SITE RESEARCH CENTER ON STABILITY, INSTABILITY, AND TURBULENCE

SITE Research Center Talk Series September 2023 to August 24

SEPTEMBER 19, 2023

[Charles Collot]

- Title: **Asymptotic stability of traveling waves for one-dimensional nonlinear Schrodinger equations**
- Abstract: We consider one dimensional nonlinear Schrodinger equations around a traveling wave. We prove its asymptotic stability for general nonlinearities, under the hypotheses that the orbital stability condition of Grillakis-Shatah-Strauss is satisfied and that the linearized operator does not have a resonance and only has 0 as an eigenvalue. As a by-product of our approach, we show long-range scattering for the radiation remainder. Our proof combines for the first time modulation techniques and the study of space-time resonances. We rely on the use of the distorted Fourier transform, akin to the work of Buslaev and Perelman and, and of Krieger and Schlag, and on precise computations and estimates of space-time resonances to handle its interaction with the soliton. This is joint work with Pierre Germain.

SEPTEMBER 26, 2023

[Slim Ibrahim]

- Title: **Local smooth resolubility of the Relativistic Vlasov-Maxwell system and applications to hot, magnetized and dense plasma**
- Abstract: The objective of the first half of my talk is to briefly discuss the problem of singularity formation for quasilinear equations, and to explain how the Relativistic Vlasov-Maxwell (RVM) does not have shocks as for Burger's equation. This offers an opportunity to introduce microlocal Radon analysis leading to this type of results. The second part of my talk will be devoted to the details of the proof of the local solvability for RVM, and to the application to hot magnetized and dense plasmas. This is a collaboration with C. Cheverry (U. Rennes).

OCTOBER 5, 2023

[Youssef Belhamadia]

- Title: **Numerical modeling of radiation-conduction heat transfer and phase change problems**
- Abstract: Phase change in materials is important in many sciences and engineering applications such as metal processing, crystal growth, industrial furnaces, among others. In all the applications that involve high temperature, radiation greatly influences the thermal features and it cannot be neglected. Experimental predictions of the impact of radiation in the heat conduction can be very demanding. Therefore, numerical modeling can play a crucial role and provide accurate and effective thermal predictions. This presentation covers two main topics. Firstly, we present accurate numerical methods for solving radiation–conduction heat transfer in participating media. The mathematical model is based on the SP_N approximations. The numerical methods are based on a mixed finite element formulation and a time-dependent mesh adaptation algorithm to enhance the accuracy of the solutions. Secondly, we present mathematical models and numerical methods developed for phase change problems. Two- and three-dimensional numerical simulations are considered to show the effectiveness of the overall methodology.

OCTOBER 10, 2023

[Giacomo Albi]

- Title: **Mathematical models and methods for crowd dynamics control**
- Abstract: In this talk we consider mathematical models and methods recently developed to control crowd dynamics, with particular emphasis on egressing pedestrians in unknown environment. We focus on special agents, called leaders, to steer the crowd towards the desired direction. Leaders can be either hidden in the crowd or recognizable as such. This strategy heavily relies on the power of the social influence (herding effect), namely the natural tendency of people to follow group mates in situations of emergency or doubt. Control strategies are designed in order to reduce as much as possible the intervention on the crowd. Ideally the natural behaviour of people is kept, and people do not even realize they are being led by an external intelligence. Mathematical models are discussed at different scales of observation, showing how macroscopic (fluid-dynamic) models can be derived by mesoscopic (kinetic) models which, in turn, can be derived by microscopic (agent-based) models.

OCTOBER 16, 2023

[Chenchen Mou]

- Title: **Minimal solutions of master equations for extended mean field games**
- Abstract: In an extended mean field game the vector field governing the flow of the population can be different from that of the individual player at some mean field equilibrium. This new class strictly includes the standard mean field games. It is well known that, without any monotonicity conditions, mean field games typically contain multiple mean field equilibria and the wellposedness of their corresponding master equations fails. In this paper, a partial order for the set of probability measure flows is proposed to compare different mean field equilibria. The minimal and maximal mean field equilibria under this partial order are constructed and satisfy the flow property. The corresponding value functions, however, are in general discontinuous. We thus introduce a notion of weak-viscosity solutions for the master equation and verify that the value functions are indeed weak-viscosity solutions. Moreover, a comparison principle for weak-viscosity semi-solutions is established and thus these two value functions serve as the minimal and maximal weak-viscosity solutions in appropriate sense. In particular, when these two value functions coincide, the value function becomes the unique weak-viscosity solution to the master equation. The novelties of the work persist even when restricted to the standard mean field games. This is based on a joint work with Jianfeng Zhang.

OCTOBER 24, 2023

[Michele Ricciardi]

- Title: **The master equation in mean field games with Neumann conditions**
- Abstract: This talk is devoted to the convergence problem of the Nash Equilibria in a N-players differential game towards the optimal strategies in the Mean Field Games framework. The novelty here is that the dynamic of the generic player includes a reflection process which guarantees the invariance of the state space Ω. This implies that the MFG system presents Neumann boundary conditions for the value function u and the density of the population m. The first part of the talk is devoted to the study of the well-posedness of the Master Equation, essential tool in order to study the convergence problem. The reflection process in the N-players game leads to two Neumann conditions in the Master Equation formulation. In the second part we analyze the convergence problem,

borrowing and readapting the ideas from the periodic case, studied by Cardaliaguet, Delarue, Lasry, Lions. The results can also be generalized in the case of Dirichlet boundary conditions.

OCTOBER 26, 2023

Nizar Demni

- Title: **Quantum states, Unitary Brownian motion and the Jacobi polynomials on the simplex**
- Abstract: After recalling basic facts from quantum information theory (density matrices, partial trace, purification), I'll present two examples of random states: the induced Gaussian state and the uniform pure one on the Euclidean sphere. Afterwards, I'll introduce the dynamical version of the latter which leads to the problem of computing the joint law of the moduli of a given set of its coordinates. The first approach to solve this problem is based on solving a heat equation in the simplex but lacks the knowledge of boundary conditions and a natural choice of basis. The second approach is rather based on direct computations by means of unitary spherical harmonics and gives rise to the co-called Jacobi polynomials in the simplex.

NOVEMBER 16, 2023

Emeric Roulley

- Title: **Dynamics of vortex cap solutions on the rotating unit sphere**
- Abstract: We provide an analytical proof for the existence of periodic vortex cap solutions for the homogeneous and incompressible Euler equations on the rotating unit 2-sphere. These solutions are piecewise constant absolute vorticity distributions, subject to the Gauss constraint and rotating uniformly around the vertical axis (rotation axis of the sphere). The emergence of such structures was numerically conjectured in the atmospheric community by Dritschel-Polvani in the 90s. Our proof is based on the bifurcation from zonal solutions given by spherical caps. This is a joint work with Claudia García (Granada) and Zineb Hassainia (NYUAD).

NOVEMBER 28, 2023

Boualem Djehiche

- Title: **On the asymptotic outcomes of generative AI**
- Abstract: Starting from a simple deterministic model, we show that the asymptotic outcomes (as time goes to infinity) of both shallow and deep neural networks such as those used in BloombergGPT to generate economic time series are exactly the Nash equilibria of a non-potential game. We then analyze deep neural network algorithms that converge to these equilibria. The approach is extended to federated deep neural networks between clusters of regional servers and on-device clients. Finally, the variational inequalities behind large language models including encoder-decoder related transformers are established.

DECEMBER 6, 2023

Sebastian Munoz

- Title: **Free boundary regularity and support propagation in mean field games and optimal transport**
- Abstract: In this talk, we present new findings on the regularity of first-order mean field games systems with a local coupling. We focus on systems where the initial density is a compactly supported function

on the real line. Our results show that the solution is smooth in regions where the density is strictly positive and that the density itself is globally continuous. Additionally, the speed of propagation is determined by the behavior of the cost function for small densities. When the coupling is entropic, we demonstrate that the support of the density propagates with infinite speed. On the other hand, when f(m) = mθ with θ > 0, we prove that the speed of propagation is finite. In this case, we establish that under a natural non-degeneracy assumption, the free boundary is strictly convex and enjoys C1,1 regularity. We also establish sharp estimates on the speed of support propagation and the rate of long time decay for the density. Our methods are based on the analysis of a new elliptic equation satisfied by the flow of optimal trajectories. The results also apply to mean field planning problems, characterizing the structure of minimizers of a class of optimal transport problems with congestion.

DECEMBER 7, 2023

Michele Dolce

- Title: **Stability threshold of the 2D Couette flow in a homogeneous magnetic field**
- Abstract: A planar incompressible and electrically conducting fluid can be described by the 2D Navier-Stokes-MHD system. One simple yet physically relevant laminar state is the Couette flow with a constant homogeneous magnetic field, given by u_E=(y,0), B_E=(b,0) in the domain TxR. The goal is to estimate how large can be a perturbation of this state while still resulting in a solution close to the laminar regime, thereby preventing the onset of turbulence. We prove that Sobolev regular initial perturbations of size O(Re^{-2/3}), with Re being the Reynolds number, remain close to u_E, B_E and exhibit dissipation enhancement. The latter quantifies the convergence towards an x-independent state on a time-scale O(Re^{-1/3}), much faster than the standard diffusive one O(Re^{-1}).

DECEMBER 8, 2023

Lingyun Ding

- Title: **Unveiling the Role of Diffusion-Driven Flow in Micro-Fluidic Transport: A Slow Manifold-Based Approach to Reduced Modeling**
- Abstract: Diffusion-driven flow, a boundary layer phenomenon resulting from the interplay of gravity and diffusion in density-stratified fluids when a gravitational field is non-parallel to an impermeable solid boundary, plays a crucial role in enhancing solute dispersion within an inclined capillary channel, as demonstrated by supporting experimental and numerical results. This study delves into the investigation of diffusion-driven flow within a nonlinearly density-stratified fluid confined between two tilted parallel plate walls. While the classical thin film equation describes cases with weak density gradients, it falls short of accurately characterizing dynamics across a broad parameter range. To address this limitation, we introduce a novel asymptotic expansion inspired by center manifold theory, expanding quantities in terms of derivatives of the cross-sectional averaged density field. This approach yields precise approximations for velocity, density, and pressure fields. Furthermore, we derive an evolution equation describing the cross-sectional averaged density field. This equation, akin to the traditional diffusion equation, replaces the constant diffusion coefficient with a positive-definite function dependent on the solution's derivative. Our exploration of the effective equation reveals that the density profile is contingent on a non-dimensional parameter denoted as γ, which correlates with the flow strength. In the large γ limit, the system mimics a diffusion process with an augmented diffusion coefficient of $1+\cot^{2}\theta$, where θ denotes the inclination angle of the channel domain. This parameter regime showcases diffusion-driven flow's heightened

mixing ability. Conversely, in the small γ regime, the density field behaves similarly to pure diffusion, characterized by distorted isopycnals.

JANUARY 16, 2024

Haithem Taha

- Title: **A variational theory of aero-hydrodynamics**
- Abstract: The Euler equation does not possess a unique solution for the flow over a two-dimensional object. This problem has serious repercussions in aerodynamics; it implies that the inviscid aero-hydrodynamic lift force over a two-dimensional object cannot be determined from first principles; a closure condition must be provided. The Kutta condition has been ubiquitously considered for such a closure in the literature, even in cases where it is not applicable (e.g. unsteady). In this talk, I will present a special variational principle that we revived from the history of analytical mechanics: Hertz' principle of least curvature. Using this principle, we developed a novel variational formulation of Euler's dynamics of ideal fluids that is fundamentally different from the previously developed variational formulations based on Hamilton's principle of least action. Applying this new variational formulation to the century-old problem of the ideal flow over an airfoil, we developed a general (dynamical) closure condition that is, unlike the Kutta condition, derived from first principles. In contrast to the classical theory, the proposed variational theory is not confined to sharp edged airfoils; i.e., it allows, for the first time, theoretical computation of lift over arbitrarily smooth shapes, thereby generalizing the century-old lift theory of Kutta and Zhukovsky. Moreover, the new variational condition reduces to the Kutta condition in the special case of a sharp-edged airfoil, which challenges the widely accepted wisdom about the viscous nature of the Kutta condition. We also generalized this variational principle to Navier-Stokes' via Gauss' principle of least constraint, thereby discovering the fundamental quantity that Nature minimizes in every incompressible flow. We proved that the magnitude of the pressure gradient over the field is minimum at every instant! We call it the Principle of Minimum Pressure Gradient (PMPG). We proved that the Navier-Stokes' equation is the necessary condition for minimizing the pressure gradient subject to the continuity constraint. Hence, the PMPG turns any fluid mechanics problem into a minimization one where fluid mechanicians need not to apply Navier-Stokes' equations, but merely need to minimize the proposed action.

JANUARY 19, 2024

Haithem Taha

- Title: Colloquium - **Some Historical and Philosophical Perspectives on Mechanics and Fluids**
- Abstract: This talk presents some prominent incidents in the history of mechanics and fluids with particular emphasis on philosophical considerations. First, I will discuss difference in views between Newton and Leibniz on Mechanics; Leibniz defines the quantity of motion differently---the Vis Viva (Living Force), which consequently requires defining the force in a different way. The search for the right mechanics between these two views (Newton's and Leibniz') divided the geometers of the 18th century for more than 30 years, which settled down for the Newtonian approach. However, Leibniz' concept of the Living Force remained actually alive. It was rejuvenated with the development of the Principle of Least Action---one of the most successful and beautiful concepts in mechanics, which turned the whole branch of mechanics into optimization. The Principle of Least Action also went through a tense polemic among scholars in the 18th century. Second, I'll discuss Navier's development

of his equations and how he obtained the viscous friction term without referring to friction at all, unlike Stokes whose objective was specifically to find the effect of friction. In fact, the community did not accept Navier's molecular hypothesis for more than 20 years; it was not until Stokes arrived at the same equation using the well-known continuum-mechanics approach. Finally, if time permits, I will discuss the mathematical war between Britain and England to develop a theory of lift during the time of their military war (the Great War of WWI). The Cambridge school of mathematical physics, led by Rayleigh, Lamb, Taylor, and Bairstow emphasized the role of viscosity. In contrast, the Gottingen school of technical mechanics, led by Prandtl, Kutta, Munk, and Betz, had developed an ideal-flow theory of lift. While the German lost the actual war, they won the mathematical war; the circulation theory of lift has dominated to become routinely taught in every single aeronautical school throughout the world including Cambridge. We will discuss how the German training in technical schools (Technische Hochschule) provided freedom/advantage in accepting assumptions about the nature of air that the Cambridge school of mathematical physics could not tolerate (e.g., neglecting viscosity). Hence, they responded differently to the same technical problems (e.g., the German invention of the Kutta condition to handle the non-uniqueness of Euler's equation). However, the role of viscosity is still emphasized today, thanks to the dominance of the Cambridge school in fluid mechanics.

JANUARY 23, 2024

Jean-Marc Delort

- Title: **Norm inflation for solutions of semi-linear one dimensional Klein-Gordon equations**
- Abstract: In space dimension larger or equal to two, the non-linear Klein-Gordon equation with small, smooth, decaying initial data has global in time solutions. This no longer holds true in one space dimension, where examples of blowing up solutions are known. On the other hand, it has been proved that if the nonlinearity satisfies a convenient compatibility condition, the ``null condition'', one recovers global existence and that the solutions satisfy the same dispersive bounds as linear solutions. The goal of this paper is to show that, in the case of cubic semi-linear nonlinearities, this null condition is optimal, in the sense that, when it does not hold, one may construct small, smooth, decaying initial data giving rise to solutions that display inflation of their L^∞ and L^2 norms in finite time.

JANUARY 30, 2024

Alpár Meszaros

- Title: **Norm inflation for solutions of semi-linear one dimensional Klein-Gordon equations**
- Abstract: In this talk I will describe how to construct global in time classical solutions to the master equation arising in mean field games. Our method works for a general class of non-separable Hamiltonians and final data that satisfy a suitable monotonicity condition. This stems from the so-called displacement convexity condition introduced and used successfully in the theory of optimal mass transportation. Our results hold true independently of the intensity of the idiosyncratic noise. The talk is based on recent works with M. Bansil, W. Gangbo, C. Mou and J. Zhang.

FEBRUARY 5, 2024

Pranava Jayanti

- Title: **Mass transfer and global solutions in a micro-scale model of superfluidity**
- Abstract: We investigate a micro-scale model of superfluidity derived by Pitaevskii in 1959 to describe the interacting dynamics between the superfluid and normal fluid phases of Helium-4. This system consists of the nonlinear Schr\"odinger equation and the incompressible, inhomogeneous Navier-Stokes equations, coupled to each other via a bidirectional nonlinear relaxation mechanism. The coupling permits mass/momentum/energy transfer between the phases, and accounts for the conversion of superfluid into normal fluid. We prove the existence of solutions in $\mathbb{T}^d$ $(d=2,3)$ for a power-type nonlinearity, beginning from small initial data. Depending upon the strength of the nonlinear self-interactions, we obtain solutions that are global or almost-global in time. The main challenge is to control the inter-phase mass transfer in order to ensure the strict positivity of the normal fluid density, while obtaining time-independent a priori estimates. We present two different approaches (purely energy based, versus a combination of energy estimates and maximal regularity) based on the dimension. The results in this talk are from recent collaborations with Juhi Jang and Igor Kukavica.

FEBRUARY 20, 2024

Marco Cirant

- Title: **On the long time behavior of equilibria in a Kuramoto Mean Field Game**
- Abstract: In a recent work, R. Carmona, Q. Cormier and M. Soner proposed a mean field game based on the classical Kuramoto model, originally motivated by systems of chemical and biological oscillators. Such MFG model exhibits several stationary equilibria, and the question of their ability to capture long time limits of dynamic equilibria is largely open. I will discuss in the talk how to show that, up to translations, there are two possible stationary equilibria only - the incoherent and the synchronised one - provided that the interaction parameter is large enough. Finally, I will present some local stability properties of the synchronised equilibrium.

FEBRUARY 22, 2024

Jakob Möller

- Title: **Nonlinear PDE in semi-classical semi-relativistic quantum physics**
- Abstract: Presentation of work on (asymptotic) analysis of nonlinear time-dependent PDEs modeling fast, self-interacting charged fermions in relativistic quantum mechanics, from the Dirac-Maxwell to Vlasov/Euler–Poisson equations. I focus on intermediate first and second order in 1/c models, such as the Pauli-Poisswell and Euler-Darwin equations, which are useful e.g. in plasma physics. This is joint work with Norbert J. Mauser (WPI c/o U. Vienna), Pierre Germain (ICL) and Changhe Yang (Caltech).

FEBRUARY 28, 2024

[Rafael Bailo]()

- Title: **Pedestrian models with congestion effects**
- Abstract: We study the validity of the dissipative Aw-Rascle system as a macroscopic model for pedestrian dynamics. The model uses a congestion term (a singular diffusion term) to enforce capacity constraints in the crowd density while inducing a steering behaviour. Furthermore, we introduce a semi-implicit, structure-preserving, and asymptotic-preserving numerical scheme which can handle the numerical solution of the model efficiently. We perform the first numerical simulations of the dissipative Aw-Rascle system in one and two dimensions. We demonstrate the efficiency of the scheme in solving an array of numerical experiments, and we validate the model, ultimately showing that it correctly captures the fundamental diagram of pedestrian flow.

FEBRUARY 29, 2024

[Hans Knüpfer]()

- Title: **Gamma--limit for zigzag walls**
- Abstract: Ferromagnets typically exhibit the formation of magnetic domains with uniform magnetization separated by thin transition layers. Thz Zigazag wall is one type of such transition layers which particularly appears in thin ferromagnetic films. in order to investigate this transition layer, we consider a sample in the form a thin strip and enforce a transition layer by suitable boundary conditions on the magnetization. In the macroscopic limit we show that the energy Gamma--converges to a limit energy where jump discontinuities of the magnetization are penalized anisotropically. In particular, there is a subcritical regime one--dimensional charged domain walls are favorable. In the supercritical regime the limit model allows for zigzaging two--dimensional domain walls.

MARCH 18, 2024

[Yoichiro Mori]()

- Title: **Inextensible interface problem in 2D Stokes fluid**
- Abstract: We consider the dynamics of a closed intextensible interface immersed in a 2D Stokes fluid, a model that has been used for 2D simulations of vesicle dynamics. In this model, a 1D closed interface exerts a bending force and the interface is subject to an inextensibility constraint. As part of the problem, one must solve for the unknown tension that ensures membrane inextensibility. Given a force exerted on the interface, we first show that the problem of determining the tension is solvable if and only if the interface is not a circle. Using this result, we prove local-in-time well-posedness for this problem. We will finally discuss open questions and future directions.

MARCH 19, 2024

[Mathieu Lauriere]()

- Title: **Deep Backward and Galerkin Methods for Learning Finite State Master Equations**

- Abstract: We propose two methods to solve the master equation for finite-state mean field games (MFGs). Solving MFGs provides approximate Nash equilibria for stochastic, differential games with finite but large populations of agents. The master equation is a partial differential equation (PDE) whose solution characterizes MFG equilibria for any possible initial distribution. The first method we propose relies on backward induction while the second one directly tackles the PDE without discretizing time. For both approaches, we prove two types of results: there exist neural networks that make the loss functions of the algorithms arbitrarily small and, conversely, if the losses are small, then the neural networks are good approximations of the master equation solution. We conclude with numerical experiments on benchmark problems from the literature in dimension up to 15, and a comparison with solutions computed by a classical method for fixed initial distributions. This is joint work with Asaf Cohen and Ethan Zell (University of Michigan, Ann Arbor).

MARCH 20, 2024

Liwei Xu

- Title**: An investigation on the structure-preserving deep learning methods for solving the radiative transport equations**
- Abstract: In this talk, we present two schemes coupling the neural network methods and the asymptotic-preserving schemes for the solution of the radiative transport equation. The first scheme is based on a micro-macro decomposition scheme, and the second one is designed through an introduction of macroscopic auxiliary equations. The schemes possess advantages on dealing with problems with high dimensionality and multiscale characteristics. Numerical examples are given to demonstrate the efficiency of numerical methods. This is joint work with Hongyan Li, Song Jiang, Wenjun Sun and Guanyu Zhou.

MARCH 26, 2024

Anna Mazzucato

- Title**: Direct and Inverse problems in monitoring of faults**
- Abstract: I will discuss a model of elastic dislocations applicable to buried faults in the Earth's crust in between seismic events. The forward problem amounts to solving a non-standard transmission problem for a system of linear PDES in elastostatics, knowing the fault and how much the rock has slipped at the fault. The inverse problem consists in determining the geometry of the fault and the slip at the fault from surface measurements, which can be obtained from GPS and satellite data. While the direct problem is well posed, the inverse problem is generally ill-posed unless assumptions are made on the fault. I will present a uniqueness result for the inverse problem and an iterative reconstruction algorithm based on a distributed shape derivative, which measures the change in the rock displacement under infinitesimal movements of the fault and the slip. I will close with some simple numerical tests from synthetic data. If time permits, I will discuss a more realistic model where the fault is assumed viscoelastic. This is joint work with Andrea Aspri (University of Milan), Elena Beretta (NYU-Abu Dhabi), and Maarten de Hoop (Rice University), and PhD student Arum Lee (Penn State).

MARCH 27, 2024

[Boyce Eugene Griffith]

- Title**: Computational Cardiac Fluid Dynamics In Vitro and In Vivo**
- Abstract: Cardiac fluid dynamics fundamentally involves interactions between complex blood flows and the structural deformations of the muscular heart walls and the thin, flexible valve leaflets. This talk will provide an overview of modern numerical methods for treating such fluid-structure interactions and detail some of their applications to cardiac fluid dynamics. I will initially focus on models of an in vitro pulse-duplicator system that is commonly used in the development and regulation of prosthetic heart valves. These models enable detailed comparisons between experimental data and computational model predictions but use highly simplified descriptions of cardiac anatomy and physiology. I will describe experimental and computational investigations on determinants of prosthetic heart valve dynamics using this platform. I will also present recent in vitro models, including a patient-specific model of transcatheter aortic valve replacement and a new comprehensive model of the human heart. This heart model includes fully three-dimensional descriptions of all major cardiac structures along with biomechanics models that are parameterized using experimental tensile test data obtained exclusively from human tissue specimens. Simulation results demonstrate that the model generates physiological stroke volumes, pressure-volume loops, and valvular pressure-flow relationships, thereby illustrating is its potential for predicting cardiac function in both health and disease.

APRIL 2, 2024

[Matteo Fornoni]

- Title**: Optimal control for non-local Cahn-Hilliard tumour growth models**
- Abstract: We consider a non-local tumour growth model of phase-field type, describing the evolution of tumour cells through proliferation in the presence of a nutrient. The model consists of a coupled system, incorporating a non-local Cahn-Hilliard equation for the tumour phase variable and a reaction-diffusion equation for the nutrient. The optimal control problem aims to identify a suitable therapy, capable of guiding the evolution of the tumour towards a predefined target. We first establish novel regularity results for the PDE system by applying maximal regularity theory in weighted L^p spaces. Such a technique enables us to prove the local existence and uniqueness of a regular solution in a quite general framework, which also includes chemotaxis effects to some extent, but restricts us to regular double-well potentials. Then, by leveraging the time-regularisation properties of the weighted spaces and some global boundedness estimates, we further extend the solution to a global one. In a second version of the model, we add a viscous regularisation term which allows us to prove the existence and uniqueness of a global regular solution under more general hypotheses. Indeed, we can also include singular double-well potentials and cross-diffusive chemotactic effects, at the expense of some additional hypotheses on the controls. These results provide the foundation for addressing the optimal control problem in both cases. Specifically, we prove the existence of an optimal therapy and then, by studying the Fr\'echet-differentiability of the control-to-state operator and introducing the adjoint system, we derive first-order necessary optimality conditions. We finally discuss some open questions and future research directions.

APRIL 16, 2024

[Maurizio Grasselli](#)

- Title**: Nonlocal Navier-Stokes-Cahn-Hilliard systems with unmatched densities**
- Abstract: H. Abels, H. Garcke, and G. Grün (2012) proposed a diffuse interface model to describe liquid-liquid phase separation in incompressible binary fluids of different densities. This model consists of the Navier-Stokes system which is nonlinearly coupled with an advective Cahn-Hilliard equation. In this talk, however, instead of taking the usual free energy functional, we consider a nonlocal version. Therefore, the resulting Cahn-Hilliard equation is a second-order (spatially) nonlocal equation. This system was already analyzed by S. Frigeri (2016) who established the existence of a global weak solution. I intend to present some further results in dimension two which I have obtained jointly with C.G. Gal, A. Giorgini, and A. Poiatti (2023). These results are mainly concerned with strong solutions, uniqueness, and convergence to a single equilibrium. Some related open issues will also be discussed.

MAY 7, 2024

[Jincheng Yang](#)

- Title**: Layer separation and energy dissipation for 3D NSE at high Reynolds number**
- Abstract: In this talk, we consider the 3D incompressible Navier-Stokes equation in a bounded domain, with a canonical example of Poiseuille flow in mind. We provide an unconditional upper bound for the boundary layer separation and energy dissipation of Leray–Hopf weak solutions, uniformly in high Reynolds numbers. We estimate layer separation by measuring the energy norm of the discrepancy between a (turbulent) low-viscosity Leray–Hopf solution and a fixed (laminar) regular Euler solution with similar initial conditions and body force. This is accomplished by a new nonlinear boundary vorticity estimate.

MAY 14, 2024

Slim Tayachi, SITE Research Center

- Title**: Asymptotic behavior for inhomogeneous nonlinear Schrödinger equations**
- Abstract: This talk will present some results on global existence and scattering for the damped and undamped inhomogeneous nonlinear Schrödinger equation. We will first discuss the local well-posedness theory that enables us to reach the critical power of the nonlinearity and to unify results for the homogenous and the inhomogeneous cases. In the absence of damping, our focus will be on discussing global existence for oscillating initial data and scattering theory in a weighted space for a new range of the nonlinearity. We give a new scattering criterion taking into account the potential (or the inhomogeneous term). For general potentials, we highlight the impact of the behavior, at the origin and at infinity, of the inhomogeneous term on the allowed range of the nonlinearity. In particular, if the potential is regular, we show that the more it decreases, the more the range of allowed nonlinearities giving scattering is wider. For instance, if the decay of the potential at infinity exceeds a critical value, then the scattering for small data occurs for the whole range of the power of the nonlinearity. For the damped case, we establish lower and upper bound estimates of the lifespan. We provide an explicit dependence of how large the damping is in terms of the initial data leading to

global existence and scattering. This fact appears to be unknown in the literature, even for the homogeneous case. These are joint works with L. Aloui and S. Jbari (University of Tunis El Manar).

MAY 21, 2024

Francisc Bozgan, SITE Research Center

- Title**: Blow-Up Dynamics for the L2 critical of the 2D Zakharov-Kuznetsov equation**
- Abstract: We investigate the blow-up dynamics for the L2 critical two-dimensional Zakharov-Kuznetsov equation with initial data u_0 slightly exceeding the mass of the soliton solution. Employing methodologies analogous to those used in the study of the gKdV equation, we categorize the solution behaviors into three outcomes: global persistence, finite-time blow-up, or divergence from the soliton's vicinity.

MAY 23, 2024

Fanze Kong, University of British Columbia

- Title**: Critical mass phenomena and blow-up behaviors of ground states to Mean-field Games systems with decreasing cost**
- Abstract: In this talk, we focus on the stationary Mean-field Games (MFG) system with the local and decreasing coupling in the whole space, where players tend to aggregate. Concerning the local coupling subject to the mass critical exponent, we discuss the existence and asymptotic profiles of ground states to the MFG system. First of all, we show the existence of minimizers to the potential-free MFG system associated with the optimal Gagliardo-Nirenberg type inequality. Then, under some mild assumptions on the unbounded potential, we show that there exists a critical mass M* such that the MFG system admits a least energy solution if and only if the total mass of population density M is less than M*. Moreover, it is proved that as M approaches M*, up to a rescaling, the energy minimizers tend to concentrate on minima of the potential, in which the profiles are captured by the potential-free MFG system. In particular, given the precise asymptotic expansions of the potential, we establish the refined blow-up behaviors of ground states. The maximal regularities of solutions to Hamilton-Jacobi equations are crucial ingredients in our arguments.

JUNE 20, 2024

Paola Goatin, Inria Centre at Univerisé Côte d'Azur

- Title**: Traffic flow models for new mobility paradigms**
- Abstract: Modern technologies have the potential to dramatically change the mobility paradigms in the next future. Indeed, they allow for extended data collection and Vehicle-to-Vehicle or Vehicle-to-Infrastructure communication, possibly providing new tools to control and optimize traffic flows. In this setting, mathematical models play an important role, allowing the design and evaluation of new management approaches. In this talk, I will present applications to road traffic regulation by means of Connected Automated Vehicles or dynamic routing. All results are based on the development of specific macroscopic models accounting for the interacting dynamics of the different classes of users. Numerical experiments show that controlling a small fraction of users is in general sufficient to consistently improve the global system performance.

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JUNE 25, 2024

Rafael Granero Belinchón, University of Cantabria

- Title**: On the dynamics of viscous internal waves: The Stokes-transport system**
- Abstract: In a couple of recent works (in collaboration with F. Gancedo and E. Salguero), we have studied the motion of an internal wave between two incompressible and viscous fluids in the Stokes regime. For such a free boundary problem we were able to establish a number of well-posedness results and also some qualitative properties of the solutions. In this talk we will briefly review some of these results.