Cultural transmission, socialization and the population dynamics of multiple-trait distributions

Alberto Bisin,* Giorgio Topa[†] and Thierry Verdier[‡]

This paper studies the population dynamics of multiple preference traits in a model of intergenerational cultural transmission. Parents socialize and transmit their preferences to their children with endogenous intensities. Populations concentrated on a single cultural group are in general not stable. There is a unique stable stationary distribution, and it supports two or more cultural groups, in particular those with greater intolerance with respect to others' traits. The larger the heterogeneity of intolerance levels across cultural groups, the smaller the number of traits that are supported in the stable stationary distribution.

Key words   socialization, population dynamics, cultural transmission

JEL classification   D10, C61, Z10

Accepted 18 April 2008

1 Introduction

The view that preferences, norms, and, more generally, cultural attitudes should be considered as endogenous with respect to socioeconomic systems has been extensively motivated in the social sciences.[1] In the present paper we consider a specific model of preference formation, based on intergenerational cultural transmission, introduced by Bisin and Verdier (2000, 2001a). The model builds on analysis of transmission and adoption of cultural traits developed by Cavalli-Sforza and Feldman (1981) and Boyd and Richerson (1985). The model in the present paper adds a parental choice of effort to affect their children's

1 See Duesenberry (1949), Leibenstein (1950), Pollak (1976), Kapteyn, Wansbeek, and Buyze (1980) and Iannaccone (1990).

socialization to their (the parents') preferred cultural trait. More specifically, the population dynamics of the distribution of preferences or cultural traits are determined in the model by a process of parental socialization: traits of children are acquired through an adaptation and imitation process which depends on their parents' socialization actions, and on the cultural and social environment in which children live. Socialization effort is chosen optimally by parents and depends on the cultural environment of the parents and the children. In particular, parental effort depends on the distribution of the population with respect to the relevant trait, which affects the socialization of children through teachers and role models.[2]

The objective of the present paper is to extend the formal analysis of Bisin and Verdier (2000, 2001a), which is limited to binary traits, to the case of multiple traits. Studying binary traits is sufficient for identifying conditions under which a unique stationary distribution of the population exists, is stable, and is heterogeneous. However, studying multiple traits is necessary to better understand the determinants of the dynamics of the distribution of the population and, in particular: (i) which cultural groups will eventually be supported at the stationary distribution (i.e. which groups will not eventually face extinction); (ii) how the distribution of group characteristics and the fragmentation of a cultural trait affect the dynamics of the distribution.[3]

We show that, in our model with multiple-trait populations, there is always a unique locally stable stationary distribution of the population over the states. Also, not necessarily *all* cultural groups are supported at the stable stationary distribution. The distribution of the population might converge to one in which only a subset of the states of the trait is sustained.

However, populations concentrated by a single cultural group are in general not stable. The intuition for this result is as follows. In our cultural transmission environment, when one cultural group comes to dominate the population the incentives for this group to socialize tend to zero, as parents essentially free-ride on the rest of society to provide the desired socialization of their children. Minority groups instead heavily socialize their children directly as society provides little help. As a consequence, the fraction of large majority groups in the population tends to decrease, that of minority groups to increase, and at least two cultural groups tend to be supported in a stationary state of the population dynamics.

When we identify the characteristics of different cultural groups with their intolerance level with respect to the other groups, we find that the groups that are supported at the stationary distribution are always the ones that are more intolerant of others. The intuition for this result is straightforward: intolerant groups have relatively larger incentives to socialize their children to their own cultural trait. As a consequence, intolerant groups will tend to dominate the population dynamics of cultural traits.

---

[2] The model has been applied, for example, to the study of preferences for social status (Bisin and Verdier, 1998) corruption (Hauk and Sáez Marti, 2002), ethnic and religious traits (Bisin, Topa, and Verdier 2004a), development and social capital (Francois, 2002), intergenerational altruism (Jellal and Wolf, 2002), globalization and cultural identities (Olivier, Thoenig, and Verdier, 2005), preferences for cooperation (Bisin, Topa, and Verdier 2004b, Tabellini 2007a,b).

[3] The dynamics of multiple religious traits in the USA has been studied empirically by Bisin, Topa, and Verdier (2004a).

Furthermore, for any $1 < k \leq N$ we derive conditions that guarantee that the unique stable stationary distribution of the population is concentrated on k traits. We show that k is inversely related to a measure of the heterogeneity of the intolerance levels across traits. In the limit, for $N \to \infty$, the distribution with full support over traits is sustained if and only if all traits have symmetric intolerance levels.

2 Model

Consider a population consisting of a large number of individuals whose total mass is normalized to 1. Each individual has exactly one of N possible cultural traits; traits are indexed by $i \in \{1, \ldots N\}$. The N-dimensional vector $q = [q^i]_{i \in \{1, \ldots, N\}}$ represents the distribution of the cultural traits in the population, and satisfies $\sum_{i=1}^{N} q^i = 1$. Let S^N denote the N-dimensional simplex. We have then $q \in S^N$.

Individuals die with hazard rate equal to 1 (this effectively normalizes 1 unit of time to the expected lifetime of a generation). Families are composed of one parent and a child, and, hence, reproduction is asexual.[4] All children are born without defined preferences or cultural traits, and are first exposed to their parent's trait. Vertical socialization to the parent's trait, say i, occurs with probability d^i. If a child from a family with trait i is not vertically socialized, which occurs with probability $1 - d^i$, he or she picks the trait of a role model chosen randomly from his or her parent's population (i.e. the child picks trait i with probability q^i and trait $j = i$ with probability q^j). In other words, oblique transmission operates by random matching within society at large, with intensity measured by q^i.

Let P^{ij} denote the probability that a child from a family with trait i is socialized to trait j. P^{ij} also denotes the fraction of children with type i parents who acquire preferences of type j. The socialization mechanism just introduced is then characterized by the following transition probabilities, for all i, j:

$$P^{ii} = d^i + (1 - d^i)q^i \tag{1}$$

$$P^{ij} = (1 - d^i)q^j. \tag{2}$$

For vertical socialization choices d^i, $i \in \{1, \ldots, N\}$, the dynamical system for the distribution of traits in the population is, in continuous time:[5]

$$\dot{q}^i = q^i \sum_{j=i} q^j (d^i - d^j) \,, \quad \forall i \in \{1, \ldots, N\}. \tag{3}$$

---

[4] In Bisin and Verdier (2000) we study a related cultural transmission model with sexual reproduction and endogenous marriage, but binary traits.

[5] Although in this paper we study the continuous time limit of the population dynamics of cultural transmission, the reader should bear in mind that the discrete time model could in general display complex dynamics not present in continuous time.

The system satisfies $\sum_{i=1}^{N} \dot{q}^i = 0$ and, hence, $\sum_{i=1}^{N} q^i(t) = 1$, for all t, if and only if $\sum_{i=1}^{N} q_0^i = 1$. As a consequence, we can restrict ourselves to the dynamical system that consists of:

$$\dot{q}^i = q^i \sum_{j=i} q^j (d^i - d^j) \ , \ \text{for } i = 1, \ldots, N-1 \tag{4}$$

$$q^N = 1 - \sum_{i=1}^{N-1} q^i \tag{5}$$

and the initial conditions $q_0^i, i \in \{1, \ldots, N\}$, such that $\sum_{i \in \{1, \ldots, N\}} q_0^i = 1$.

We now study a cultural transmission mechanism in which parents take costly actions to socialize their children and, hence, endogenously determine vertical socialization, d^i, for all i.

Let V^{ij} denote the utility to a type i parent of a type j child, $i, j \in \{1, \ldots, N\}$. The expected lifetime utility (abstracting from socialization costs) of a family of type i is then:

$$P^{ii} V^{ii} + \sum_{j=i} P^{ij} V^{ij},$$

where P^{ii} and P^{ij} are the transition probability defined in (1)–(2).

We assume that for all $i, j \in \{1, \ldots, N\}$, with $i = j$, $V^{ii} > V^{ij}$. This assumption can be rationalized as a form of myopic or paternalistic altruism: parents, while altruistic, prefer children to adopt their own cultural trait and, hence, try to socialize them to this trait.[6] The intensity of the parents of type i's preferences for having children with their own cultural trait is measured by $\frac{1}{N-1} \sum_{j=i} \Delta V^{ij}$, where $\Delta V^{ij} = V^{ii} - V^{ij}$. We refer to ΔV^{ij} as to the *intolerance* of agents of cultural group i towards j.

We also assume that socialization is costly. Let $H(d^i)$ denote socialization costs: for any $i \in \{1, \ldots, N\}$: the map $H : [0, 1] \to \Re_+$ is C^2, strictly increasing and strictly convex; moreover, $H(0) = 0$, $\frac{\partial H(0)}{\partial d^i} = 0$.

Parents of type i choose $d^i \in [0, 1]$ to maximize:

$$-H(d^i) + P^{ii} V^{ii} + \sum_{j=i} P^{ij} V^{ij} \ \text{subject to (1)}-\text{(2)}. \tag{6}$$

Under our assumptions, the socialization choice problem satisfies the following necessary and sufficient first-order conditions, for all $i = j$:

$$H'(d^i) = \sum_{j=i} q^j (V^{ii} - V^{ij}) = \sum_{j=i} q^j \Delta V^{ij}. \tag{7}$$

For any i, let $\Delta V^i = [\Delta V^{ij}]_{j=1}^N$. Let $d(q, \Delta V^i)$ denote the solution to (7). It follows that $d(q, \Delta V^i)$ is increasing in each element of ΔV^i: naturally, the more parents prefer having children with their own cultural trait, the larger are their incentives to socialize their children

---

[6] See Bisin and Verdier (2001b) for an evolutionary justification of paternalistic altruism.

to their own trait. The dynamics of the fraction of the population with cultural trait i is then determined by (4)–(5) evaluated at $d^i(q) = d(q, \quad V^i)$. If $N = 2$ the dynamics is:

$$\dot{q}^i = q^i(1 - q^i)(d^i(q^i) - d^j(1 - q^i)), \quad i = j,$$

which is a logistic with an added nonlinear term $(d^i(q^i) - d^j(1 - q^i))$. In this case it is immediate to see that if, as in most of the previous published literature, we assume that vertical transmission is exogenously determined, then d^i and d^j are independent of q^i, and in the limit one cultural group will generically dominate (the group with the higher vertical socialization rate; i.e. group i if $d^i > d^j$). If instead vertical socialization results from the parents' rational effort choice, under our assumption, d^i is decreasing in q^i and the dynamics of cultural traits will robustly look as in Figure 1: a unique stable steady state of the population dynamics, q^*, appears in which both traits are represented, whereas the dominant steady states, $q^i = 0, q^i = 1$, are not stable.

Characterizing the dynamic behavior of the distribution of traits in the population is more complicated in the multiple-traits case ($N > 2$), and we will make extra assumptions. However, even in the general case it is easy to see that any homogeneous population constitutes an unstable stationary state of the dynamics of (4)–(5) evaluated at $d^i = d(q, \quad V^i)$.

Proposition 1 *Under Assumptions 1 and 2, any degenerate distribution, that is, any distribution q such that, for some $i \in \{1, \ldots, N\}, q^i = 1$ (and, hence, $q^j = 0, \forall j = i$), is a locally unstable stationary distribution.*

PROOF: Pick an arbitrary $i \in \{1, \ldots, N\}$. Differentiating (4)–(5) at the stationary state q such that $q^i = 1, q^j = 0, \forall j = i$, gives:

$$\left. \frac{\partial \dot{q}^i}{\partial q^i} \right|_q = -[d(q, \quad V^i) - d(q, \quad V^N)] = (H)^{-1}(\quad V^{N1}) > 0 \tag{8}$$

as $d(q, \quad V^i) = (H)^{-1}(0) = 0$. Also, $\left(\frac{\partial \dot{q}^i}{\partial q^k} \right)_q$

$$= [d(q, \quad V^i) - d(q, \quad V^k) - (d(q, \quad V^i) - d(q, \quad V^N))]$$
$$= (H)^{-1}(\quad V^{N1}) - (H)^{-1}(\quad V^{k1})$$

and

$$\left. \frac{\partial \dot{q}^k}{\partial q^k} \right|_q = [d(q, \quad V^k) - d(q, \quad V^i)] = (H)^{-1}(\quad V^{k1}) > 0 \quad \text{for} \quad k = i, N \tag{9}$$

$$\left. \frac{\partial \dot{q}^k}{\partial q^h} \right|_q = 0 \quad \text{for} \quad k = i, N \quad \text{and} \quad h = N;$$

(8) and (9) then readily imply local instability of q. □

Therefore, degenerate homogeneous distributions of traits are unstable. We will next characterize (Propositions 2, 3) and study the stability properties (Proposition 4) of heterogeneous distributions of preferences for N state trait populations. We shall do this

(a)

$\dot{q}$

$q = 0$ $q = 1$

(b)

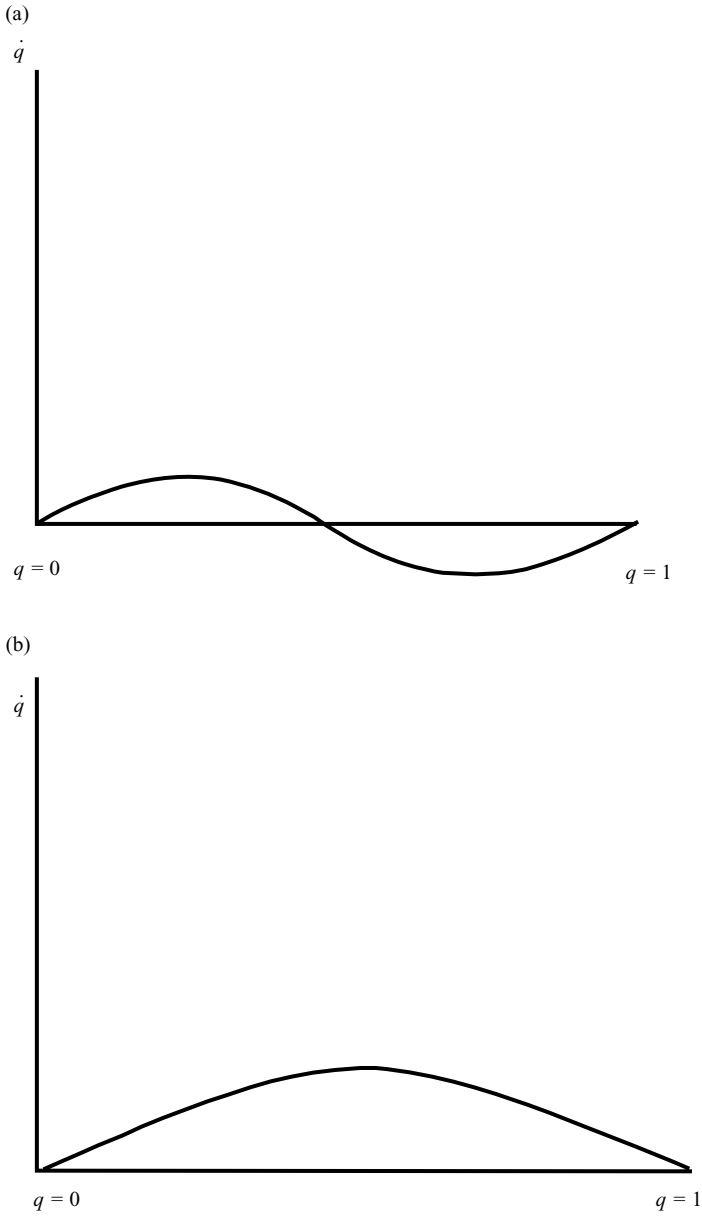$\dot{q}$

$q = 0$ $q = 1$

Figure 1 $N = 2$ traits, where (a) parents choose socialization rates and (b) socialization rates are exogenous.

by simplifying the symmetry assumption on preferences, as characterized by intolerance levels. We shall assume that all groups, while differentially favoring socialization to their own state of the cultural trait, disfavor all the other groups symmetrically. Formally,

Assumption 1 *1.* [**Symmetric Intolerances**] *For any $i \in \{1, \ldots, N\}$: (i) $V^{ij} = V^{ik}, \forall j$, $k = i$ (abusing notation, we let $V^{ij} \equiv V^i$).*

   2. [**Quadratic Costs**] $H(d^i) = \frac{1}{2}(d^i)^2$.
   Under this, condition (7) becomes:

$$d(q^i, \quad V^i) \equiv (1 - q^i) \quad V^i \tag{10}$$

and the dynamic system (4)–(5), evaluated at (10), can be written as:

$$\dot{q}^i = q^i \quad (1 - q^i) \quad V^i - \sum_{j=1}^{N} q^j(1 - q^j) \quad V^j] \quad \text{for all } i. \tag{11}$$

   Let $\mathcal{F}_k$, with $1 \le k \le N$, denote the set of all k-dimensional subsets of $\{1, \ldots, N\}$; $\mathcal{F}_k$ contains $\binom{N}{N-k} \equiv \frac{N!}{(N-k)!}$ different subsets of $\{1, \ldots, N\}$. We say that a stationary distribution supports the k traits $F_k \in \mathcal{F}_k$, and we denote it $q(F_k)$, if it is contained in the appropriate simplex:

$$q(F_k) \in S^{F_k} \equiv \{q \in S^N \mid q^i = 0, \text{ for } i \in F_k\}.$$

   Essentially, without loss of generality, we order the cultural groups so that

$$V^1 >\Delta V^2 > \cdots >\Delta V^N. \tag{12}$$

Ties between intolerance levels are in fact non-generic and can be easily dealt with at some notational cost.

Proposition 2 *Under our assumptions, a stationary distribution that supports F_k exists iff*

$$V^i > [k - 1]G^{F_k}, \quad \forall i \in F_k, \tag{13}$$

where $\frac{1}{G^{F_k}} \equiv \sum_{i \in F_k} \frac{1}{V^i}$. Moreover, a stationary distribution $q(F_k)$, which supports F_k, is defined by:

$$q^i(F_k) = 1 - \frac{k-1}{V^i}G^{F_k} \quad \text{for} \quad i \in F_k \quad \text{and} \quad q^j = 0 \quad \text{for} \quad j \notin (F_k). \tag{14}$$

   G^{F_k} is a measure of the cultural intolerance of the traits belonging to F_k. Note in fact that, for example, if $V^i = V$ for all $i \in F_k$, $G^{F_k} = \frac{V}{k}$. Moreover, $q^i(F_k)$ increases in V^i and decreases in V^j, for $j = i$. Proposition 2 should then be interpreted to imply that a cultural group i is not supported by a stationary state if it is not sufficiently intolerant relative to the other groups.

PROOF: (**If**) All $F_1 \in \mathcal{F}_1$ and all $F_2 \in \mathcal{F}_2$ are supported by a stationary distribution. The fact that all $F_1 \in \mathcal{F}_1$ belong to C is trivial. As for all $F_2 \in \mathcal{F}_2$, we need to show that for any

arbitrary i, $j \in F_2$, and any arbitrary $F_2 \in \mathcal{F}_2$, $V^i > G^{F_k}$. However, $G^{F_2} = \frac{V^i \ V^j}{V^i + \ V^j}$ and $V^i > \frac{V^i \ V^j}{V^i + \ V^j} = G^{F_2}$.

Let

$$C \equiv \bigcup_{k=1}^{N} \{F_k \in \mathcal{F}_k : \quad V^i > [k-1]G^{F_k}, \ \forall i \in F_k\}.$$

Note that $C = \varnothing$ because $\mathcal{F}_1$ and all $\mathcal{F}_2$ belong to C.

For any $F_k \in C$, with $k > 1$, any q that solves (14) belongs to the interior of S^{F_k} and, hence, it satisfies $q^i = 0$ for $i \in F_k$ and

$$(1 - q^i) \ V^i - \sum_{j \in F_k} q^j(1 - q^j) \ V^j = 0 \quad \text{for} \ i \in F_k,$$

which implies: $(1 - q^i) \ V^i = (1 - q^j) \ V^j$, for $i = j \in F_k$; that is, q is a stationary distribution.

(**Only if**) For any arbitrary $F_k \in \mathcal{F}_k$, a stationary state, q, in the interior of S^{F_k} satisfies: $q^i = 0$, for $i \in F_k$, and

$$(1 - q^i) \ V^i - \sum_{j \in F_k} q^j(1 - q^j) \ V^j = 0 \quad \text{for} \ i \in F_k$$

or:

$$\frac{1 - q^i}{\frac{1}{V^i}} = \frac{1 - q^j}{\frac{1}{V^j}} = \frac{\sum_{h \in F_k}(1 - q^h)}{\sum_{h \in F_k} \frac{1}{V^h}} = (k-1)G^{F_k}, \quad \text{for} \ i = j \in F_k$$

and, therefore,

$$q^i = 1 - \frac{k-1}{V^i} G^{F_k}, \quad \text{for} \ i \in F_k; \tag{15}$$

that is, q satisfies (14) for F_k.

Obviously, (15) can be satisfied only if $F_k \in C$. If $F_k \in C$, then no stationary state exists in the interior of S^{F_k}. Finally, a stationary state on the boundary of S^{F_k} is a stationary state in the interior of $S_{F_{k-1}}$ for some F_{k-1} and, hence, satisfies (14) for such F_{k-1}. □

Proposition 3 *Under our assumptions, there exists a $k^* \geq 2$ such that:*

A unique $F_{k^*} \in \mathcal{F}_{k^*}$ is supported by a stationary distribution.
All $F_k \in \mathcal{F}_k$, for $k < k^*$ are supported by a stationary distribution.
No $F_k \in \mathcal{F}_k$, for $k > k^*$ is supported by a stationary distribution.

Furthermore, F_{k^*} contains the traits with highest intolerance; that is, if $V^1 > \Delta V^2 > \cdots > \Delta V^N$, then $F_{k^*} = \{1, \ldots, k^*\}$.

F_{k^*} is the largest subset of cultural groups $\{1, \ldots, N\}$ that is supported by a stationary distribution, and the cultural groups with highest intolerance are supported.

PROOF: Recall that essentially, without loss of generality, we assume

$$V^1 >_\Delta V^2 > \cdots >_\Delta V^N.$$

For any $k \in \{1, \ldots, N\}$, let $\hat{F}_k \equiv \{1, \ldots, k\}$. Recall that $V^1 >_\Delta V^2 > \cdots >_\Delta V^N$, and construct k^* has follows:

$$k^* \equiv \max k \in \{1, \ldots, N\} \text{ such that } V^k > (k-1)G^{\hat{F}_k}.$$

Let $q(\hat{F}_k)$ denote a stationary state in the interior of $S^{\hat{F}_k}$. By construction of $\hat{F}_k$, and using (12), if such a stationary state exists, it is unique. For $k = k^*$, existence follows from Proposition 2 and the construction of k^*. Finally, $\hat{F}_{k^*} = F_{k^*}$, by (12).

By the ordering in (12), $V^k > (k-1)G^{\hat{F}_k}$ implies that,

$$V^k > (k-1)G^{\hat{F}_k}, \quad \text{for any } k < k, \tag{16}$$

$$V^k < (k-1)G^{\hat{F}_k}, \quad \text{for any } k > k. \tag{17}$$

As a consequence: for $k > k^*$, no stationary state exists in the interior of $S^{\hat{F}_k}$ (from (16)); whereas, for any $k \le \hat{k}$, there exists a stationary state q_k in the interior of $S^{\hat{F}_k}$ (from (17)).

Proposition 4 *Under our assumption, the stationary distribution $q(F_{k^*})$, which supports F_{k^*}, is locally stable. Moreover, any stationary distribution $q(F_k)$, which supports F_k, is locally unstable for $k < k^*$.*

PROOF: See the Appendix.

The population dynamics in the case of three state locally stable traits ($k^* = N = 3$) is illustrated in Figure 2.

We obtain the following simple corollary.

Corollary *If*

$$\sum_{i=1}^{N} \frac{1}{V^i} > \frac{N-1}{Min_i\{V^i\}},$$

there is a unique stationary state in the interior of S^N:

$$q^i = 1 - \frac{N-1}{V^i} \left(\sum_{i=1}^{N} \frac{1}{V^i} \right)^{-1}, \quad \forall i \in \{1, \ldots, N\}.$$

Moreover, this stationary state is locally stable.

Note that (18) is stricter for larger N (in the limit, for $N \to \infty$, it requires symmetric preferences for children of their own type across cultural groups: V^i independent of i). This Corollary then identifies symmetry of the parents' preferences for children as a factor that facilitates the stability of heterogeneous stationary distributions of traits in the population.
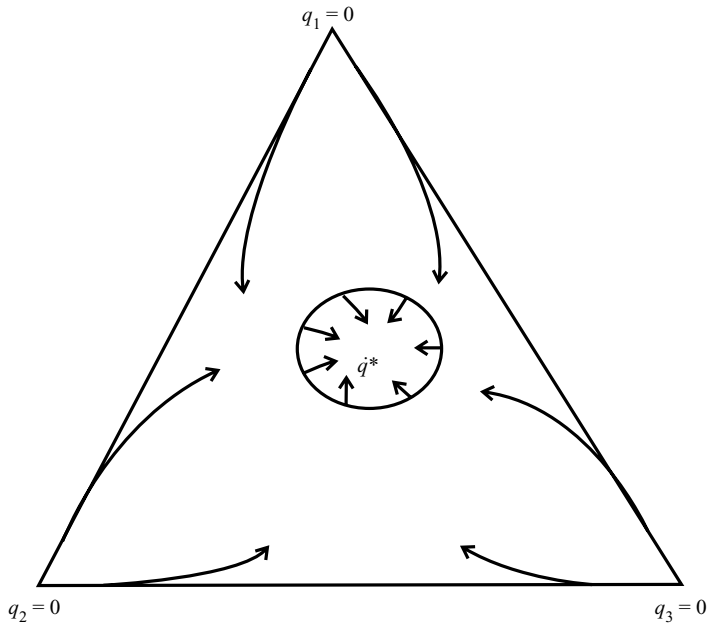
Figure 2 The dynamics of three trait populations with a stable stationary state q^* in S^3.
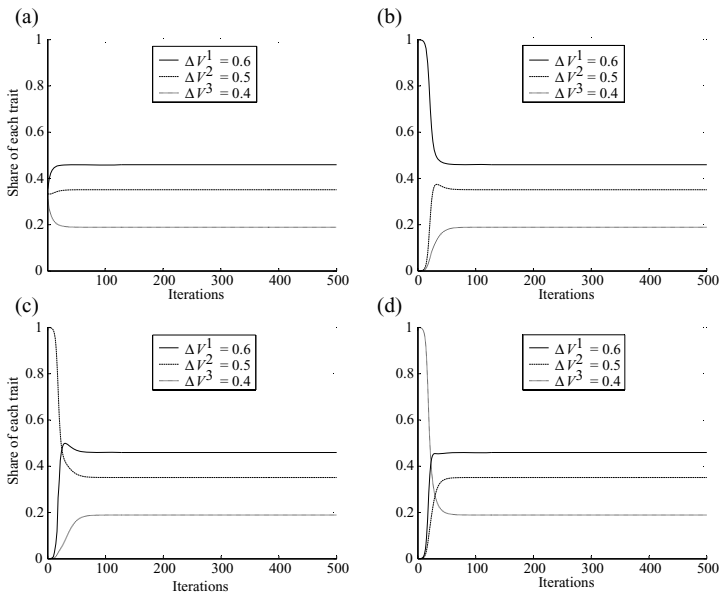

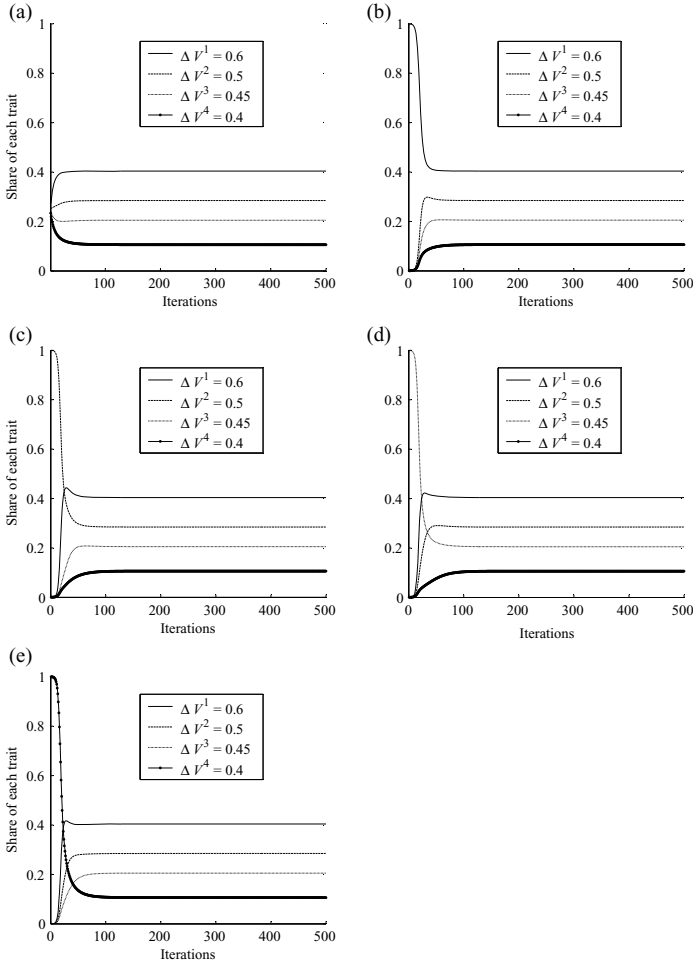**Figure 3** Simulations with $N = 3$; and V^i such that (13) is satisfied.

Figure 4 Simulations with $N = 4$; ΔV^i such that $k^* = 4$.

3 More on the stability properties

Because our theoretical analysis of population dynamics only produces local stability results, we proceed to simulate the dynamical system, with the objective of gaining a better understanding of the global stability properties of $q(F_{k^*})$ under our assumptions. Starting with $N = 3$, we simulate the process choosing values for ΔV^i that satisfy condition (13), $\forall\, i \in F_N$. The simulations are performed in discrete time for 500 iterations, which are more than sufficient for reaching the stationary distribution (see Figure 3). As initial conditions, we choose a set A_0 of points in the interior of (but close to) the simplex S^N, starting with the vertices $(1 - 2\varepsilon, \varepsilon, \varepsilon)$, $(\varepsilon, 1 - 2\varepsilon, \varepsilon)$, $(\varepsilon, \varepsilon, 1 - 2\varepsilon)$ and moving along the segments that join these points.[7]

---

[7] We use $\varepsilon = 0.001$ and a step size equal to 0.001 in performing this grid search.
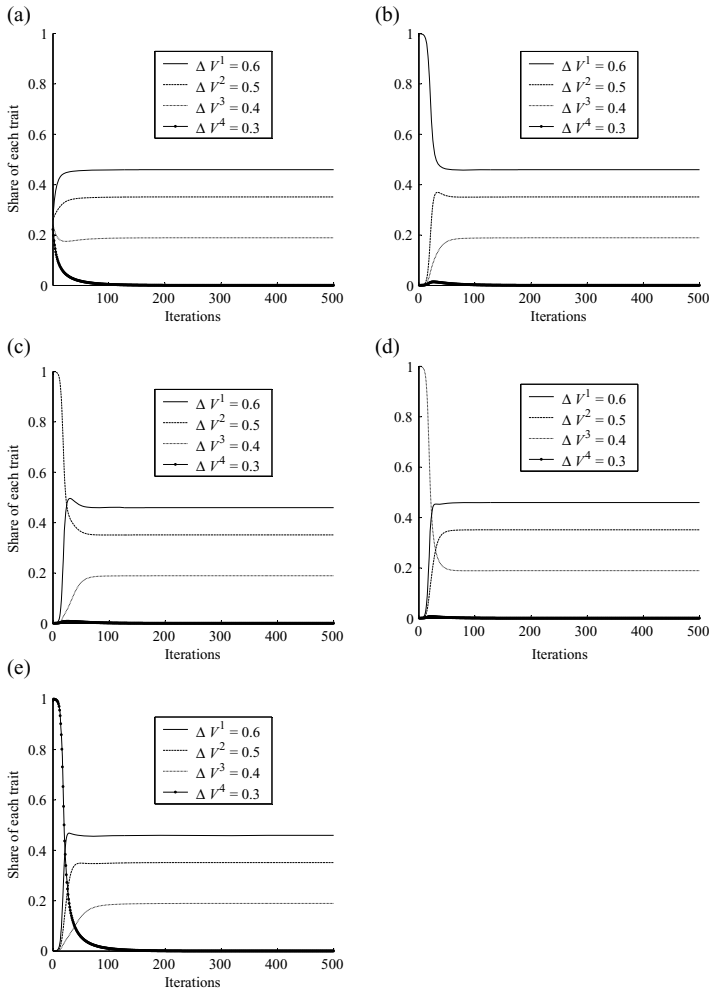
Figure 5 Simulations with $N = 4$; ΔV^i such that $k^* = 3$.

The result of the simulations is that for every possible initial condition in this set, the process converges to the stationary distribution $q(F_{k^*})$, defined in (14). A few typical simulation runs are reported in Figure 3, using different initial conditions. Essentially, in the simulations, we check that the vector field of the system is inward pointing on the boundary of A_0, and, moreover, that the dynamical system does not converge to a limit cycle from any initial condition on the boundary of A_0. This is a strong indication that the basin of attraction of $q(F_{k^*})$ is in fact the whole interior of the simplex S^N, and, therefore, that the stationary distribution $q(F_{k^*})$ is indeed globally stable.

The same results hold with an $N = 4$ state traits population. In this case, we study both the case in which the values for ΔV^i are such that: (i) condition (13) is satisfied for $k^* = 4$ $= N$ (see Figure 4) and (ii) condition (13) is satisfied for $k^* = 3 < N$ (see Figure 5).

4 Conclusion

In this paper we have studied a simple stylized model of intergenerational cultural transmission of multiple traits. Cultural transmission is the result of vertical and oblique transmission, and the intensity of vertical transmission is determined by the rational choice of parents. Many simplifying assumptions have made the formal analysis tractable: for example, asexual reproduction, no horizontal transmission (through peers), quadratic socialization costs, symmetric intolerances, and parental preferences for children of their own trait independent of the cultural environment. All these assumption can be relaxed without changing the nature of the analysis.

Our analysis implies that intolerant groups have relatively larger incentives to socialize their children to their own cultural trait. As a consequence, intolerant groups will tend to dominate the population dynamics of cultural traits. Because this is historically not the case, our analysis must miss some countervailing forces of cultural dynamics. For instance, intolerant groups often require higher participation costs on the part of their members (e.g. to signal purity of intentions and motivations).

Most importantly, we have studied the population dynamics of traits determined by cultural transmission in isolation, with no attempt to analyze the genetic and cultural co-evolution of traits. Several authors have built on the work of Cavalli-Sforza and Feldman (1981) and Boyd and Richerson (1983) to study co-evolutionary models (see e.g. Bowles and Gintis 2003; Gintis 2003a,b). The integration of co-evolution with rational parental choice has yet to be developed; but, for a first attempt, see Bisin and Verdier (2001a,b).

Appendix

Proof of Proposition 4:   Without loss of generality, one can order groups so that

$$V^1 >_\Delta V^2 > \cdots >_\Delta V^N.$$

Suppose $k^* = N$. Then, by (13),

$$\sum_{i=1}^{N} \frac{1}{V^i} > \frac{N-1}{Min_i\{ V^i\}}. \tag{18}$$

By (12), $V^N = \min_{i \in \{1,\dots,N\}} V^i$. Consider then the system

$$\dot{q}^i = q^i \left[(1-q^i) V^i - \sum_{j=1}^{N} q^j(1-q^j) V^j \right], \quad i = 1, \dots, N-1, \tag{19}$$

$$q^N = \sum_{i=1}^{N-1} q^i, \tag{20}$$

which is equivalent to (11). The Jacobian matrix of this dynamical system evaluated at q is given by $[a^{ik}]_{i,k \in \{1, N-1\}}$, where:

$$a^{ii} \equiv \left. \frac{\partial \dot{q}^i}{\partial q^i} \right|_q = -\left(1 - \frac{N-1}{V^i} G^N\right) V^N < 0 \quad \text{for} \quad i = N$$

$$a^{ik} \equiv \left. \frac{\partial \dot{q}^i}{\partial q^k} \right|_q = \left(1 - \frac{N-1}{V^i} G^N\right)(V^k - V^N) \geq 0 \quad \text{for} \quad i, k = N \quad \text{and} \quad i = k.$$

We now introduce the following result.

Lemma 1 (Tambs-Lyche, 1928)   *Suppose the $n \times n$-dimensional (real) matrix $A \equiv [a^{ik}]$ satisfies the following conditions:*

$a^{ik} \geq 0$ for all $i, k(i = k)$,
there exists positive numbers $t_1, \ldots, t_n$ such that $\sum_{j=1}^{n} t_j a^{ij} < 0$, for $i = 1, \ldots n$

   Then the real parts of all the characteristic roots of A are non-positive.

For a proof of this result, see Marcus and Minc (1964).

Using Lemma 1 we then need to find positive numbers $t_1, \ldots, t_{N-1}$ such that

$$\sum_{k=1}^{N-1} t_k \frac{\partial \dot{q}^i}{\partial q^k} < 0, \quad \text{for } i = 1, \ldots N-1.$$

Let $F_{N-1} = \{1, \ldots, N-1\}$, and consider $t_k = \frac{G^{F^{N-1}}}{V^k} > 0$. Then, $\sum_{k=1}^{N-1} t_k \left(\frac{\partial \dot{q}^i}{\partial q^k} \right)$

$$= \sum_{k=1, k=i}^{N-1} \frac{G^{F^{N-1}}}{V^k} \left(1 - \frac{N-1}{V^i} G^N\right)(V^k - V^N) - \frac{G^{F^{N-1}}}{V^i} \left(1 - \frac{N-1}{V^i} G^N\right) V^N,$$

which has the sign of

$$\sum_{k=1, k=i}^{N-1} \frac{G^{F^{N-1}}}{V^k} (V^k - V^N) - \frac{G^{F^{N-1}}}{V^i} V^N = (N-2) G^{F^{N-1}} - V^N \sum_{k=1}^{N-1} \frac{G^{F^{N-1}}}{V^k}$$

$$= (N-2) G^{F^{N-1}} - V^N$$

(the last equality follows from $\frac{1}{G^{F^{N-1}}} = \sum_{k=1}^{N-1} \frac{1}{V^k}$). We need to show then that

$$(N-2) G^{F^{N-1}} <_\Delta V^N. \tag{21}$$

From (18),

$$\frac{1}{G^{F_N}} = \sum_{i=1}^{N} \frac{1}{V^i} > \frac{N-1}{Min_i\{V^i\}} = \frac{N-1}{V^N}.$$

However, $\frac{1}{G^{F_N}} = \frac{1}{G^{F^{N-1}}} + \frac{1}{V^N}$. As a consequence, $\frac{1}{G^{F^{N-1}}} + \frac{1}{V^N} > \frac{N-1}{V^N}$, which implies $V^N > (N-2) G^{F^{N-1}}$.

This proves the local stability of the $q(F_N)$ (note that $F_N = \{1, \ldots, N\}$); moreover, $q(F_N)$ is uniquely defined by

$$q^i = 1 - \frac{N-1}{V^i} \left(\sum_{i=1}^{N} \frac{1}{V^i} \right)^{-1}, \quad \forall i \in \{1, \ldots, N\}. \tag{22}$$

As noted, $q(F_N)$ exists if (18) is satisfied.

We next prove the local stability result for heterogeneous distributions of preferences whenever condition (18) does not hold.

Note that

$$V^k > (\text{ resp. } <) (k-1) G^{\hat{F}_k}$$

implies that

$$V^k > (\text{ resp. } <) (1 - q_k^i) \ V^i, \quad \forall i \in \hat{F}_k;$$

and, from (10), $V^i = d(0, \ V^i)$. In particular,

$$V^k < (1 - q_k^i) \ V^i, \quad \forall i \in \hat{F}_k, \ k > \hat{k} \tag{23}$$

and

$$V^k > (1 - q_k^i) \ V^i, \quad \forall i \in \hat{F}_k, \ k \leq k^*. \tag{24}$$

Equation (23) then implies that the same argument used to prove local uniqueness of the stationary state in the interior of S^N, under condition (18), in Lemma 1, proves then local uniqueness of q_{k^*}; while (24) implies that any stationary state in the interior of S_{F_k}, for $k < k^*$, is locally unstable (see Figure 1 for the phase diagram in the case $k^* = 3$).

References

Bisin, A., and T. Verdier (1998), "On the cultural transmission of preferences for social status," *Journal of Public Economics* **70**, 75–97.

Bisin, A., and T. Verdier (2000), "Beyond the melting pot: Cultural transmission, marriage, and the evolution of ethnic and religious traits," *Quarterly Journal of Economics* **115**, 955–88.

Bisin, A., and T. Verdier (2001a), "Cultural transmission, socialization, and the dynamics of preferences," *Journal of Economic Theory* **97**, 298–319.

Bisin, A., and T. Verdier (2001b), "Agents with imperfect empathy may survive natural selection," *Economics Letters* **71**, 277–85.

Bisin, A., G. Topa, and T. Verdier (2004a), "Religious intermarriage and socialization in the U.S," *Journal of Political Economy* **112–13**, 615–64.

Bisin, A., G. Topa, and T. Verdier (2004b), "Cooperation as a transmitted cultural trait," *Rationality and Society* **16**, 477–507.

Boyd, R., and P. Richerson (1985), *Culture and the Evolutionary Process*, Chicago: University of Chicago Press.

Bowles, S., and H. Gintis (2003), "Origins of human cooperation," P. Hammerstein, ed., *Genetic and Cultural Evolution of Cooperation*, 429–43, Cambridge, MA: MIT Press.

Cavalli-Sforza, L., and M. Feldman (1981), *Cultural Transmission and Evolution: A Quantitative Approach*, Princeton, NJ: Princeton University Press.

Duesenberry, J. (1949), *Income, Saving and the Theory of Consumer Behavior*, Cambridge, MA: Harvard University Press.

Francois, P. (2002), *Social Capital and Economic Development*, New York: Routledge.

Gintis H. (2003a), "Solving the puzzle of prosociality," *Rationality and Society* **15**, 155–87.

Gintis H. (2003b), "The hitchhiker is guide to altruism: Genes, culture, and the internalization of norms," *Journal of Theoretical Biology* **220**, 407–18.

Hauk, E., and M. Sáez-Marti (2002), "On the cultural transmission of corruption," *Journal of Economic Theory* **107**, 311–35.

Jellal, M., and F. Wolff (2002), "Cultural evolutionary altruism: Theory and evidence," *European Journal of Political Economy* **18**, 241–62.

Iannaccone, L. (1990), "Religious practice: A human capital approach," *Journal for the Scientific Study of Religion* **29**, 297–314.

Kapteyn, A., T. Wansbeek, and J. Buyze (1980), "The dynamics of preference formation," *Journal of Economic Behavior and Organization* **1**, 123–57.

Leibenstein, H. (1950), "Bandwagon, Snob and Veblen effects in the theory of consumers' demand," *Quarterly Journal of Economics* **64**, 183–207.

Marcus, M., and H. Minc (1964), *A Survey of Matrix Theory and Matrix Inequalities*, New York: Dover.

Olivier, J., M. Thoenig, and T. Verdier (2005), "Globalization and the dynamics of cultural identity," mimeo, Paris-Jourdan Sciences Economiques, Paris.

Pollak, R. (1976), "Habit formation and long run utility functions," *Journal of Economic Theory* **13**, 272–97.

Tabellini, G. (2007a), "The scope of cooperation: Values and incentives," mimeo, Universita' Bocconi, Milano.

Tabellini, G. (2007b), "Institutions and culture," mimeo, Universita' Bocconi, Milano.

Tambs-Lyche, R. (1928), "*Un Théorème sur les déterminants*," *Det Kongelige Videnskabernes Selskab* **1**, 119–120.