

Managerial hedging, equity ownership, and firm value

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Suppose risk-averse managers can hedge the aggregate component of their exposure to firm's cash-flow risk by trading in financial markets but cannot hedge their firm-specific exposure. This gives them incentives to pass up firm-specific projects in favor of standard projects that contain greater aggregate risk. Such forms of moral hazard give rise to excessive aggregate risk in stock markets. In this context, optimal managerial contracts induce a relationship between managerial ownership and (i) aggregate risk in the firm's cash flows, as well as (ii) firm value. We show that this can help explain the shape of the empirically documented relationship between ownership and firm performance.

1. Introduction

■ The interests of managers and entrepreneurs are not necessarily aligned with those of the claimants of their firm. This is the case, for instance, when costly unobservable effort on their part is required to manage the firm or when they can divert part of the firm's cash flow to their private accounts. Incentive compensation schemes are hence devised to induce managers and entrepreneurs to act efficiently in the interests of their firm's claimants. Such schemes determine the share of their own firm that managers must retain in their portfolios. Accordingly, these schemes restrict managers from freely trading their firm. Similarly, a diverse set of regulations in financial markets also restricts the ability of managers and entrepreneurs to trade their own firm's

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stock.¹ Nonetheless, no regulation restricts or imposes disclosure on the portfolios of managers and entrepreneurs in dimensions other than the ownership of the managed firm. Also, rarely do boards impose direct contractual limitations on managerial hedging, a phenomenon that Schizer (2000) documents on the basis of off-the-record interviews with investment bankers, and that some authors, most notably Bebchuk, Fried, and Walker (2002), consider a manifestation of managerial rent extraction.

Given the lack of such contractual restrictions, risk-averse managers and entrepreneurs can (and do) to an extent enter financial markets in order to privately hedge their risk exposure to the firm. Evidence of managerial hedging is provided in the law literature by Easterbrook (2002) and in the finance literature by Bettis, Bizjak, and Lemmon (2001). Recent empirical evidence shows, however, that managers appear to be able to hedge aggregate-risk exposure more effectively than firm-specific risk. For instance, Jin (2002) and Garvey and Milbourn (2003) find that the pay-performance sensitivity of incentive contracts falls with the idiosyncratic risk of firm's cash flows but is invariant to the market risk. This finding is consistent with managers and entrepreneurs hedging their aggregate-risk exposure, for example, by trading in market indices or basket products, but being restricted from trading in their own firms.

If the restrictions imposed on managers' and entrepreneurs' trading in financial markets principally concern trading in their own firms (as we argued above), then risk-averse managers have an incentive to substitute the unhedgeable, firm-specific risk of their firm's cash flows for hedgeable, aggregate risks. For example, they may pass up innovative projects with firm-specific risk in favor of standard projects that have greater aggregate risk.² Such risk substitution enables managers to be better diversified, but has perverse implications for aggregate risk sharing in a general equilibrium context: if all managers in the economy engage in such risk substitution, then the correlation of cash flows of different firms is enhanced, as is, in turn, the aggregate risk in stock markets.

We study an economy in which managers and entrepreneurs face incentive compensation schemes and can only hedge the aggregate-risk exposure of their firm (but not firm-specific risk) by trading in capital markets. In this economy we study risk-substitution moral hazard, which arises when managers and entrepreneurs can affect the risk composition of their firms' cash flow for example, through investment activities which cannot be *ex ante* contracted upon. We cast risk-substitution moral hazard in a general-equilibrium setting in order to address the efficiency of endogenous risk composition. We show that in equilibrium, the level of aggregate risk in the stock market exceeds the first-best level. Nonetheless, it is constrained (second-best) efficient. We study the positive aspects of this moral hazard by characterizing the optimal incentive contract designed to address it. We show that such an optimal incentive compensation scheme might require a "dampening" of pay-performance sensitivity, whereby managerial ownership is smaller than in the absence of the risk-substitution moral hazard. We also characterize the resulting equilibrium relationship between managerial equity ownership and (i) the extent of aggregate risk in the

¹ Since 1994, in the United States such trades must be disclosed to the Securities and Exchange Commission. Disclosure rules regarding own stock trading have also become stricter with the Sarbanes-Oxley Act of 2002. Furthermore, additional regulation is often imposed by the law of the firm's state of incorporation, by the rules of the stock exchange where the firm is listed, and by the firm's articles of incorporation. Needless to say, managers and entrepreneurs might be able to circumvent such restrictions; in this regard, see the discussion and the references in footnote 9.

² Consider, for example, the following choice faced by the CEO of a pharmaceutical company: the CEO can invest the company's funds in R&D activities directed toward the invention of a new drug. Alternatively, the CEO can invest these funds in boosting sales of drugs that already proliferate in the market. The risk from R&D activities is firm specific. In contrast, the risk from sales of existing drugs is more aggregate in nature: it depends upon factors, such as global demand for drugs, which also affect the profits of other pharmaceutical companies.

The media has often expressed the view that managers tend to pass up risky projects when exposed to the cash-flow risk of their firms, although not always as giving up of idiosyncratic projects for more standard ones. For example, the article *Do Large Stakes Inhibit CEOs?—Big Holdings May Curb Risk-Taking* (Business Week, May 6, 1996) reports that "[W]hile it's good for top executives to have equity stakes in their company, they may grow excessively cautious if their stakes become too large."

firm's cash flows, as well as (ii) the firm's performance as measured by firm value. This analysis provides a structural model of the relationships between managerial ownership, risk composition, expected returns, and firm value, and has important empirical implications. In particular, we show that these endogenous relationships help explain various important cross-sectional relationships documented in corporate finance.

A detailed summary of our analysis follows. We study firms in an incomplete-markets, general-equilibrium capital asset pricing model (CAPM) economy. Our analysis can be applied equivalently to *owner-managed firms* and to *corporations* run by managers. Let us consider in this introduction the case of corporations, for concreteness. The fraction of their firm that managers retain in their portfolios, that is, their equity ownership of the firm, is determined contractually. Contractual agreements cannot, however, restrict their trades in aggregate indices. Once the ownership structure of firms is designed, agents trade in financial markets and prices are determined. Subsequently, managers choose the technology of the firm. Firms can produce a given expected cash flow with a given total risk through the use of different technologies: some technologies are standard and have greater betas with respect to the aggregate risk factor and thus have greater aggregate risk; others are innovative and have lower betas with respect to the aggregate risk factor and thus have greater firm-specific risk. Technological innovation (modifying the "intrinsic" or the initial aggregate-risk beta of each firm's project) is costly for managers. The resulting aggregate-risk beta is not observed by the firm's investors.

The choice of the firm's technology introduces risk-substitution moral hazard. In equilibrium, managers retain a positive share of their own firm in their portfolios. But, because they are risk averse and they can hedge only the aggregate-risk exposure by trading in market indices, managers have an incentive to increase the aggregate-risk beta of their firm's cash flows: by loading their firm's projects on aggregate risk, managers can reduce their own exposure to unhedgeable firm-specific risks. Such risk substitution by managers is aimed at diversification of their personal portfolios and it occurs at the cost of reducing the firm's market value: under CAPM pricing, the market price of the firm's shares decreases in its aggregate-risk beta, for given mean and variance of its cash flow.

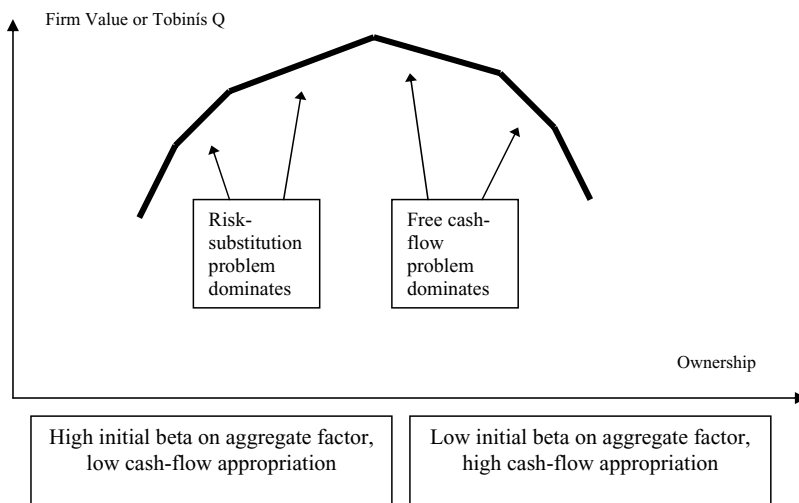
We characterize the optimal ownership structure of firms in the face of risk-substitution moral hazard and the induced equilibrium risk composition of firms' cash flows. We show that if the firm's technology is intrinsically more loaded on aggregate risk factors (for example, in procyclical industries), then the optimal ownership scheme provides managers with a lower equity holding of their firms. Risk-substitution moral hazard is particularly severe for firms with high intrinsic aggregate-risk loadings. Thus, in equilibrium, a smaller managerial ownership share is optimal for these firms. Indeed, it may even be the case that it is optimal for these firms to choose equity holdings for managers that are smaller than the optimal contractual holdings in the absence of moral hazard, a situation we refer to as "dampening" of pay-performance sensitivity.

Our analysis has rich empirical implications. First, firms whose entrepreneurs or managers hold a larger share of equity in equilibrium are characterized by less aggregate risk in equilibrium, and hence by low expected returns. This implies, other things being equal, a negative relationship between managerial ownership and expected returns. To our knowledge, such a relationship has yet to be explored empirically.

Second, the risk-substitution moral hazard we study, when explicitly combined with an alternate moral hazard, for example, Jensen's (1986) free cash-flow agency problem, can help explain the hump-shaped cross-sectional relationship between managerial ownership and firm performance, measured by the ratio of the firm's market value to book value (documented by Morck, Shleifer, and Vishny, 1988, and McConnell and Servaes, 1990, among others). In particular, all else being equal, as the risk-substitution moral hazard becomes more severe, a positive equilibrium relationship is obtained between managerial ownership and performance. In contrast, an increase in the severity of the free cash-flow problem induces a negative relationship between ownership and performance (as also found empirically by Bizjak, Brickley, and Coles, 1993). Thus, a possible structural explanation of the hump-shaped relationship consists of

FIGURE 1

STRUCTURAL EXPLANATION FOR HUMP-SHAPED RELATIONSHIP BETWEEN FIRM PERFORMANCE (FIRM VALUE OR TOBIN'S Q) AND OWNERSHIP



This figure illustrates our conceptual explanation for the empirically documented hump-shaped relationship between firm performance (measured as firm value in the article) and ownership. The explanation relies on a negative correlation between the severity of risk-substitution moral hazard ($\bar{\beta}_1^f$) and free cash-flow moral hazard (η), as illustrated in the figure for low and high levels of ownership.

recognizing that at low levels of ownership, the dominant moral hazard problem is the risk-substitution one, whereas at high levels of ownership, traditional moral hazard problems like the free cash-flow problem dominate (see Figure 1).

Importantly, this proposed distribution of the relative severity of different moral hazard problems—the dominance of risk substitution at low ownership levels and of the free cash-flow problem at high ownership levels—has independent implications regarding the shape of the relationship between managerial ownership and diversification. In particular, because risk substitution implies a negative relationship between ownership and diversification, and the free cash-flow problem a positive one, the proposed distribution implies a U-shaped relationship between diversification and ownership. Thus, our analysis of the risk-substitution moral hazard has the potential to simultaneously explain, as equilibrium relationships, the hump-shaped relationship between firm performance and inside ownership, and the U-shaped relationship between diversification (R^2) and inside ownership.

Finally, our analysis also contributes to the understanding of the issue of relative performance evaluation (RPE). The literature (starting with Holmstrom, 1982) that assumes managers cannot affect the risk composition of their firms has argued that managers' compensation schemes should be independent of the aggregate component of their firms. In the context of our model, however, incentive schemes providing for explicit RPE clauses may in fact be inefficient. Because managers hedge their aggregate-risk position to hold the market share of such risk, but do so at a cost, in equilibrium managers reduce the aggregate-risk component of their firms (just not up to first-best levels). This is, in fact, in the interest of the firms' claimants and of efficient provision of incentives. Introducing RPE clauses would induce resistance in capital budgeting toward innovation because managers no longer bear the cost of having aggregate risk in their firm's cash flows.

The choice of risk composition of firms' cash flows by managers also endogenously affects the level of risk sharing in the economy. We show that, in equilibrium, managers choose aggregate risk in their firm's cash flows that exceeds the first-best level. However, market prices and the optimal ownership structure of the firm induce a level of aggregate risk in firms that is constrained

(second-best) efficient.³ That is, the ownership structure is efficient from the point of view of a planner who cannot internalize the externality of managerial activity aimed at substituting firm-specific risk of the firm's cash flows with aggregate risk. Prices in financial markets are not only market clearing but also efficiently align the objectives of management and stockholders with those of the constrained social planner: managers recognize that increasing the aggregate risk of the firm reduces the equilibrium price of the firm's shares; and, in equilibrium, the fraction of the firm's shares that managers retain induces them to choose the constrained-efficient firm loadings.

We extend our analysis by considering multiple sectors, whereby the aggregate risk factor can be interpreted as a stock market index. In this setting, we argue that the risk-substitution moral hazard also gives rise to an excessive loading of the firm's stock returns on the index returns and, in turn, that it generates an excessive correlation of returns across sectors. Next, we show that the risk-substitution moral hazard is more severe the greater the extent of purely idiosyncratic risk in the firm's cash flows.

The remainder of the article is structured as follows. After a discussion of some related literature, Sections 2 and 3 contain the model and analysis of the risk-substitution moral hazard. Section 4 discusses empirical implications and Section 5 addresses the efficiency of equilibrium choices. Section 6 establishes the isomorphism between owner-managed firms and corporations. Section 7 considers various extensions. Section 8 concludes. Appendices A and B contain the closed-form expressions for the competitive equilibrium and the proofs of Propositions 1 and 2, respectively. More detailed appendices, containing a formal analysis of a more general CAPM economy, the expression for the welfare criterion, and complete proofs of all the propositions in the article are available upon request.

□ **Related literature.** The design of entrepreneurial ownership and managerial compensation under asymmetric information and moral hazard has been examined extensively in the corporate finance literature. Diamond and Verrecchia (1982) and Ramakrishnan and Thakor (1984) were the first to analyze moral hazard when the firm's returns have systematic and idiosyncratic risks. These papers are cast in partial-equilibrium settings. Our principal theoretical contribution is rather to embed the agency-theoretic approach of Fama and Miller (1972) and Jensen and Meckling (1976) into a general equilibrium model of the price of risk, such as the CAPM.⁴

Few general equilibrium analyses of the ownership structure of firms have been developed. Allen and Gale (1988, 1991) study the capital structure of firms in general equilibrium. However, they do not study economies with moral hazard. Magill and Quinzii (2002) and Ou-Yang (2002) do in fact consider the issue of moral hazard between entrepreneurs and investors in a general equilibrium setting. In the setup of these papers, entrepreneurs can affect the variance of their firm's cash flows and/or their levels, rather than their correlation with aggregate risk, as in our case.

The moral hazard we concentrate on, risk substitution, is induced by incentive compensation. Specifically, it arises because incentive compensation might give managers incentives to substitute from unhedgeable, firm-specific risk in the firm's cash flows toward hedgeable aggregate risks. To our knowledge, this form of moral hazard has not been directly studied in previous work. Theoretical and empirical literature in corporate finance has concentrated instead on the incentives of managers to inefficiently alter *only* the firm-specific variance by means of diversification activities (Amihud and Lev, 1981; Lambert, 1986), to reduce the firm's expected cash flow by expropriation of the firm's assets and diversion of cash flow (Jensen, 1986), or to reduce the effort provided in the management of the company.

³ At the first-best, the social planner can choose both the technology of firms and their ownership structure. In contrast, at the second-best, the social planner designs the firm's ownership structure, but must let managers and entrepreneurs make technology decisions.

⁴ In particular, we follow Willen (1997) in introducing incomplete financial markets and restricted participation in the CAPM economy. In addition, we introduce assets in positive net supply to capture a stock market economy.

The interaction between incentive compensation and hedging opportunities of managers has also been studied mostly in the context of economies in which managers affect the expected returns of their companies by their choice of effort (Jin, 2002; Garvey and Milbourn, 2003; Ozerturk, 2006; Bisin, Gottardi, and Rampini, 2008). In this context, the managers' ability to hedge the risk exposure in their compensation need be restricted because, by hedging unrestrictedly in capital markets, managers could completely undo the incentives in their compensation. In this article, we instead model risk-substitution moral hazard. For simplicity, we do not directly model other forms of moral hazard, for example effort choice, but rather we take incentive compensation and hedging restrictions as primitives of the analysis, in fact justified by such other forms of moral hazard.⁵

Our structural modelling approach is in the spirit of important antecedents such as Demsetz and Lehn (1985) and, more recently, Himmelberg, Hubbard, and Love (2002). Specifically, from the standpoint of providing a structural model linking managerial ownership and firm value, our article is closest to the recent work of Coles, Lemmon, and Menschke (2006). These authors provide a different structural explanation of the hump-shaped empirical relationship between ownership and performance. We discuss the relationship of our analysis to theirs in Section 4.

Finally, Core and Guay (1999) and Coles, Daniel, and Naveen (2006), among others, examine the role of nonlinear incentive compensation schemes such as executive stock options (ESOs) to improve risk-taking incentives of top-level management. Coles, Daniel, and Naveen (2006) document that a higher sensitivity of CEO wealth to firm volatility (the "vega") results in riskier policy choices in terms of more R&D expense, less investment in property, plants, and equipment, fewer lines of business, and higher leverage. Our article ignores the provision of such convex compensation schemes for reasons of analytical tractability, although these have become increasingly important over time.

2. The model

■ We study a perfectly competitive two-period equilibrium economy in which the CAPM pricing rule can be derived.

A subset of the agents in the economy, entrepreneurs and managers, make capital budgeting choices: at a private cost, they can choose their firm's technology and affect the risk composition of cash flows and, hence, stock returns. The CAPM setting enables us to cast the capital budgeting choice faced by entrepreneurs and managers in terms of a choice of betas (i.e., the loadings of cash flows) onto traded risk factors: by choosing the betas of firm cash flows, entrepreneurs and managers determine the proportion of aggregate and firm-specific components in the total cash-flow risk of firms.

Capital budgeting choices are affected by the equity ownership structure of the firms. We maintain the assumption that entrepreneurs and managers are prohibited from trading the stock of their own firms and others in the same sector, but they can trade other financial assets.⁶ This endows entrepreneurs and managers with a preference to substitute projects whose cash-flow risk cannot be hedged easily with projects whose cash-flow risk is readily hedgeable by trading in financial markets. This creates the possibility of there being a risk-substitution moral hazard in the capital budgeting choices of entrepreneurs and managers.

The ownership structure is, in turn, the result of an optimal contracting problem between entrepreneurs and investors, or between managers and stockholders. We consider different corporate governance structures and the contracting problems induced under these structures. A governance structure determines whether the firm is originally held by entrepreneurs, as in *owner-managed firms*, or by stockholders, as in *corporations*. In the case of a corporation, the firm

⁵ In Section 4, however, we briefly introduce an economy with moral hazard in the form of both risk substitution and cash-flow diversion.

⁶ As already noted, this assumption is necessary with moral hazard in the form of hidden action or cash-flow diversion, which we do not introduce explicitly here for simplicity (but see Section 4).

is run by managers, that is, the firm is management controlled. We concentrate on owner-managed firms for most of the article for concreteness, but we show in Section 6 that our results extend isomorphically to corporations.

An owner-managed firm is owned *ex ante* by an entrepreneur. If the firm’s cash-flow betas are observable and the entrepreneur can credibly commit to a choice of these betas when the firm is sold in the stock market, then no moral-hazard concerns arise. Consequently, the entrepreneur’s choice of ownership structure and the cash-flow betas are both optimal. If, instead, the cash-flow betas are not observed by the market (i.e., they are private information of the entrepreneur) and the choice of these betas occurs *after* the firm is sold in the stock market, then the issue of moral hazard arises.⁷ In this case, the proportion of the firm that the entrepreneur retains determines the choice of the firm’s cash-flow betas. Investors in the market rationally anticipate the mapping between the entrepreneur’s holding of the firm and the choice of betas. Thus, the market price of shares depends upon the publicly observed ownership structure of the firm. Entrepreneurs also realize that the firm’s value will depend on its ownership structure, understanding that discounted prices will be associated with ownership structures that impart incentives to increase the aggregate risk of cash flows.

We introduce formally the simplest version of the model with a representative firm, relegating technical details to Appendix A.

□ **The CAPM economy with a firm.** The economy is populated by H agents, who live for two periods, 0 and 1. Agent h ’s preferences are represented by a constant absolute risk aversion (CARA) utility function,

$$u^h(c_0^h, c_1^h) \equiv -\frac{1}{A}e^{-Ac_0^h} - \frac{1}{A}e^{-Ac_1^h}, \tag{1}$$

where c_0^h and c_1^h denote consumption at time 0 and 1, respectively; $A > 0$ is the absolute risk-aversion coefficient, which is assumed to be the same for all agents.

Agent 1 in the economy is the representative entrepreneur. The remaining agents, $h = 2, \dots, H$, are the investors. The entrepreneur owns a firm, which has a technology that produces a random, normally distributed cash flow at time 1, y_1^f , of the unique consumption good. To emphasize that this is the firm’s cash flow, we will often refer to it as y_1^f . The entrepreneur has a private endowment at time 0, y_0^f , but no private endowment at time 1 save his ownership of the firm. Each investor $h = 2, \dots, H$ has an endowment y_0^h in period 0, and a random, normally distributed endowment y_1^h in period 1.

The economy’s risks are spanned by N orthogonal normally distributed factors, x_n , $n = 1, \dots, N$, $N \geq 2$. The firm’s cash flow is driven by an aggregate risk factor, x_1 , that is positively correlated with the aggregate endowment of investors, $\sum_{h=2}^H y_1^h$, and by a second risk factor, x_2 , that is orthogonal to x_1 and to the aggregate endowment of investors. The second factor is interpreted as the “corporate sector-specific” risk in the economy⁸:

$$y_1^f - E y_1^f \equiv \beta_1^f x_1 + \beta_2^f x_2. \tag{2}$$

Without loss of generality, we adopt the normalizations $E(x_i) = 0$, $\text{var}(x_i) = 1$ for $i = 1, 2$. The firm’s betas, β_1^f and β_2^f , measure the covariance of the firm’s earnings, y_1^f , with risk factors x_1

⁷ The difficulty in estimating a firm’s stock-return betas is ubiquitous in corporate finance and asset pricing and, in fact, is the primary reason for the portfolio-based approach to tests involving firm betas. Hence, it is reasonable to assume that a firm’s cash-flow betas are also not perfectly observed by investors. The literature starting with Amihud and Lev (1981) that focuses on the alteration of firm-specific risk only also tacitly assumes that either the firm’s volatility or its betas are not perfectly observed by shareholders.

⁸ Risk factor x_1 is common to both the stock market (the “corporate sector”) and agents’ endowments (for instance, private business income and returns to human capital). For instance, x_1 could represent a general aggregate productivity index. Extending the analysis to account for multiple industrial sectors, as in Section 7, allows us to interpret x_1 more naturally as a general stock market index, with x_2 (and x_3, x_4, \dots) as the additional risk components of specific sectors.

and x_2 , respectively:

$$\beta_j^f = \text{cov } y_1^f, x_j, \quad j = 1, 2. \quad (3)$$

For simplicity, we suppose that $\beta_1^f, \beta_2^f > 0$. The betas of investor h , β_1^h and β_2^h , are defined similarly.

There are three financial markets: a riskless bond market, where asset 0 with a deterministic payoff of 1 is traded, a market where the aggregate factor x_1 is directly traded, and the stock market where shares of the representative firm f are traded. The bond and the asset paying off the aggregate factor x_1 are in zero net supply. The fraction w of the firm sold in the stock market constitutes the positive supply of the stock. The remaining fraction $(1 - w)$ constitutes the equity ownership of the entrepreneur. If an N -dimensional factor structure drives risk where $N > 2$, then the economy is one of incomplete markets. Trading in financial and stock markets is restricted. In particular, we assume that the entrepreneur, after having placed w shares on the market, cannot trade the stock of his own firm.⁹ However, all agents can trade the riskless bond.

We treat the entrepreneur as a price-taker and the economy as competitive. In particular, we abstract from the ability of entrepreneurs to strategically affect the equilibrium prices. One can interpret the representative entrepreneur as one of a continuum of entrepreneurs. Furthermore, for ease of exposition, we assume a firm's cash flows are driven only by the aggregate and the corporate sector-specific risk factors, and not by any firm-specific risk factor. That is, we treat the representative firm as equivalent to the corporate sector composed of a continuum of identical firms. In the second section of Section 7, we distinguish between the firm and the sector by allowing the cash flows of each firm to contain both a sector-specific and a purely firm-specific risk factor. Crucial in these contexts is that either the entrepreneur cannot hedge his sector-specific risk in financial markets (in the model we analyze below), or else he cannot hedge his firm-specific risk (in the second section of Section 7).

□ **Equilibrium.** Our analysis proceeds recursively. First, given arbitrary equity ownership structures and cash-flow betas on risk factors, we solve for the market equilibrium and induced CAPM pricing rule. Then, given the ownership structure, we analyze the capital budgeting problem, that is the entrepreneur's choice of betas. Finally, we study the optimal contracting problem, which determines the ownership structure of the firm.

Competitive equilibrium of the CAPM economy. Given the price of the riskless bond, π_0 , the price of the aggregate factor, π_1 , and the price of the representative firm, p^f , each agent chooses (i) a consumption allocation at time 0, c_0^h , (ii) portfolio positions in the risk-free bond, θ_0^h , in the aggregate factor, θ_1^h , and in the firm, θ_f^h , and (iii) a consumption allocation at time 1, a random variable c_1^h , to maximize

$$E u^h(c_0^h, c_1^h) \equiv -\frac{1}{A} e^{-Ac_0^h} + E \left[-\frac{1}{A} e^{-Ac_1^h} \right]. \quad (4)$$

The budget constraints faced by the investor h , $h > 1$, are

$$c_0^h = y_0^h - \pi_0 \theta_0^h - \pi_1 \theta_1^h - p^f \theta_f^h \quad (5)$$

$$c_1^h = y_1^h + \theta_0^h + \theta_1^h x_1 + \theta_f^h y_1^f. \quad (6)$$

⁹ This assumption is admittedly stark but it is necessary to avoid undoing of incentives when other forms of moral hazard such as effort aversion or cash-flow diversion are present. We acknowledge that recent evidence in Bettis, Coles, and Lemmon (2000) and Ofek and Yermack (2000) suggests that managers might be able to partly circumvent such trading restrictions. Indeed, Bettis, Coles, and Lemmon (2000) document that about 40% of firms in their sample do not impose trading restrictions or blackout periods on managers, and another 45% allow managers to trade in their company stock for 10 days in each quarter, starting with the third day following earnings announcement.

The entrepreneur, agent $h = 1$, faces the additional constraint that he cannot trade his firm ($\theta_f^1 \equiv 0$), once he sells fraction w at date 0:

$$c_0^1 = y_0^1 + wp^f - \pi_0\theta_0^1 - \pi_1\theta_1^1 \quad (7)$$

$$c_1^1 = \theta_0^1 + \theta_1^1 x_1 + (1 - w)y_1^f. \quad (8)$$

Note that the entrepreneur receives proceeds wp^f from selling fraction w of the firm at the market price of p^f .

A *competitive equilibrium* of the economy is a consumption allocation (c_0^h, c_1^h) , for all agents $h = 1, \dots, H$, that solves the problem of maximizing (4) subject to (5) and (6) for $h > 1$, and the problem of maximizing (4) subject to (7) and (8) for $h = 1$, and prices (π_0, π_1, p^f) such that consumption and financial markets clear

$$\sum_{h=1}^H c_0^h - y_0^h \leq 0, \quad (9)$$

$$\sum_{h=1}^H c_1^h - y_1^h \leq 0 \quad (\text{with probability 1 over possible states at } t = 1), \quad \text{and} \quad (10)$$

$$\sum_{h=1}^H \theta_j^h = 0, \quad j = 0, 1; \quad \sum_{h=2}^H \theta_f^h = w. \quad (11)$$

Given the equity ownership structure of the firm, w , and its cash-flow betas $\beta_j^f, j = 1, 2$, a competitive equilibrium is uniquely determined. We discuss below the salient features of the competitive equilibrium that we exploit in our analysis. Closed-form solutions for equilibrium allocations and prices are reported in Appendix A.

The factor structure of the firm's cash flow, equation (2), implies that the equilibrium price of the firm can be written as the composition of price of the deterministic component, the price of the aggregate-risk component, and the implicit price of the corporate sector-specific risk of its cash flow:

$$p^f = \pi_0 E y_1^f + \pi_1 \beta_1^f + \pi_2 \beta_2^f, \quad (12)$$

where π_2 is the equilibrium price of a portfolio paying off x_2 . Given our assumptions, a portfolio paying off x_2 can be replicated through the trading of available assets by all agents except the entrepreneur; the price π_2 can therefore be determined by no arbitrage from π_0, π_1 , and p^f . It is convenient to express the properties of equilibrium pricing in terms of the factor prices, (π_0, π_1, π_2) .

At the competitive equilibrium, each agent holds three "funds": the bond, the portion of aggregate endowment that is exposed to traded risk factors (subject to the restricted participation constraints), and the unhedgeable component of the personal endowment. The positive supply of the firm's stock also translates into positive supplies $s_j, j = 0, 1, 2$, of the riskless bond and risk factors:

$$s_0 = w E y_1^f, \quad (13)$$

$$s_j = w \beta_j^f, \quad j = 1, 2. \quad (14)$$

This follows also from the factor structure of the firm's cash flow (equation (2)).

Under this representation, a version of the cross-sectional beta pricing relationship holds: the price of factor j relative to the price of a bond is proportional to the covariance of the factor with the aggregate endowment of the economy and to the positive supply of factor j . The aggregate endowment relevant for the pricing of factor j is the sum of the endowments of the agents who

can trade factor j . Formally,

$$\frac{\pi_1}{\pi_0} = E(x_1) - \frac{A}{H} \operatorname{cov} \left((1-w)y_1^1 + \sum_{h=2}^H y_1^h, x_1 + w\beta_1^f \right) = -\frac{A}{H} \sum_{h=1}^H \beta_1^h, \quad (15)$$

$$\frac{\pi_2}{\pi_0} = E(x_2) - \frac{A}{H-1} \operatorname{cov} \left(\sum_{h=2}^H y_1^h, x_2 + w\beta_2^f \right) = -\frac{A}{H-1} \sum_{h=2}^H \beta_2^h + w\beta_2^f, \quad (16)$$

where we have employed the normalization that $E(x_j) = 0, j = 1, 2$. Because the entrepreneur cannot trade the stock of his firm, he (effectively) cannot trade sector-specific risk factor x_2 . The relevant aggregate endowment for the price of factor x_2 thus excludes his holding of this risk $(1-w)\beta_2^f$. Recall also that asset x_1 is positively correlated with the aggregate endowment of investors, $\sum_{h=2}^H y_1^h$, the firm endowment y_1^f is positively loaded on asset x_1 ; and asset x_2 is orthogonal to the aggregate endowment of investors. Thus,

$$\sum_{h=1}^H \beta_1^h > 0, \quad \sum_{h=2}^H \beta_2^h = 0. \quad (17)$$

Finally, in equilibrium, the expected utility of agent h is

$$E u^h c_0^h, c_1^h = -\frac{(1 + \pi_0)}{A} e^{-Ac_0^h w, \beta_1^f, \beta_2^f, p^f}, \quad (18)$$

where we stress the fact that the equilibrium time 0 consumption depends on the ownership structure of the firm, its technology, and the price of the firm. This expected utility also depends on the induced equilibrium prices, $(\pi_j), j = 0, 1, 2$, that we omit for parsimony.

□ **Capital budgeting and equity ownership structure.** The entrepreneur can, at a private nonpecuniary cost, choose the risk composition of the firm’s cash flows. Formally, the entrepreneur can choose the betas, β_1^f and β_2^f , the respective loadings of the firm’s cash flows on the aggregate risk and the corporate sector-specific risk.¹⁰ We assume the entrepreneur’s choice only affects the distribution of the variance of cash flows between the aggregate and the sector-specific risks, but does not alter their expected value or the total variance. That is, the entrepreneur’s choice consists of substituting between projects which are innovative and projects that are otherwise identical but are standard and more exposed to aggregate risk.¹¹ That is, we assume that

$$\beta_1^{f^2} + \beta_2^{f^2} = \bar{V}, \quad (19)$$

where \bar{V} , the total variance of the cash flow of the firm, is held constant.

The entrepreneur must exert a nonpecuniary costly *effort* to change the intrinsic composition of the cash-flow risk. We assume that the cost is nonpecuniary, and is measured in terms of the time 0 consumption good. More specifically, this cost enters the entrepreneur’s expected utility according to the multiplicative factor $e^{AC(\beta_1^f - \bar{\beta}_1^f)^2}, C > 0$, where $\bar{\beta}_1^f > 0$ denotes the intrinsic level of β_1^f (only changes in β_1^f from its intrinsic level need be considered in the costs, because the associated changes in β_2^f are determined via equation (19)). These assumptions on the cost structure are made for analytical tractability. They imply that the quadratic cost, $C(\beta_1^f - \bar{\beta}_1^f)^2$, is subtracted from the certainty equivalent of the entrepreneur’s time 1 consumption, as in typical CARA-normal principal-agent setups, such as Holmstrom and Milgrom (1987) and Laffont and

¹⁰ Note that the equilibrium price of the firm is affected by the capital budgeting choice. In turn, the expected stock return on the firm is affected as well even though the expected cash flows are not.

¹¹ In practice, in fact, managers do affect firm-specific risk per se, for example, they diversify their firm’s cash flow by acquiring unrelated businesses. We concentrate on risk substitution in the interest of focus and simplicity. In Section 4, we discuss the case in which the manager can also affect expected cash flow, that is, the case in which a free cash-flow problem is added to the risk-substitution moral hazard.

TABLE 1 The Sequence of Events under Different Governance Structures

Governance Structure		Sequence of Events		
Benchmark	Entrepreneurs choose fraction w to sell		Entrepreneurs choose aggregate-risk loading β_1^f , β_1^f is <i>observable</i>	\Rightarrow All agents including entrepreneurs trade, markets clear, prices are determined
Owner-managed firms	Entrepreneurs choose fraction w to sell	\Rightarrow	All agents including entrepreneurs trade, markets clear, prices are determined	\Rightarrow Entrepreneurs choose aggregate-risk loading β_1^f , β_1^f is <i>unobservable</i>
Corporations (management-controlled firms)	Investors choose fraction w to retain, managers are awarded fraction $(1 - w)$	\Rightarrow	All agents including managers trade, markets clear, prices are determined	\Rightarrow Managers choose aggregate-risk loading β_1^f , β_1^f is <i>unobservable</i>

Martimort (2002). Formally, net of capital budgeting costs, the entrepreneur’s expected utility at equilibrium (equation (18)) is given by

$$-\frac{(1 + \pi_0)}{A} e^{-A} e_0^1 w, \beta_1^f, \beta_2^f, p^f - C \beta_1^f - \bar{\beta}_1^f{}^2. \tag{20}$$

Finally, entrepreneurs choose their equity ownership share $(1 - w)$ optimally. Table 1 details the exact sequence of events (for the analysis of corporations, see Section 6).

□ **Benchmark: no moral hazard.** We first study the determination of the ownership structure and the firm’s technology in the benchmark case in which (i) the entrepreneur owns the firm *ex ante*, and (ii) investors observe the choice of technology, $\beta_j^f, j = 1, 2$, so that the entrepreneur can commit to a technology choice when choosing the share w of the firm to sell. Because there is no moral hazard, the choices of β_j^f and w are effectively simultaneous. Given that the firm trades as a composite and not in a piecemeal manner for its different risk loadings, it is a strong assumption that investors observe the risk composition of firm cash flows. Nevertheless, this case serves as a useful benchmark.

When choosing w , entrepreneurs rationally anticipate the unit price p^f at which they can sell this share:

$$p^f = \pi_0 E y_1^f + \pi_1 \beta_1^f + \pi_2 \beta_2^f.$$

Each entrepreneur can affect the price of his own single firm, p^f , by his choice of β_1^f through this mapping, but he cannot affect the bond prices or risk factor prices: these prices are determined at equilibrium by the aggregate choices of the continuum of entrepreneurs. That is, markets are competitive, and all agents including entrepreneurs are price-takers: all agents rationally anticipate that the price of a single firm depends on its cash-flow betas β_j^f , given the prices of traded assets in the economy.¹²

¹² As discussed in Section 2 and assumed in the first section of Section 2, the entrepreneur takes as given the price of the riskless bond, π_0 , the price of the aggregate risk factor, π_1 , and the price of the representative firm, p^f . The composition of p^f , equation (12), implies that, in addition to π_0 and π_1 , the entrepreneur effectively takes as given the price of the sector-specific risk factor, π_2 .

The price of the entrepreneur’s own firm is also denoted as p^f for parsimony of notation. The entrepreneur recognizes that this price depends on the risk composition of his firm’s cash flows, for given prices of risk factors. In equilibrium, the price of each entrepreneur’s firm equals the price of the representative firm.

Formally, the representative entrepreneur chooses the share w of the firm to sell, as well as its technology β_1^f to maximize expected utility net of the exerted effort

$$\max_{w, \beta_1^f, \beta_2^f} -\frac{(1 + \pi_0)}{A} e^{-A c_0^1 w, \beta_1^f, \beta_2^f, p^f - C \beta_1^f - \bar{\beta}_1^f{}^2} \tag{21}$$

subject to

$$p^f = \pi_0 E y_1^f + \pi_1 \beta_1^f + \pi_2 \beta_2^f, \tag{22}$$

$$\beta_1^f{}^2 + \beta_2^f{}^2 = \bar{V}, \tag{23}$$

given the equilibrium prices of the bond and the risk factors π_0, π_1 , and π_2 , respectively.

□ **Moral hazard.** In contrast to this benchmark case, consider now owner-managed firms where the technology choice is not observed by capital market investors.¹³ As a result, entrepreneurs cannot commit their technology choice, β_j^f , at the moment they choose the fraction w of their firm to sell in the market; they choose β_j^f after they choose w , and after agents have traded and markets have cleared. Although the specific timing of the choice of β_j^f and trading in capital markets is somewhat arbitrary, crucial for our analysis is that the chosen β_j^f are not observed by investors in competitive markets.¹⁴

Proceeding recursively, we first study the capital budgeting problem of entrepreneurs which determines β_j^f for a given w . Because w is observed by investors, but β_j^f is not, entrepreneurs anticipate that the price of their own firm p^f will depend only on w and not on their specific choice of β_j^f . Therefore, for given w and p^f , the choice of cash-flow betas maximizes the entrepreneur’s expected utility net of the exerted effort

$$\max_{\beta_1^f, \beta_2^f} -\frac{(1 + \pi_0)}{A} e^{-A c_0^1 w, \beta_1^f, \beta_2^f, p^f - C \beta_1^f - \bar{\beta}_1^f{}^2} \tag{24}$$

subject to

$$\beta_1^f{}^2 + \beta_2^f{}^2 = \bar{V}. \tag{25}$$

Because the price of the firm p^f does not affect the solution of this capital budgeting problem, we denote the solution simply as $\beta_j^f(w)$.

We now consider the choice of equity ownership by entrepreneurs. An entrepreneur’s proceeds from selling share w of his firm are $w p^f$. Hence, he perceives a direct effect of the choice of w on his proceeds. In addition, the entrepreneur expects investors to rationally anticipate the equilibrium map between ownership structure and the risk composition of the firm, $\beta_j^f(w)$, which results from the solution of the capital budgeting problem. The entrepreneur therefore also perceives an indirect effect of his choice of w on the price of the firm p^f (equation (12)) through its effect on his future choice of β_j^f via the map $\beta_j^f(w)$.¹⁵

¹³ In particular, the lack of observability of a firm beta rules out institutions or organizational features such as proxy statements or prospectuses in fundraising that bind the entrepreneur to a specific beta in that there is at least partial commitment to maintaining the nature of the assets. Ability to contract such would limit the severity of the moral-hazard problem we study.

¹⁴ It appears that the assumption that the beta of an individual firm is not observed by investors is rather natural. In fact, the difficulty in observing firm-level betas manifests itself most tellingly in the asset-pricing literature where the link between expected returns and beta is tested only at the level of portfolios, rather than at the level of individual firms, and in the corporate finance literature where betas are averaged across industry peers to calculate firm-level cost of capital. In both cases, betas are aggregated across a set of firms to reduce the “error in variables” problem in estimation of firm-level betas.

¹⁵ This equilibrium concept is related to the one introduced in the context of general equilibrium theory with asymmetric information by Prescott and Townsend (1984). The formulation we adopt is, however, Magill and Quinzii’s (2002) who, in a related setting, explicitly formulate the anticipatory behavior of entrepreneurs as “rational conjectures.”

Formally, the entrepreneur chooses w to maximize the expected utility net of effort

$$\max_w -\frac{(1 + \pi_0)}{A} e^{-A c_0^h w, \beta_1^f, \beta_2^f, p^f - C \beta_1^f - \bar{\beta}_1^f} \quad (26)$$

subject to

$$p^f = \pi_0 E y_1^f + \pi_1 \beta_1^f + \pi_2 \beta_2^f, \quad (27)$$

$$\beta_j^f = \beta_j^f(w), \quad j = 1, 2, \quad (28)$$

given $\pi_j, j = 0, 1, 2$.

3. Equilibrium equity ownership and risk

■ We characterize below (i) the entrepreneurial choice of the aggregate-risk beta of the firm’s cash flows, β_1^f , and (ii) the optimal equity ownership of firms, measured by the fraction $(1 - w)$ retained by entrepreneurs. We first consider the benchmark case when investors can observe the firm’s risk loadings and hence there is no moral hazard.

Proposition 1. For owner-managed firms with no moral hazard, in equilibrium, the loading on the aggregate risk factor, denoted β_1^* , is reduced from its initial value $\bar{\beta}_1^f$,

$$\beta_1^* = \bar{\beta}_1^f - \frac{A\pi_0}{2CH(1 + \pi_0)} \beta_1^h < \bar{\beta}_1^f. \quad (29)$$

Each entrepreneur sells fraction w^* of the firm, retaining fraction

$$(1 - w^*) = \frac{1}{H}. \quad (30)$$

In the absence of risk substitution moral hazard, each entrepreneur simply owns the market fraction of the firm. The entrepreneur rationally anticipates that decreasing the aggregate risk of the firm, and hence increasing the firm-specific risk, increases the equilibrium value of its shares (equation (12)).¹⁶ Hence, in equilibrium, the entrepreneur optimally reduces the aggregate-risk loading of the firm, choosing $\beta_1^* < \bar{\beta}_1^f$.

Now consider owner-managed firms when investors do not observe the firm’s risk loadings. In this case, entrepreneurs do not fully internalize the cost borne by the rest of the economy due to an increase in their firm’s aggregate-risk beta. Because entrepreneurs are restricted not to trade the firm-specific risk component of their firm’s cash flow, they privately prefer to increase their firm’s aggregate-risk beta in order to reduce the fraction of their own wealth that is composed of unhedgeable firm-specific risk. In equilibrium, they will trade such aggregate risk and re-balance their portfolios. However, such risk substitution is costly for investors: investors’ endowments are exposed to aggregate risk, but not to firm-specific risk. The result is that investors can bear the firm-specific risk supplied by the stock market at a lower welfare loss than they can bear the aggregate risk.

Entrepreneurs can, however, design the ownership structure to reduce the extent of inefficient risk substitution, that is to create an incentive to decrease the aggregate-risk beta of cash flows. We characterize the equilibrium loading on aggregate risk, β_1^f , and also the condition on the initial loading $\bar{\beta}_1^f$ that guarantees the equilibrium level of ownership retained by the entrepreneur is smaller than the market share.

Bisin and Gottardi (1999) study a different equilibrium concept appropriate when the equity ownership structure is also not observable.

¹⁶ If the entrepreneurs did not change β_1 the marginal cost of an infinitesimal reduction of β_1 (and related increase in β_2) would be 0, whereas the marginal benefit for investors of such a change would be positive and proportional to the relative price of the factors.

Proposition 2. For owner-managed firms with moral hazard, in equilibrium, the loading on the aggregate risk factor, β_1^{**} , is such that

$$\beta_1^* < \beta_1^{**} < \bar{\beta}_1^f. \tag{31}$$

The fraction of the firm retained by the entrepreneur, $(1 - w^{**})$, is such that

$$(1 - w^{**}) < (1 - w^*) \tag{32}$$

if

$$\bar{\beta}_1^f > K \prod_{h=2}^H \beta_1^h, \quad \text{where} \quad K = 1 + \frac{A}{4CH^2}. \tag{33}$$

At equilibrium, the optimal choice of w induces entrepreneurs to decrease the aggregate cash-flow beta of their firms, $\beta_1^* < \bar{\beta}_1^f$, but not fully to the level without moral hazard, $\beta_1^* < \beta_1^{**}$. When the intrinsic aggregate-risk beta of the firm $\bar{\beta}_1^f$ is sufficiently high and/or the aggregate-risk beta of investors' endowments $\prod_{h=2}^H \beta_1^h$ is sufficiently low, condition (33) is satisfied and entrepreneurs hold a smaller fraction of the firm compared to the benchmark case, $(1 - w^{**}) < (1 - w^*)$.

This result is important in the context of our analysis. It demonstrates that, under certain conditions, the optimal contract designed to mitigate the risk-substitution moral hazard requires entrepreneurs to hold a smaller fraction of the firm than they would hold if such moral hazard were not to be present. We interpret this as a sort of dampening of pay-performance sensitivity: ownership can in fact have adverse incentive effects on entrepreneurs by providing incentives for risk substitution. As a consequence, firms where the risk-substitution problem is most severe, for example, pro-cyclical firms which intrinsically have high aggregate risk, should optimally design contracts offering a smaller equity ownership to entrepreneurs. There exists no closed-form characterization for the equilibrium dependence of equity ownership on the firm's intrinsic aggregate-risk loading. Hence, we have confirmed this implication numerically; see Figure 2.

Before we discuss the empirical implications of Proposition 2, we discuss condition (33) underlying the dampening of pay-performance sensitivity. We present its intuitive interpretation and discuss its reasonableness from an empirical standpoint.¹⁷

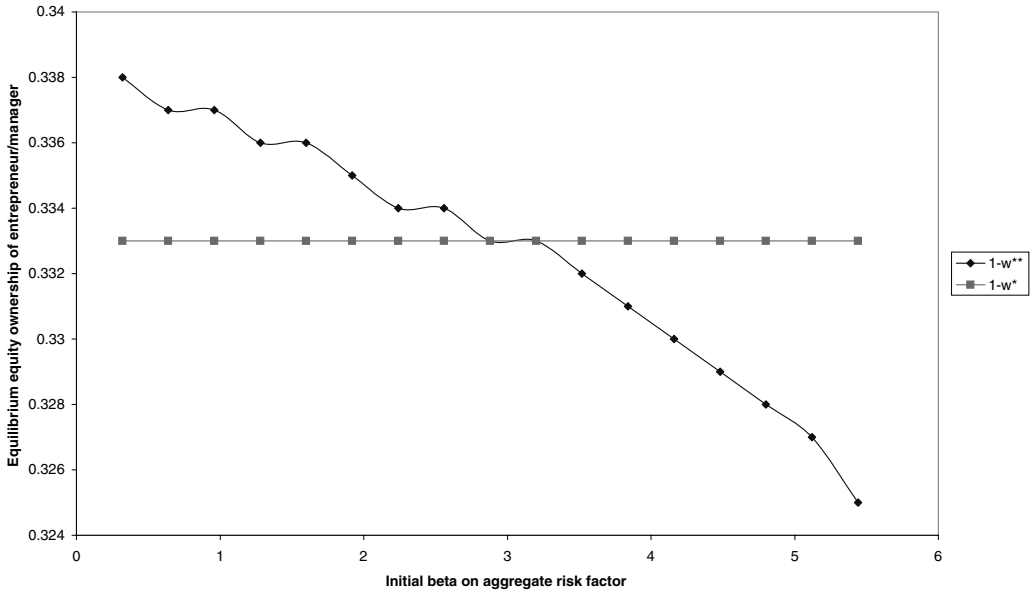
On the one hand, entrepreneurs benefit from increasing aggregate risk of firm cash flows because this reduces their exposure to unhedgeable, firm-specific risk. On the other hand, entrepreneurs also face a cost from doing so. In equilibrium, entrepreneurs diversify their personal portfolio by trading the aggregate-risk component of their wealth, $(1 - w)\beta_1^f$, at the given price π_1 , and retain only the average market component of this risk, $\frac{1}{H} \prod_{h=1}^H \beta_1^h$. Because aggregate risk is disliked by agents, it is sold at a negative price and its re-balancing is costly for entrepreneurs (by increasing the loading of β_1 and then re-balancing their portfolio, the entrepreneur, depending on the fraction of the firm he owns due to the incentive compensation contract, either increases the amount of aggregate risk he sells or else decreases the amount he buys, at a negative price). In other words, the cost of trading aggregate risk to the market counteracts the entrepreneurial incentives to increase the aggregate risk of cash flows.

The effectiveness of using ownership structure to pre-commit a reduction in the aggregate cash-flow beta depends upon the relative strengths of these two conflicting effects. The price of aggregate-risk π_1 increases (in magnitude) with the aggregate-risk beta of investors' endowments, $\prod_{h=2}^H \beta_1^h$. When the aggregate-risk exposure of the investors' endowment $\prod_{h=2}^H \beta_1^h$ is high, it is relatively more costly for entrepreneurs to sell aggregate risk in capital markets. The optimal pre-commitment device offered by an incentive compensation scheme is then one where the entrepreneur retains a fraction of the firm that exceeds the market share. This induces the

¹⁷ The formal argument is based on the mixed partial derivative of entrepreneurial objective (equation (24)) with respect to the aggregate-risk beta of cash flows, β_1^f , and the share retained, $(1 - w)$; see Appendix B for the formal proof.

FIGURE 2

EQUILIBRIUM OWNERSHIP AND INITIAL BETA OF THE FIRM



This figure plots $(1 - w^{**})$, the equilibrium ownership in the moral-hazard economy of Section 3, as a function of $\bar{\beta}_1^f$, the initial or intrinsic aggregate-risk loading of the firm. The benchmark ownership in the case of no moral hazard, $(1 - w^*)$, is also plotted. The numerical values are based on an example economy with the following parameter values: $A = 0.25$, $H = 3$, $J = 2$, $y_0 - E(y_1) = -5.0$, $\beta_1^2 = 5.0$, $\beta_2^2 = 5.0$, $\beta_1^3 = -2.5$, $\beta_2^2 = -5.0$, $\bar{V} = 64.0$, and $C = 0.06$. Note that $\frac{H}{h=2} \beta_1^h = 2.5 > 0$ AND $\frac{H}{h=2} \beta_1^h = 0$, consistent with the assumptions. The initial beta of the firm on aggregate risk, $\bar{\beta}_1^f$, is varied from 0.32 to 5.44. The competitive equilibrium is computed by a numerical fixed-point algorithm using the analytical expressions in Appendix A.

entrepreneur to diversify by trading in capital markets: because the quantity of aggregate risk the entrepreneur has to sell increases in the aggregate-risk beta of the firm, the entrepreneur is incentivized to choose a smaller aggregate beta. Formally, a sufficient condition for this case to arise is $\bar{\beta}_1^f < \frac{H}{h=2} \beta_1^h$ ¹⁸

However, when $\frac{H}{h=2} \beta_1^h$ is sufficiently low relative to $\bar{\beta}_1^f$, the cost of hedging aggregate risk is not too high and entrepreneurs could diversify easily by trading in the market. In this case, the only feasible pre-commitment device is one where the entrepreneur retains a smaller fraction of the firm than the market share, and pay-performance sensitivity is dampened. This exposes entrepreneurs to less firm-specific unhedgeable risk than in the benchmark case. By reducing the aggregate-risk component of the firm's cash flow, the entrepreneur gains (i) by buying more of this risk in the market (at a negative price) and (ii) by increasing his exposure to unhedgeable, firm-specific risk.

¹⁸ It is important to notice that in this case, the ability of entrepreneurs to hedge their aggregate-risk exposure is in fact instrumental in inducing them to reduce in equilibrium their firms' loading on aggregate risk. In particular, it is the cost of hedging aggregate risk at market prices which counteracts the entrepreneurial incentive to reduce the firm-specific risk of cash flows. Indeed, in the context of risk-substitution moral hazard, if entrepreneurs and managers are restricted from trading in *any* capital market (aggregate or firm-specific risk), then they would have no incentives whatsoever to lower their firms' aggregate risk. However, restrictions on the entrepreneurs' position in firm-specific risks are necessary to provide incentives to entrepreneurs facing moral hazard due, for example, to hidden effort provision or cash-flow diversion. Therefore, throughout our analysis, we consistently impose restriction on entrepreneurs on trading in firm-specific risks but not in aggregate risks. As such, restrictions on entrepreneurs' trading of aggregate risks in capital markets are hard—perhaps impossible—to implement, as they can be circumvented by resorting, for example to anonymous private accounts or family accounts.

To better understand condition (33) from an empirical standpoint, suppose that the aggregate risk factor x_1 is perfectly correlated with $\sum_{h=2}^H y_1^h$, the non-corporate sector (investors') endowment of the economy. Then, x_1 could be interpreted as the gross domestic product (GDP) minus the corporate sector output, but normalized to have unit variance. Thus, $\sum_{h=2}^H \beta_1^h$ equals $\frac{\bar{V}_{nc}}{\sqrt{\bar{V}}}$, where \bar{V}_{nc} is the variance of the non-corporate sector endowment. Furthermore, $\bar{\beta}_1^f$ equals $\rho\sqrt{\bar{V}}$, where \bar{V} is the variance of the corporate sector endowment, and ρ is the correlation between corporate sector and non-corporate sector endowments. Finally, let H go to infinity, keeping \bar{V}_{nc} and ρ constant.¹⁹ Then, K tends to unity, and condition (33) requires that $\rho^2 \bar{V} > \bar{V}_{nc}$ or, in other words, that the correlation of corporate sector cash flows and non-corporate sector endowments be high and that the variability of corporate sector cash flows be large relative to the variability of non-corporate sector endowments. Empirical evidence suggests that the corporate sector output of economies is highly correlated with the non-corporate sector output, and is much more variable.²⁰

4. Empirical implications

■ We discuss in this section the empirical implications of Proposition 2.²¹

The equilibrium relationship between equity ownership ($1 - w^{**}$) and the firm's intrinsic aggregate-risk loading $\bar{\beta}_1^f$ is negative, as implied by Proposition 2. This relationship, illustrated in Figure 2, represents an interesting theoretical result of our analysis. Although intrinsic aggregate-risk loadings are exogenous parameters in our model, they are not directly observable. However, our analysis also identifies structural relationships between managerial ownership, risk composition, expected returns, and firm value. These relationships have several important empirical implications.

First, consider the equilibrium relationship between managerial ownership ($1 - w^{**}$) and the firm's risk composition $\beta_1^f(w^{**})$. This relationship is numerically illustrated in Figure 3, which shows that firms whose managers have larger equity ownership in equilibrium are characterized by less aggregate risk in equilibrium. This is a structural relationship between two endogenous variables of our model: any combination of ownership and risk results from the solution of the optimal contracting problem.²² In our economy, aggregate-risk loading of the firm is linked to the firm's expected return through the CAPM pricing rule. Therefore, this analysis implies, other things being equal, a negative relationship between managerial ownership and expected returns. To our knowledge, this agency-theoretic implication for asset prices and returns has not yet been explored empirically.

The second and most important implication of our results concerns the widely documented cross-sectional relationship between managerial ownership and the firm's performance, measured by the ratio of the firm's market value to book value, that is, the empirical proxy for Tobin's Q. The uncovered relationship between performance and inside ownership is non-monotonic: the market-to-book ratio first increases (Tobin's Q increases) with inside ownership for low ownership

¹⁹ This can be achieved, for example, by distributing investors into a continuum of cohorts that are ranked by the correlation of investors' endowment with corporate sector endowment, the correlations ranging from a minimum negative value to a maximum positive value.

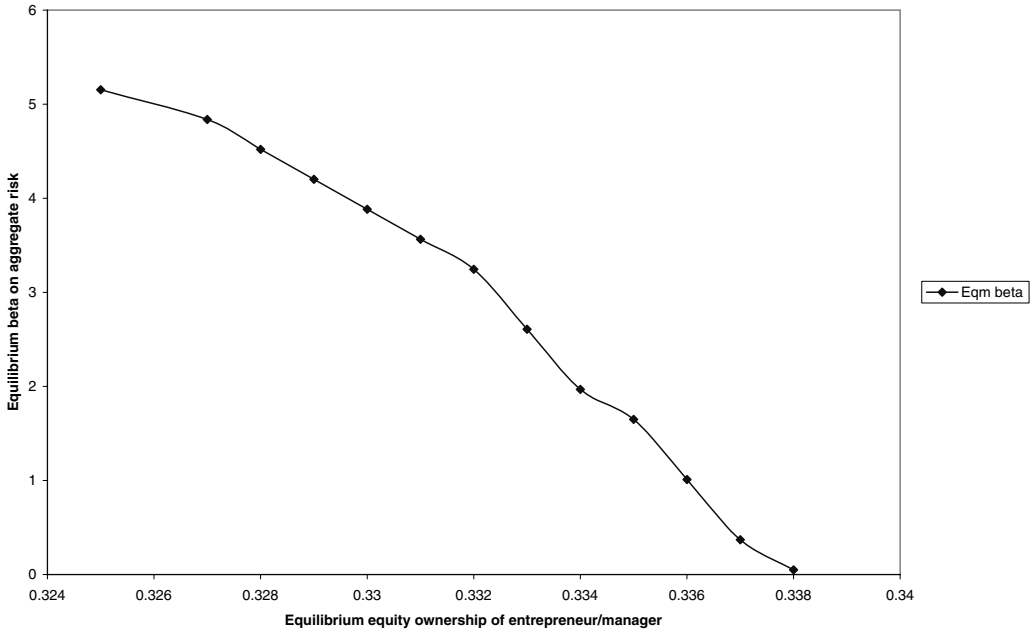
²⁰ For example, based on data from the National Income and Product Accounts Table, the de-trended corporate sector output (growth rate) in the United States during 1946–2003 is approximately 1.6 (1.3) times as variable as the de-trended non-corporate sector output (growth rate), where the non-corporate sector output is measured as the difference between the gross domestic product and the corporate sector output. The corporate and non-corporate sector outputs are almost perfectly correlated for the United States. These calculations suggest that the condition $\rho^2 \bar{V} > \bar{V}_{nc}$ is satisfied for the United States.

²¹ In this discussion, we use "managers" and "entrepreneurs" interchangeably. Recall that in Section 6, we show formally that the analysis of owner-managed firms extends isomorphically to the case of "corporations," where investors hire a manager to run the firm, and optimally design his incentive compensation.

²² This structural modelling approach is similar in spirit to important antecedents such as Demsetz and Lehn (1985), Himmelberg, Hubbard, and Love (2002), Core, Guay, and Larcker (2003), and, especially in the context of this article, Coles, Lemmon, and Meschke (2006).

FIGURE 3

EQUILIBRIUM BETA OF THE FIRM AND ENDOGENOUS EQUITY OWNERSHIP



This figure plots $\beta_1^f(w^{**})$, the equilibrium beta of the firm on aggregate risk in the moral-hazard economy of Section 3, as a function of $(1 - w^{**})$, the equilibrium ownership in the moral-hazard economy, when $\bar{\beta}_1^f$, the initial or intrinsic aggregate-risk loading of the firm, is varied. The numerical values are based on an example economy with the following parameter values: $A = 0.25$, $H = 3$, $J = 2$, $y_0 - E(y_1) = -5.0$, $\beta_1^2 = 5.0$, $\beta_2^2 = 5.0$, $\beta_1^3 = -2.5$, $\beta_2^3 = -5.0$, $\bar{V} = 64.0$, and $C = 0.06$. Note that $\sum_{h=2}^H \beta_1^h = 2.5 > 0$ and $\sum_{h=2}^H \beta_1^h = 0$, consistent with the assumptions. The initial beta of the firm on aggregate-risk $\bar{\beta}_1^f$ is varied from 0.32 to 5.44.

levels, and it decreases for higher ownership levels. Early evidence of this relationship includes Morck, Shleifer, and Vishny (1988), McConnell and Servaes (1990, 1995), and Hermalin and Weisbach (1991), and McConnell, Servaes, and Lins (2004) provide a more recent reassessment confirming this evidence. See, for instance, Figure 1 in Morck, Shleifer, and Vishny (1988).

It is a theoretical challenge to explain this nonmonotonic relationship between performance and inside ownership as an equilibrium relationship. This is because in many agency-theoretic problems (though not all), the endogenous relationship between firm value and ownership is negative: higher ownership is required only to address a more severe agency problem. In contrast, our result regarding the dampening of pay-performance sensitivity implies that as the risk-substitution problem becomes more severe, ownership is in fact lowered in equilibrium. These two facts put together can provide a structural explanation for the nonmonotonicity: in particular, our analysis of ownership and risk-substitution moral hazard can explain the positive relationship between firm value and ownership.

To see this implication, it is useful to consider a more general model of managerial choice than the one we have studied so far. In particular, we add to our model a version of Jensen's (1986) free cash-flow problem. The manager, besides choosing the firm's risk loadings, can also divert part of the firm's cash flow into private benefits. Notable examples of this agency problem include entrenchment, empire building, as well as a forthright diversion of cash flow into private accounts.²³ Formally, the manager can give up consuming a fraction η of the firm's expected cash

²³ Empirical studies documenting versions of the free cash-flow problem include Lang, Stulz, and Walking (1991), Mann and Sicherman (1991), and Blanchard, Lopez-de-Silanes, and Shleifer (1994).

flow, $E(y_1^f)$, at a nonpecuniary cost $C_2(\eta E(y_1^f))^2$ to be added to the cost of his choice of betas.²⁴ The parameter η thus captures the severity of the free cash-flow problem.

As noted before, in an economy with only the free cash-flow problem, inside ownership increases as the severity of the cash problem grows, as has also been documented empirically by Bizjak, Brickley, and Coles (1993). Simultaneously, firm value decreases because of greater cash-flow expropriation and greater cost of incentive compensation. As a consequence, the free cash-flow agency problem could explain the negatively sloped part of the empirical relationship between ownership and performance, but cannot explain the positive slope observed for small ownership levels.²⁵

Consider now the economy with both free cash flow and risk-substitution moral hazard. In this economy, an increase in managerial ownership has potentially two contrasting effects: increased ownership ameliorates the free cash-flow problem while inducing a greater substitution from firm-specific risk toward aggregate risk. Keeping constant the extent of the free cash-flow problem η , the higher the initial loading on the aggregate risk factor $\bar{\beta}_1^f$, the lower the inside ownership and the lower the firm value in equilibrium. The resulting positive relationship between performance and ownership is illustrated in Figure 4 for the parameters of our basic calibration, where performance is measured by market value (book value is implicitly assumed constant in our whole analysis).

Next, postulate that the severe free cash-flow problem (high η) tends to induce relatively large inside ownership levels at equilibrium. Then, we can explain the positive relationship between ownership and performance as illustrated in Figure 1²⁶: at low ownership levels, the relationship between ownership and performance is driven by the optimal ownership contract that trades off a small free cash-flow problem with a predominant risk-substitution problem. In contrast, high inside ownership levels would be an optimal contracting response to a relatively dominant free cash-flow agency problem. In turn, this explains the negative relationship between ownership and performance at high ownership levels. Note that, in this range, the excessive substitution of firm-specific risk for aggregate risk reinforces the reduction in market value of the firm by raising the expected return.

In terms of underlying structural parameters, this explanation relies on the thesis that the free cash-flow problem (η) and risk-substitution problem ($\bar{\beta}_1^f$) are negatively correlated. In the left of Figure 1, where the relationship between performance and ownership slopes upward, the initial beta on the aggregate risk factor, $\bar{\beta}_1^f$, is large but severity of the free cash-flow problem, η , is low. The relationship thus slopes upward until $\bar{\beta}_1^f$ declines sufficiently and η increases enough to dominate. Then, moving to the right in Figure 1, the aggregate-risk beta is low but cash-flow appropriation is high and increases further, so that equilibrium ownership rises but firm value declines.

²⁴ The manager's utility function becomes

$$\frac{(1 + \pi_0)}{A} e^{-A c_0^{\frac{1}{\alpha}} w, \beta_1^f, \beta_2^f, \eta, p^f - C \beta_1^f - \bar{\beta}_1^f{}^2 - C_2 \eta E y_1^f{}^2}$$

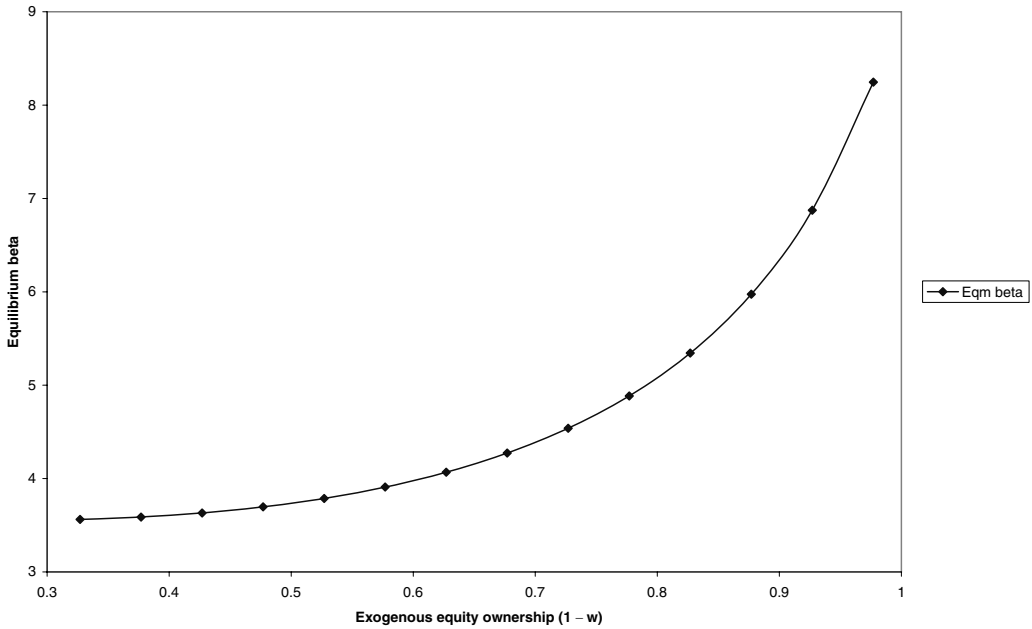
Note that consumption at time 0 now depends on η . We omit the straightforward, even if notationally cumbersome, analysis required to study this economy. It should be pointed out that one advantage of adding the free cash-flow problem is to dispose of the (counterfactual) literal implication of Proposition 2 that the fraction of the firm awarded to the manager might be smaller than the market share.

²⁵ Depending on the specific form of the costs of corporate control, the optimal contract in the free cash-flow problem could be such that no incentive compensation is provided for small enough agency problems. In this case, we would observe a mass of firms with no ownership and relatively low market-to-book ratios, but not the documented positive relationship.

²⁶ A fully structural calibration of the model is beyond the scope of the current article. Figures 1–4 are thus illustrative in nature. In particular, the equilibrium levels of ownership observed would depend on the relative magnitudes of the risk-substitution moral hazard (which induces the endogenous ownership in Figure 3) and the alternative rent-seeking moral hazard (which is unmodelled in our article and induces the exogenous ownership in Figure 4). Furthermore, the piecewise linear feature of Figure 1 is just an approximation.

FIGURE 4

EQUILIBRIUM BETA OF THE FIRM AND EXOGENOUS EQUITY OWNERSHIP



This figure plots the equilibrium market value of the firm, $(1 - w^{**})p^f(w^{**})$, as a function of $(1 - w^{**})$, the equilibrium ownership in the moral-hazard economy of Section 3, when $\bar{\beta}_1^f$, the initial or intrinsic aggregate-risk loading of the firm, is varied. The numerical values are based on an example economy with the following parameter values: $A = 0.25$, $H = 3$, $J = 2$, $E(y_1) = 10$, $y_0 - E(y_1) = -5.0$, $\beta_1^2 = 5.0$, $\beta_2^2 = 5.0$, $\beta_1^3 = -2.5$, $\beta_2^3 = -5.0$, $\bar{V} = 64.0$, and $C = 0.06$. Note that $\frac{H}{h=2} \beta_1^h = 2.5 > 0$ and $\frac{H}{h=2} \beta_2^h = 0$, consistent with the assumptions. The initial beta of the firm on aggregate-risk $\bar{\beta}_1^f$ is varied from 0.32 TO 5.44.

The positive relationship between managerial ownership and performance as measured by firm value for low levels of ownership (Figure 4) is also intimately tied to the negative relationship between aggregate risk and ownership obtained in our model (Figure 3). In other words, firms with a low managerial ownership in equilibrium should display low valuation, high aggregate risk, and, hence, also high expected returns. Evidence for these implications thus constitutes indirect evidence in support of our structural explanation of the hump-shaped relationship. Such supporting evidence is provided, for example, by Lamont and Polk (2001), who find that “diversification discount,” the low valuation of diversified firms, reflects in substantial part high expected returns in addition to low expected cash flows.

Finally, evidence from the literature that links managerial ownership to diversifying activities also bears on our analysis. In fact, our proposed distribution of the relative severity of different moral-hazard problems—the dominance of risk substitution at low ownership levels and of the free cash-flow problem at high ownership levels—has precise implications regarding the shape of the relationship between managerial ownership and diversification.

Typically, in the empirical literature, the diversifying activities of managers are quantified as R^2 in a regression of firm-level stock returns on market returns. By Figure 3, risk substitution implies a negative endogenous relationship of inside ownership with the aggregate risk of firm cash flows and, by implication, with the R^2 .²⁷ The free cash-flow moral hazard implies instead a positive relationship between inside ownership and aggregate risk. Therefore, if risk substitution is dominant at low levels of ownership and the free cash-flow problem is dominant at high levels

²⁷ A higher R^2 can result from a substitution of firm-specific risk with aggregate risk, as we consider, but also from a reduction of firm-specific risk with no effect on aggregate risk.

of ownership, then we should observe in data a nonlinear U-shaped relationship between R^2 and ownership.

The extant literature has derived inconclusive findings about the relationship between a firm's risk composition and managerial incentive compensation. Amihud and Lev (1981) document a significant negative relationship between R^2 from equity accounting returns, a proxy for a firm's diversification or systematic risk, and the equity ownership of officers and directors. May (1995) reports that a firm's diversification, measured using the covariance of a firm's stock returns with market returns around acquisitions, is positively correlated with the ratio of the manager's value of share ownership to total wealth. Denis, Denis, and Sarin (1997) identify a negative relationship similar to that of Amihud and Lev (1981) between several measures of diversification (fraction of firms with multiple segments, number of business segments reported by management, number of four-digit Standard Industrial Classification (SIC) codes assigned to a firm, and revenue-based and asset-based Herfindahl indices across business segments) and equity ownership (OWN) of officers and directors. Denis, Denis, and Sarin also find that this negative relationship holds at low to moderate levels of OWN, whereas very high levels of OWN yield a positive relationship between diversification and OWN. The positive relationship is, however, weaker and only marginal once industry fixed effects are allowed for. In a different approach, Aggarwal and Samwick (2003) test a structural model where diversification can arise either due to managerial risk aversion or to a managerial desire to "build empires." Their model determines incentive compensation endogenously, and their empirical results show a positive relationship between firm diversification and the extent of the manager's incentive compensation. They conclude that managers diversify in response to changes in empire-building motives rather than to reduce exposure to risk.²⁸

Because conclusive evidence linking managerial ownership to diversification based on existing theories has been elusive, we conclude that our analysis of the risk-substitution moral hazard has at least the potential to explain, as equilibrium relationships, various important cross-sectional relationships documented in corporate finance. Specifically, it offers a simultaneous explanation of the hump-shaped relationship between firm performance and inside ownership, and the U-shaped relationship between diversification (R^2) and inside ownership.

Note that a different explanation of the hump-shaped relationship between managerial ownership and performance is provided by Coles, Lemmon, and Meschke (2006). Their analysis is also based on a structural model of agency, but it exploits the variation of the optimal incentive compensation contract with the productivity of firm capital and the productivity of managerial effort rather than the relative dominance of different moral-hazard components, as in our case. They support their analysis by calibrating the model to U.S. firm data. Whereas their approach assumes that the cross-sectional variation in observed ownership and investment levels is entirely explained by exogenous heterogeneity in the productivity parameters mentioned above, our article assumes that the entire variation in ownership is driven by the risk-substitution moral hazard. Integrating the two setups and teasing out the relative importance of these two different assumptions is an open issue for future research.

5. Welfare properties

■ In this section, we address the following welfare questions. Do entrepreneurs hold too much or too little of their firms? Is there efficiency in the induced equilibrium loading of the firms' cash flows on the aggregate risk factor? Does the stock market contribute additional risk to the aggregate endowment risk of the economy? Is such additional risk inefficient? Not surprisingly, the presence of moral hazard implies that, in equilibrium, entrepreneurs diversify inefficiently by

²⁸ However, Aggarwal and Samwick (2003) do not consider the presence of rent-extraction problems other than empire-building motives. For instance, managers can affect expected cash flows by seeking private benefits. This is a form of moral hazard that does not lead to diversification as does building an empire. Importantly, if compensation has been designed to address such other rent-extraction problems, then a positive relationship between diversification and incentive compensation is perfectly consistent with diversification caused by managerial risk aversion.

overloading their firms on aggregate risk factors, relative to the first-best. However, the relevant welfare question is as follows: could a social planner regulate the firms' equity ownership structure so as to improve aggregate welfare, given the constraint that entrepreneurs will then choose technology to maximize their expected utility?

In CAPM economies, it is convenient to measure the welfare associated with the equilibrium of an economy relative to a benchmark. We take the welfare of the autarkic economy as this benchmark, where agents only trade the bond (see Willen, 1997, and Acharya and Bisin, 2006) and no capital budgeting takes place. The welfare of our economy, which we denote μ , is defined as the minimal aggregate transfer, in terms of time 0 consumption, needed to equate an agent's expected utility at equilibrium with his expected utility at autarky. Formally, let $[c_{0,c_1}^0] \equiv [c_0^h, c_1^h]_{h \in H}$ denote the competitive equilibrium allocation in the economy, and let $[c_0^a, c_1^a]$ be the equilibrium allocation at autarky. Let π_0 be the equilibrium price of the bond, and π_0^a the price of the bond at autarky. Let $U^h(c_0^h, c_1^h)$ denote the expected equilibrium utility of agent h , and let $U^{ah}(c_0^{ah}, c_1^{ah})$ be the corresponding expected utility at autarky. The aggregate compensating transfer, μ , is defined as

$$\mu = \sum_{h=1}^H \mu^h, \quad (34)$$

where the individual compensating transfer, μ^h , is given by the solution to

$$U^{a1}(c_0^{a1}, c_1^{a1}) + \mu^1 = U^1(c_0^1, c_1^1) - C \beta_1^f - \bar{\beta}_1^f{}^2, \quad \text{and} \quad (35)$$

$$U^{ah}(c_0^{ah}, c_1^{ah}) + \mu^h = U^h(c_0^h, c_1^h), \quad \text{for } h = 2, \dots, H. \quad (36)$$

We show in Appendix D (available upon request) that

$$\mu = -\frac{H}{A} \ln \frac{1 + \pi_0}{1 + \pi_0^a} - C \beta_1^f - \bar{\beta}_1^f{}^2. \quad (37)$$

Therefore, an economy is more efficient with a low equilibrium price of the risk-free asset and a correspondingly high risk-free return. This is because the risk-free rate increases when precautionary savings fall. This occurs when financial markets serve to hedge away the majority of agents' risk exposures.

□ **Efficiency of equity ownership and risk loadings.** The fraction w of the firm held by capital market investors, and the loadings β_j^f of the firm's cash flows on the economy's risk factors, are *first-best efficient* if they maximize the aggregate welfare index μ , taking into account the effects of w and β_j^f on competitive equilibrium prices. Formally, the first-best efficient choices of w and β_j^f maximize μ ,²⁹

$$\max_{w, \beta_1^f, \beta_2^f} -\frac{H}{A} \ln \frac{1 + \pi_0}{1 + \pi_0^a} - C \beta_1^f - \bar{\beta}_1^f{}^2 \quad (38)$$

subject to

$$\beta_1^f{}^2 + \beta_2^f{}^2 = \bar{V}, \quad (39)$$

where π_0 , the equilibrium price of the risk-free asset, is given by equation (A1) in Appendix A.

Proposition 3. For owner-managed firms with no moral hazard, the equilibrium fraction of the firm held by investors, w^* , and aggregate-risk loading, β_1^* , are first-best efficient.

In the absence of moral hazard, this result on the first-best efficiency is intuitive. Consider now the situation in which a moral hazard arises: owner-managed firms for which risk loadings are not observed by investors. In this case, first-best efficiency is too strong a welfare requirement.

²⁹ Note that the solution to the first-best problem as well as the constrained-efficiency problem is independent of π_0^a , so that the choice of benchmark in the definition of μ is arbitrary.

For the equilibrium to satisfy constrained efficiency, (i) the maps $\beta_j^f(w)$, defined in equations (24) and (25), determine the risk factor loadings of the firm’s cash flows, whereas (ii) the fraction of the firm held by capital market investors w maximizes the aggregate welfare index μ , given $\beta_j^f(w)$ and taking into account the effects of w and β_j^f on competitive equilibrium prices. Formally, the constrained-efficient choice of w maximizes μ ,

$$\max_w \quad -\frac{H}{A} \ln \frac{1 + \pi_0}{1 + \pi_0^a} - C \beta_1^f - \bar{\beta}_1^f \quad (40)$$

subject to

$$\beta_j^f = \beta_j^f(w), \quad j = 1, 2, \quad (41)$$

where π_0 , the equilibrium price of the risk-free asset, is given by equation (A1).

Proposition 4. For owner-managed firms with moral hazard, the equilibrium fraction of the firm held by investors, w^{**} , and the aggregate-risk loading, β_1^{**} , are constrained efficient.

To summarize, the private choice of entrepreneurs leads to socially optimal (second-best) outcomes. That is, the price mechanism efficiently aligns the objectives of entrepreneurs with those of the (constrained) social planner when the former designs the equity ownership structure to pre-commit capital budgeting choices. Entrepreneurs, although price-takers for prices of the risk factors, nevertheless face a price schedule for the firms they own and manage. They recognize that increasing the aggregate risk of the firm reduces the equilibrium value of its shares. In equilibrium, motivated by the capital gains from reducing this aggregate-risk component, entrepreneurs choose equity ownership structures that enable a pre-commitment of the (constrained) efficient choices of cash-flow loadings on risk factors.

6. Corporations

■ We define a corporation as a governance structure in which it is stockholders who hire a manager and choose the fraction $(1 - w)$ of equity with which to endow the manager. For a corporation, it is natural to interpret this stock grant as “incentive compensation.” For the sake of consistency, however, we refer to it as the firm’s ownership structure. In addition, the manager must be given a time 0 compensation W (in terms of time 0 consumption good units), such that the manager’s utility from time 0 compensation and the stock grant equals his reservation utility value of \bar{W} . We assume that the payment of this time 0 compensation is borne equally by all stockholders. The manager chooses the firm’s cash-flow betas after receiving the stock award and after trading has taken place.

The analysis of corporations mirrors the analysis of owner-managed firms. The capital budgeting problem again determines a map at equilibrium between managerial equity ownership, w , and the manager’s choice of risk composition, $\beta_j^f(w)$, $j = 1, 2$.³⁰ Stockholders then choose w and W to maximize the sum of their individual welfares, $\sum_{h=2}^H \mu^h$, where the individual compensating transfer, μ^h , is defined in equation (36) and characterized in Appendix D (available upon request),

$$\max_{w,W} \quad \sum_{h=2}^H \mu^h \equiv \sum_{h=2}^H c_0^h - c_0^{ah} - \frac{1}{A} \ln \frac{1 + \pi_0}{1 + \pi_0^a} \quad (44)$$

³⁰ The budget constraints of investors and managers, (5) and (7), respectively, are modified as

$$c_0^h = y_0^h - \frac{W}{H-1} - \pi_0 \theta_0^h - \pi_1 \theta_1^h - p^f \theta_f^h, \quad h > 1 \quad (42)$$

$$c_0^1 = y_0^1 + W - \pi_0 \theta_0^1 - \pi_1 \theta_1^1. \quad (43)$$

However, for parsimony, we use the same notation $\beta_j^f(w)$ as that for owner-managed firms with moral hazard. We show in Appendix E (available upon request) that the entrepreneur’s choice β_j^f for a given w in an owner-managed firm is identical to the manager’s choice of β_j^f in a corporation for that same w .

subject to the manager's ($h = 1$) reservation utility constraint

$$-\frac{(1 + \pi_0)}{A} e^{-A c_0^1 w, \beta_1^f, \beta_2^f, p^f - c \beta_1^f - \bar{\beta}_1^f} = \bar{W}, \tag{45}$$

and subject to

$$p^f = \pi_0 E y_1^f + \pi_1 \beta_1^f + \pi_2 \beta_2^f, \tag{46}$$

$$\beta_j^f = \beta_j^f(w), \quad j = 1, 2, \tag{47}$$

given $\pi_j, j = 0, 1, 2$.

Thus, as with owner-managed firms, corporations can use w to pre-commit to the *ex post* choice of technology characterized by $\beta_j^f(w)$. Entrepreneurs in owner-managed firms and stockholders in corporations both rationally anticipate the effect of technology choice on the value of the firm. As a result, all else equal, the proportion of the firm awarded to managers and the cash-flow betas in equilibrium are the same as those under the equilibrium for owner-managed firms. The two settings are in fact isomorphic.

Proposition 5. In the case of corporations, stockholders choose to retain for themselves the same fraction of the firm that an entrepreneur sells to the stock market in an owner-managed firm with moral hazard w^{**} . As a consequence, at equilibrium, managers hold a fraction $(1 - w^{**})$ of the firm and choose the same loading on the aggregate risk factor as entrepreneurs would in an owner-managed firm with moral hazard β_1^{**} .

It follows from Propositions 4 and 5 that, in the case of corporations too, the equilibrium fraction of the firm held by investors and the induced cash-flow betas are constrained efficient.

Our analysis of incentive compensation in the case of corporations also contributes to the understanding of the issue of relative performance evaluation (RPE). Holmstrom (1982) suggests that managers' compensation schemes should be independent of the aggregate component of their firms, which they cannot affect. In fact, payment schemes which explicitly insulate managers from aggregate risk, that is, which provide for explicit RPE clauses, are rarely observed in practice (see, e.g., Aggarwal and Samwick, 1999; Coles, Suay, and Woodbury, 2000; Deli, 2002). It has been argued in the literature, however, that this puzzle can be explained if managers can trade in capital markets, because in this case they can directly hedge the market risk in their portfolio and hence explicit RPE clauses in incentive schemes turn out to be irrelevant (see, e.g., Garvey and Milbourn, 2001; Jin, 2002; Core, Guay, and Larcker, 2003).

In the context of our article, because the moral-hazard problem that incentive compensation schemes must address includes risk substitution, incentive schemes providing for explicit RPE clauses may in fact be inefficient. An implication of Proposition 5 (and of Proposition 2) is that, when managers can affect the relative loading of aggregate and firm-specific risk, it is important that they directly trade the aggregate-risk component of their firm in capital markets at equilibrium prices. Because managers in equilibrium will hedge their aggregate-risk position to hold the market share of such risk, their risk-substitution choice can influence the amount of risk they trade and hence the hedging costs they face (see the discussion of the first case following Proposition 2). This, in turn, can provide managers with an incentive to reduce the aggregate-risk component of their firms. This is, in fact, in the interest of the firms' claimants and of efficient provision of incentives.³¹

7. Extensions

■ We have analyzed thus far a simple economy with two risk factors. We extend this analysis to consider different risk factors underlying the risk composition of firm's cash flows and agents'

³¹ For another explanation of the puzzle of the lack of RPE clauses in incentive compensation schemes, see Himmelberg and Hubbard (2000).

endowments. For this analysis, we more fully exploit the generality of the CAPM economy, formally stated in Appendix C (available upon request).³² We continue to interpret the firm as a representative sector, and thus often refer to firms as sectors.

□ **Multi-sector economy.** Consider an economy and a stock market with two sectors, f and g . The economy's factor structure is composed of a common risk factor, x_1 , and two additional risk factors, x_2 and x_3 , which are orthogonal to the common factor. In this multi-sector economy, the common factor can be interpreted as a "stock market index," and the additional risk factors can be interpreted as the "sector-specific" risks. The cash flows of the two sectors in terms of this basic factor structure of the economy are as follows:

$$y_1^f - E y_1^f \equiv \beta_1^f x_1 + \beta_2^f x_2, \quad (48)$$

$$y_1^g - E y_1^g \equiv \beta_1^g x_1 + \beta_3^g x_3. \quad (49)$$

Entrepreneurs cannot trade the shares of their own firms, but can trade otherwise in the stock market: entrepreneurs in sector f (respectively in sector g) can trade factors x_1 and x_3 (respectively x_1 and x_2).

At equilibrium, entrepreneurs load their firms' cash flows on x_1 , the component of cash flows that is common with the stock market index and is correlated with the aggregate endowment risk. Consequently, the cash flows of firms traded in the stock market, and by implication the stock returns of these firms, are excessively correlated across sectors, in addition to being correlated with the index returns and the aggregate portfolio.

More formally, entrepreneurs in sector f would want to trade the stock of sector g only as a way to hedge a part of the endowment risk. Entrepreneurs in sector f do not have incentives to trade factor x_3 , which is uncorrelated with their wealth. They trade in the stock market index x_1 only.³³ The argument is symmetric for entrepreneurs in sector g . Again, in general, the excessive correlation of stock market returns across sectors is enhanced for firms and economies that employ high-powered incentive compensation schemes to address alternative agency problems.

□ **Purely idiosyncratic risk in the stock market.** Consider a firm in our single-sector economy as in fact a continuum of identical firms of measure 1, indexed by $s \in (0, 1)$, and facing independent and identically distributed (i.i.d.) shocks.³⁴ We perturb our basic decomposition of stock market returns as follows:

$$y_1^{f,s} - E y_1^{f,s} \equiv \beta_1^f x_1 + \beta_2^f x_2 + x_2^s, \quad s \in (0, 1). \quad (50)$$

Factor x_2^s represents firm s 's purely idiosyncratic component: it is i.i.d. over s , uncorrelated with x_1 and x_2 , and satisfies $E(x_2^s) = 0$ and $\text{var}(x_2^s) = \sigma$. An entrepreneur cannot trade the shares of his own firm and of other firms in his sector, but can trade in the stock market otherwise. Specifically, entrepreneurs can trade factors x_1 , but cannot trade $(x_2 + x_2^s)$: the entrepreneur in firm s must hold the sector-specific component of his firm, x_2 , as well as the purely idiosyncratic risk component, x_2^s .

Entrepreneurs have incentives to overload their firms' risk onto the hedgeable component x_1 and away from the unhedgeable component $(x_2 + x_2^s)$. The resources entrepreneurs employ to reduce the loading of the firm on x_2^s are wasted from the point of view of the economy: each unhedgeable unit of the firm carries a variance of $(1 + \sigma)$ when held by the entrepreneur; the

³² For sake of expositional simplicity, we do not state our results in this section as formal propositions. Formal statements and proofs are available from the authors upon request.

³³ In fact, the equilibrium entrepreneurial ownership $(1 - w^{**})$ and the equilibrium loading β_1^{**} are chosen exactly as in Section 3 if the cost function for technology changes is commensurate.

³⁴ Working with a continuum of firms that face i.i.d. shocks requires abusing the law of large numbers. Working instead with a countable infinity of firms would avoid this abuse with no change to our analysis, but at a notational cost. See, for example, Al-Najjar (1995).

same unit, when sold to investors, carries an effective variance of 1, as investors can diversify away x_2^s across the continuum of firms. Thus, in equilibrium, the fraction of their firms that entrepreneurs hold decreases in σ , and the induced loading of each firm on the common stock market component x_1 increases in σ . In particular, for any $\sigma > 0$, this loading is greater than β_1^{**} , the equilibrium loading for $\sigma = 0$.

8. Conclusions

■ In this article, we examine implications for capital budgeting arising from a manager’s ability to hedge the aggregate risk of his exposure to firm cash flows. In particular, we focus on the incentives of managers to substitute the firm-specific risk of cash flows for aggregate risk, for example, by passing up entrepreneurial activity in favor of more prosaic projects. We show that such risk-substitution moral hazard increases aggregate risk in stock markets and reduces the ability of investors to share risks. We characterize the optimal ownership structure designed to counteract this moral hazard and study its welfare properties.

Our objective has been to identify managerial incentives as an endogenous determinant of the extent of aggregate or economy-wide risk. Our integrated model could be of more general use in financial economics. Potential applications include analysis of cross-sectional and time-series variation in firm-level and market-level volatility, and an in-depth study of changes in the risk composition of firms following corporate mergers and acquisitions.

Appendix A

■ **Capital asset pricing model economy and its competitive equilibrium.** Consider the economy described Section 2 and its competitive equilibrium defined in Section 2.

The competitive equilibrium of the two-period CAPM economy defined by equations (4)–(11) is characterized by prices of assets (π_j), portfolio choices (θ_j^h), and consumption allocations (c_j^h), given below.

$$\pi_0 = \exp \left\{ A(y_0 - E y_1) + \frac{A^2}{2H} \sum_{h=1}^H \left[1 - R_h^2 \text{var } y_1^h + \sum_{j \in J^h} \beta_j + \frac{1}{|H_j|} s_j \right]^2 \right\}, \tag{A1}$$

where

$$y_0 = \frac{1}{H} \sum_{h=1}^H y_0^h, \quad y_1 = \frac{1}{H} \sum_{h=1}^H y_1^h, \tag{A2}$$

$$\beta_1 = \frac{1}{|H_1|} \text{cov} \left((1-w)y_1^1 + \sum_{h=2}^H y_1^h, x_1 \right), \quad \beta_2 = \frac{1}{|H_2|} \text{cov} \left(\sum_{h=2}^H y_1^h, x_2 \right), \tag{A3}$$

$$s_0 = w E y_1^f, \quad s_j = w \beta_j^f, \quad j = 1, 2, \tag{A4}$$

$$\frac{\pi_j}{\pi_0} = E(x_j) - A \left(\beta_j + \frac{1}{|H_j|} s_j \right), \quad j = 1, 2, \tag{A5}$$

and for $h > 1$ (non-entrepreneurs),

$$R_h^2 \equiv \frac{\sum_{j=1}^2 \beta_j^h{}^2}{\text{var } y_1^h}, \tag{A6}$$

$$\theta_j^h = \beta_j + \frac{1}{|H_j|} s_j - \beta_j^h, \quad j = 1, 2, \tag{A7}$$

$$\theta_0^h = \frac{1}{1 + \pi_0} \left(y_0^h - E y_1^h - \sum_{j=1}^2 \pi_j \theta_j^h + \frac{A}{2} \text{var } c_1^h - \frac{1}{A} \ln(\pi_0) \right), \tag{A8}$$

$$c_1^h = \theta_0^h + \sum_{j=1}^2 \left(\beta_j + \frac{1}{|H_j|} s_j \right) x_j + y_1^h - \sum_{j=1}^2 \beta_j^h x_j, \tag{A9}$$

$$\text{var } c_1^h = \text{var } y_1^h - \sum_{j=1}^2 \beta_j^h{}^2 + \sum_{j=1}^2 \beta_j + \frac{1}{|H_j|} s_j{}^2, \tag{A10}$$

$$c_0^h = -\frac{1}{A} \ln \frac{1}{\pi_0} + E y_1^h + \theta_0^h - \frac{A}{2} \text{var } c_1^h, \tag{A11}$$

and, finally, for $h = f = 1$ (entrepreneur),

$$R_h^2 \equiv \frac{(1-w)^2 \beta_1^f{}^2}{\text{var } y_1^h}, \tag{A12}$$

$$\theta_1^h = \beta_1 + \frac{1}{|H_1|} s_1 - (1-w)\beta_1^f, \text{ and } \theta_2^h = 0, \tag{A13}$$

$$\theta_0^h = \frac{1}{1+\pi_0} y_0^h + w p^f - (1-w)E y_1^f - \pi_1 \theta_1^h + \frac{A}{2} \text{var } c_1^h - \frac{1}{A} \ln(\pi_0), \tag{A14}$$

$$c_1^h = \theta_0^h + \beta_1 + \frac{1}{|H_1|} s_1 x_1 + (1-w) y_1^f - \beta_1^f x_1, \tag{A15}$$

$$\text{var } c_1^h = (1-w)^2 \text{var } y_1^f - (1-w)^2 \beta_1^f{}^2 + \beta_1 + \frac{1}{|H_1|} s_1{}^2, \tag{A16}$$

$$c_0^h = -\frac{1}{A} \ln \frac{1}{\pi_0} + (1-w)E(y_1^f) + \theta_0^h - \frac{A}{2} \text{var } c_1^h. \tag{A17}$$

A derivation of these competitive equilibrium properties is a special case of the derivation for the general CAPM economy outlined in Appendix C, available upon request.

Appendix B

■ Proofs of propositions 1 and 2

Proof of Proposition 1. Under the benchmark case in equation (21), the entrepreneur simultaneously chooses the ownership structure, w , and the cash-flow betas, β_1^f and β_2^f . We first consider the choice of β_1^f for a given w , and next the choice of optimal w , taking into account the choice of β_1^f ; β_2^f is determined by the constraint (23).

The entrepreneur is a price-taker and trades only in asset 0 and asset 1, but rationally anticipates the effect of cash-flow betas on the price of the firm p^f (equation (22)). Using the competitive equilibrium outcomes (A13)–(A17) from Appendix A, we obtain

$$\frac{\partial}{\partial \beta_1^f} c_0^1(w, \beta_1^f, \beta_2^f, p^f) = \pi_0 E y_1^f + \pi_1 \beta_1^f + \pi_2 \beta_2^f - C \beta_1^f - \bar{\beta}_1^f{}^2 \tag{B1}$$

$$= \frac{\partial}{\partial \beta_1^f} \theta_0^1 - \frac{A}{2} \text{var } c_1^1 - 2C \beta_1^f - \bar{\beta}_1^f \tag{B2}$$

$$\begin{aligned} &= \frac{\partial}{\partial \beta_1^f} \frac{1}{1+\pi_0} [w\pi_1 \beta_1^f + w\pi_2 \beta_2^f + (1-w)\pi_1 \beta_1^f - \frac{A\pi_0}{2} \text{var } c_1^1] - 2C \beta_1^f - \bar{\beta}_1^f \\ &= \frac{\pi_1}{1+\pi_0} - \frac{\pi_2 w}{1+\pi_0} \frac{\beta_1^f}{\beta_2^f} + \frac{A\pi_0}{1+\pi_0} (1-w)^2 \beta_1^f - 2C \beta_1^f - \bar{\beta}_1^f \end{aligned} \tag{B3}$$

$$= \frac{A\pi_0}{1+\pi_0} \left[(1-w)^2 + \frac{w^2}{H-1} - \frac{1}{H} \beta_1^f - \frac{1}{H} \sum_{h=2}^H \beta_1^h \right] - 2C \beta_1^f - \bar{\beta}_1^f, \tag{B4}$$

where, to obtain equation (B3), we substitute constraints (22) and (23) and the restricted participation constraints $\theta_2^1 = 0$, $|H_1| = H$, and $|H_2| = H - 1$. Finally, to obtain equation (B4) from equation (B3), we substitute equilibrium prices and aggregate supplies using equations (A3)–(A5) and the maintained assumption (equation (17)) that $\beta_2 = 0$. The optimal β_1^f for a given w sets the partial derivative in equation (B4) to zero.

Consider next the choice of w given the choice of β_1^f .

$$\frac{d c_0^1 - C \beta_1^f - \bar{\beta}_1^f{}^2}{dw} = \frac{\partial c_0^1 - C \beta_1^f - \bar{\beta}_1^f{}^2}{\partial \beta_1^f} \frac{d \beta_1^f}{dw} + \frac{\partial c_0^1 - C \beta_1^f - \bar{\beta}_1^f{}^2}{\partial w} \tag{B5}$$

$$= \frac{\partial c_0^1 - C \beta_1^f - \bar{\beta}_1^f{}^2}{\partial w} \quad \text{by the envelope theorem.} \quad (\text{B6})$$

Because the entrepreneur is a price-taker, using equations (A13)–(A17), we obtain

$$\frac{\partial c_0^1 - C \beta_1^f - \bar{\beta}_1^f{}^2}{\partial w} = \frac{\partial c_0^1}{\partial w} = \frac{\partial}{\partial w} (1-w)E y_1^f + \theta_0^1 - \frac{A}{2} \text{var } c_1^1, \quad \text{where} \quad (\text{B7})$$

$$\begin{aligned} \theta_0^1 - \frac{A}{2} \text{var } c_1^1 &= \frac{1}{1+\pi_0} y_0^1 - (1-w)E y_1^f + \pi_0 w E y_1^f + \sum_{j=1}^2 \pi_j w \beta_j^f - \pi_1 \theta_1^1 \\ &\quad - \frac{A\pi_0}{2(1+\pi_0)} \text{var } c_1^1 - \frac{1}{A(1+\pi_0)} \ln(\pi_0). \end{aligned} \quad (\text{B8})$$

Simplifying using constraints (22) and (23), the maintained assumptions, and the expressions for equilibrium prices and aggregate supplies in equations (A3)–(A5), we obtain

$$\begin{aligned} \frac{\partial c_0^1 - C \beta_1^f - \bar{\beta}_1^f{}^2}{\partial w} &= -E y_1^f + \frac{1}{1+\pi_0} (1+\pi_0)E y_1^f + \pi_2 \beta_2^f + A\pi_0(1-w) \beta_2^f{}^2 \\ &= \frac{1}{1+\pi_0} \pi_2 \beta_2^f + A\pi_0(1-w) \beta_2^f{}^2 \end{aligned} \quad (\text{B9})$$

$$= \frac{A\pi_0}{1+\pi_0} \beta_2^f{}^2 \left(1-w - \frac{w}{H-1}\right). \quad (\text{B10})$$

This first-order derivative is set to zero at $w = w^* \equiv (1 - \frac{1}{H})$. Furthermore,

$$\frac{d^2 c_0^1 - C \beta_1^f - \bar{\beta}_1^f{}^2}{dw^2} = \frac{d}{dw} \left(\frac{\partial c_0^1 - C \beta_1^f - \bar{\beta}_1^f{}^2}{\partial w} \right) = \frac{\partial^2 c_0^1 - C \beta_1^f - \bar{\beta}_1^f{}^2}{\partial w^2} \quad (\text{B11})$$

$$= -\frac{HA\pi_0}{(H-1)(1+\pi_0)} \beta_2^f{}^2 < 0, \quad \text{because} \quad (\text{B12})$$

$$\frac{\partial^2 c_0^1 - C \beta_1^f - \bar{\beta}_1^f{}^2}{\partial \beta_1^f \partial w} = \frac{\partial}{\partial \beta_1^f} \frac{A\pi_0}{1+\pi_0} \beta_2^f{}^2 \left(1-w - \frac{w}{H-1}\right) \quad (\text{B13})$$

$$= -\frac{2A\pi_0}{1+\pi_0} \beta_1^f \left(1-w - \frac{w}{H-1}\right) = 0 \quad \text{at } w = w^*. \quad (\text{B14})$$

The equity ownership retained by the entrepreneur is thus given by $(1-w^*) = \frac{1}{H}$. Substituting $w = w^*$ in equation (B4) and setting it to zero, we find that the aggregate-risk beta chosen by the entrepreneur is $\beta_1^* = \bar{\beta}_1^f - \frac{A\pi_0}{2CH(1+\pi_0)} \sum_{h=2}^H \beta_1^h < \bar{\beta}_1^f$. Note that at $w = w^*$, $\frac{\partial^2}{\partial \beta_1^f} [c_0^1 - C(\beta_1^f - \bar{\beta}_1^f)^2] = -\frac{2A\pi_0}{1+\pi_0} < 0$, satisfying the optimality of β_1^* .

Proof of Proposition 2.

Sequence of steps. First, we characterize the technology choice β_1^{**} and the ownership structure choice w^{**} for the case of the owner-managed firm with moral hazard (Proposition 2). Next, we prove that the equilibrium aggregate-risk loading in the moral-hazard case exceeds the benchmark aggregate-risk loading, that is, $\beta_1^* < \beta_1^{**}$.

Step 1. First consider the entrepreneur's technology choice for owner-managed firms with moral hazard as specified in equation (24). The analysis differs from the proof of Proposition 1 (the case of no moral hazard) as follows: from the entrepreneur's standpoint, the firm's proceeds cannot be affected by a choice of betas, because the betas are not observed by investors. Formally, this implies that the constraint $p^f = \pi_0 E(y_1^f) + \pi_1 \beta_1^f + \pi_2 \beta_2^f$ (equation (22)) does not affect the capital budgeting problem in the case of moral hazard. Define $\hat{c}_0^1 \equiv c_0^1(w, \beta_1^f, \beta_2^f, p^f)$, where p^f is treated as a lump-sum constant in order to distinguish it from $c_0^1 \equiv c_0^1(w, \beta_1^f, \beta_2^f, p^f) = \pi_0 E(y_1^f) + \pi_1 \beta_1^f + \pi_2 \beta_2^f$.

Using the competitive equilibrium outcomes (A13)–(A17), we obtain

$$\frac{\partial}{\partial \beta_1^f} c_0^1(w, \beta_1^f, \beta_2^f, p^f) - C \beta_1^f - \bar{\beta}_1^f{}^2 \quad (\text{B15})$$

$$= \frac{\partial}{\partial \beta_1^f} \left(\theta_0^1 - \frac{A}{2} \text{var } c_1^1 - 2C \beta_1^f - \bar{\beta}_1^f{}^2 \right) \quad (\text{B16})$$

$$= \frac{\partial}{\partial \beta_1^f} \frac{1}{1 + \pi_0} (1 - w)\pi_1 \beta_1^f - \frac{A\pi_0}{2} \text{var } c_1^1 - 2C \beta_1^f - \bar{\beta}_1^f \tag{B17}$$

$$= \frac{\pi_1(1 - w)}{1 + \pi_0} + \frac{A\pi_0}{1 + \pi_0} (1 - w)^2 \beta_1^f - 2C \beta_1^f - \bar{\beta}_1^f \tag{B18}$$

$$= \frac{A\pi_0}{1 + \pi_0} (1 - w) \left[1 - w - \frac{1}{H} \beta_1^f - \frac{1}{H} \sum_{h=2}^H \beta_1^h \right] - 2C \beta_1^f - \bar{\beta}_1^f, \tag{B19}$$

where, to obtain equation (B18), we substitute only the constraint (25) and the restricted participation constraints $\theta_2^1 \equiv 0$, $|H_1| = H$, and $|H_2| = H - 1$; finally, to obtain equation (B19) from equation (B18), we substitute equilibrium prices and aggregate supplies using equations (A3)–(A5) and the maintained assumption that $\beta_2 = 0$.

Let $\beta_1^f(w)$ be the choice of β_1^f that sets the partial derivative in equation (B19) to zero. The solution to the capital budgeting problem in (24) is thus given by

$$\beta_1^f(w) = K_1(w)\bar{\beta}_1^f - K_2(w) \sum_{h=2}^H \beta_1^h, \quad \text{where} \tag{B20}$$

$$K_1(w) = 1 - \frac{A\pi_0(1 - w)}{2C(1 + \pi_0)} \left[1 - w - \frac{1}{H} \right], \quad \text{and} \tag{B21}$$

$$K_2(w) = \frac{A\pi_0(1 - w)}{2CH(1 + \pi_0)} K_1(w). \tag{B22}$$

Next, consider the choice of w by the entrepreneur of the owner-managed firm, as specified in equation (26). While choosing w , the rational expectation constraints apply both for the firm value (equation (27)) and for the effect of w on β_1^f (equation (28)). Thus, we obtain

$$\frac{d c_0^1 - C \beta_1^f - \bar{\beta}_1^f{}^2}{dw} = \frac{\partial c_0^1 - C \beta_1^f - \bar{\beta}_1^f{}^2}{\partial \beta_1^f} \frac{d\beta_1^f(w)}{dw} + \frac{\partial c_0^1 - C \beta_1^f - \bar{\beta}_1^f{}^2}{\partial w}, \tag{B23}$$

where $c_0^1 \equiv c_0^1(w, \beta_1^f(w), \beta_2^f(w), p^f) = \pi_0 E(y_1^f) + \pi_1 \beta_1^f + \pi_2 \beta_2^f$ as distinct from $\hat{c}_0^1 \equiv c_0^1(w, \beta_1^f, \beta_2^f, p^f)$. We examine successively each of the three terms in the above equation.

(i) First, note that

$$\frac{\partial c_0^1 - C \beta_1^f - \bar{\beta}_1^f{}^2}{\partial w} = \frac{A\pi_0}{1 + \pi_0} \beta_2^f{}^2 \left[1 - w - \frac{w}{H - 1} \right] = 0 \quad \text{at } w = w^*, \tag{B24}$$

as in the case of no moral hazard (see equation (B10) in the proof of Proposition 1, above).

(ii) Second, we examine $\frac{\partial [c_0^1 - C(\beta_1^f - \bar{\beta}_1^f)^2]}{\partial \beta_1^f}$. Note that, unlike the case of no moral hazard, $\frac{\partial [c_0^1 - C(\beta_1^f - \bar{\beta}_1^f)^2]}{\partial \beta_1^f} = 0$ (in general), because $\beta_1^f(w)$ is chosen treating p^f as a lump-sum constant. That is, $\beta_1^f(w)$ is chosen such that $\frac{\partial [c_0^1 - C(\beta_1^f - \bar{\beta}_1^f)^2]}{\partial \beta_1^f} = 0$, which is not generally the same as $\frac{\partial [c_0^1 - C(\beta_1^f - \bar{\beta}_1^f)^2]}{\partial \beta_1^f} = 0$. Thus, when $\beta_1^f = \beta_1^f(w)$,

$$\frac{\partial c_0^1 - C \beta_1^f - \bar{\beta}_1^f{}^2}{\partial \beta_1^f} = \frac{\partial \hat{c}_0^1 - C \beta_1^f - \bar{\beta}_1^f{}^2}{\partial \beta_1^f} - \frac{\partial \hat{c}_0^1 - c_0^1}{\partial \beta_1^f} = - \frac{\partial \hat{c}_0^1 - c_0^1}{\partial \beta_1^f}, \tag{B25}$$

$$\text{where } \frac{\partial \hat{c}_0^1 - c_0^1}{\partial \beta_1^f} = \frac{\partial}{\partial \beta_1^f} \frac{1}{1 + \pi_0} [w p^f - \pi_0 w E y_1^f - \pi_1 w \beta_1^f - \pi_2 w \beta_2^f] \tag{B26}$$

$$= \frac{1}{1 + \pi_0} [-\pi_1 w - \pi_2 w] - \frac{\beta_1^f}{\beta_2^f} \tag{B27}$$

$$= \frac{A\pi_0 w}{1 + \pi_0} \frac{1}{H} \sum_{h=2}^H \beta_1^h + \beta_1^f - \frac{w}{H - 1} \beta_1^f \tag{B28}$$

$$= \frac{A\pi_0 w}{H(1 + \pi_0)} \sum_{h=2}^H \beta_1^h + \left[1 - \frac{w}{w^*} \right] \beta_1^f, \tag{B29}$$

where we have employed the expressions for equilibrium prices and aggregate supplies in equations (A3)–(A5). It follows that at $w = w^*$, $\frac{\partial [c_0^1 - C(\beta_1^f - \bar{\beta}_1^f)^2]}{\partial \beta_1^f} < 0$, because $\frac{H}{h=2} \beta_1^h > 0$.

(iii) Finally, we examine $\frac{d\beta_1^f(w)}{dw}$. Because $\frac{\partial [c_0^1 - C(\beta_1^f - \bar{\beta}_1^f)^2]}{\partial \beta_1^f} = 0$ and $\frac{\partial^2 [c_0^1 - C(\beta_1^f - \bar{\beta}_1^f)^2]}{\partial \beta_1^{f2}} < 0$ at $\beta_1^f = \beta_1^f(w)$ by the optimality of $\beta_1^f(w)$ under moral hazard, it follows that

$$\frac{\partial^2 \hat{c}_0^1 - C \beta_1^f - \bar{\beta}_1^f{}^2}{\partial \beta_1^{f2}} \frac{d\beta_1^f(w)}{dw} + \frac{\partial^2 \hat{c}_0^1 - C \beta_1^f - \bar{\beta}_1^f{}^2}{\partial w \partial \beta_1^f} = 0 \tag{B30}$$

$$\Rightarrow \text{sign} \frac{d\beta_1^f(w)}{dw} = \text{sign} \left(\frac{\partial^2 \hat{c}_0^1 - C \beta_1^f - \bar{\beta}_1^f{}^2}{\partial w \partial \beta_1^f} \right). \tag{B31}$$

From equation (B19), we obtain

$$\frac{\partial^2 \hat{c}_0^1 - C \beta_1^f - \bar{\beta}_1^f{}^2}{\partial w \partial \beta_1^f} = \frac{A\pi_0}{1 + \pi_0} - 2(1 - w)\beta_1^f + \frac{1}{H} \sum_{h=2}^H \beta_1^h + \beta_1^f, \tag{B32}$$

where $\beta_1^f = \beta_1^f(w)$. It follows that at $w = w^* \equiv (1 - 1/H)$,

$$\frac{\partial^2 \hat{c}_0^1 - C \beta_1^f - \bar{\beta}_1^f{}^2}{\partial w \partial \beta_1^f} = \frac{A\pi_0}{H(1 + \pi_0)} \sum_{h=2}^H \beta_1^h - \beta_1^f(w). \tag{B33}$$

Thus, $\frac{d\beta_1^f(w)}{dw} < 0$ at $w = w^*$ if and only if $\beta_1^f(w^*) > \sum_{h=2}^H \beta_1^h$. Substituting $w = (1 - 1/H)$ in equation (B19) and equating it to zero yields

$$\beta_1^f(w^*) = \bar{\beta}_1^f - \frac{A\pi_0}{2CH^2(1 + \pi_0)} \sum_{h=2}^H \beta_1^h < \bar{\beta}_1^f. \tag{B34}$$

It follows from (i), (ii), and (iii) above that at $w = w^*$, $\frac{d[c_0^1 - C(\beta_1^f - \bar{\beta}_1^f)^2]}{dw} > 0$ iff $\bar{\beta}_1^f > K(\pi_0) \sum_{h=2}^H \beta_1^h$, where $K(\pi_0) = A\pi_0 / (2CH^2(1 + \pi_0)) + 1$. If the cost parameter C is sufficiently high, then the function $[c_0^1 - C(\beta_1^f - \bar{\beta}_1^f)^2]$ is globally concave.³⁵ Then, the optimal ownership structure under moral hazard is w^{**} , where $w^{**} > w^*$ iff $\bar{\beta}_1^f > K(\pi_0) \sum_{h=2}^H \beta_1^h$.

Furthermore, because $\beta_1^f(w^*) < \bar{\beta}_1^f$, and in both cases above w^{**} is chosen to reduce the aggregate-risk loading from its value at w^* , it follows that $\beta_1^{f**} = \beta_1^f(w^{**}) < \beta_1^f(w^*) < \bar{\beta}_1^f$. Because $\pi_0 \in [0, 1]$, $K(\pi_0) \in [1, K]$, where $K \equiv 1 + A/(4CH^2)$.

Step 2. Next, we prove that the choice of the risk loading, β_1^{f**} , is greater than the benchmark case (first-best), β_1^* . Because w^{**} is constrained efficient (as proved below in the proof of Proposition 4), it follows that at $w = w^{**}$,

$$\frac{d\mu}{dw} = \frac{\partial \mu}{\partial \beta_1^f} \frac{d\beta_1^f(w)}{dw} + \frac{\partial \mu}{\partial w} = 0 \Rightarrow \frac{\partial \mu}{\partial \beta_1^f} = - \frac{\frac{\partial \mu}{\partial w}}{\frac{d\beta_1^f}{dw}}. \tag{B36}$$

Now, consider the case where $\beta_1^f(w^*) > \sum_{h=2}^H \beta_1^h$. Then, for the moral-hazard economy, $\frac{d\beta_1^f(w)}{dw} < 0$ at $w = w^{**} > w^*$. By the first-best efficiency of w^* , we have $\frac{\partial \mu}{\partial w} = 0$ and $\frac{\partial^2 \mu}{\partial w^2} < 0$ at $w = w^*$. This, in turn, implies that $\frac{\partial \mu}{\partial w} < 0$ at $w = w^{**}$. From equation (B36), we conclude that $\frac{\partial \mu}{\partial \beta_1^f} < 0$ at $w = w^{**}$. That is, the choice of β_1^f under

³⁵ Define function $g(w, \beta_1^f(w)) \equiv c_0^1(w, \beta_1^f, \beta_2^f, p^f = \pi_0 E(y_1^f) + \pi_1 \beta_1^f + \pi_2 \beta_2^f) - C(\beta_1^f - \bar{\beta}_1^f)^2$, where $\beta_1^f = \beta_1^f(w)$ and $(\beta_1^f)^2 + (\beta_2^f)^2 = \bar{V}$. Then,

$$\frac{d^2 g}{dw^2} = \frac{\partial^2 g}{\partial w^2} + 2 \frac{\partial^2 g}{\partial \beta_1^f \partial w} \frac{d\beta_1^f(w)}{dw} + \frac{\partial^2 g}{\partial \beta_1^{f2}} \left(\frac{d\beta_1^f(w)}{dw} \right)^2 + \frac{d^2 \beta_1^f(w)}{dw^2} \frac{\partial g}{\partial \beta_1^f}. \tag{B35}$$

Note the following: (i) from equation (B12), $\frac{\partial^2 g}{\partial w^2} < 0, \forall w$; this is the global concavity of the entrepreneur’s objective as a function of w under the benchmark case. (ii) $\frac{\partial^2 g}{\partial \beta_1^{f2}} = \frac{\partial^2 c_0^1}{\partial \beta_1^{f2}} - C < 0$ for C sufficiently large. (iii) Using equation (B20), it can be shown that $\frac{d\beta_1^f(w)}{dw} \rightarrow 0$ as $C \rightarrow \infty$. (iv) Similarly, it can be shown that $\frac{d^2 \beta_1^f(w)}{dw^2} \rightarrow 0$ as $C \rightarrow \infty$. (v) From equation (B13), $\frac{\partial^2 g}{\partial \beta_1^f \partial w}$ is independent of C . Finally, (vi) from equation (B4), $\frac{\partial g}{\partial \beta_1^f}$ is bounded. It follows that for C sufficiently large, $\frac{d^2 g}{dw^2} < 0, \forall w$, that is if the moral-hazard problem is not “too severe,” then as a function of w the objective of the entrepreneur under the moral-hazard case is a “perturbation” around the globally concave objective under no moral hazard.

moral hazard, β_1^{**} , exceeds the first-best (for which $\frac{\partial \mu}{\partial \beta_1^f} = 0$). The proof for the case where $\beta_1^f(w^*) < \frac{H}{h-2} \beta_1^h$ follows analogously.

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