Some Historical and Philosophical Perspectives on Mechanics and Fluids

Math colloquium

NYU Abu Dhabi

Jan 19, 2024

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Previous Lectures on History of Science

1. History of Mechanics: From Aristotle to Einstein (Oct 18)
   https://www.youtube.com/watch?v=31JmS8hkork

2. History of the Principle of Least Action (Nov 19)
   https://www.youtube.com/watch?v=Xg6bGErqh8k&list=PLCheZLRn7G_wBBvP82qVmnZ3X2eHz1L7o&index=7

3. History of Fluid Mechanics I: From Archimedes to Stokes (May 20)
   https://www.youtube.com/watch?v=aImsn3lEcAI

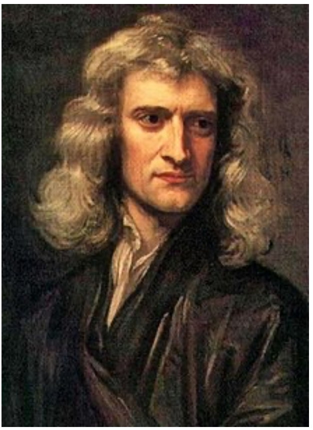
4. History of Aerodynamics II: The Science that Enabled Flight (May 20)
   https://www.youtube.com/watch?v=Pfre2_dvq74

5. History of the Theory of Lift (Mar 23)
   A Mathematical War in the Background of the Great War
   Part I: https://www.youtube.com/watch?v=ECEB2RJnCuY
   Part II: https://www.youtube.com/watch?v=MUSnno-FX2w&t=2401s

The Formation of Classical Mechanics

Dynamics: Cause (Force) → Effect (Motion)

Newton (1642-1727)



Quantity of motion $\equiv mv$

Force = Change in mv

Principia 1687:

"II. The alteration of [the quantity of] motion is ever **proportional** to the motive force"

Leibniz (1646 -1716)

Vis Viva (Living Force) $\equiv mv^2$

Force = Change in mv^2

Work = Change in Kinetic Energy

Leibniz (1668)

"A short demonstration of **a famous error of Decartes** … concerning the claimed natural law according to which God always preserves the same quantity of motion"

The Controversy About Living Forces

Newton (1642-1727) Leibniz (1646 -1716)



Académie des sciences(1724)

“Communication of Motion for Competition”

<u>Principia 1687:</u>

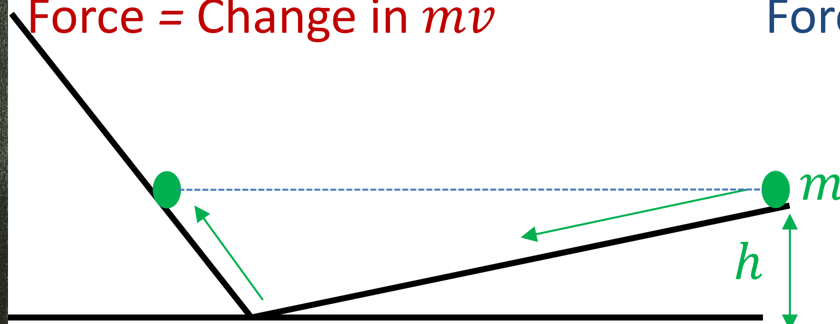
Quantity of motion $\equiv mv$

Force = Change in mv

<u>Leibniz (1668)</u>

Vis Viva (Living Force) $\equiv mv^2$

Force = Change in mv^2



Force $\propto m, h$

Force $\neq$ Change in mv

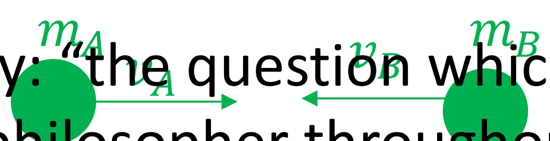
$$4m, h \equiv m, 4h \longrightarrow 4m\sqrt{h} \neq m\sqrt{4h}$$

MacLaurin, Stirling, Clarke
Maziere, Abbe de Catelan, de Mairan

Bernoulli, Gravesande, Wolf, Bulfinger,
Herman, Koenig

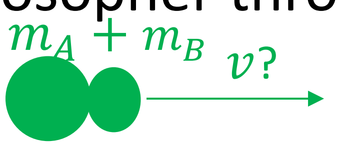
Tokaty: “the question which has been encountered by every scientist and philosopher throughout the entire history of civilization: what is force?”

Elastic Collision



$\sum mv \neq$constant But $\sum mv^2$ =constant

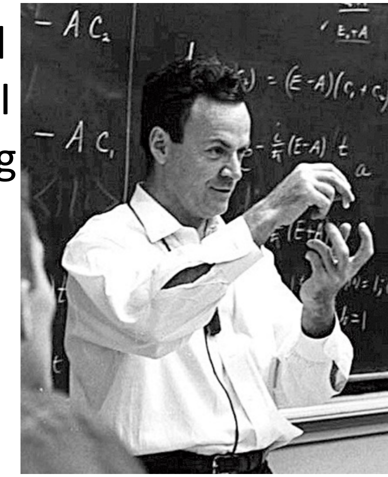
Inelastic Collision



$\sum mv$ =constant But $\sum mv^2 \neq$constant

The Principle of Least Action

"Then, he told me about something which I found absolutely fascinating, and have, since then, always found fascinating. Every time the subject comes up, I work on it. In fact, when I began to prepare this lecture I found myself making more analyses on the thing. Instead of worrying about the lecture, I got involved in a new problem."



1662 Fermat's Principle of Shortest Time

1668 Leibniz

1687 Newton's Principia

1696 Bernoulli's Brachistochrone

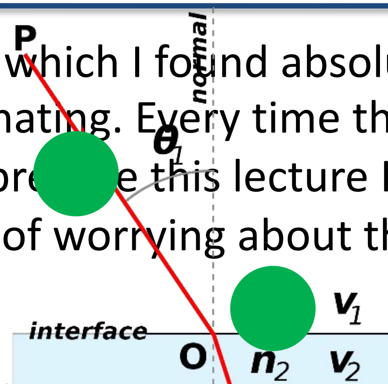
1724 Academy Competition

1736 Euler's Mechanica

1740 Maupertuis

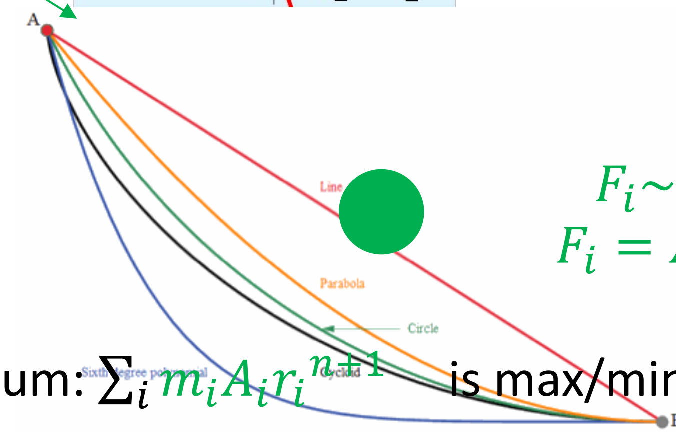
1744 Maupertuis

1747 Maupertuis



$$\frac{v_1}{v_2} = \frac{\sin\theta_1}{\sin\theta_2}$$

m_i F_i



$F_i \sim r_i{}^n$

$F_i = A_i r_i{}^n$

Equilibrium: $\sum_i m_i A_i r_i{}^{n+1}$ is max/min

The agreement between the different laws of Nature that had until now, seemed incompatible.

Action:$=mvs$

On the Laws of Motion and of Rest

m_1 v_1 m_2 v_2

$m_1 + m_2$ $v?$

Action:$=m_1(v-v_1)^2+m_2(v-v_2)^2$

$$\frac{d\text{Action}}{dv} = 0$$

The Principle of Least Action

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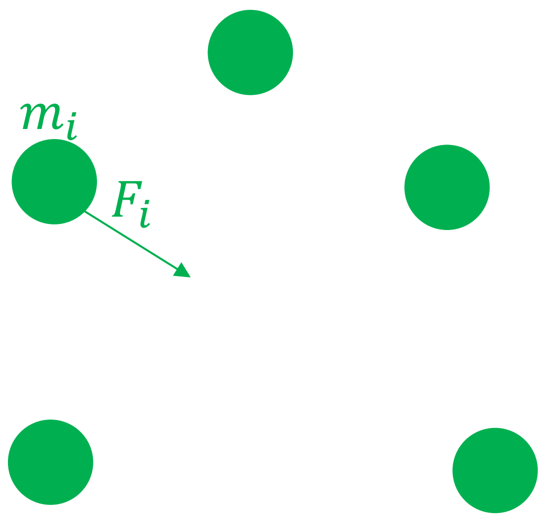

Koenig (1712-1757)

Maupertuis (1698-1759)

Equilibrium: $\sum_i m_i A_i r_i^{\,n+1}$ is max/min

Action:=mvs Action =$\int mv\,ds = \int mv^2 dt$

"After so many great men have worked on this matter, I hardly dare say that I have discovered the principle on which all the laws of motion are founded."

The Principle of Least Action

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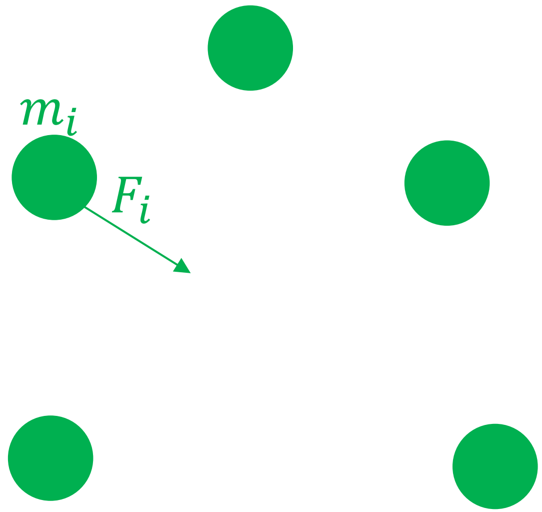


Koenig (1712-1757)

Maupertuis (1698-1759)

Equilibrium: $\sum_i m_i A_i {r_i}^{n+1}$ is max/min

Action:=mvs

The trajectory of a projectile extremizes $\int mvds$

The Principle of Least Action

1662 Fermat's Principle of Shortest Time

1668 Leibniz

1687 Newton's Principia

1696 Bernoulli's Brachistochrone

1724 Academy Competition

1736 Euler's Mechanica

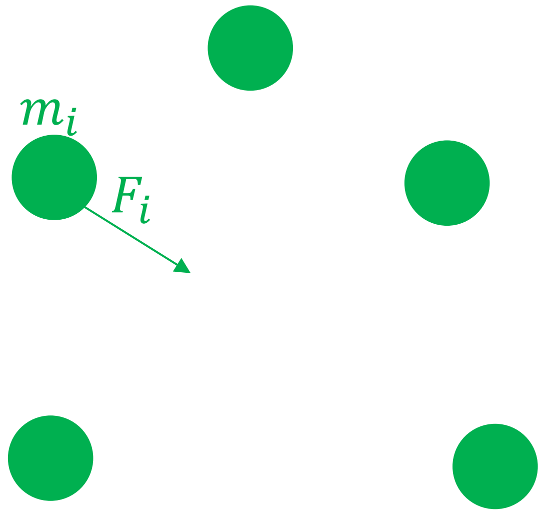
1740 Maupertuis

1744 Maupertuis

1747 Maupertuis

1788 Lagrange

1834 Hamilton



$F_i \sim {r_i}^n$

$F_i = A_i {r_i}^n$

Maupertuis (1698-1759)

Equilibrium: $\sum_i m_i A_i {r_i}^{n+1}$ is max/min

Action:= mvs Action = $\int mvds = \int mv^2 dt$

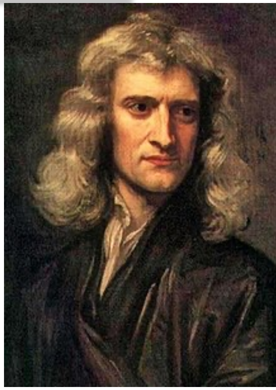
$$\delta \int \sum_i m_i {v^2}_i \, dt = 0$$

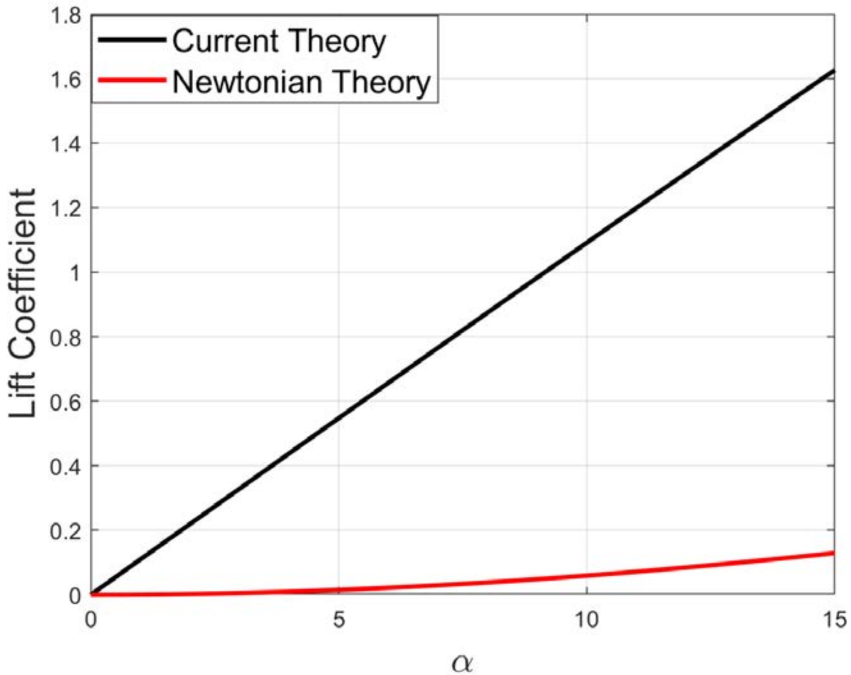
Time-Varying, Velocity-Dependent Potential (Non-conservative)

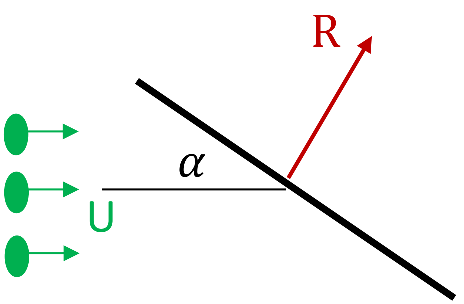
Sociology of Fluid Mechanics and the Role of Viscosity

The Fluid Force Problem

- First Theory:
- Second Theory:
  - a stream of fluid to be a continuous chain of particles, which can move relative to each other.
  - If one part of the fluid moves, this motion gradually communicates to the rest of the fluid: ***defectus lubricitatis*** (lack of slipperiness).
  - Proposition 34: the resistance on a solid cylinder ($\frac{\pi}{16}\rho D^2 U^2$) and sphere







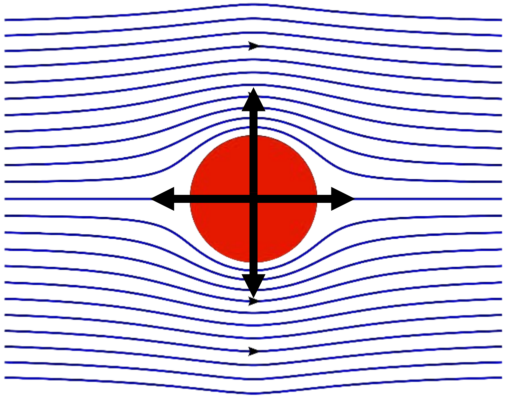
$$R = \rho A U^2 \sin^2\alpha$$

D'Alembert Paradox

- The Academy of Berlin competition on the resistance of fluids (1750)
  - No awards!
  - D'Alembert participated and did not like the decision.

D'Alembert (1717 -1783)

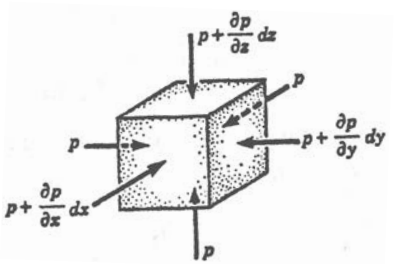
Essai d'une nouvelle theorie de la resistance des fluides (1752):
"I must therefore confess that I do not know how the resistance of fluids can be explained by the theory in a satisfactory way. On the contrary, it seems to me that this theory, handled with all possible rigour, yields a resistance which is absolutely nothing in at least several situations. I bequeath this strange paradox to the geometers, that they may explain it."



D'Alembert Paradox!

Euler’s Continuum Fluid Mechanics

- <u>Euler (1707-1783):</u>
  - Lagrange: Euler “did not contribute to Fluid Mechanics but created it”.
  - Equilibrium of Fluids:
    Principes généraux de l'état d'équilibre des fluides (1755)
  - Euler’s Equations of Motion:
    Principes généraux du mouvement d'équilibre des fluides (1755)
  - Quite General (compressible, nonhomogeneous):
  - Crystal Clear!
  - Dugas: “So perfect is this paper that not a line has aged.”
  - Lagrange: “By the discovery of Euler the whole mechanics of fluids was reduced to a matter of analysis alone, ... Unfortunately, they are so difficult that, up to the present, it has only been possible to succeed in very special cases”.



Haithem Taha

Navier molecular Hypothesis (Equilibrium)

- <u>Navier (1785-1836):</u>
    - Tokaty: “Euler was the creator of Hydrodynamics; but the beautiful trousers he tailored had no buttons, they failed to include viscosity”.
    - 3 papers to Academie des Sciences:
      Elasticity (1821), hydrostatics (1822), hydrodynamics (1822)
    - Molecular Hypothesis: ***Repulsive Forces*** $f = f(r)$

- Two Fluid particles: $M = (x, y, z)$, $M' = (x+\alpha, y+\beta, z+\gamma)$ → $r = \sqrt{\alpha^2+\beta^2+\gamma^2}$
- If *M* is displaced by $(\delta x, \delta y, \delta z)$, *M'* will be displaced by $(\delta x+\delta\alpha, \delta y+\delta\beta, \delta z+\delta\gamma)$
- $\delta r = \frac{\alpha\delta\alpha+\beta\delta\beta+\gamma\delta\gamma}{r}$ → *moment of the mutual actions* (Virtual Work): $\iiint f(r)\delta r\,dxdydz$
- $\iiint f(r)\delta r\,dxdydz = \underbrace{\frac{4\pi}{3}\int_0^\infty r^3 f(r)dr}\left[\frac{\partial\delta x}{\partial x}+\frac{\partial\delta y}{\partial y}+\frac{\partial\delta z}{\partial z}\right] \equiv 2Term\left[\frac{\partial\delta x}{\partial x}+\frac{\partial\delta y}{\partial y}+\frac{\partial\delta z}{\partial z}\right]$
- Principle of Virtual Work: $\iiint\left[Term\left[\frac{\partial\delta x}{\partial x}+\frac{\partial\delta y}{\partial y}+\frac{\partial\delta z}{\partial z}\right]+F_x\delta x+F_y\delta y+F_z\delta z\right]dxdydz = 0$
- $F_x = \frac{\partial Term}{\partial x}, F_y = \frac{\partial Term}{\partial y}, F_z = \frac{\partial Term}{\partial z}$ Clairaut/Euler’s Equations

$$p = Term = \frac{2\pi}{3}\int_0^\infty r^3 f(r)dr$$

- **Navier (1785-1836):**

  - Fluids in Motion (1822):
    - "it is necessary to assume the existence of **new molecular forces** which are produced by the state of **motion**".
    - This force is proportional to the relative velocity between *M, M'* projected on *M-M'*: $\mathrm{V} = \frac{\alpha\delta u+\beta\delta v+\gamma\delta w}{r}$
    - *moment of the mutual actions* (Virtual Work): $\iiint \boldsymbol{f(r)V\delta V}\,dxdydz$
    - $\iiint f(r)V\delta V dxdydz = \underbrace{\frac{8\pi}{30}\int_0^\infty r^4 f(r)dr}_{2\epsilon}\left[3\frac{\partial u}{\partial x}\delta\left(\frac{\partial u}{\partial x}\right)+\frac{\partial u}{\partial y}\delta\left(\frac{\partial u}{\partial y}\right)+\cdots+3\frac{\partial v}{\partial y}\delta\left(\frac{\partial v}{\partial y}\right)+\cdots\right]$
    - Principle of Virtual Work, assuming incompressible $\left(\frac{\partial\delta x}{\partial x}+\frac{\partial\delta y}{\partial y}+\frac{\partial\delta z}{\partial z}=0\right)$
    - $F_x-\frac{\partial p}{\partial x}=\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}\right)-\epsilon\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)$
  - Criticism from Poisson: *Annales de Chimie et de Physique* for 1828 and 1829
  - "Poisson's equations, having come seven years too late, might be said to be of the same form as the equations that had appeared first.... In order to rob me of the merit of having given the differential equations concerned, it would be necessary to show that my principles are contradictory in themselves or with the natural facts. It is not sufficient to say that the same equations have been found in another way to claim, without proof, that this way is better than mine."
  - Arago settled the dispute to Navier.

Stokes' Equations

Navier's work went to oblivion and got resurrected after Stokes.

XXII. *On the Theories of the Internal Friction of Fluids in Motion, and of the Equilibrium and Motion of Elastic Solids.* *By* G. G. STOKES, M.A., *Fellow of Pembroke College.*

[Read *April* 14, 1845.]

In reflecting on the principles according to which the motion of a fluid ought to be calculated when account is taken of the tangential force, and consequently the pressure not supposed the same in all directions, I was led to construct the theory explained in the first section of this paper, or at least the main part of it, which consists of equations (13), and of the principles on which they are formed. I afterwards found that Poisson had written a memoir on the same subject, and on referring to it I found that he had arrived at the same equations. The method which he employed was however so different from mine that I feel justified in laying the latter before this Society*. The leading principles of my theory will be found in the hypotheses of Art. 1, and in Art. 3.

* The same equations have also been obtained by Navier in the case of an incompressible fluid, (*Mém. de l'Institut,* T. VI.) but his principles differ from mine still more than do Poisson's.

Navier-Poisson-Stokes Equations?

The Arnold Principle: If a notion bears a personal name, then this name is NOT the name of the discoverer.

Stokes' Equations

XXII. *On the Theories of the Internal Friction of Fluids in Motion, and of the Equilibrium and Motion of Elastic Solids.* *By* G. G. STOKES, M.A., *Fellow of Pembroke College.*

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Poiseuille's Experiment (1846) & Helmholtz' Derivation (1860): Flow in Capillary Tubes.

Sociology of Science

MacLaurin, Stirling, Clarke
Maziere, Abbe de Catelan, de Mairan

D'Alembert Paradox

Navier Equations

Kutta Condition

Helmholtz Projection

Von Karman Street

Bernoulli, Gravesande, Wolf, Bulfinger,
Herman, Koenig

Dirichlet Paradox

Stokes Equations

Zhukovsky Condition

Leray Projection

Avenue de Henri Benard

Sociology of Fluid Mechanics: Navier/Stokes Equations

XXII. *On the Theories of the Internal Friction of Equilibrium and Motion of Elastic Solids.* *low of Pembroke College.*

[Read *April* 14, 1845.]

Poiseuille's Experiment (1846) & Helmholtz' Derivation (1860): Flow in Capillary Tubes.

- Lamb (1910):

"It was however pointed out by Reynolds that the equations … have been put to a very severe test in the experiments of Poiseuille and others."

"we can hardly hesitate to accept the equations … as a **complete statement of the laws of viscosity**."

- Bloor: "immense authority behind this judgement"

J. Fluid Mech. (1988), *vol.* 187, *pp.* 61–98 61
Printed in Great Britain

Direct simulation of a turbulent boundary layer up to $R_\theta = 1410$

By PHILIPPE R. SPALART

The Dawn of Aviation

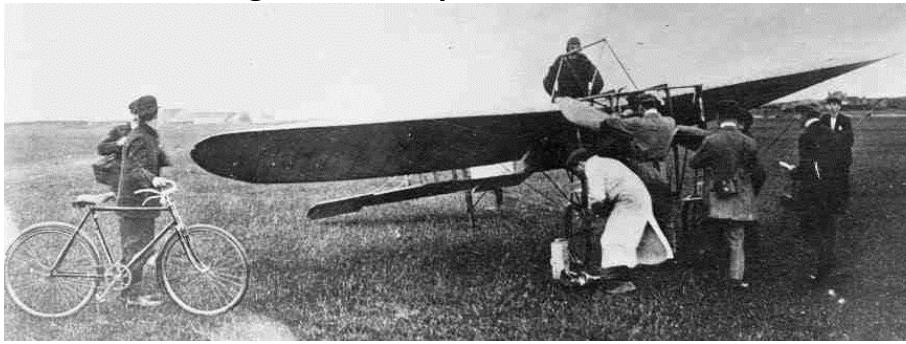
- Dec 17, 1903: Wright Brothers Historic Flight

- No Principles/No Theory

- Lanchester $\xrightarrow{03/03/1909}$ Col. Fullerton:

"I think it was a mistake of the Aeronautical Society giving the Wrights a medal for their contribution to aeronautical science, I agree with their having the medal but it should have been for what they have done."

- Jul 1909: 1st cross-channel flight France → England by Louis Bleriot



Britain is no longer an Island!

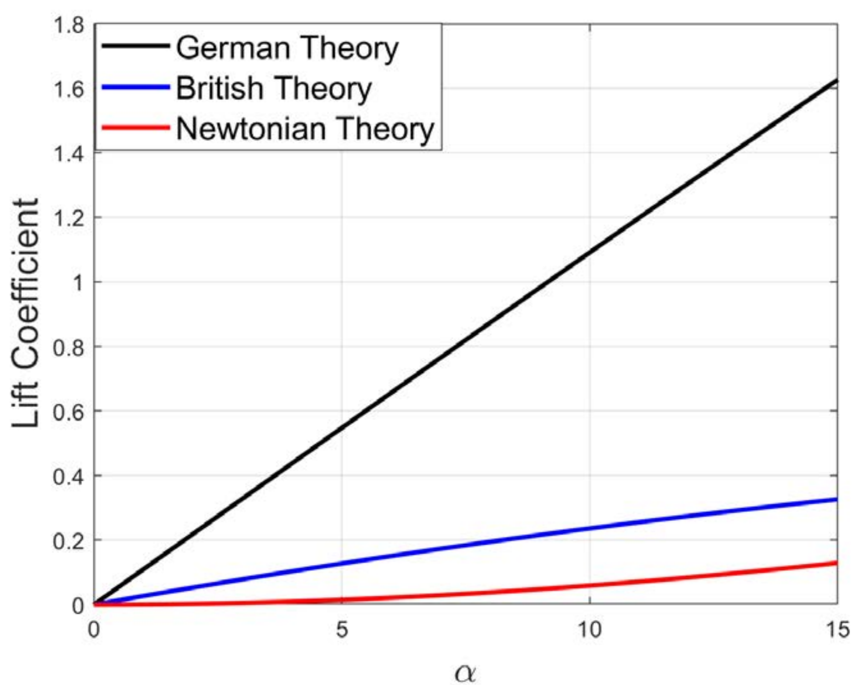
The nation's basic line of defense is breached.

The channel is no longer a moat that makes the island impregnable fortress.

Mathematical War: Need for a Theory of Lift

- George Bryan: “the Germans are probably putting their best brains into improving their aeroplanes.”
- Greenhill (1914 Nature): “Mathematical War”

Two Competing Theories of Lift



Sociology of Fluid Mechanics: The Role of Viscosity

- G. I. Taylor's Adams Prize Essay (1914):
  - "in searching for an explanation of the forces which act on solids moving through fluids, it is **useless** to confine one's attention to irrotational motion."

- Cowley & Levy "*Aeronautics in Theory and Experiment*" (1918):
  - "the failure of the various treatments of the problem...is evidently due to the supposition that the fluid...is perfect."
  - Need for a viscous theory: "will clarify at one stroke the whole problem of aerodynamics."

- Bairstow's "Applied Aerodynamics" (1920):

"[it] appears to be **fundamentally impossible** to represent the motion of a real fluid accurately by any theory relating to an **inviscid** fluid."

- G. I. Taylor (1921 Wright Memorial Lecture):

"One must seek for the explanation of the forces that are observed in these cases in the action of the eddying region on the flow."

- Bairstow (1923 RAeS Meeting):
  - "without mentioning a fundamental property of air on which its motion depends, viz., its **viscosity**."
  - Stokes' "equations were sufficient to account for the phenomena, whether it was a steady flow or an eddying flow. These equations did not appear in the Prandtl theory."

Two Different Perspectives

British Mathematical Physics* Vs *German Technical Mechanics

British

- Ideal fluid is fiction.

"was regarded purely as an exercise for the amusement of students."

- N-S is the truth

- Lift is viscous. Solve N-S.

Germans

- Ideal fluid is a good approximation of the average flow at a high Re.

- Both ideal flow and N-S flow are idealizations.

- Ideal fluid theory → reasonable estimate of the lift.

- Low (1923 RAeS Meeting):

"I have no objection to providing scientists with endowments and facilities to allow them to pursue their 'strictly abstract studies'. But who knows when, if ever, these studies will bear fruit? As an engineer, I do not intend to wait for them on this occasion."

- Jul 03, 1923: ACA (Aerodynamics Subcommittee):

Experimental investigation of Prandtl theory

Victory of the Circulation Theory

- Taylor (1926):
  - "Bryant and Williams show that the flow round a certain model aerofoil placed in a wind tunnel is not very different from an irrotational flow with circulation."
- 1927: Prandtl's Wright Memorial Lecture and Gold Medal of RAeS

"The Generation of Vortices in Fluids of Small Viscosity"

"no serious error will be made if in the case of flow behind sharp edges viscosity is totally neglected."

- 1930 Taylor → Prandtl: You deserve Nobel Prize in Physics.
- Bairstow's "Applied Hydrodynamics" (1939)

"From a consideration of all available experimental results it may be concluded that the main effects … can be reproduced by potential flow theory"

- Bairstow's: "Twenty-One Years' Progress of Aerodynamic Science" (1930)

  "Aerodynamic theory is now rather like the physical theory of light … physicists use the electron theory on Mondays, Wednesdays and Fridays, and the wave theory on alternate days. Both have uses but reconciliation of the two ideas has not yet been achieved. So it is in aeronautics. In our experimental work we assume that viscosity is an essential property of air … The practically useful theory of Prandtl comes from considering air as frictionless or inviscid."

Some Historical and Philosophical Perspectives on Mechanics and Fluids

Thank You!

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Newtonian Mechanics Vs Variational Mechanics

Mechanics

Newtonian ($F = ma$)

$$\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u} \cdot \boldsymbol{\nabla} \boldsymbol{u} = -\boldsymbol{\nabla} p$$

$$\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u} \cdot \boldsymbol{\nabla} \boldsymbol{u} = -\boldsymbol{\nabla} p + \boldsymbol{\nabla} \cdot \boldsymbol{\tau}$$

Mandatory upon all graduate students in fluid mechanics

Search:

stokes Search

Word search of titles and abstracts only.

330 Results Found.

“why Hamilton’s principle is not more widely used in the field of fluid mechanics?”

Variational/Lagrangian

$$\partial J = 0$$

Rarely Demonstrated

Search:

variational Search

Word search of titles and abstracts only.

19 Results Found.

THE PHYSICS OF FLUIDS

VOLUME 9, NUMBER 6 JUNE 1966

Hamilton’s Principle for Fluids

PAUL PENFIELD, JR.

Newtonian Mechanics Vs Variational Mechanics

A Historical Perspective

Mechanics

Newtonian ($F = ma$)

Variational

Lagrangian
Hamilton's Principle

Gauss' Principle

Newton's Principia (1687)
$F = ma$

Maupertuis' Least Action (1744)
Lagrange's
M´ecanique Analytique (1788)
Hamilton's Formulation (1834)

Gauss' Principle (1829)
Jacobi's Lectures (1848)

Euler (1755)

$$\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u} \cdot \boldsymbol{\nabla} \boldsymbol{u} = -\boldsymbol{\nabla} p$$

Lagrange (1788)
In Lagrangian coordinates

In Eulerian Coordinates (1960s)

This Work (2023)

Navier (1822) - Stokes (1845)

$$\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u} \cdot \boldsymbol{\nabla} \boldsymbol{u} = -\boldsymbol{\nabla} p + \boldsymbol{\nabla} \cdot \boldsymbol{\tau}$$

???

The Generation and Decay of Vorticity

B. R. MORTON†

National Center for Atmospheric Research, Boulder, Colorado, U.S.A.‡

(*Received December 14, 1983*)

Vorticity, although not the primary variable of fluid dynamics, is an important derived variable playing both mathematical and physical roles in the solution and understanding of problems. The following treatment discusses the generation of vorticity at rigid boundaries and its subsequent decay. It is intended to provide a consistent and very broadly applicable framework within which a wide range of questions can be answered explicitly. The rate of generation of vorticity is shown to be the relative tangential acceleration of fluid and boundary without taking viscosity into account and the generating mechanism therefore involves the tangential pressure gradient within the fluid and the external acceleration of the boundary only. The mechanism is inviscid in nature and independent of the no-slip condition at the boundary, although viscous diffusion acts immediately after generation to spread vorticity outward from boundaries. Vorticity diffuses neither out of boundaries nor into them, and the only means of decay is by cross-diffusive annihilation within the fluid.

The generation and conservation of vorticity: deforming interfaces and boundaries in two-dimensional flows

S. J. Terrington[1,†], K. Hourigan[1] and M. C. Thompson[1]

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This article presents a revised formulation of the generation and transport of vorticity at generalised fluid–fluid interfaces, substantially extending the work of Brøns *et al.* (*J. Fluid Mech.*, vol. 758, 2014, pp. 63–93). Importantly, the formulation is effectively expressed in terms of the conservation of vorticity, and the latter is shown to hold for arbitrary deformation and normal motion of the interface; previously, vorticity conservation had only been demonstrated for stationary interfaces. The present formulation also affords a simple physical description of the generation of vorticity in incompressible, Newtonian flows: the only mechanism by which vorticity may be generated on an interface is the inviscid relative acceleration of fluid elements on each side of the interface, due to pressure gradients or body forces. Viscous forces act to transfer circulation between the vortex sheet representing the interface slip velocity, and the fluid interior, but do not create vorticity on the interface. Several representative example flows are considered and interpreted under the proposed framework, illustrating the generation, transport and, importantly, the conservation of vorticity within these flows.