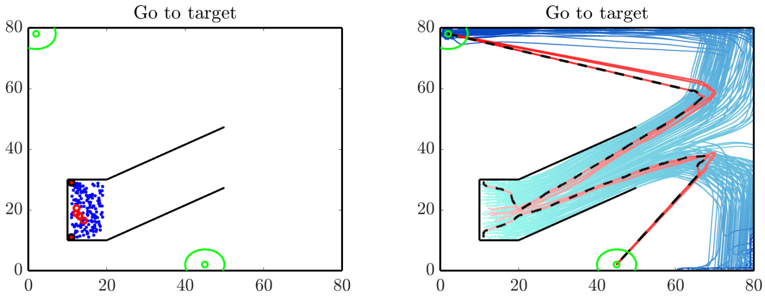


Figure – Go to target strategy.

Mass maximization : microscopic case

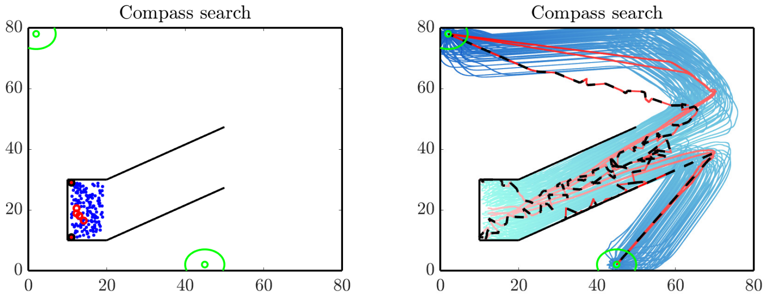


Figure – Optimized compass search strategy.

Mass maximization : microscopic case

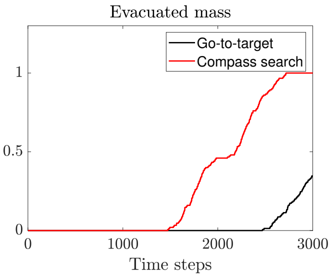


Figure – Mass maximization in presence of obstacles : evacuated mass.

	go-to-target	CS (3 it)
Evacuation time	>3000	2718
Evacuated mass	34%	100%

Table – Performance of leader strategies.

Mass maximization : mesoscopic case

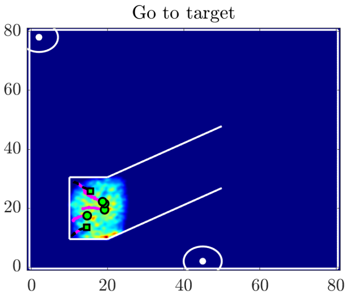


Figure – Go to target strategy.

Mass maximization : mesoscopic case

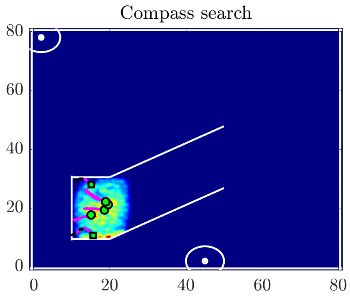


Figure – Optimized compass search strategy.

Mass maximization : mesoscopic case

	go-to-target	CS (5 it)
Evacuation time	>3000	2380
Evacuated mass	78.8%	100%

Table – Performance of leader strategies.

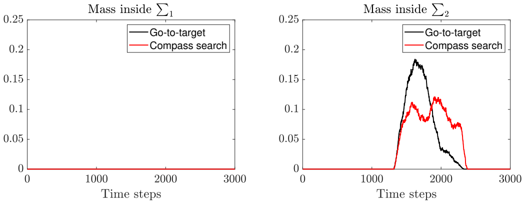


Figure – Mass maximization in presence of obstacles : occupancy of the visibility area Σ_1 (left) and Σ_2 (right) as a function of time for go-to-target and optimal compass search strategies.

Mass splitting in presence of obstacles

- Microscopic case

	go-to-target	CS (50 it)
Evacuation time	>3000	2704
Evacuated mass from x_1^τ	0%	45%
Evacuated mass from x_2^τ	34%	55%
Total mass evacuated	34%	100%

Table – Performance of leader strategies.

- Mesoscopic case

	go-to-target	CS (50 it)
Evacuation time	>3000	>3000
Evacuated mass from x_1^τ	0%	49%
Evacuated mass from x_2^τ	78.8%	50%
Total mass evacuated	78.8%	99%

Table – Performance of leader strategies.

Mass splitting in presence of obstacles

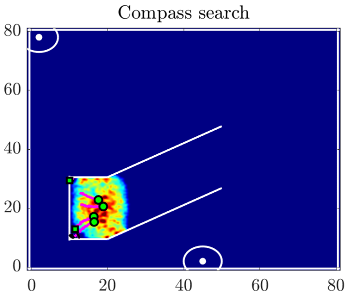


Figure – Optimized compass search strategy.

Conclusions and perspectives

Conclusions :

- We have shown how it is possible to influence the whole crowd by introducing few informed agents and by optimizing their strategies.
- We have considered different scenarios to create more complex situations.

Some perspectives

- Optimal position and number of leaders to control a certain configuration of followers.
- Cooperative strategies to optimally distribute the followers among different exists.
- Improve optimization of leaders trajectories (e.g. CBO, PSO) with theoretical guarantees of convergence to global minimum.

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Thank you !

Conclusions and perspectives

References

1. G. A., M. Bongini, E. Cristiani, D. Kalise, Invisible Control of Self-Organizing Agents Leaving Unknown Environments, *SIAM Journal of App. Math.*, 2016.
2. G. A., E. Cristiani, L. Pareschi, D. Peri, Mathematical models and methods for crowd dynamics control, Crowd Dynamics, Vol. 2, 2020.
3. G. A., F. Ferrarese, C. Segala, Optimized leaders strategies for crowd evacuation in unknown environments with multiple exits, Crowd Dynamics, Vol. 3, 2021.
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Projects :

1. **Data-driven discovery and control of multi-scale interacting artificial agent systems**. MUR PRIN-PNRR 2022
2. **Efficient numerical schemes and optimal control methods for time-dependent partial differential equations**. MUR PRIN 2022

Macroscopic limits

- The derivation of a macroscopic model is a very difficult task. The main problem to pass to macroscopic equations is that there is no classical Maxwellian equilibrium.
- There are derivations of macroscopic limits for flocking and swarming dynamics assuming a mono-kinetic ansatz and that the fluctuations are negligible, i.e. $f(x,v,t) = \rho(x,t)\delta(v - V(x,t))$ and $T(x,t) = 0$, for example for the Cucker-Smale model in this case we have [13]

$$\partial_t \rho + \nabla_x \cdot (\rho V) = 0,$$

$$\rho \partial_t V + \nabla_x \cdot (\rho V \otimes V) = \int_{\mathbb{R}^d} H(|x-y|)(V(t,y) - V(t,x))\rho(t,y)\rho(t,x)\, dy.$$

- For the self-alignment particles of Vicsek type, with fixed speed, a phase transition study in presence of noise has been proposed by (P. Degond, A. Frouvelle J.-G. Liu '12), investigating the hyperbolicity of the macroscopic system.

13. J.A. Carrillo, M. Fornasier, J. Rosado, G. Toscani '10

Modified compass search algorithm

- Choose an initial leaders strategy given by the go to target one.
- Focus on the optimized leaders strategy $u^* = u^{\text{opt},(0)}$.
- For $j = 1, \dots, j_{MAX}$
  - Select $\pi(t)$ points on each leaders trajectory, $t \in S_M = \{t_1, t_2, \dots, t_M\}$ and define
  $$\pi^*(t_m) = \pi(t_m) + B_m, \quad m = 1, \dots, M$$
  where $B_m \in Unif([-1,1]^d)$ is a random perturbation.
  - Set
  $$u^{\text{opt},(j)}(t) = \frac{\pi^*(t_{m+1}) - \pi^*(t_m)}{\|\pi^*(t_{m+1}) - \pi^*(t_m)\|}, \quad t \in [t_m, t_{m+1}].$$
  - Compute the functional $\mathcal{J}(\mathbf{x}, \mathbf{u}^{\text{opt},(j)})$.
  - If $\mathcal{J}(\mathbf{x}, \mathbf{u}^{\text{opt},(j)}) \leq \mathcal{J}(\mathbf{x}, \mathbf{u}^*)$ then set $u^* \leftarrow u^{\text{opt},(j)}$ and $\mathcal{J}(\mathbf{x}, \mathbf{u}^*) \leftarrow \mathcal{J}(\mathbf{x}, \mathbf{u}^{\text{opt},(j)})$.