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Present-bias, quasi-hyperbolic discounting, and fixed costs ☆

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ABSTRACT

In this paper we elicit preferences for money–time pairs via experimental techniques. We estimate a general specification of discounting that nests exponential and hyperbolic discounting, as well as various forms of *present bias*, including quasi-hyperbolic discounting. We find that discount rates are high and decline with both delay and amount, as most of the previous literature. We also find clear evidence for present bias. When identifying the form of the present bias, little evidence for quasi-hyperbolic discounting is found. The data strongly favor instead a specification with a small present bias in the form of a fixed cost, of the order of $4 on average across subjects. With such a fixed cost the curvature of discounting is imprecisely estimated and both exponential and hyperbolic discounting cannot be rejected for several subjects.

1. Introduction

A vast literature in experimental psychology has studied time preferences by eliciting preferences over various alternative rewards obtained at different times, that is, over reward–time pairs.[1] Representations of such time preferences include a specification of discounting. This literature has documented various behavioral regularities with regards to discounting. The most important of such regularities is called "reversal of preferences." It occurs, for example, when a subject prefers $10 now rather than $12 in a day, but he/she prefers $12 in a year plus a day rather than $10 in a year. Reversals of preferences are not consistent with exponential discounting. Psychologists (e.g., Herrnstein, 1961; de Villiers and Herrnstein, 1976; Ainslie and Herrnstein, 1981; see also Ainslie, 1992, 2001) and also behavioral economists (e.g., Elster, 1979; Laibson, 1997; Loewenstein and Prelec, 1992; O'Donoghue and Rabin, 1999) have noted that reversals of preferences are instead consistent with a rate of time preference which declines with time. Various specifications of discounting with this property, notably *hyperbolic discounting* and *quasi-hyperbolic discounting* have been suggested.[2]

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[1] The axiomatic study of representation of preferences in reward–time space has been pioneered by Fishburn and Rubinstein (1982). For a recent general representation theorem, see Ok and Masatlioglu (2003).

[2] Of course, a declining rate of time preference is not the only possible explanation of these "anomalies" of time preference. Rubinstein (2003) shows how reversals of preferences might be induced by a specific form of procedural rationality; Read (2001) shows instead how reversals can be the outcome of discounting over a given period of time being greater than the sum of discounting over subdivisions of the period, a form of subadditivity. Also, most of the documented anomalies are consistent in principle with preferences over sets of actions, under standard rationality assumptions; see Gul and Pesendorfer (2001). Finally, various specifications of psychological models of strategic interactions between multiple selves at each time period may rationalize such anomalies; see e.g., Thaler and Shefrin (1981), Bernheim and Rangel (2004), Benhabib and Bisin (2005).

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Another important regularity pertaining to time preferences, referred to as the *magnitude effect*, is the observation that discounting declines with the amount to be discounted. It has been quite extensively documented, starting with Thaler (1981), and then notably e.g., by Benzion et al. (1989) and Green et al. (1997).

Time preferences allowing for reversals and/or for a magnitude effect might require a fundamental paradigm change in the study of dynamic choice in economic theory. For instance, specifications of discounting allowing for declining rate of time preference lack time-consistency (see Strotz, 1956 for an early discussion of such issues).[3] In this case dynamic choice problems are not determined by the solution of a simple maximization problem, and require the postulation of an equilibrium notion.[4] Specification allowing for discount rates declining with amounts also are problematic for dynamic choice theory, as they are ruled out by the axioms underlying the available representation results of time preferences in reward–time space.[5]

While experimental psychologists have collected an impressive amount of data on time preference in support of declining discount rates with respect to time and amounts, some of this data is not without problems. Often experiments have been conducted with hypothetical rewards, or with "points" redeemable at the end of the experiment (thereby eliminating any rationale for time preference); the design of the experiments is seldom immune to issues of strategic manipulability, or of framing effects. Most importantly, rarely have the data been analyzed with proper econometric instruments.[6]

Formal statistical procedures are necessary to identify behavioral regularities with noisy data; this is important in the context of experimental data on discounting which, the literature has repeatedly observed, are particularly noisy (Kirby and Herrnstein, 1995; Kirby, 1997; see also Frederick et al., 2002).

In view of some of these problematic aspects of the available evidence on preference reversal and on the magnitude effect and in view of their important theoretical implications, in this paper we study time preferences with a novel methodology and a new specification of discounting. We shall show that our analysis and results are consistent with, and perhaps even have the ability of systematizing, the wealth of data collected on discounting in the literature. We first elicit preferences for pairs of alternative monetary rewards obtained at different times, money–time pairs, via experimental techniques. We then estimate a general specification of the induced discount curves. Our specification nests exponential and hyperbolic discounting, as well as various forms of *present bias* in discounting, in which subjects associate a discrete cost to any future, as opposed to present, monetary reward. A classic example of present bias is quasi-hyperbolic discounting, as adopted e.g., by Laibson (1997) and O'Donoghue and Rabin (1999), in which the cost associated with future rewards is variable, that is, is proportional to the amount of the reward. The specification of present bias we estimate in this paper allows also for fixed costs (possibly in addition to variable costs) associated to future rewards. Such a fixed cost component of present bias is important in principle because it induces a magnitude effect by implying unit costs associated to future rewards which diminish with the amount of the reward.[7]

We find that exponential discounting is rejected by the data. Discount rates vary with the time delay and with the amount of the reward. A specification with present bias fits the data best. Regarding the form of the present bias, the data do not seem to support a quasi-hyperbolic specification. In fact the fixed cost component of present bias is significantly different from 0 (and estimated as of the order of $4 on average across agents), while a positive variable cost is not present for most of our subjects and, when it is, it is very small. In other words, a simple fixed costs associated to any future reward, independently of its amount, is sufficient to capture both the dependence of discounting on time delay and on amount as it is observed in our data. An additional parameter, the quasi-hyperbolic (variable cost) component of the present bias, which independently controls the dependence of discounting on time delay, adds little to the fit of our data.

The curvature of discounting (exponential vs. hyperbolic), in the fixed cost specification, is not precisely estimated with our data, and is consistent for several of the subjects with both exponential as well as hyperbolic discounting.

The paper is laid out as follows. In Section 2 we present a brief review of the main specifications of discount curves in the literature. In Section 3 we present our experimental design and econometric procedure. The results are in Section 4, while Section 5 offers some conclusions.

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[3] Most experiments in the literature, however, elicit preferences over reward–time pairs. This procedure, avoiding the elicitation of preferences over consumption streams, provides no direct evidence for time inconsistent behavior; see Rubinstein (2001) for a discussion and Casari (2005) for an exception.

[4] Furthermore, when preferences are time inconsistent, the scope for normative statements and welfare analysis is of course greatly limited.

[5] While a quasi-hyperbolic specification of discounting is consistent at least with the most general axiomatic theory of time preferences (Ok and Masatlioglu, 2003, Example 2), the fixed cost specification we'll study in this paper which induces the magnitude effect is not (Ok and Masatlioglu, 2003, Example 9).

[6] By design, reversal of preference experiments cannot identify discount rates. In their comprehensive survey of the empirical literature on time preference, Frederick et al. (2002) tabulate a total of 41 experimental studies on time preferences for which an estimate of the annualized discount rate is either directly reported or easily computed from the reported statistics (see Table 1 at the end of their paper). Of these studies, only 10 report results on time preference based on data obtained through experiments conducted with real, as opposed to hypothetical, money; we discuss these studies in some detail in Section 4.

[7] A theoretical model generating a fixed costs specification of present bias is studied by Benhabib and Bisin (2005); see especially Benhabib and Bisin (2007) for a focused discussion of this issue.

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2. Discounting

Consider a subject's time preferences over monetary rewards and times pairs (y,t), to be interpreted as y dollars obtained at time t (equivalently, t periods from now). Let subjects' preferences over monetary rewards be linear.[8] The discount function $D(y,t)$ is defined so that the subject is indifferent between the pair (y,t) and the pair $(yD(y,t),0)$. We say that "the value of y at time (equivalently, with delay) t is $yD(y,t)$." Notice that we allow the discount factor $D(y,t)$ to depend on amount to be discounted, y, as well as the delay t.

$D(y,t)$ represents *exponential discounting* if

$$D(y,t)=\exp\{-rt\},\quad r>0 \tag{1}$$

and it represents *hyperbolic discounting* if

$$D(y,t)=\frac{1}{1+rt},\quad r>0 \tag{2}$$

Both exponential and hyperbolic discounting are independent of the amount to be discounted, y. But, in contrast to exponential discounting, preferences that display hyperbolic discounting induce declining subjective discount rates. More precisely, let the subjective discount rate of y be defined in general as $|\frac{\frac{\partial}{\partial t}D(y,t)}{D(y,t)}|$. The subjective interest rate associated with exponential discounting is then r, a constant, while the subjective interest rate associated with hyperbolic discounting is $\frac{r}{1+rt}$, and hence it is declining in the delay t.

Another specification of discounting which has been extensively studied in psychology and economics is *quasi-hyperbolic discounting*:

$$D(y,t)=\begin{cases}1 & \text{if } t=0\\ \alpha\exp\{-rt\} & \text{if } t>0\end{cases} \tag{3}$$

This specification displays a *present bias* for $\alpha<1$, that is, a discontinuous discounting of all rewards obtained at any future time, $t>0$. Note that the present bias in quasi-hyperbolic discounting is larger the smaller the parameter α, and takes the form (the interpretation) of a *variable cost* associated to future payoffs: any reward y is valued at most at $\alpha y=y-(1-\alpha)y$ when received in the future. The cost $(1-\alpha)y$ is variable in the sense that it increases linearly with the amount y.[9] This interpretation induces us to consider another possible specification of discounting, in which the present bias is represented by a *fixed cost* b rather than by a variable cost. In this case, any reward y is valued at most $y-b$ when received in the future. Formally, the discount curve is[10,11]:

$$D(y,t)=\begin{cases}1 & \text{if } t=0\\ \exp\{-rt\}-\frac{b}{y} & \text{if } t>0\end{cases} \tag{6}$$

These specifications of discounting are flexible enough to account for the regularities documented by the experimental literature on time preferences cited in the Introduction. Consider first of all "reversal of preferences." It occurs if a subject prefers a reward–time pair $(x,0)$ to (y,t) where $y>x$, as well as the pair $(y,t+\tau)$ to (x,τ), for some delay $\tau>0$. It is straightforward to verify that both hyperbolic discounting (specification (2)), and present bias (either with variable or fixed costs, specifications (3) and (6), respectively) are consistent with reversal of preferences. A "magnitude effect," i.e. the strength of the preference of $(x,0)$ over (y,t) is greater than the strength of preference of $(x+k,0)$ over $(y+k,t)$, however, is only consistent with specification (6) for which present bias is represented by a fixed cost. In this case, in fact, the discount factor $D(y,t)$ declines with the amount y.

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8 This assumption is discussed and partly relaxed in Section 5.3.

9 This formulation has been introduced by Phelps and Pollak (1968) to study interpersonal discounting, and has been adopted by behavioral economists to study intrapersonal discounting and preferred over the hyperbolic specification; see Laibson (1997), O'Donoghue and Rabin (1999).

10 More generally, the specification with fixed costs is

$$yD(y,t)=\begin{cases}y & \text{if } t=0\\ \exp\{-rt\}(y+be^{-mt})-b & \text{if } t>0\end{cases} \tag{4}$$

where the parameter m controls the distribution of costs over time. This specification reduces to (6) for $m=\infty$, where the cost b is not discounted. For $m=0$, where the cost is discounted, it reduces instead to:

$$yD(y,t)=\begin{cases}y & \text{if } t=0\\ y\exp\{-rt\}-(1-\exp\{-rt\})b & \text{if } t>0\end{cases} \tag{5}$$

11 Note that with a fixed cost, small amounts offered in the future, say a dollar, may be worth a negative amount today because of the psychological disutility of the transaction, so individuals would refuse to entertain such transactions. This is consistent with the magnitude effect on discounting since in our experiments we find the average fixed cost to be about $4.

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3. Elicitation of time preferences and discounting

Under the assumption that "reversals" are due to hyperbolic (or quasi-hyperbolic) discounting, the delay at which a reversal occurs contains some information about r in Eq. (2) or (3). Nonetheless, individual discount rates cannot be properly estimated with the data generated by preference reversal experiments. It is then hard to evaluate results statistically, e.g., to formally distinguish consistent empirical regularities from the effects of noise. In this paper we instead elicit time preferences and discounting directly via experimental techniques.

3.1. Experimental design

A total of 27 inexperienced subjects were recruited from the undergraduate population of New York University to engage in our experiment. Each subject did the experiment on two different days. During both sessions they were asked a set of questions whose aim was to elicit their discount rates for money using a version of the Becker–DeGroot–Marschak mechanism to be described later. The two sessions differed by the types of questions asked which we will also describe in more detail later in this section.[12]

The paper-and-pencil experiment took place at the Center for Experimental Social Science (CESS) at New York University. When subjects arrived in the lab they were seated at tables and separated from each other for the duration of the experiment. They were then given a set of instructions which were read out loud to them after they had a chance to read them individually. Each experimental session lasted about 1/2 an hour and subjects earned on average approximately $28 in each session. (These are undiscounted amounts; some of these earnings were paid to the subjects at later times.)

In Session 1 subjects were asked to reply to a set of 30 questions of the following form:

What amount of money, \$$x$, if paid to you today would make you indifferent to \$$y$ paid to you in t days.

We refer to this framing of the time elicitation question as [*Q-present*]. In the actual experiment y and t were specified so a typical question [*Q-present*] would be:

What amount of money, \$$x$, if paid to you today would make you indifferent to $10 paid to you in 1 month.

The amounts \$$y$ varied from $10, to $20, $30, $50, to $100 while t varied from 3 days to 1 week, 2 weeks, 1 month, 3 months, to 6 months. For each amount we asked therefore six questions involving the six different time frames; with five different amounts this totalled 30 questions.

To give the subjects an incentive to answer these questions truthfully we used a version of the Becker–DeGroot–Marschak mechanism to determine what amount would be paid to the subjects and when. This mechanism was employed on one of the thirty questions drawn at random. For example, say that at the end of the experiment we drew the question that asks the subject what amount, \$$x$, he would require today to make him indifferent between that amount today and $50 to be paid to him in one month. Assume he says $x = \$40$. In that case we would draw a random number uniformly from the interval $[0, \$50]$. If the number drawn was less than the $40 indifference amount stated by the subject then he or she would have to wait for one month at which time the $50 would be paid. If the number drawn was greater than the $40 indifference amount stated, the number drawn would be paid immediately. To insure waiting the subject need only state an indifference amount of $50 while to insure receiving some money today the subjects need only state an indifference amount of $0. Under risk neutrality it is a dominant strategy to report the true indifference amount in this procedure and this fact was explained to the subjects. We had no doubt that the subjects understood the incentive properties of the mechanism.[13]

In this experimental session, therefore, subjects received either money today or money in the future. If money today were to be paid subjects were handed a check. If future money were to be paid subjects were asked to supply their mailing address and were told that on the day promised a check would arrive at their campus mailboxes with the promised amount. This was done to minimize any possible transaction costs involved in waiting, i.e., when paid in the future no subject had to travel to the lab to pick up his money etc., it would just arrive at their door.

Session 2 was identical to Session 1 except the question asked adopted a reversed framing. Here we asked:

What amount of money, \$$y$, would make you indifferent between \$$x$ today and \$$y$ t days from now ($y^{Large} = w$).

We refer to this framing of the time elicitation question as [*Q-future*].

Note that in this question instead of asking what amount of money a subject would need today to make him/her indifferent to a given amount of money in the future, we ask him/her what amount of money if given at a specific time in the future would make him/her indifferent to a fixed amount today. When answering question of the [*Q-future*] format

[12] A set of experimental instructions are provided at the end of the paper. We modified the notation to make it consistent with the one used in the paper.

[13] For some skeptical evidence on the workings of the Becker–DeGroot–Marschak mechanism, see e.g., Ariely et al. (2004), Harrison (1992), Plott and Zeiler (2005).

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subjects were not allowed to state an amount larger than some pre-determined quantity, y^{Large}. Here y^{Large} varied from $10, to $20, to $30, to $50 and finally to $100 while the time horizons varied from 3 days to 1 week, 1 month, to 3 months and finally to 6 months, just as in Session 1. The $$x$ amounts given them were derived from the answers to questions received in Session 1 and were the minimum of the amounts stated there.[14]

In summation, we employed a within-subject design using 27 subjects and two treatments where the treatments varied according to the type of question asked.

3.2. Econometric specification

Consider the following two-parameter class of functions:

$$d(t;\theta,r) = \big(1-(1-\theta)rt\big)^{\frac{1}{1-\theta}} \tag{7}$$

The parameter θ measures the curvature of the function $d(t;\theta,r)$, which is hyperbolic for $\theta = 2$ and exponential for $\theta = 1$[15]:

$$d(t;\theta=1,r) = \exp\{-rt\}, \qquad d(t;\theta=2,r) = \frac{1}{1+rt}$$

More generally, we study the following four-parameter specification of discounting which nests exponential and hyperbolic curvatures as well as quasi-hyperbolic (variable cost) and fixed cost specification of present bias:

$$D(y,t;\theta,r,\alpha,b) = \begin{cases} 1 & \text{if } t=0 \\ \alpha d(t;\theta,r) - \frac{b}{y} & \text{if } t>0 \end{cases} \tag{8}$$

A quasi-hyperbolic component of present bias is present if $\alpha < 1$, while a fixed cost component is present if $b > 0$.

Our data consists, for each subject $h = 1, 2, \ldots,$ of answers to a battery of questions such as [*Q-present*] and [*Q-future*]. For each agent h and each of the two different formulations of the questions, therefore, we have a series of 30 observations in the form of pairs (y,t), $(x,0)$ which leave agent h indifferent. Assuming risk neutrality, (y,t), $(x,0)$ are then such that, for agent h,

$$x = yD(y,t) \tag{9}$$

For each agent h and each of the two different formulations of the questions, we estimate (9) under various special cases of the general discount specification (8). For each of the 27 agents, therefore we estimate 2 equations (one for each battery of questions), with 30 data points each.

Our econometric procedure is as follows. Consider for example the estimate of (9) under the most general specification of discounting, (8), with data obtained from [*Q-future*]. Let $y^h(x,t)$ denote the answer given by subject h to question [*Q-future*] for amount x and delay $t > 0$. We assume that $y^h(x,t)$, the data, is generated by

$$x = y^h(x,t)D\big(y^h(x,t),t;\theta^h,r^h,\alpha^h,b^h\big)\varepsilon^h(x,t)$$

where the error $\varepsilon^h(x,t)$ is i.i.d. with respect to subjects h and questions (x,t). Moreover, we assume $\varepsilon^h(x,t)$ is lognormally distributed.[16] Since we estimate individual discount curves, independently across subjects, we allow the parameters of the discount curve, $(\theta^h, r^h, \alpha^h, b^h)$, to be indexed by the subject. Estimates are by *non-linear least squares*.

4. Comparison with data from earlier studies

In our data the annual discount rate is of the order of 472%. Furthermore, a common feature of our data is that the annual discount rate is declining with time and with amount. In other words, our data display *present bias* as well as a *magnitude effect*, as documented in most experimental studies of discounting. This is immediately apparent from Fig. 1 where we report the implied annual discount rates for 3 typical subjects in our experiment. The time preference data from our experiments is generally consistent with data from previous studies. For instance, Thaler (1981), for corresponding monetary rewards and times, reports a yearly discount rate, obtained for amounts of the order of $15 and time-delays of about month, of the order of 345%.

[14] For example, one question was, "What amount of money, $$y$, would make you indifferent between $14 today and $$y$ 3 days from now? ($y^{Large} = 20$)." Here the subject was forced to give an answer of $20 or less. The introduction of the upper bound y^{Large} amounts to censoring of the data. It has been chosen so that it would never bind for the time preferences reported in the answers for Session 1. For instance, we had not observed an amount less than $14 being asked for $20 in 3 days. However, some censoring did nonetheless appear to occur in our data; see the discussion in footnote 23.

[15] Eq. (7) is a generalized hyperbola. It is mentioned by Loewenstein and Prelec (1992) and Prelec (2004) (which refers to an unpublished 1989 paper), in the context of time preferences, but never adopted in empirical work to nest the hyperbolic and exponential specifications. For instance, Kirby (1997), as well as Kirby and Marakovic (1995), Myerson and Green (1995), Rachlin et al. (1991), and others also estimate exponential and hyperbolic specifications of discounting. But, they limit themselves to estimating exponential and hyperbolic discounting independently and then to comparing R^2's. Kirby (1997), for instance, concludes that the hyperbolic discount specification fits better that the exponential, in the sense that the R^2 is higher, for 19 out of 23 participants.

[16] A specification with additive rather than multiplicative errors does not produce significative differences in the estimates.

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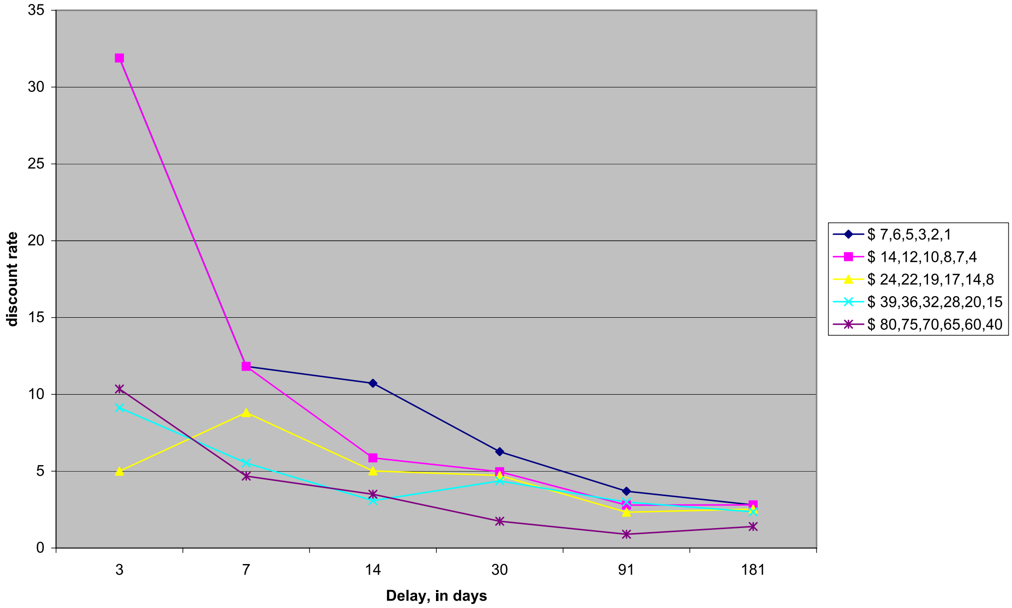
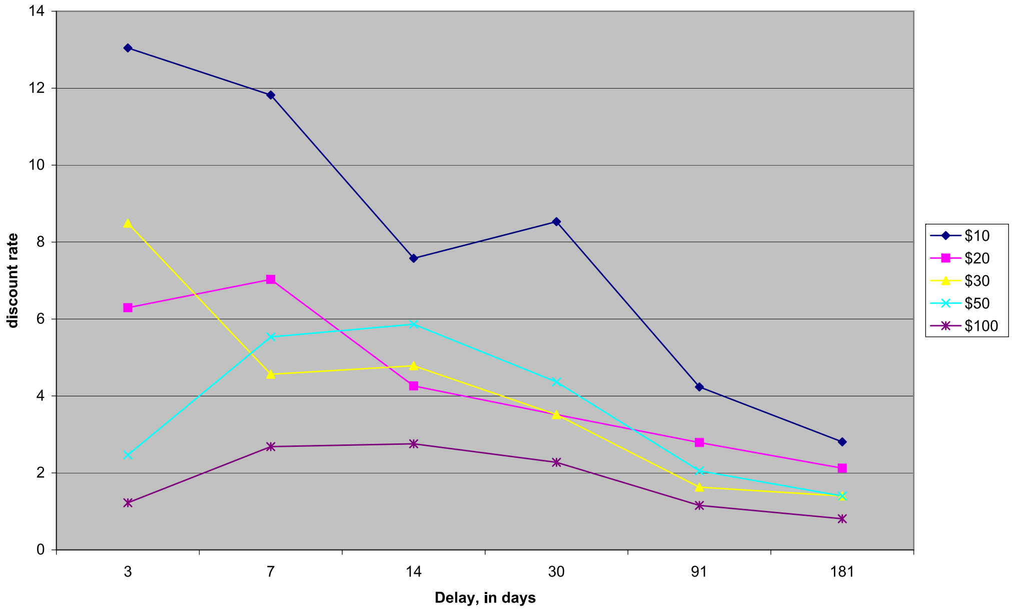


(a)



(b)

Fig. 1. Annual discount rate, subject 4. (a) [*Q-future*], (b) [*Q-present*].

More specifically, we can compare our data with that of previous studies which report direct estimates of either hyperbolic of exponential discount functions by fitting the same functional specification for discounting and comparing estimates.[17] Kirby et al. (1999) in the hyperbolic specification of discounting, Eq. (2), estimate r[18] equal to 4.745 on average in annual terms.[19] If we fit a hyperbolic discounting specification through our data, we obtain estimates of r which are smaller than Kirby et al.'s (1999), 2.21 on average. But the distribution of Kirby–Petry–Bickel's estimates across agents

[17] We restrict to the studies tabulated by Frederick et al. (2002) which conducted experiments with real, as opposed to hypothetical, money.

[18] The parameter r, in a hyperbolic discount function, is obviously not to be interpreted literally as a discount rate. However note that any sum discounted hyperbolically with $r = 1$ is worth $1/2$ when received with a 1 year delay; if instead r were equal to 4, any sum received with a 1 year delay would be worth just $1/5$.

[19] The estimate refer to the sub-sample of controls in a sample which include also heroin addicts, with significantly higher discount rates, $r = 9.125$.

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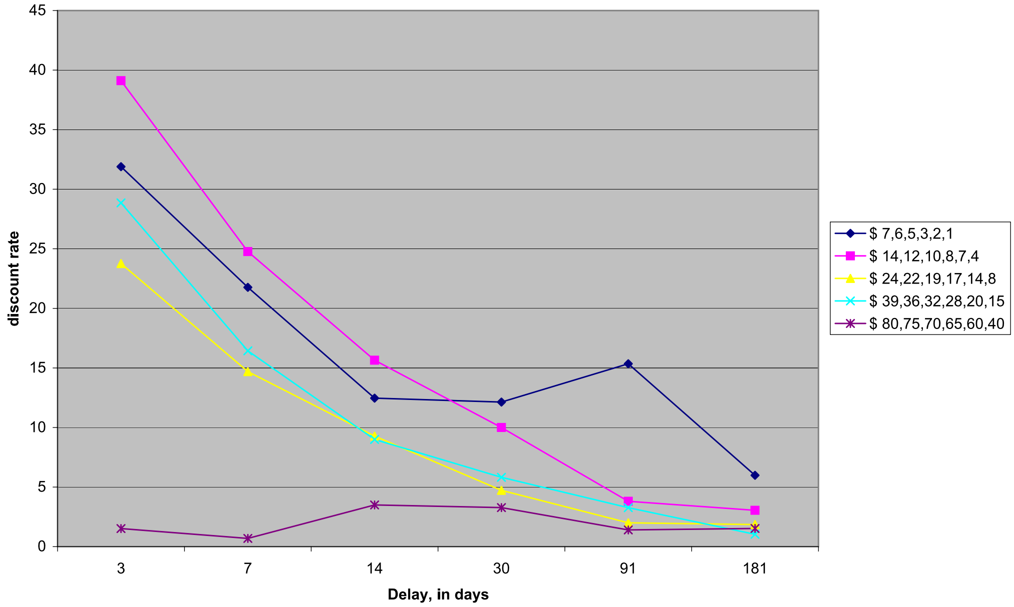
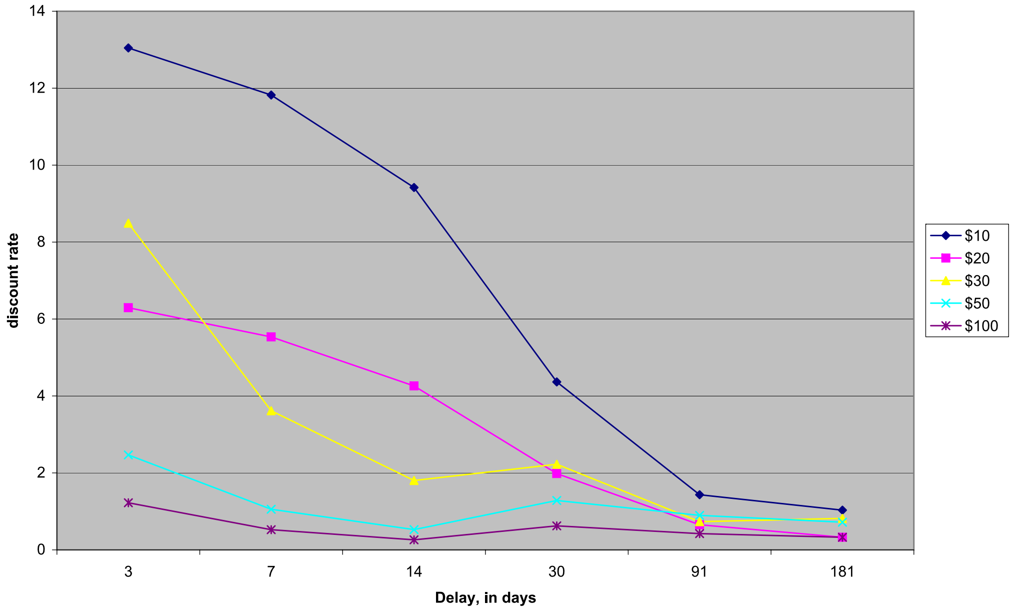


(c)



(d)

Fig. 1. Annual discount rate, subject 10. (c) [*Q-future*], (d) [*Q-present*].

(Fig. 2 in their paper) displays a substantial mass of agents with r between 1 and 2, as is the case for our data. In fact, substantial differences in estimates of hyperbolic discount functions, for different experimental treatments in single studies as well as across studies, are to be expected if the annualized discount rates are obtained from different amounts and time delays. Consistently, for the same specification, the estimates of Kirby and Marakovic (1996) are much closer to ours: r is on average equal to 1.825 in the treatment with amounts comparable to ours, of the order of $70–85.[20] Finally, while we do not pursue here a detailed comparative analysis of our data with that obtained from experiments with hypothetical rewards, we note however that the two methodologies tend to produce generally consistent data in the case of time preference experiments (see Kirby and Marakovic, 1995 for a detailed comparison).

[20] Both Kirby (1997) and Kirby and Marakovic (1995) estimate instead a much higher discount with smaller amounts and shorter delays, as high as $r = 18$. In one treatment Kirby (1997) estimates $r = 32.12$ on average; but this estimate is an outlier and it appears to be due to the elicitation method adopted.

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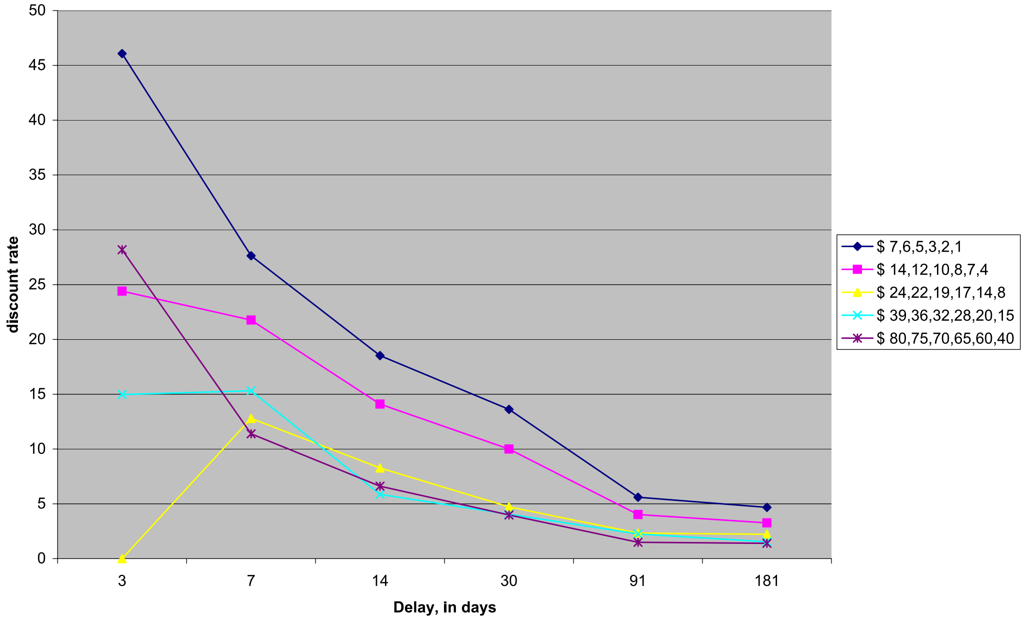

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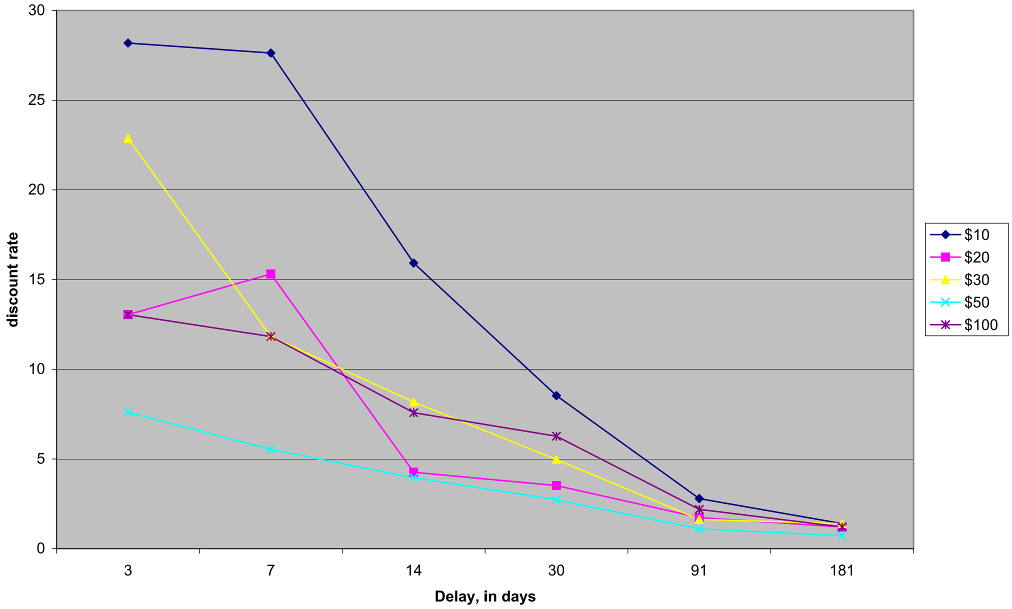

(f)

Fig. 1. Annual discount rate, subject 27. (e) [*Q-future*], (f) [*Q-present*].

While our data, as we documented, is consistent with previous data from laboratory experiment, it is less in line with field data. In particular, our data implies substantially higher discounts than e.g., the data in Coller and Williams (1999), who estimate discount rates, from an exponential discount function specification, of the order of 28–41% across subjects.[21] This difference can be explained however by the much larger amounts (over $500) used by Coller and Williams (1999) and by various framing effects; for instance, subjects are not allowed to choose their indifference amounts, as in our design, but

[21] Some of the treatments in Coller and Williams (1999) elicit time preferences only through delayed alternatives, a design referred to as *front-end delay*. Because this design eliminates present-bias, it is not surprising that lower discounting is documented in this case, between 22 and 28% across subjects. We discuss front-end delay experiments in more detail in the last section.

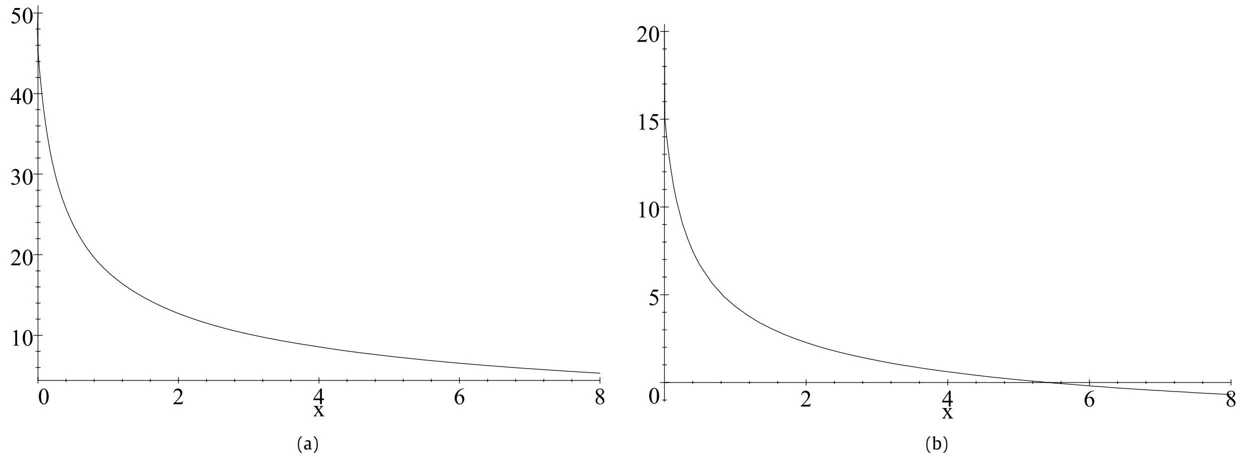

Fig. 2. Discount curve. Subject 10, with median θ (specification (8) – [*Q-future*]). Parameters values: $\theta = 3.4$, $r = 2.37$, $b = 4.68$. (a) Evaluated at $y = 50$. $D(x) = \begin{cases} 50 & \text{if } x = 0 \\ \frac{50}{(1+5.69x)^{0.42}} - 4.68 & \text{if } x > 0 \end{cases}$. (b) Evaluated at $y = 20$. $D(x) = \begin{cases} 20 & \text{if } x = 0 \\ \frac{20}{(1+5.69x)^{0.42}} - 4.68 & \text{if } x > 0 \end{cases}$.

rather choose between pre-specified options which in turn are designed to reflect relatively small implied interested rates (below 100% annually).[22]

5. Results

In this section we report the results of our econometric study of the time preference data we have elicited in the experimental lab. All results in Section 5.1 refer to data from [*Q-future*]. We analyze the data from [*Q-present*] in the context of framing in Section 5.2.

5.1. Discount curves and present bias

We start our estimations restricting specification (8) so that $\alpha = 1$, $b = 0$. This is to statistically document declining subjective discount rates. Note that this specification, corresponding to Eq. (7) includes exponential discounting for $\theta = 1$ and hyperbolic discounting for $\theta = 2$. Table 1 shows the estimated values of θ, all but one of which are above 1.

Estimation results for this restricted specification are collected in Table 1.[23] For 23 of the 27 agents the exponential specification, $\theta = 1$, is rejected by the data. Nonetheless the estimates do not appear particularly appealing, essentially because the point estimates for r are extremely high, in 17 cases of the order of thousands of percentage points.[24] Even though when discounting is not exponential r does not represent the discount rate, it is still the case that one dollar with no delay is worth more than 5 in a year, for more than half of the agents in the sample at the point estimates. The point estimates of r are in only 1 case less than 100%.[25]

We turn then to a second specification. In this formulation discounting is allowed to be hyperbolic, as in the previous specification. But we include a fixed cost component to the preference for the present, that is we estimate (8) under the restriction that $\alpha = 1$. Results are reported in Table 2. The fixed cost b is estimated to be significantly different than 0 for all the subjects (except subject 19 for which the estimate does not converge). It is, on average, about \$4 (with a minimum value of \$.31 and a maximum value of \$5.38). The estimates of r are also more reasonable when we include fixed costs. For instance, the point estimates of r are less than 100% for 15 subjects and less than 30% for 9 subjects. The estimates of θ, the curvature of the discounting function at delays t different than zero, are however very imprecise when a fixed cost

[22] Estimated discount rates from natural and field experiments also vary greatly. Hausman (1979) estimates exponential discount rates between air conditioner buyers which are very sensitive e.g., to income, and average 25%. Gately (1980) finds discount rates varying from 45 to 300% between refrigerator buyers. More recently, Warner and Pleeter (2001) estimate discount rates above 20% from a military downsizing program.

[23] Seven of the subjects consistently chose the amounts y^{Large} for the full set of 30 questions, that is, they favor present payments over the future ones possibly more than we allow to represent with their choices. This is the case even though y^{Large} has been chosen so that it would never bind for the time preferences reported in the answers for Session 1. The choices of these seven agents may therefore reflect a framing effect that affects the estimates of the parameters of their discounting curves for Session 2. However, we fail to identify overall large framing effects between the results of Session 1 and Session 2; see Section 5.2 below. Moreover, we have estimated a logistic specification to account formally for censoring, without notable differences in the results.

[24] The numbers for r in Table 1 should be multiplied by 100 to convert them to percentages. For example $r = 0.5$ would correspond to 50%.

[25] This is by no means only a property of our data. Similar discount rates have been generally imputed from experimental data; see Frederick et al. (2002, Table 1.1).

Table 1
Question [*Q-future*]. Specification with no present bias.

Person	θ	se(θ)	r	se(r)
1	3.88	0.61	33.64	14.50
2	−2.00	4.40	0.50	0.37
3	2.85	1.52	4.16	2.69
4	2.26	0.48	4.99	1.34
5	1.96	0.83	2.88	1.05
6	2.65	0.41	7.24	1.78
7	4.67	3.55	2.66	2.29
8	4.25	0.86	14.22	6.19
9	3.88	0.61	33.64	14.50
10	3.15	1.16	18.36	15.41
11	16.62	5.27	93.63	125.12
12	4.14	0.84	16.42	7.45
13	3.88	0.61	33.64	14.50
14	3.55	0.92	13.06	6.93
15	2.46	0.56	6.37	2.12
16	2.33	0.91	4.08	1.80
17	1.88	2.72	1.20	0.73
18	3.88	0.61	33.64	14.50
19	4.61	2.42	10.45	10.93
20	4.86	1.20	13.95	7.47
21	3.92	0.61	33.81	14.44
22	2.62	0.49	7.03	2.03
23	3.88	0.61	33.64	14.50
24	6.53	1.77	66.44	60.55
25	3.62	0.60	26.63	11.19
26	3.88	0.61	33.64	14.50
27	3.91	0.91	20.25	11.14
Median	3.88		14.22	
Mean	3.86 (2.95)		21.11 (21.2)	

Note: Standard deviations of means across agent in this and subsequent tables reported in parentheses.

Table 2
Question [*Q-future*]. Specification with fixed cost.

Person	θ	se(θ)	r	se(r)	b	se(b)
1	6.27	0.75	14.27	4.30	5.38	0.96
2	−22.74	16.10	0.08	0.04	0.44	0.15
3	−4.94	3.72	0.28	0.11	2.38	0.44
4	−1.02	1.14	0.73	0.22	4.46	0.83
5	−1.39	1.30	0.56	0.14	2.26	0.47
6	1.81	0.52	2.07	0.42	3.51	0.69
7	−6.10	24.75	0.12	0.09	1.33	0.23
8	2.69	1.24	1.53	0.48	4.81	0.71
9	6.27	0.75	14.27	4.30	5.38	0.96
10	3.45	1.31	2.37	0.93	4.68	1.00
11	−0.90	1.41	0.66	0.20	2.17	0.65
12	5.46	1.11	3.00	0.85	4.51	0.59
13	6.27	0.75	14.27	4.30	5.38	0.96
14	4.77	0.69	3.74	0.79	3.40	0.51
15	−1.24	2.22	0.62	0.30	5.31	1.14
16	−0.13	0.83	0.78	0.15	2.45	0.42
17	−4.95	5.83	0.23	0.10	0.52	0.31
18	6.27	0.75	14.27	4.30	5.38	0.96
19						
20	30.78	18.44	309.07	979.21	3.30	1.43
21	6.28	0.75	14.44	4.37	5.32	0.96
22	1.92	0.62	2.01	0.46	3.78	0.74
23	6.27	0.75	14.27	4.30	5.38	0.96
24	41.34	23.05	9370.38	45832.09	4.16	1.07
25	4.35	0.55	6.49	1.56	5.37	0.83
26	6.27	0.75	14.27	4.30	5.38	0.96
27	8.47	1.47	9.80	3.92	4.42	0.89
Median	3.8		2.68		4.44	
Mean	4.06 (11.46)		377.48 (1835.71)		3.88 (1.58)	
Median w/o agents 20, 24	3.07		2.22		4.48	
Mean w/o agents 20, 24	1.39 (6.62)		5.63 (6.07)		3.89 (1.64)	

Note: Blank entries in this and subsequent tables correspond to cases where estimates did not converge.

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Table 3
Question [*Q-future*]. Specification with fixed cost and quasi-hyperbolic component.

Person	θ	se(θ)	r	se(r)	b	se(b)	α	se(α)
1	1.24	0.80	1.36	0.35	4.67	0.77	1.21	0.03
2	−8.13	9.66	0.13	0.05	0.73	0.17	0.98	0.01
3	−0.92	2.19	0.48	0.15	3.51	0.47	0.94	0.01
4	−1.28	1.38	0.68	0.24	4.22	1.02	1.01	0.03
5	−0.62	1.31	0.66	0.18	2.62	0.58	0.98	0.02
6	1.70	0.64	1.97	0.52	3.42	0.80	1.01	0.03
7	21.74	24.80	0.41	0.48	1.68	0.28	0.98	0.01
8	−0.51	1.28	0.68	0.18	3.41	0.60	1.09	0.02
9	1.24	0.80	1.36	0.35	4.67	0.77	1.21	0.03
10	4.39	1.70	3.43	2.28	4.94	1.16	0.97	0.05
11	−3.21	2.39	0.42	0.17	1.27	0.69	1.06	0.03
12	5.60	1.52	3.15	1.55	4.53	0.66	1.00	0.03
13	1.24	0.80	1.36	0.35	4.67	0.77	1.21	0.03
14	6.21	0.78	8.90	4.12	3.87	0.52	0.92	0.04
15	−0.06	2.06	0.83	0.43	6.17	1.52	0.96	0.04
16	0.21	0.92	0.85	0.19	2.66	0.52	0.99	0.02
17	−2.66	5.84	0.27	0.13	0.72	0.39	0.99	0.01
18	1.24	0.80	1.36	0.35	4.67	0.77	1.21	0.03
19	−11.59	19.25	0.15	0.21	6.17	1.43	0.93	0.03
20	−48.75		0.04	0.00	3.04	1.28	1.20	0.04
21	1.21	0.80	1.35	0.35	4.62	0.77	1.21	0.03
22	4.57	0.75	1.74	0.50	3.54	0.85	1.02	0.03
23	1.24	0.80	1.36	0.35	4.67	0.77	1.21	0.03
24	41.33	23.02	17944.30		4.16	1.07	0.97	0.13
25	3.95	0.79	5.11	2.20	5.29	0.89	1.02	0.05
26	1.24	0.80	1.36	0.35	4.67	0.77	1.21	0.03
27	−0.02	1.44	0.74	0.23	3.79	0.70	1.16	0.03
Median	1.24		1.35		4.16			
Mean	0.763 (13.73)		666.09 (3453.09)		3.79 (1.44)			
Median w/o agent 24	1.22		1.1		4.04		1.01	
Mean w/o agent 24	−0.797 (11.3)		1.54 (1.88)		3.77 (1.47)		1.06 (0.109)	

is added. For 5 subjects the hypothesis of exponential discounting is not rejected at the 95% confidence interval, and for 1 of these neither is the hypothesis of hyperbolic discounting. For 9 subjects the confidence interval of θ lies in the region smaller than 2, while for 16 subjects it is in the region greater than 1. In other words, the data seem to be consistent with a present bias which we postulated in the form of a fixed cost, but do not have much power to distinguish exponential from hyperbolic discounting.

The final specification we estimate is (8), unrestrictedly. In this case both a fixed cost and a quasi-hyperbolic (variable cost) component of present bias are allowed for. Note that we can identify fixed versus variable costs components since we have data for different amounts x.[26] Results are reported in Table 3. For 16 subjects we estimate a value of α greater than one. For 5 of the remaining 11 subjects, α is estimated significantly smaller than 1 (that is, we reject $\alpha \geqslant 1$). Finally, the point estimates of α are never smaller than .92, and the lower bound of the 95% interval never lower than .84.[27] The estimates we obtain for the fixed cost b in this specification are not much different from those obtained in the previous specification, and still around \$4 on average across agents. The estimates for the curvature of discounting, θ, are still quite imprecise but seem now to favor, at the margin, the exponential specification: for 15 subjects the hypothesis of exponential discounting is not rejected at the 95% confidence interval, and for 11 of these neither is the hypothesis of hyperbolic discounting; but for all 15 subjects exponential discounting is not rejected at the 97% level, while for 12 of these the hypothesis of hyperbolic discounting is rejected at the 97%.

In summary, we interpret our result to imply that the data favor a specification of present bias containing a fixed cost component, and not a significant quasi-hyperbolic (variable cost) component. As an illustration, in Fig. 2 we report the estimated discounting curve for the subject which, in the specification favored by our data, (8) with $\alpha = 1$, has median value of θ.

As we argued in Section 4, our data does not appear to differ in a significant manner from the other experimental data in the literature. As a consequence, our results do not appear to be the outcome of some specific or peculiar characteristic of the data. We would rather expect a present bias in the form of a fixed cost component to fit generally well the available experimental data on discounting. As discussed in the Introduction, this specification apparently contradicts most axiomatic

[26] We have also estimated the different functional forms for present bias discussed in footnote 10. In particular we have estimated Eq. (4). Results (not reported) are not significantly different from those obtained with specification (8) illustrated in Table 3. The estimates of m are large enough so that (8) appears to represent a good approximation, with one less parameter.

[27] The imputed value of α obtained in a consumption-saving model by Laibson et al. (2004) is lower, around .6.

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Table 4
Question [*Q-present*]. Specification with no present bias.

Person	θ	se(θ)	r	se(r)
1				
2	3.20	10.24	0.10	0.05
3	−80.90	156.93	0.00	0.05
4	3.90	0.98	7.10	2.91
5	6.90	2.49	1.00	0.27
6	7.40	0.53	31.10	6.24
7	−4.40	8.24	0.30	0.19
8	16.80	3.34	16.50	9.11
9	48.70	24.78	9.10	14.05
10	14.60	4.59	7.90	5.54
11	2.70	0.55	6.30	1.90
12	11.50	1.85	20.30	8.86
13	3.00	0.83	12.10	6.61
14	9.10	3.29	3.10	1.69
15	1.30	3.58	0.70	0.35
16	24.00	5.77	18.80	14.00
17	334.40	4.72E+08	0.00	0.00
18	177.70	433.96	2000.00	44794.41
19				
20	7.70	1.13	4.40	1.05
21	−0.60	5.32	0.30	0.16
22	2.70	4.58	0.80	0.51
23	334.40	4.72E+08	0.00	0.00
24	2.00	0.65	6.70	3.10
25	73.20	45.04	1.20	1.48
26	−0.80	1.66	0.60	0.18
27	10.40	2.00	48.90	31.27

treatments of time preferences. Of course strong caution in interpreting our results is necessary, because our estimates are derived from a sample which does not include amounts greater than $100.

Finally in the last rows in Tables 1–3, even though averaging and aggregating across subjects must be interpreted with caution because of preference heterogeneities, we also present the median and mean of our estimates, with the standard deviations of the means across the 27 agents. One set includes all subjects, while a second set excludes two outlier subjects in case of Table 2 and one outlier subjects in case of Table 3. Consistently with our basic results, fixed costs b are always around $4, with small standard deviations across agents. Mean and median returns are lower when fixed costs are included, but at the level of monetary rewards used in our experiment, they remain high, consistent with other estimates in the literature. The parameter for hyperbolicity, θ shows significant variability across agents, though point estimates suggest it is above 1, the level for exponential discounting. Lastly the mean and median of α, the quasi-hyperbolicity (variable cost) parameter in Table 3 is tightly close to 1 as it is in the cases of the individual subjects, indicating the presence of hyperbolic rather than quasi-hyperbolic discounting.

5.2. Framing

Do results depend on how the question is posed? Frederick et al. (2002) survey an extensive literature in experimental psychology documenting framing in discounting experiments.[28] To address the issue of framing, in this paper we estimate the same specifications of discounting with the data obtained from question [*Q-present*], and compare the results with those just discussed, that is, with the estimates with the data from [*Q-future*].

The results are reported in Tables 4–6 for our three specifications of the discounting curves, respectively.[29]

We do find statistical evidence for framing in our data. Consider in fact the cross-subject distribution of the estimates of the parameters of our most general specification, (8), for each of the two question formats, [*Q-future*] and [*Q-present*]. For each of the parameters we have run the Kolmogorov–Smirnov test that the cross-subject distributions are the same across the two frames. For parameters θ, α, and b the p-value of the test is $< .005$, rejecting the hypothesis that the distributions are the same. For r the p-value is instead .153 and we cannot reject the hypothesis.[30] Nonetheless,[31] the

[28] See also Frederick (2003).

[29] Estimates for individual subjects are not reported when the non-linear least square algorithm did not converge. It should also be noted that subjects 1, 9, 16, 18 possibly misunderstood this question (for instance, they claimed to be willing to accept an amount y in the present to avoid waiting 30 days for $15, but to require an amount $y > y$ in the present to avoid waiting 60 days for the same $15; subjects 3, 17, 19 and 23 essentially did not discount.

[30] In the last page of the paper we report for illustration the cumulative cross-subject distributions for b (the dotted line refers to [*Q-future*] and the solid line to [*Q-present*]).

[31] To identify and measure framing effects we also check, parameter by parameter, if the confidence intervals obtained with the data generated by [*Q-future*] and [*Q-present*] overlap. Framing is less evident according to this methodology. Consider for instance the specification which best fits the data,

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Table 5
Question [*Q-present*]. Specification with fixed cost.

Person	θ	se(θ)	r	se(r)	b	se(b)
1						
2	4.06	12.35	0.14	0.06	−0.02	0.16
3						
4	5.58	1.29	2.55	0.76	2.80	0.71
5	4.33	1.57	0.62	0.11	0.22	0.22
6	8.83	0.69	24.47	5.35	1.34	0.61
7	−17.04	24.08	0.09	0.06	0.69	0.37
8	18.31	3.07	11.76	5.76	0.55	0.69
9	161.48	626.71	0.25	1.81	1.99	0.89
10	2.27	4.45	0.40	0.16	1.91	0.44
11	1.39	0.61	1.64	0.34	3.28	0.74
12	16.75	2.43	115.38	72.81	−0.86	0.81
13	1.27	0.71	1.75	0.45	3.31	1.01
14	17.51	6.44	0.95	0.48	1.66	0.41
15	−42.61	60.27	0.05	0.06	1.11	0.49
16	21.57	3.57	2.41	0.83	1.46	0.33
17	8.17		0.00	0.00	0.00	0.00
18	−192.27	164.16	−6.12	21.76	2.14	0.50
19						
20	7.48	1.19	3.36	0.87	0.26	0.57
21	0.12	17.10	0.14	0.09	0.41	0.29
22	4.28	2.70	0.84	0.31	−0.74	0.58
23	29.37		0.00	0.00	0.00	0.00
24	2.28	0.57	2.49	0.52	1.95	0.78
25	57.46	17.25	2.56	2.11	−0.31	0.37
26	−41.53		0.05	0.00	0.99	0.46
27	8.92	1.15	30.11	11.27	−0.78	1.03

Table 6
Question [*Q-present*]. Specification with fixed cost and quasi-hyperbolic component.

Person	θ	se(θ)	r	se(r)	b	se(b)	α	se(α)
1								
2	10.80	12.29	0.20	0.09	0.18	0.20	0.99	0.01
3								
4	8.01	1.27	7.72	4.47	3.64	0.69	0.91	0.04
5	4.52	1.87	0.63	0.14	0.24	0.27	1.00	0.01
6								
7	−3.45	17.53	0.15	0.11	1.15	0.48	0.98	0.01
8								
9	163.32	865.87	0.26	4.36	2.00	0.96	1.00	0.05
10	7.37	4.04	0.78	0.38	2.69	0.51	0.97	0.01
11	1.87	0.67	1.99	0.49	4.24	0.88	0.96	0.02
12	1.94	2.90	0.68	0.30	−0.89	0.64	1.24	0.02
13	2.47	0.57	3.05	0.75	5.91	0.91	0.89	0.02
14								
15	−12.57	13.95	0.12	0.08	2.09	0.61	0.97	0.01
16	25.68	4.69	8.04	13.09	1.56	0.34	0.97	0.04
17	9.62		0.00	0.00	0.00	0.00	1.00	0.00
18								
19								
20	9.57	1.17	10.40	6.33	0.92	0.52	0.92	0.04
21	11.29	12.75	0.28	0.18	1.02	0.34	0.98	0.01
22	5.38	2.84	1.09	0.52	−0.12	0.68	0.97	0.02
23	9.62		0.00	0.00	0.00	0.00	1.00	0.00
24	3.57	0.48	4.84	1.11	3.61	0.68	0.90	0.02
25								
26	−16.31	10.87	0.12	0.07	2.05	0.69	0.97	0.01
27								

qualitative implications we have derived from the analysis of the data regarding [*Q-future*] continue to apply to the data regarding [*Q-present*]. In particular, we find that i) exponential discounting is rejected by the data, ii) the data do not seem

Eq. (8) with fixed costs but no quasi-hyperbolic component. It is described by 3 parameters. In this case we obtain 3 overlapping confidence intervals for 7 out of 24 subjects, 2 overlapping intervals for 9 subjects, 1 overlapping interval for 7 subjects, and finally distinct estimates for only 1 subject.

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to support a quasi-hyperbolic specification while the fixed cost component of present bias is significantly different from 0 (but smaller than for *Q-future*, of the order of $2 on average across agents).

5.3. Robustness: Some follow-up experiments

In this section we discuss some limitations of our experimental design and of the specification of discounting we estimate. We report on various attempts we have pursued to address the issue of robustness and to overcome such limitations.

Front end delay. All the questions used in our experiment to elicit time preferences over money–time pairs involve a reward at time 0, that is, an immediate reward. In *Q-forward* the reward at time 0 is given as part of the question to the subject, while in *Q-present* we look for a reward at time 0 as the subject's answer. This design is useful to generate data with power to identify the form of present bias in discounting, that is, e.g., to distinguish between fixed and variable costs components of present bias. Unfortunately the cost of this design is that the data thus generated have less power to identify the form of discounting at any time $t > 0$, that is, e.g., to distinguish between the hyperbolic and the exponential specification. In this respect we have noted in Section 5 that our estimates of θ, the curvature of the discounting curve, are in fact somewhat imprecise.

A series of time preference experiments, e.g., Coller and Williams (1999) and Coller et al. (2003) have been designed with a complementary intent: only questions comparing an amount y at time $t > 0$ with another amount y at time $t > 0$ are asked (this design is said to involve "front end delay," in the sense that no reward are ever obtained without some minimal delay). Naturally, when the front end delay design is adopted the form of present bias is not identified but the curvature of discounting curve is estimated more precisely. It is noteworthy that in these experiments the exponential discounting specification is not rejected; see also Harrison et al. (2002) and the brief survey in Harrison and Lau (2005).

Concavity of preferences over rewards. Most of the experimental literature (not only with regards to time preferences) assumes risk neutrality for monetary rewards on account of the fact that small rewards are involved. We nevertheless need to deal in some detail with the issue of concavity of preferences since it could represent an alternative explanation for the magnitude effect. A smaller discount rate for larger rewards (the magnitude effect), when utility over rewards is assumed linear, could simply spuriously capture the concavity of utility over rewards (larger rewards valued less than proportionally so). While the confounding effects of concavity and the magnitude effect might be impossible to identify in general absent data on choice over lotteries, we have repeated our econometric analysis allowing for Constant Relative Risk Aversion preferences over rewards; that is, we have estimated, for each agent and for both [*Q-present*] and [*Q-future*], equations of the form

$$(x)^\beta = (y)^\beta D(y,t)$$

for the same specification of discounting, (8).[32]

We find for most subjects h estimates of β close to 1 and, most importantly, estimates of the fixed cost b significantly different from 0; see Tables 7 and 8, where we only report the estimates for the specifications with and without fixed costs. We conclude that while possibly important,[33] risk aversion does not appear to explain in a determinant manner the magnitude effect.

Fungibility of money. Most of the experimental literature on time preferences adopts monetary rewards.[34] The underlying assumption of all time preferences experiments eliciting preferences on money–time pairs is that the subject's preference ordering for evaluating monetary reward obtained at time t is independent of discounting, that is, that the reward is "consumed" immediately when received at time t (or at least that the subjects thinks of the reward as to be "consumed" upon receipt). Money is fungible, however; it can be easily stored and spent in the future. If monetary rewards are stored, the subject's preference ordering for evaluating rewards at time t is not independent of discounting. If amounts obtained at time t are only in part "consumed" at t, it turns out, our identification and estimation procedures i) might underestimate the quasi-hyperbolic (variable cost) component of present bias (that is, our estimates of α might be biased upward), and ii) it might produce a spurious magnitude effect (that is, our estimates of the fixed cost component of present bias, b, might be biased upward).

To clearly illustrate this point, consider the following example. Time t is discrete; but one period is a short amount of time (e.g., 3 days, the minimal delay in the our experiment). Suppose each agent receiving a monetary reward y at t "consumes" a fraction γ of it immediately at t and a fraction $1-\gamma$ at time $t+1$ (in the near future). Suppose the agents discounts rewards by $D(y,t)$. In this case, the subject is indifferent between (y,t) and $(x,0)$ if

$$\gamma x + (1-\gamma) x D(x,1) = \gamma y D(y,t) + (1-\gamma) y D(y,t+1) \tag{10}$$

[32] It should be noted that the Becker–DeGroot–Marschak mechanism, which we adopt, is only designed to elicit preferences truthfully under risk neutrality.
[33] For instance, Andersen et al. (2005) find that allowing for risk aversion has sizable effects on the estimates of the instantaneous discount rate r^h.
[34] Notable exceptions are a few field studies, as e.g., Shapiro (2005), which studies food stamp recipients.

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Table 7
Concave preferences; question [Q-*future*]. Specification with no fixed costs.

Person	θ	se(θ)	r	se(r)	β	se(β)
1	1.55	1.20	1.73	0.79	0.94	0.01
2	−5.78	9.82	0.16	0.07	1.00	0.00
3	−0.10	2.87	0.61	0.30	1.00	0.01
4	−0.67	1.42	0.84	0.35	0.98	0.01
5	−0.11	1.45	0.79	0.27	1.00	0.01
6	2.02	0.76	2.55	0.93	0.99	0.01
7	61.38	21.16	795354.08	5257970.00	1.05	0.02
8	−0.03	1.65	0.83	0.33	0.97	0.01
9	1.55	1.20	1.73	0.79	0.94	0.01
10	7.54	0.75	32417.58	53217.81	1.27	0.05
11	−2.43	2.10	0.49	0.19	0.98	0.01
12	9.41	0.80	38781.97	54440.42	1.21	0.03
13	1.55	1.20	1.73	0.79	0.94	0.01
14	7.57	0.51	1292.26	1255.03	1.16	0.03
15	0.71	2.00	1.17	0.73	0.99	0.01
16	0.44	1.13	0.97	0.29	0.99	0.01
17	−2.44	5.42	0.29	0.14	1.00	0.00
18	1.55	1.20	1.73	0.79	0.94	0.01
19	−4.14	7.70	0.34	0.34	1.00	0.01
20	−15.70	37.66	0.12	0.26	0.95	0.01
21	1.54	1.20	1.72	0.78	0.94	0.01
22	1.96	0.88	2.30	0.90	0.99	0.01
23	1.55	1.20	1.73	0.79	0.94	0.01
24	−63.62	0.00	0.03	0.00	0.93	0.01
25	5.83	0.39	6318.12	6490.85	1.26	0.05
26	1.55	1.20	1.73	0.79	0.94	0.01
27	0.34	1.82	0.89	0.40	0.95	0.01

Table 8
Concave preferences; question [Q-*future*]. Specification with fixed costs.

Person	θ	se(θ)	r	se(r)	β	se(β)	b	se(b)
1	1.30	0.83	1.34	0.36	0.96	0.01	5.80	0.76
2	−9.43	10.32	0.12	0.05	1.00	0.00	0.64	0.16
3	−1.11	2.28	0.46	0.15	1.01	0.00	3.16	0.43
4	−1.22	1.37	0.69	0.24	1.00	0.01	4.34	0.94
5	−0.57	1.30	0.67	0.18	1.00	0.00	2.53	0.53
6	1.78	0.63	2.04	0.55	1.00	0.01	3.50	0.74
7	16.50	25.54	0.31	0.34	1.00	0.00	1.55	0.26
8	−0.46	1.30	0.68	0.19	0.98	0.00	3.94	0.58
9	1.30	0.83	1.34	0.36	0.96	0.01	5.80	0.76
10	5.12	1.63	4.95	3.78	1.01	0.01	4.74	1.07
11	−2.93	2.32	0.44	0.17	0.99	0.01	1.64	0.66
12	6.05	1.48	3.74	1.96	1.00	0.01	4.51	0.61
13	1.30	0.83	1.34	0.36	0.96	0.01	5.80	0.76
14	6.08	0.75	8.57	3.93	1.02	0.01	3.41	0.50
15	0.05	2.02	0.86	0.45	1.01	0.01	5.93	1.35
16	0.20	0.92	0.85	0.19	1.00	0.00	2.59	0.47
17	−3.16	6.00	0.26	0.13	1.00	0.00	0.64	0.35
18	1.30	0.83	1.34	0.36	0.96	0.01	5.80	0.76
19	−12.19	20.84	0.15	0.21	1.02	0.01	5.65	1.26
20	−1816635.30	0.00	0.00	0.00	0.96	0.01	4.19	1.31
21	1.28	0.84	1.34	0.36	0.96	0.01	5.76	0.76
22	1.68	0.75	1.81	0.53	1.00	0.01	3.70	0.79
23	1.30	0.83	1.34	0.36	0.96	0.01	5.80	0.76
24	41.70	24.72	146.27	3040.28	0.98	0.10	4.75	3.09
25	4.32	0.77	6.34	3.03	1.00	0.01	5.38	0.84
26	1.30	0.83	1.34	0.36	0.96	0.01	5.80	0.76
27	−0.18	1.39	0.71	0.21	0.97	0.00	4.50	0.63

Suppose discounting is quasi-hyperbolic, $D(y,t) = \begin{cases} 1 & \text{if } t = 0 \\ \alpha e^{-rt} & \text{if } t > 0 \end{cases}$. Then estimating $x = yD(y,t)$ implies that we identify $\alpha e^{-rt} = \frac{x}{y}$. If instead the correct specification is (10), then in fact

$$\frac{\alpha\gamma e^{-rt} + \alpha(1-\gamma)e^{-r(t+1)}}{\gamma + \alpha(1-\gamma)e^{-r}} = \frac{x}{y}$$

Table 9
Non-separable preferences; question [Q-*future*]. Specification with fixed costs.

Person	θ	se(θ)	r	se(r)	b	se(b)	m	se(m)	n	se(n)
1	4.01	0.67	7.32	3.31	3.53	0.68	0.93	0.05	−0.11	0.17
2										
3	2.56	2.30	1.19	0.65	3.81	0.76	1.14	0.04	0.01	0.20
4	0.55	0.53	1.50	0.35	2.96	0.51	0.99	0.04	−0.60	0.07
5										
6	1.92	0.51	3.10	0.89	2.10	0.55	0.99	0.04	−0.41	0.13
7										
8	2.83	0.95	2.12	0.72	2.24	0.51	0.93	0.03	−0.07	0.17
9	4.01	0.67	7.32	3.31	3.53	0.68	0.93	0.05	−0.11	0.17
10	6.07	1.85	9.43	11.14	4.70	0.92	1.07	0.12	−0.50	0.11
11	−7.45	19.47	0.19	0.27	1.56	0.58	0.86	0.04	−0.46	0.15
12	6.03	1.01	17.78	17.34	3.44	0.65	1.16	0.13	−0.27	0.13
13	4.01	0.67	7.32	3.31	3.53	0.68	0.93	0.05	−0.11	0.17
14	5.71	0.87	11.25	7.21	1.37	71.95	1.11	0.08	1.36	71.52
15	1.63	0.81	2.64	1.14	3.32	0.93	1.05	0.07	−0.56	0.13
16	3.63	1.05	4.02	2.08	1.14	2.62	1.13	0.06	0.68	2.14
17										
18	4.01	0.67	7.32	3.31	3.53	0.68	0.93	0.05	−0.11	0.17
19	−14.42	62.67	0.12	0.38	10.09	2.50	1.19	0.10	−0.39	0.17
20										
21	3.90	0.70	6.79	3.11	3.34	0.70	0.91	0.05	−0.10	0.18
22	1.97	0.69	3.01	1.14	0.84	1.53	0.95	0.05	0.18	1.04
23	4.01	0.67	7.32	3.31	3.53	0.68	0.93	0.05	−0.11	0.17
24	−5.70	9.30	0.26	0.27	1.31	24.48	0.79	0.04	1.23	23.77
25	4.00	0.57	8.50	3.59	4.30	0.65	1.01	0.06	−0.22	0.12
26	4.01	0.67	7.32	3.31	3.53	0.68	0.93	0.05	−0.11	0.17
27	3.34	1.28	2.86	1.50	4.21	0.91	0.99	0.05	−0.01	0.22

Since $\frac{\alpha\gamma e^{-rt}+\alpha(1-\gamma)e^{-r(t+1)}}{\gamma+\alpha(1-\gamma)e^{-r}} > \alpha e^{-rt}$ for $\gamma < 1$, α is over-estimated by using $x = yD(y,t)$ rather than (10).

Moreover, it is conceivable that the fraction of a monetary reward y which is immediately "consumed" upon receipt is declining with the amount y. This would be the case, for instance, if the subject's preferences over pairs (y,t) were non-separable on the component of the reward consumed at time t and the component consumed in the future (at time $t+1$ in the example). As an illustration, suppose that an agent receiving an amount y at time t "consumes" $y_o < x$ at t and $y - y_o$ at $t+1$. Suppose that discounting $D(y,t)$ is quasi-hyperbolic and that the utility of pair (y,t) is

$$y_o D(t) + (y - y_o)H(y)D(t+1)$$

where the term $H(y)$ captures the non-separability of preferences. If $H(y)$ is e.g., increasing in y, the higher is the reward y, the higher is at the margin the utility from the component of "consumption" in the future. In this case, the appropriate specification to estimate would have the form

$$x = y\Gamma(y)D(y,t)$$

for some function $\Gamma(y)$, decreasing in y. This specification, in particular, allows for $\frac{x}{y}$ to depend on y even if $b = 0$. Estimating instead $x = yD(y,t)$ (as we have done in Section 5.1) might induce us to spuriously reject the quasi-hyperbolic model if, given t, $\frac{x}{y}$ is not constant in y. In particular, we could be attributing to the magnitude effect (and hence to a positive fixed cost b) the decline in the immediate consumption component y_o, which is instead a consequence of a non-separability factor of preferences, $H(y)$ increasing in y.

We have attempted a thorough analysis of these issues as they directly question our identification procedure. First of all we have in fact estimated the equation:

$$x = y\Gamma(y)D(y,t)$$

for

$$\Gamma(y) = \frac{my}{y+n} \tag{11}$$

and $D(y,t)$ defined by (8). (11) captures in a flexible but parsimonious manner preferences which induce the subject i) to "consume" a monetary reward only in part upon receipt, ii) to "consume" upon receipt a smaller fraction of a monetary rewards the larger the reward itself. Note also that the specification reduces to (8) for $n = 0$. Estimates for $(\theta^h, r^h, m^h, b^h, n^h)$ for each subject h are reported in Table 9. Interestingly, for 15 subjects out of the 22 for which we obtained an estimate, we cannot reject that $n = 0$. Furthermore, estimates of m are very close to 1, that is, $\Gamma(y) = 1$. The conclusion that $\Gamma(y) = 1$ severely undermines the possibility of relevant forms of non-separability in preferences. Nonetheless, it does not imply that

Table 10
Total amount chosen to be received immediately.

Amount earned	10% 1 week	20% 2 weeks	5% 1 week	10% 2 weeks
26	0	0		
17.98	0	0		
14	14	14		
23	0	0		
20.64	0	0		
18	0	0		
23	0	0		
20.75	20.75	20.75		
22	22	22		
21.98	21.98	21.98		
23	3	3		
16.50	0	0		
17	0	0		
25	0	0		
16.70	16.70	16.70		
17.71	0	0		
17.75	0	0		
18	18	18		
18.04	18.04	18.04		
20	20	5		
20	20	20		
15.03	0	0		
42.20	42.20	42.20		
38.20	38.20	38.20		
41	41	41		
47.25	47.25	47.25		
44.20	0	0		
38.70	0	0		
46.20	46.20	0		
33.20	0	0		
44.70	44.70	44.70		
42	42	42		
37.20	37.20	37.20		
46.70	46.70	46.70		
50.20	50.20	50.20		
42.50	42.50	42.50		
38.25	0	0		
47.45	47.45	0		
46.20			46.20	46.20
35.70			35.70	35.70
40.20			40.20	40.20
42.20			42.20	42.20
45.20			45.20	45.20
36.20			36.20	36.20
31			31	31
45			45	45
37.95			37.95	37.95
44.20			0	0
30.20			0	0
48.70			48.70	48.70

$\alpha = 1$. If subjects do indeed "consume" only part of monetary rewards at receipt, as illustrated above, $\frac{\alpha\gamma e^{-rt}+\alpha(1-\gamma)e^{-r(t+1)}}{\gamma+\alpha(1-\gamma)e^{-r}} > \alpha e^{-rt}$, for $\gamma < 1$, and our estimates of α might be biased upward.

It is impossible to identify independently α with our experimental data if γ is unobservable and potentially < 1. To tackle this issue in somewhat general terms we therefore designed a new experiment to obtain some direct evidence on γ. Far from being conclusive, this experiment sheds light on the possibility that monetary rewards obtained by subjects in experiments are not perceived as immediate rewards (that is, are not thought of as to be "consumed" immediately).

In this experiment we arrived in the lab after other investigators had finished performing their experiments and offered their subjects an opportunity to either be paid immediately what they had earned or to postpone a fraction of the payment (to "save" a fraction of the amount earned) and be paid that fraction later with interest. We ran two treatments. In the first treatment subjects indicated which fraction of the amount they earned they were willing to receive in a week at 10% interest as well as in two weeks at 20% interest. For example, a subject choosing a fraction 1/2 of $40 earned in a week at 10% implies that he/she received $20 the day of the experiment plus $22 seven days from the experiment. In the second treatment subjects indicated which fraction of the amount they earned they were willing to receive in a week at 5% interest as well as in two weeks at 10% interest. Once each subject had indicated how much he/she was willing to "save" in each of the two options offered, we flipped a coin to determine whether he/she was going to receive the early (one week) or the late (two weeks) payment option. Those who chose to "save" nothing were paid fully on the spot. Those who chose to save a positive amount filled out self-addressed envelopes and were told that the money would be delivered at their addresses the agreed-upon day. Note that since we used subjects from other experiments the amount they had earned varied considerably from experiment to experiment and from subject to subject.

Results are collected in Table 10. In the 5% in a week and in the 10% in two weeks option no subject chose to receive a fraction of the reward immediately (10 subjects chose to keep the whole amount at the end of the experiment, while 2 subjects chose to receive the whole amount plus interests in the future). In the 10% in a week option only 1 subject out of 38 did in fact choose to receive only a fraction of the reward immediately (21 kept the whole amount and 16 chose to receive the whole amount plus interests in the future). Finally, in the 20% in two weeks option 2 subjects out of 38 did in fact choose to receive only a fraction of the reward immediately (18 kept the whole amount and 18 chose to receive the whole amount plus interests in the future). We conclude that we find no evidence that subjects plan not to "consume" immediately the monetary rewards obtained in the experiments, nor that they do so more for larger rewards.[35]

6. Conclusions

In this paper we designed an experiment to examine the manner in which people discount future monetary payments. Using experimental methods that allow subjects to truthfully state amounts of money that make them indifferent between money offered them today and other amounts in the future, we use the data generated to estimate the subjects' discount factors by using a specification which nests both hyperbolic and exponential discounting. As such, this experiment is one of the few that generates data that is then rigorously estimated econometrically.

Our estimation provides clear experimental evidence against exponential discounting in that it exhibits a present bias. This present bias, however, is in the form of a fixed cost with no quasi-hyperbolic component. Moreover, in the fixed cost specification, the curvature of discounting (exponential vs. hyperbolic) is not precisely estimated with our data, and for several of the subjects, it is consistent with exponential discounting.

These results are qualitatively robust to various changes in the frame used to elicit indifference amounts from our subjects. For example, whether we ask subjects to envision a future amount of money that will make them indifferent to money today, or an amount of money today that will make them indifferent to money tomorrow, we still support a fixed cost version of a present bias. Quantitatively, however, framing does matter as the estimated parameters of our discounting function can change.

Our results also appear robust to specifications where subjects are allowed to be risk averse in the sense that introducing a constant relative risk aversion utility function and estimating its risk coefficient, we find that the coefficient is generally not significantly different from 1.

Caution should be exercised, however, in drawing general conclusions as they require extrapolating outside of our sample. Experiments with relatively large rewards are needed to confirm the fixed cost representation of present bias that we identify in the experimental data.

Supplementary material

The online version of this article contains additional supplementary material.
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[35] Surprisingly, subjects seem to be less willing to postpone receiving the reward when the reward is relatively larger rather than smaller. For instance, in the 10% treatment, with the largest number of subjects, the average reward that is chosen to be obtained at the end of the experiment is $35.66, while the average amount that is chosen to be obtained in the future $25.91.

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