

# MANAGERIAL HEDGING AND PORTFOLIO MONITORING

---

**Alberto Bisin**  
New York University

**Adriano A. Rampini**  
Duke University

**Piero Gottardi**  
University of Venice

## Abstract

Incentive compensation induces correlation between the portfolio of managers and the cash flow of the firms they manage. This correlation exposes managers to risk and hence gives them an incentive to hedge against the poor performance of their firms. We study the agency problem between shareholders and a manager when the manager can hedge his compensation using financial markets and shareholders can monitor the manager's portfolio in order to keep him from hedging, but monitoring is costly. We find that the optimal incentive compensation and governance provisions have the following properties: (i) the manager's portfolio is monitored only when the firm performs poorly, (ii) the manager's compensation is more sensitive to firm performance when the cost of monitoring is higher or when hedging markets are more developed, and (iii) conditional on the firm's performance, the manager's compensation is lower when his portfolio is monitored, even if no hedging is revealed by monitoring. Moreover, the model suggests that the optimal level of portfolio monitoring is higher for managers of firms whose performance can be hedged more easily, such as larger firms and firms in more developed financial markets. (JEL: G30, D82)

---

## 1. Introduction

The objective of incentive compensation is to induce a correlation between managers' compensation and the cash flow of the firms they manage so as to induce

---

Acknowledgments: We thank Radhakrishnan Gopalan, Michael Fishman, Kathleen Hagerty, Narayana Kocherlakota, Corrado Rampini, Sonje Reiche, Ilya Strebulaev, and seminar participants at the London School of Economics, UCL, NUS, Northwestern, Pompeu Fabra, Carlos III, Alicante, Melbourne, Zurich, the Federal Reserve Bank of Richmond, the 2002 SED Annual Meeting, the 2005 SAET Conference, the 2006 AFA Annual Meeting, and the 2006 WFA Annual Meeting, as well as the editor, Patrick Bolton, and two anonymous referees for comments. Piero Gottardi gratefully acknowledges financial support from MIUR.

E-mail addresses: Bisin: [alberto.bisin@nyu.edu](mailto:alberto.bisin@nyu.edu); Gottardi: [gottardi@unive.it](mailto:gottardi@unive.it); Rampini: [rampini@duke.edu](mailto:rampini@duke.edu)

them to work diligently and increase firm performance.<sup>1</sup> But this correlation exposes managers to risk and hence gives them an incentive to trade in financial markets so as to hedge against the poor performance of their firms. In the 1990s several financial instruments were developed which allow managers to hedge the firm specific risk in their compensation packages. Examples of such instruments include zero-cost collars, equity swaps, and basket hedges. Although little data exist, off-the-record interviews with investment bankers reported in the press suggest that the market for executive hedging instruments is sizable and that most large investment banks offer such instruments.<sup>2</sup>

Many legal and financial commentators have argued that managerial hedging undermines incentives in executive pay schemes, significantly alters the executives' effective ownership of the firm, and hence has adverse effects on performance.<sup>3</sup> But as boards and shareholders recognize that managers might have the opportunity to hedge their incentive compensation packages, one should expect them to take this into account when designing their managers' incentive compensation and their firm's governance provisions. If shareholders were able to perfectly observe the managers' transactions, they could explicitly rule out the possibility that managers trade any hedging instruments. In practice, managers' portfolios are not publicly disclosed and they are difficult and costly to monitor. For one, disclosure rules regarding managerial transactions of hedging instruments are relatively lax,<sup>4</sup> and only few trades are effectively disclosed to investors and shareholders.<sup>5</sup> Moreover, financial markets have proved quite effective in designing instruments which overcome regulation, governance provisions, and tax laws. For instance, equity swaps have been substituted with collars when swaps became subject to more stringent tax treatment (see Schizer 2000).

---

1. For evidence on the relationship between managerial incentives and firm performance see, Morck, Shleifer, and Vishny (1988), and Jensen and Murphy (1990). See Murphy (1999) for a survey on incentive compensation.

2. See, for example, the *Economist* (1999a), Puri (1997), Smith (1999), and Lavelle (2001).

3. In the legal profession, see Easterbrook (2002), Schizer (2000), Bank (1995); in the financial press, see the *Economist* (1999a,b,c, 2002), Ip (1997), Lavelle (2001), Puri (1997), and Smith (1999).

4. Since September 1994 equity swaps and similar instruments must be reported to the Securities and Exchange Commission (SEC): on Table II of Form 4; Release No. 34-34514, and Release No. 34-347260. But the back page of Table II of Form 4 is not included in the electronic filing used by analysts; see Bolster, Chance, and Rich (1996) and Lavelle (2001). Finally, non-insiders and CEOs of non-U.S. firms are not obligated to disclose their trades. Recently, though, the Sarbanes-Oxley Act of 2002 introduced more stringent rules regarding the electronic filing of transactions involving such instruments and has substantially reduced the delay in disclosure, when disclosure is required.

5. In 1994 only one hedging transaction was disclosed to the SEC, Autotote's CEO equity swap, the case studied by Bolster, Chance, and Rich (1996). The number of transactions reported in subsequent years increased to 15 transactions in 1996, 39 in 1997, and 35 in 1998 (the whole 90 transactions are studied by Bettis, Bizjak, and Lemmon 2001), 31 transactions in 2000 (Lavelle 2001). No evidence is yet available about the effects of the Sarbanes-Oxley Act of 2002 on disclosures.

Although costly, monitoring of managers' portfolios can nonetheless help to align shareholders' and managers' objectives within an optimal incentive compensation contract. Managers are not restricted by law from trading derivatives on stocks of their own firm<sup>6</sup> but may be subject to derivative suits brought by shareholders for violation of fiduciary duty if financial transactions to hedge their incentive compensation are revealed.<sup>7</sup> For transactions disclosed to the SEC, shareholders can force executives to satisfy their burden of establishing the validity of the transaction. When instead monitoring reveals evidence of breach of disclosure, action can be pursued under securities law, which is easier than under corporate law (see Fox 1999).<sup>8</sup> Successful legal action allows a monetary recovery to the firm at least in the amount of the managers' gains on the hedging positions that are detected.<sup>9</sup>

In this paper we study the optimal contracts when managers have access to anonymous hedging instruments in financial markets and when shareholders can monitor the portfolios of managers. Optimal contracts include incentive compensation as well as governance provisions regarding the monitoring of managers' portfolios. Because, as we argued, managers' portfolios are difficult to monitor, we consider the case where monitoring is possible but costly and thus less than perfect. Hence, we study executive compensation with costly corporate governance. Also, in accordance with the limited possibilities for legal action by shareholders discussed previously, we assume that whenever hedging by a manager is detected, only the payoffs that the manager would receive from this activity can be seized by the shareholders. We will show, however, that our main results carry over to the

---

6. Under Section 16(c) of the Securities and Exchange Act of 1934, and Rule 16c-4, managers are only prohibited from selling their firm's stock short.

7. For a discussion of the fiduciary principle and derivative suits see, for example, Easterbrook and Fischel (1991), chapter 4, and Klausner and Litvak (2000). Of course, under Rule 10b-5 of the Securities Exchange Act of 1934, it is illegal for insiders to trade while in possession of material value-relevant information (insider trading). Although there is some evidence that the observed hedging transactions of executives might in part constitute insider trading (see Bettis, Coles, and Lemmon 2000), we concentrate in this paper on the pure hedging motives.

8. Derivative suits are more easily brought against executives whose compensation contracts explicitly state trading limitations. In practice this is still fairly rare, and when firms do have trading policies, they are usually not disclosed to minority shareholders; for a detailed discussion of such restrictions see Schizer (2000) and Bettis, Bizjak, and Lemmon (2001). This contractual practice could be motivated by the aim of protecting the firm against "frivolous" actions of shareholders; this is consistent with the practice of providing executives with insurance policies against such actions; see Klausner and Litvak (2000) for a discussion. Bebchuk, Fried, and Walker (2002) interpret the limited contractual restrictions of hedging instead as evidence of managerial rent extraction. See also Bebchuk and Fried (2003).

9. Only for actions brought by the SEC for violations of the securities law can courts grant "any equitable relief that may be appropriate or necessary for the benefit of investors" (Sarbanes-Oxley Act of 2002, Section 305, 5). In the case of insider trading during black-out periods, for example, it is "profit realized by a director or executive officer" that shall "be recoverable by the issuer" (Sarbanes-Oxley Act of 2002, Section 306, 2A). The Sarbanes-Oxley Act of 2002 does not explicitly state any provision for hedging in violation of fiduciary duty.

case where additional monetary penalties can be imposed on the manager when hedging is detected.

The main implication of our analysis concerning governance provisions is that monitoring of a manager's portfolio optimally occurs only when the performance of the firm is poor. Because for incentive reasons the manager's compensation is low when the firm does poorly, if the manager were to hedge he would buy claims which pay off when the firm does poorly. The fact then that shareholders could seize the payoffs of managerial hedging, if detected, because it violates fiduciary duty, implies that shareholders will monitor the manager's portfolio when such hedging positions would pay off, namely, when the firm performs poorly.

Moreover, conditional on the firm performing poorly, the optimal compensation of the manager is lower when the manager is monitored, and hence his portfolio scrutinized, than when the manager is not monitored. This is so even if monitoring does not reveal any hedging transactions of the manager. In other words, managers strictly prefer not to be monitored at the optimal contract, despite the fact that at the optimal contract they choose not to hedge their compensation. The manager's compensation both when he is monitored and when he is not monitored in states when the firm does poorly affects his incentive to work diligently. But the compensation when the manager is not monitored also affects his desire to hedge his compensation risk. To reduce the manager's desire to hedge his compensation, it is thus optimal to pay him more when he is not monitored, than when he is monitored. Consequently, in our model investigations regarding the managers' conduct are associated with reductions in their pay and benefits. This is in accord with the common perception that in practice agents who are monitored are worse off even if they did nothing wrong. The key for the result is that we assume that when the manager is monitored and hedging is detected his pay cannot be reduced (or at most can be reduced by a fixed amount), that is, managerial pay cannot be fully recovered if a violation of fiduciary duty is found.

The main implication of our analysis for incentive compensation is that when monitoring is costly or hedging markets are more developed, the incentives provided by shareholders to the manager are steeper. Thus, worse corporate governance implies that shareholders have to make managers' compensation more sensitive to the firm's performance. The intuition is as follows: When managerial hedging is costly to monitor, managers have to be induced to refrain from hedging by the structure of the compensation scheme rather than being forced to refrain by monitoring. Thus, shareholders have to make it expensive for managers to hedge. This is achieved by paying the manager more in states where the firm does well. We consider the case where the hedging market understands that, given that a manager is hedging, he will work less diligently and hence states with good performance are less likely, which is reflected in the price at which the manager can sell claims contingent on such states. In short, claims contingent on good performance trade at a discount in the hedging market. Thus, an increase

in the steepness of compensation decreases the present value of the manager's compensation in the hedging market and makes it more expensive for the manager to hedge. Thus, if the development of financial markets increases managers' ability to hedge, this, according to our analysis, may increase the optimal level of incentive pay as well as the optimal level of monitoring of managers' portfolios. Indeed, in countries where hedging markets have developed earlier, say the US and the UK, monitoring and disclosure requirements have appeared earlier than in countries where such hedging markets have developed more recently. And the development of hedging markets may have further increased the extent of incentive pay in these countries. Moreover, monitoring of managerial hedging is more of a concern, both in practice as well as according to theory, for the managers of larger firms who can hedge their compensation more easily using the contingent claims traded on their firms. Our model also predicts that the higher the level of monitoring as dictated by legal disclosure requirements or corporate governance rules, the less steep incentive contracts should be. Thus, the recent increase in disclosure requirements may bring a reduction in the steepness of incentive compensation and hence reduce the amount of stocks and options granted.

Finally we show that the managers' incentives are also affected by the possibility of trading claims whose payoff does not depend on the firm specific risk and hence whose fluctuations are not attributable to the manager's choice of effort. One example is the managers' ability to borrow and lend, that is, to trade a riskless asset. Similar considerations apply to the trade of market indices and basket hedges, where the derivative's value is based not only on the stock price of the employer but also on a basket of correlated stocks, which allow the manager to hedge the systematic risk in his compensation. Our analysis shows that imposing restrictions also on the trade of such claims would be beneficial, although this benefit is quantitatively smaller. Financial innovation that allows managers to trade claims contingent on their firms' specific risk makes the problem caused by hedging more severe and increases the optimal level of portfolio monitoring.

From the standpoint of the theory of optimal contracts, this paper introduces and studies a new class of principal agent problems, with stochastic monitoring of the agent's portfolio which is not otherwise observable. This class of problems has a wide range of applications that we do not explicitly explore in this paper. For example, consider a credit market where a borrower (the agent) has access to a primary lender (the principal), as well as to a secondary market for credit, and hence his total liabilities are not observable. In this context the stochastic monitoring technology represents the institution of bankruptcy, and an important component of the design of the optimal contract are the properties of such an institution.<sup>10</sup>

---

10. Bisin and Rampini (2006) study bankruptcy in a related environment, but without an explicit stochastic monitoring technology. Parlour and Rajan (2001) study a model in which the borrower may accept more than one loan contract and the borrower's incentives to default depend on the total amount borrowed.

We should also point out that not all hedging activity is undesirable and constitutes a violation of fiduciary duty. As discussed in Section 4, in the presence of tax advantages for incentive compensation shareholders may choose to give managers an excessive level of incentives while allowing at the same time partial hedging of the incentive compensation.

*Related literature.* In contrast to the set-up considered here, the theoretical literature on principal–agent problems has studied either the case in which the agent’s trades are perfectly observable (e.g., Prescott and Townsend 1984 and Bisin and Gottardi 2006), or the case in which they are unobservable (see Allen 1985; Arnott and Stiglitz 1991; Kahn and Mookherjee 1998; Pauly 1974; also Admati, Pfleiderer, and Zechner 1994; Bisin and Gottardi 1999; Bisin and Guitoli 2004; Bizer and DeMarzo 1992, 1999; Cole and Kocherlakota 2001; Park 2004). More specifically with regard to the application to managerial incentive compensation, Jin (2002), Acharya and Bisin (2005), and Garvey and Milbourn (2003) study the case where executives can anonymously trade market indices. Garvey (1993, 1997) and Ozerturk (2006) study the case where managers can hedge (without any monitoring) in financial markets by trading a single—exclusive—contract. However, this assumes that contracts traded in the hedging market exhibit stronger enforceability properties than the compensation contract itself, which seems counterintuitive, and implies that it should be optimal to have non-zero trade in the hedging market and that the possibility of engaging in unmonitored hedging entails no efficiency loss. On the other hand, we consider the case where managers can hedge their compensation by trading non-exclusive contracts (with costly monitoring); our conclusions are also rather different as we find that this possibility affects the form of the optimal compensation and entails an efficiency loss.

Costly monitoring has been introduced in the study of principal agent problems by, for instance, Townsend (1979), Gale and Hellwig (1985), and Mookherjee and Png (1989). They analyze situations where it is the realization of a privately observed state, rather than private hedging activity as in our paper, which can be monitored at a cost (costly state verification).<sup>11</sup> This class of models has different implications than our analysis of portfolio monitoring. In particular, in contrast to the findings of our paper, costly state verification models imply that managers strictly prefer to be monitored at the optimal contract, as their compensation is higher when they are monitored and found to have told the truth. This result is often considered

---

11. In addition, Winton (1995) studies costly state verification with multiple investors. Baiman and Demski (1980) and Dye (1986) study environments where it is the agents’ privately observed effort which can be monitored at a cost. To our knowledge, the only previous analysis of a principal–agent problem with limited observability of trades, through bankruptcy procedures, is in Bisin and Rampini (2006).

counterintuitive and we show that with our alternative assumptions about the feasible punishments, we obtain the empirically more plausible result that being monitored is considered bad news even by agents who did not violate any rules.

*Reader's guide.* The paper proceeds as follows. Section 2 studies the one period case, where firms have cash flow and managers are compensated at only one point in time. Most of the intuition and main results can be obtained in this case. Section 3 extends the analysis to two periods, which introduces intertemporal considerations. We consider both the case where managers can trade any claim contingent on the firms' specific risk as well as the case where they have access only to risk free borrowing and lending, which allows us to study the effect of financial innovation. Section 4 provides a discussion and Section 5 concludes. All proofs are in the appendices.

## 2. Incentive Compensation and Portfolio Monitoring: Static Case

### 2.1. Overview

Our analysis will be developed in the context of a simple standard agency environment with hidden effort (see, e.g., Grossman and Hart 1983). A (risk-neutral) principal owns a production process, whose outcome is uncertain, and has to hire a (risk-averse) agent to manage it. The agent's effort level in this task is not observable and affects the probability distribution of the process' outcome.

In this paper the principal and the agent are, respectively, the shareholders (or the board) and the manager of a firm. We study the optimal incentive compensation contract shareholders can write to align their objective with that of the manager when his effort is not observable and when (i) the manager can engage in trades in financial markets to hedge his risk, which may adversely affect his incentives, and (ii) shareholders can monitor the manager's trades in financial markets but monitoring is costly.

We consider first the case where there is a single period where production and payments take place. In the following section the analysis will be extended to allow for more production and payment dates.

*The manager and the shareholders.* Let  $S = \{H, L\}$ , with generic element  $s$ , describe the possible realizations of the uncertainty. The cash flow of the firm is  $y_H$  in state  $H$  and  $y_L$  in state  $L$ , with  $y_H > y_L > 0$ . The probability of each state  $s \in S$  depends on the effort level  $e \in \{a, b\}$  undertaken by the manager and is denoted  $\pi_s(e)$ .

The shareholders' income coincides with the firm's cash flow, less the compensation paid to the manager. We assume that shareholders are risk-neutral (for instance because the risk of the firm is idiosyncratic and can be fully diversified by shareholders). On the other hand, the manager is risk-averse. We assume he has no resources other than his ability to work and has Von Neumann–Morgenstern preferences defined over his level of consumption (equal to the compensation received) in every state as well as over his effort level:

$$\sum_{s \in \{H,L\}} \pi_s(e) u(z_s) - v(e).$$

More precisely, we require the utility index  $u(\cdot)$  to satisfy the following assumption.

ASSUMPTION 1.  $u: \mathbb{R}_+ \rightarrow \mathbb{R}$  is strictly increasing, strictly concave, and  $\lim_{z \rightarrow 0} u(z) = \infty$ .

The last part of the assumption implies that the manager's compensation has to ensure him a strictly positive level of income in every state.

The term  $v(e)$  in the manager's utility function describes his disutility for effort. We assume that  $v(a) > v(b) > 0$  and  $\pi_H(a) > \pi_H(b)$ . Thus,  $a$  should be viewed as the high effort level, which entails a larger disutility but also a higher probability for state  $H$ , in which the firm's cash flow is larger.

The realization of the uncertainty, that is, of  $s$ , is commonly observed. However, the effort undertaken by the manager is his private information and cannot be monitored. As usual, we will assume that the gains from eliciting high effort are always sufficiently big relative to its cost,  $v(a) - v(b)$ , so that in designing the optimal contract we face a non-trivial incentive problem. In particular, we will assume that the manager, when his compensation equals the firm's entire cash flow, prefers to exert high effort rather than low effort even when, in this second case, he has the opportunity to fully hedge his risk (at prices  $\pi(b)$ , fair contingent on low effort):

ASSUMPTION 2. The manager's preferences  $u(\cdot)$  and the parameters  $v(e)$ ,  $\pi(e)$  are such that

$$\pi_H(a)u(y_H) + \pi_L(a)u(y_L) - v(a) > u(\pi_H(b)y_H + \pi_L(b)y_L) - v(b).$$

*Markets.* The manager and the shareholders have access to competitive financial markets where they can trade, at the beginning of the period, claims contingent on each possible realization of the uncertainty. In particular the manager can trade any derivative contract on the firm's cash flow, thereby hedging any incentive



component of his compensation.<sup>12</sup> Because the probability distribution of the firm's cash flow depends on the manager's effort, such derivative markets are characterized by the presence of moral hazard.

Because of moral hazard, the competitive prices in such derivative markets will depend on what the observable component of the manager's trades is insofar as this affects or conveys information about the manager's effort (and hence the firm's cash flow). We consider here the case in which the contracts traded in these markets are non-exclusive, that is, the case in which a market maker trading with a manager does not know whether the manager engages in other trades in the market.<sup>13</sup> The price of these contracts cannot therefore depend on the manager's total portfolio or the level of his trades (because nobody except the manager observes them), though it may vary with the sign of each transaction, which is observable (i.e., it can depend on whether a contract involves a purchase or a sale of insurance). The dependence of prices on the sign of each manager's transaction may then give rise to a bid-ask spread in the markets for derivative contracts traded by managers, which is similar to the bid-ask spread that arises in Glosten and Milgrom (1985) when some traders have private information about payoffs or to the price impact of informed trading in Kyle (1985).<sup>14</sup>

In our environment managerial trading results in equilibrium prices in the financial markets which exhibit the following properties: The price of a hedging contract is fair conditionally on low effort being exerted, that is, it is evaluated with state prices  $p_s^+ = \pi_s(b)$ ,  $s \in S$ ; the price for bets on the firm is on the other hand fair conditionally on high effort being exerted, that is, is evaluated with state prices  $p_s^- = \pi_s(a)$ ,  $s \in S$  (see also Bisin and Gottardi 1999). Such prices reflect the fact that, at the optimal compensation contract, if the manager hedges in the market, he will have no incentives to choose the high effort;<sup>15</sup> the price will therefore take this into account, and hedging will be costly (in particular, fair conditional on low effort). Betting on the firm's performance, in contrast, will not induce the manager to switch from the high effort level, and hence the price faced by the manager for betting on his firm will be fair.<sup>16</sup>

12. Equivalently, we could model such derivative contracts as being intermediated in competitive markets by market makers, for example, investment banks, who are then hedging their position in the financial markets.

13. This is in accordance with the flexible institutional setting of these markets: Managers can trade different contracts with different investment banks, as well as construct basket hedges or simply trade using family members' accounts.

14. In the absence of a moral hazard problem, there would instead be a unique vector of state prices and a unique equivalent martingale measure pricing both sales and purchases of insurance as is standard in the frictionless case with complete markets.

15. Note that in equilibrium, the manager exerts high effort and does not hedge. The price of a hedging contract is determined by the off-equilibrium beliefs that when the manager hedges, exerting high effort is no longer incentive compatible.

16. At these prices the financial market is arbitrage free, because the prices for purchases of state-contingent claims,  $\pi_H(a)$  and  $\pi_L(b)$ , exceed the prices for sales,  $\pi_H(b)$  and  $\pi_L(a)$ , for both states.

We are assuming for simplicity that there are no liquidity traders in our model which implies that prices are fair conditional on the effort level which is consistent with the direction of trade. However, even in the presence of liquidity traders we would obtain similar results as managerial trading would still have some price impact. Although an explicit analysis of the problem with liquidity traders is beyond the scope of the present paper, one would expect that the more liquidity trading there is, the lower the bid–ask spread as the inference about the manager’s effort level from the observed direction of trades becomes harder. This would make hedging less expensive for the manager and, in turn, the agency problem due to managerial hedging more severe. However, as long as the size of liquidity traders is not too large, a positive bid ask spread would still be present and our main qualitative findings remain valid.<sup>17</sup>

*Monitoring.* Whether the agent’s trades in the market are observed by the principal or not plays an important role in the determination of the optimal contract between the two parties in the presence of asymmetric information. If not detected, such trades may in fact undo the incentives provided by the contract. We examine the case where a monitoring technology may be used to detect the manager’s trades in financial markets. Monitoring takes place ex post, namely, not when trades are actually made (at the beginning of the period), but rather when the payments associated with such trades are made (at the end of the period, in a given state). We assume that the shareholders can commit to a stochastic level of monitoring.<sup>18</sup> In particular, there is a randomization device which allows to observe with some probability  $m_s$  the payments due to or from the manager in state  $s \in S$ .<sup>19</sup> The intensity of monitoring in each state  $s$  will be measured by  $m_s$ .

Monitoring is costly and hence will not typically occur with probability 1. More precisely, we assume that the cost of exerting monitoring in each state  $s$  with intensity  $m_s$  is given by  $\varphi(\bar{m})$ , where  $\bar{m} = \sum_{s \in S} \pi_s(a) m_s$  and  $\varphi$  is a positive and increasing function of  $\bar{m}$ .<sup>20</sup> The monitoring cost is assumed to be a disutility cost

Furthermore, if we think of dealers as offering derivative contracts to managers and trading stocks or other claims in financial markets to hedge their positions, then, at the above prices, such dealers would make zero-profits.

17. In fact the effects of more liquidity trading are somewhat analogous to those of a higher financial development discussed in Section 3.

18. The importance of commitment has been noted in the literature (see, e.g., Krasa and Villamil 2000). It turns out that commitment is somewhat less of a concern in our model, because shareholders are better off when monitoring occurs (conditional on the cash flow realization), as we will discuss in Section 2.3. The same considerations, however, do not extend to renegotiation-proofness.

19. Stochastic monitoring dominates deterministic monitoring, but is at times considered unrealistic. However, one can interpret stochastic monitoring instead as follows: The manager produces a report on his portfolio in state  $s$ , which is informative only with probability  $m_s$ ; at an increasing cost, the manager can increase the probability with which his report is informative.

20. Notice that we are evaluating the probabilities  $\pi_s(a)$ ,  $s \in S$ , at the high effort level  $a$  because, given our assumptions, the optimal contract always implements high effort.

incurred by the manager, similar to the effort cost, which enters the manager's utility function in an additively separable way (we can think of the disutility cost as the cost to the manager of producing reports and documents to disclose his portfolio). This assumption simplifies the analysis but is not essential.<sup>21</sup>

Furthermore, we need to specify which punishment can be inflicted on the manager if he is found to have traded in the financial markets. We assume the punishment can only take a monetary form. As discussed in the Introduction, the punishment which can be inflicted is limited. Given the specification of the monitoring technology it seems natural to consider the case where punishments consist in the seizure of the payments due to the manager from his trades in the financial market. Thus, if the manager is monitored in state  $s$ , all the payoffs of any hedging transactions that are due to him in this state will be seized, and the manager will still have to make all the payments due from him for his hedging trades. We will also discuss the case where additional penalties, for example, a reduction, up to a maximum level  $k$ , of the compensation paid to the manager, can be imposed on the manager and show that our main results extend to this case (see Section 2.4).

## **2.2. The Contracting Problem**

We are now ready to describe the optimal contracting problem between the manager and the shareholders in this framework. A contract specifies the compensation due to the manager in every contingency that is commonly observed by the parties: the firm's cash flow realization and whether or not monitoring occurs. The contract also specifies the monitoring probabilities in each of the possible realizations of the firm's cash flow. Finally, the contract contains a recommendation concerning the manager's level of effort and the trades he is allowed to make in the financial markets.

The level of trades in financial markets can be set equal to zero without any loss of generality, because the outcome of any trade can always be replicated by appropriate changes in the net payments. In practice, of course, firms might have incentives to design compensation packages composed mostly of equity derivatives, for example, of stock options because of their advantageous tax treatment (see Murphy 1999), and then let the manager partially hedge his compensation in the market. In this case, the managerial hedging transactions that are observed in practice might be viewed, explicitly or implicitly, as part of the

---

21. In particular, this assumption allows us to proceed in two steps, by first determining the optimal contract for given monitoring probabilities and then determining the optimal level of monitoring. Assuming instead that monitoring involves a resource cost borne by the shareholders would yield similar results but would make the analysis more cumbersome.

firms' compensation packages. Our analysis can be readily extended to deal with such cases.

We will first characterize the properties of the optimal compensation scheme for any given monitoring probabilities  $(m_H, m_L)$ , and then discuss the determination of the optimal level of monitoring when monitoring costs are explicitly taken into account. Let then  $\mathbf{z}^{nm}(e) = (z_H^{nm}(e), z_L^{nm}(e)) \in \mathbb{R}_+^2$  (respectively,  $\mathbf{z}^m(e) \in \mathbb{R}_+^2$ ) denote the payment to the manager in each state when no monitoring (respectively, monitoring) occurs and effort  $e$  is recommended. Under Assumption 2, as we will see, shareholders are always able to implement a high level of effort  $e = a$  by the manager, whatever is  $(m_H, m_L)$ , and this is optimal. As a consequence, to keep the notation simpler in what follows, whenever possible, we will avoid to explicitly write the dependence of  $\mathbf{z}$  on  $e$ .

The optimal compensation contract for the manager in the presence of moral hazard and random monitoring of side trades, when monitoring occurs in the two states with probability  $m_H$  and  $m_L$ , respectively, is then obtained as a solution to the following program (and prescribes a high effort level):

$$\max_{(\mathbf{z}^m, \mathbf{z}^{nm}) \in \mathbb{R}_+^4} \sum_{s \in \{H, L\}} \pi_s(a) ((1 - m_s)u(z_s^{nm}) + m_s u(z_s^m)) - v(a), \quad (\mathcal{P}_{\text{MON}})$$

subject to

$$\sum_{s \in \{H, L\}} \pi_s(a) (y_s - (m_s z_s^m + (1 - m_s) z_s^{nm})) \geq 0, \quad (1)$$

and

$$\begin{aligned} & \sum_{s \in \{H, L\}} \pi_s(a) ((1 - m_s)u(z_s^{nm}) + m_s u(z_s^m)) - v(a) \\ & \geq \sum_{s \in \{H, L\}} \pi_s(e) ((1 - m_s)u(z_s^{nm} - \tau_s) + m_s u(z_s^m - \max\{\tau_s, 0\})) - v(e) \quad (2) \end{aligned}$$

for all  $e \in \{a, b\}$ ,  $(\tau_H, \tau_L) \in \mathcal{F}$ , where  $\tau_H$  and  $\tau_L$  are the manager's trades in financial markets and

$$\mathcal{F} \equiv \left\{ (\tau_H, \tau_L) \in \mathbb{R}^2 : \begin{array}{l} \text{either } \tau_H \geq 0, \tau_L \leq 0, \text{ and } \sum_{s \in \{H, L\}} \pi_s(b) \tau_s = 0; \\ \text{or } \tau_H \leq 0, \tau_L \geq 0, \text{ and } \sum_{s \in \{H, L\}} \pi_s(a) \tau_s = 0. \end{array} \right\}$$

is the set of admissible trades in these markets, as explained more in detail in the next two paragraphs.

This program requires maximizing the manager's utility subject to the shareholders' participation constraint, given by equation (1), and the incentive compatibility constraint (2). We choose this formulation, rather than the maximization of the shareholders' expected utility subject to a participation constraint

for the manager, because it simplifies the analysis and, at the same time, the results obtained are clearly unaffected. The term appearing on the left-hand side of (1) is the shareholders' expected utility (equivalently expected net income, given the shareholders' risk neutrality) when compensation  $(\mathbf{z}^m, \mathbf{z}^{nm})$  is paid to the manager in the various states. On the right-hand side the shareholders' reservation utility is set at zero.<sup>22</sup> The participation constraint amounts to setting an upper bound on the expected payments to the manager.

Equation (2) describes the incentive constraints in our set-up, where both effort and trades in financial markets are private information of the manager. They require the manager to be unable to achieve a higher utility level not only by choosing a different effort level ( $b$ ), but also by engaging in some trades  $(\tau_H, \tau_L) = 0$ . We adopt the convention that  $\tau_s$  is the amount that the manager promises to pay in state  $s$ . A negative value of  $\tau_s$  denotes thus the purchase of a claim (contingent on state  $s$ ) and hence the right to receive a payment in state  $s$ . In the event of monitoring, when  $\tau_s < 0$ ,  $-\max\{\tau_s, 0\} = 0$  and hence no payment is received. This is a reflection of our assumption that positive payoffs of managerial hedging can be seized when they are detected. On the other hand, when  $\tau_s > 0$ ,  $-\max\{\tau_s, 0\} = -\tau_s$ , that is, the manager has to make a payment  $\tau_s$  whether or not monitoring occurs. Thus trades such that  $\tau_H > 0$ ,  $\tau_L < 0$  correspond to the purchase of insurance and are priced at  $\pi_s(b)$ , whereas trades such that  $\tau_H < 0$ ,  $\tau_L > 0$  correspond to the sale of insurance and are priced at  $\pi_s(a)$ . Note that the manager faces no restriction in his trades in the financial markets except his budget constraint; hence any self-financing trade is admissible.<sup>23</sup>

Because the manager is risk-averse and shareholders are risk-neutral, the solution of  $\mathcal{P}_{\text{MON}}$  yields the compensation scheme with minimal risk that is compatible with incentives. The tightness of the incentives, and hence the specific form of the compensation, depends, as we will see, on the values of  $(m_H, m_L)$ .

### 2.3. The Optimal Contract

We provide here a characterization of the solution to the optimal contracting problem described in the previous section. We first determine in which of the states (i.e., for which realizations of the firm's cash flow) monitoring should optimally occur. Next, we characterize the manager's optimal compensation scheme.

22. This is without loss of generality because cash flows can always be redefined to be net of a fixed payment to shareholders. To see this note that if  $\tilde{U}$  is the reservation utility of shareholders and  $Y_s$ ,  $s \in S$ , are the gross cash flows, then we can obtain (1) by setting the net cash flows to  $y_s \equiv Y_s - \tilde{U}$ ,  $s \in S$ .

23. Given the specification of the program  $\mathcal{P}_{\text{MON}}$ , at the optimal contract managers never choose to engage in side trades. Hence there is no need to specify what happens to the payments seized from them since no payments are ever seized.

*When should monitoring occur?* Our first result shows that the optimal compensation contract does not depend on the monitoring probability in the high state,  $m_H$ .

PROPOSITION 1. *The optimal compensation paid to the manager (that is, the solution to  $\mathcal{P}_{\text{MON}}$ ) is independent of  $m_H$ .*

From this it follows that, if monitoring is costly, as we assume, it should never occur in state  $H$ , but only in state  $L$ , that is, when the realized cash flow of the firm is low. The intuition for the result is clear. At the prices  $\pi(a)$  the manager never wishes to engage in hedging trades involving a sale of insurance; hence, given the form of the punishment considered, it never pays to monitor the manager in state  $H$ .<sup>24</sup>

In what follows we can hence set  $m_H = 0$  and, to simplify the notation,  $m \equiv m_L$ . We will consider the contracting problem as a function of  $m$ .

*Optimal compensation.* In this section, we characterize the optimal compensation scheme  $\mathbf{z}(m) = (z_H(m), z_L^{nm}(m), z_L^m(m))$  for any  $m, 0 \leq m \leq 1$ . We consider first two benchmark cases:

1. perfect observability of trades/perfect monitoring ( $m = 1$ );
2. non-observability of trades/no monitoring ( $m = 0$ ).

If monitoring takes place with probability  $m = 1$ , trades are perfectly observed by the shareholders. In this case the manager is unable to profit from any trade in the financial market (because their proceeds will be seized with certainty). We can support then the incentive efficient (or second best) contract  $(z_H^*, z_L^*)$ , which is the solution to

$$\max_{(z_H, z_L) \in \mathbb{R}_+^2} \sum_{s \in \{H, L\}} \pi_s(a) u(z_s) - v(a), \tag{\mathcal{P}_{\text{SB}}}$$

subject to

$$\sum_{s \in \{H, L\}} \pi_s(a) (y_s - z_s) \geq 0, \tag{3}$$

$$\sum_{s \in \{H, L\}} \pi_s(a) u(z_s) - v(a) \geq \sum_{s \in \{H, L\}} \pi_s(b) u(z_s) - v(b), \tag{4}$$

---

24. This result is however more general and obtains, under certain conditions, even if other forms of punishment than the seizure of the payments due for side trades were allowed. See the discussion of alternative punishments in Section 2.4.

where in the incentive compatibility constraint (4) we are only checking for deviations concerning the effort level, and the compensation only depends on the realized state.<sup>25</sup> The solution to  $\mathcal{P}_{SB}$  is given by the values of  $z_H, z_L$  satisfying (3) and (4) as equalities.<sup>26</sup>

On the other hand, if  $m = 0$ , shareholders do not engage in any monitoring of the manager's trades. Thus the manager can always trade in financial markets without any risk of being detected. It is easy to see that in this case the best the manager can do by trading in the market is to fully insure (at the price  $\pi(b)$ ) against the fluctuations in his income (and in that case he would switch to low effort). Under Assumption 2 the high level of effort can still be implemented in this case; the optimal compensation scheme is then the one that makes the manager just indifferent between making such trades and not making them (incentive compatibility); that is,

$$\pi_H(a)u(z_H) + \pi_L(a)u(z_L^{nm}) - v(a) = u(\pi_H(b)z_H + \pi_L(b)z_L^{nm}) - v(b) \quad (5)$$

and satisfies the participation constraint (3) as equality.<sup>27</sup> We will denote by  $(z_H(0), z_L^{nm}(0))$  the solution to (3) and (5), which describes the optimal compensation scheme when  $m = 0$ . The incentive constraint is now clearly more restrictive and we can show that the optimal compensation is characterized by a higher level of risk than when trades are fully observed (i.e., at the second best  $(z_H^*, z_L^*)$  the manager's compensation is less steep):<sup>28</sup>

**PROPOSITION 2.** *Comparing the optimal compensation scheme with no monitoring and with full monitoring, we have  $z_H(0) > z_H^* > z_L^* > z_L^{nm}(0)$ .*

From Proposition 2 we obtain

$$z_H(0) - z_L^{nm}(0) > z_H^* - z_L^*.$$

Because  $(z_H(0), z_L^{nm}(0))$  and  $(z_H^*, z_L^*)$  are characterized, as we said, by the same expected value of the payments to the manager, we conclude that the variance of the manager's compensation is higher with zero than with full monitoring of his trades. The intuition for why increasing the variance of the manager's compensation allows to preserve the incentive to exert high effort is as follows:

25. When there is no uncertainty over monitoring, that is, when  $m = 1$  or  $m = 0$ , the participation constraint (1) simplifies as in (3).

26. Under our assumption that preferences are separable in consumption and effort, it is known (see, e.g., Bennardo and Chiappori 2003), that at any incentive efficient allocation the participation constraint binds.

27. For sufficient conditions implying that the participation constraint binds in this case, see Lemma A.2 in Appendix A.

28. Garvey (1993) studies a similar problem with continuous effort choice.

Insurance can be purchased in the hedging market, but at a high cost (at the prices  $\pi(b)$ ), hence the higher the variability of the compensation the lower the full insurance level.

We proceed now to the characterization of the optimal compensation scheme for any given intermediate value of  $m \in (0, 1)$ . When  $m = 1$ , as we saw, both the incentive and the participation constraints hold as equality at an optimum so that, because there are only two states, the optimal compensation in each state is simply obtained by solving these constraints. In fact, we can show that, whatever  $m$  is, at an optimal contract the incentive constraint still holds as equality (Lemma A.1) and provide some sufficient conditions for the participation constraint to also bind (Lemma A.2). We will assume in what follows that the participation constraint binds.

To characterize the level of steepness that is required in the manager's compensation to satisfy incentive compatibility, we have to determine the maximum utility the manager can attain, for any given compensation  $z$ , by switching to low effort and hedging his risk in the market. This is the maximal value of the term on the right-hand side of the inequality in the incentive compatibility condition (2). As argued in the proof of Proposition 1 (because at the optimal compensation scheme the manager can never gain by selling insurance and maintaining a high effort level), it suffices to look at trades involving the purchase of insurance; thus, we have to consider the problem:

$$\max_{(\tau_H, \tau_L) \in \mathbb{R}^2} \pi_H(b)u(z_H - \tau_H) + \pi_L(b)(mu(z_L^m) + (1 - m)u(z_L^{nm} - \tau_L)) - v(b),$$

such that  $\tau_H \geq 0$ ,  $\tau_L \leq 0$ , and  $\sum_{s \in \{H, L\}} \pi_s(b)\tau_s = 0$ .

Its first-order conditions are

$$u(z_H - \tau_H) \geq (1 - m)u\left(z_L^{nm} + \tau_H \frac{\pi_H(b)}{\pi_L(b)}\right), \quad (6)$$

$$\tau_H \geq 0.$$

Therefore, if

$$u(z_H) < (1 - m)u(z_L^{nm}),$$

(i.e., if  $z_H$  is considerably larger than  $z_L^{nm}$ ), then the maximal utility (by deviating to low effort) is attained with a non-zero level of trade in the market, whereas if

$$u(z_H) \geq (1 - m)u(z_L^{nm}), \quad (7)$$

then the manager prefers not to engage in trades in the market.

On this basis we can show that if the probability of monitoring  $m$  is sufficiently high (though less than 1), the optimal contract is the same as the one with perfect observability of trades ( $m = 1$ ):



PROPOSITION 3. *Let  $m^* \equiv 1 - u(z_H^*)/u(z_L^*) < 1$ . Then, for any  $m \geq m^*$ , the second best contract  $z_H^*$ ,  $z_L^*$  can be implemented (satisfies (2)) and hence constitutes the optimal compensation scheme (for given  $m$ ):  $z_H(m) = z_H^*$  and  $z_L^{nm}(m) = z_L^m(m) = z_L^*$ .*

To better understand this finding, notice that by trading in the market the manager can freely transfer income from state  $H$  to state  $L$  when no monitoring occurs (he is obviously unable to transfer income to state  $L$  when monitoring occurs because all the proceeds from any trade will be seized). The relative price at which such a transfer can occur is  $\pi_L(b)/\pi_H(b)$ , and the odds of these states are  $\pi_L(b)(1 - m)/\pi_H(b)$ . Thus monitoring implies that the manager can hedge (some of) his risk but at a price which is less than fair. When  $m$  is sufficiently close to 1, the cost of hedging becomes so high that the manager prefers not to do any of it.

For any  $m < m^*$  the second best contract is not implementable: The manager can in fact attain a higher utility by switching to low effort and making non-zero trades in the market than by exerting high effort. To sustain incentives the optimal compensation scheme will hence have to depart from  $z^*$ , but in which direction? A first answer is provided by the following.

PROPOSITION 4. *For any  $m < m^*$  the optimal compensation scheme (for given  $m$ )  $\mathbf{z}(m)$  is such that*

$$z_L^{nm}(m) > z_L^m(m),$$

*and, if the manager were to deviate to low effort, he would choose to buy insurance,  $\tau_H > 0$ .*

This result shows that, when the manager wishes to engage in side trades, it is optimal to condition his compensation on whether or not monitoring occurs. To gain some intuition for this, notice first that the contract must provide incentives to exert high effort: The compensation in the high state has to be sufficiently higher than the compensation in the low state. But the contract must also provide incentives not to engage in trades in the market. Such trades, as we said, allow the manager to transfer income from the high state to the low state when monitoring does not occur. Hence the possibility to engage in these trades will be more valuable to the manager the larger is the difference between his income in these two states,  $z_H$  and  $z_L^{nm}$ . On the other hand, his compensation in the low state when monitoring does occur,  $z_L^m$ , plays no role for this. As a consequence, by setting  $z_L^{nm}$  relatively high we can enhance the manager's incentives not to engage in side trades and can sustain his incentive to exert high effort with a sufficiently low level of  $z_L^m$ .

Therefore, at the optimal contract managers are always better off when they are not monitored than when they are monitored (even though at the optimum they never choose to engage in hedging trades).

It is interesting to point out that the property  $z_L^{nm}(m) > z_L^m(m)$  we find is in contrast to the finding in the costly state verification literature that the agent is rewarded if he is monitored and did tell the truth (see in particular Lemma 2 in Mookherjee and Png 1989). In our model, when the agent is monitored his compensation is low even if he did nothing wrong. Being monitored is then always considered bad news, which seems an empirically more plausible result because in practice rewards are rare. Indeed, managers, or agents more generally, typically express concern when their activities are scrutinized even when they abide by the rules.<sup>29</sup>

To understand the source of these different results, notice that in our model there is a link between the compensation of the manager when he is monitored and found not to have engaged in hedging trades, given by  $z_L^m$ , and the compensation when he is monitored and did engage in such trades, which is  $z_L^m - \max\{\tau_L, 0\}$ . Increasing  $z_L^{nm}$  reduces the benefits of hedging because the agent would enjoy these in state  $L$  when he is not monitored in which case he would consume  $z_L^{nm} - \tau_L$ . Furthermore, reducing  $z_L^m$  increases the penalty in utility terms that the seizure of the payoffs from the hedging trades imposes and thus increases the penalty for hedging. In the standard costly state verification model, in contrast, there is no link between what the agent gets paid when he is monitored and announced the cash flow truthfully and what he is paid when he is monitored and found to have understated the cash flow. Mookherjee and Png (1989), for example, assume that the agent is paid 0 in that case, that is, penalties give the agent his lower bound on utility. Without a link between the compensation when a deviation is detected and when monitoring occurs and no deviation is detected, it is then optimal to reward the agent when he is monitored and no deviation occurred. His compensation in that state affects only the objective and the left-hand side of the incentive compatibility constraint, whereas the compensation when he is not monitored also affects the right-hand side of the incentive compatibility constraint—namely, the agent's incentives to understate cash flow. The analysis of alternative specifications of penalties in the next section provides additional discussion of this point.

Although it is often observed that, to exert monitoring after the agent has taken his action, the principal has to credibly commit to do so *ex ante*, in our set-up this problem may be less of a concern. This is because at an optimum the

---

29. The conventional wisdom that managers dislike audits may also be explained by the fact that they are not compensated for the costs, for example in terms of time, effort, and soon, associated with complying. Note, however, that our model takes such costs into account and nevertheless predicts that the compensation of managers who are monitored is lower.

compensation paid to the agent/manager is lower when monitoring is exerted, and this provides an incentive for the principal to indeed monitor.<sup>30</sup>

EXAMPLE 1. Consider the case in which the manager has logarithmic preferences, that is,  $u(z_s) = \ln z_s$ . In this case, we can explicitly compute the level of trade  $\tau_H$  the manager would choose if he were to undertake low effort when his compensation is  $\mathbf{z}$ :

$$\tau_H = \max \left\{ 0, \frac{(1-m)z_H - z_L^{nm}}{(1-m) + \pi_H(b)/\pi_L(b)} \right\}.$$

Note that  $\tau_H$  varies linearly with  $\mathbf{z}$  and is larger the larger is the difference between  $z_H$  and  $z_L^{nm}$  (i.e., the larger are the gains from insurance).

Consider then the following parameter values:

$$\begin{aligned} y_H &= 5/4, & \pi_H(a) &= 3/4, & v(a) &= 1/4, \\ y_L &= 1/4, & \pi_H(b) &= 1/4, & v(b) &= 0. \end{aligned}$$

The manager's optimal compensation for different values of  $m$  are reported in Panel (A) of Table 1 and in Figure 1. The optimal compensation with perfect observability ( $z_H^*$ ,  $z_L^*$ ) (dotted) lies between the optimal compensation with no monitoring ( $z_H(0)$ ,  $z_L(0)$ ) (dashed), and thus the compensation contract is steeper without monitoring (see Proposition 2). The solid line graphs the compensation contract ( $z_H(m)$ ,  $z_L^{nm}(m)$ ,  $z_L^m(m)$ ) as a function of  $m$ . When the monitoring probability exceeds  $m^* \approx 39\%$ , the compensation schedule is the same as when hedging is perfectly observed (see Proposition 3). Moreover, the manager's utility increases monotonically as  $m$  is increased from 0 to  $m^*$ . Also, the steepness in the manager's compensation decreases as  $m$  rises; in particular the compensation in the good state  $H$  goes down while the one in the bad state  $L$  when monitoring occurs goes up. Moreover, because the expected compensation is independent of  $m$  and the compensation in state  $H$  is decreasing in  $m$ , the expected compensation in state  $L$  is increasing in  $m$ . Thus, the steepness in terms of the difference between  $z_H(m)$  and the expected compensation in state  $L$  is decreasing in  $m$ . On the other hand, in this example the compensation in state  $L$  when no monitoring occurs varies non-monotonically with  $m$ : as  $m \rightarrow 0$ ,  $z_L^{nm}(m) \rightarrow z_L^{nm}(0) < z_L^*$ , but for  $m$  close to but less than  $m^*$ ,  $z_L^{nm}(m)$  is even higher than the second-best level  $z_L^*$ . Here, the effect that higher  $z_L^{nm}(m)$  reduces the incentives to hedge dominates. Finally, for all  $m < m^*$ ,  $z_L^{nm}$  is strictly greater than  $z_L^m$  (which is optimal as we

30. This may actually give the shareholders an incentive to monitor too much. If shareholders were unable to make any commitment with regard to monitoring, the compensation would have to be such that  $\mathbf{z}^{nm} = \mathbf{z}^m$ ; we conjecture however that the other properties of the optimal compensation contract, as in Propositions 1 through 3, remain valid.

TABLE 1. Managerial compensation with portfolio monitoring.

(A) Optimal compensation: One-period case				
	Monitoring probability			Perfect observability
	$m = 0$	$m = 0.2$	$m \geq m^*$	
$z_H$	1.1210	1.1116	1.1091	1.1091
$z_L^{nm}$	0.6370	0.6748	0.6727	0.6727
$z_L^m$	n.a.	0.6268	0.6727	n.a.
$\tau_H$	0.3630	0.1893	0	n.a.
$\tau_L$	-0.1210	-0.0631	0	n.a.
$V$	-0.2771	-0.2727	-0.2715	-0.2715

(B) Optimal compensation: Two-period case				
	Monitoring probability			Perfect observability
	$m = 0$	$m = 0.2$	$m \geq m^+$	
$z_0$	0.6041	0.6214	0.6271	0.6250
$z_H$	0.7280	0.6989	0.6911	0.6932
$z_L^{nm}$	0.3999	0.4262	0.4184	0.4204
$z_L^m$	n.a.	0.3840	0.4184	n.a.
$\tau_0$	0.0611	0.0183	-0.0128	n.a.
$\tau_H$	0.1850	0.0958	0.0512	n.a.
$\tau_L$	-0.1431	-0.0563	0	n.a.
$V$	-1.2214	-1.2129	-1.2116	-1.2115

(C) Optimal compensation: Two-period case with hidden risk-free borrowing and lending only				
	Monitoring probability			Perfect observability
	$m = 0$	$m = 0.2$	$m \geq m^f$	
$z_0$	0.6084	0.6206	0.6250	0.6250
$z_H$	0.7156	0.6985	0.6932	0.6932
$z_L^{nm}$	0.4197	0.4257	0.4204	0.4204
$z_L^m$	n.a.	0.4079	0.4204	n.a.
$\tau_0$	0.0689	0.0246	0	n.a.
$\tau_1$	-0.0689	-0.0246	0	n.a.
$V$	-1.2150	-1.2118	-1.2115	-1.2115

argued because it reduces the manager’s incentive to engage in hedging activity; see Proposition 4).

We have studied so far the optimal contracting problem for given monitoring probability  $m$ . By introducing the consideration of monitoring costs the optimal intensity of monitoring can also be determined.

Let  $V(m)$  denote the manager’s expected utility (gross of the disutility cost of monitoring) at the optimal contract for given  $m$ ,  $\mathbf{z}(m)$ , obtained as a solution to  $\mathcal{P}_{\text{MON}}$ . We can show that this value is increasing in  $m$ .

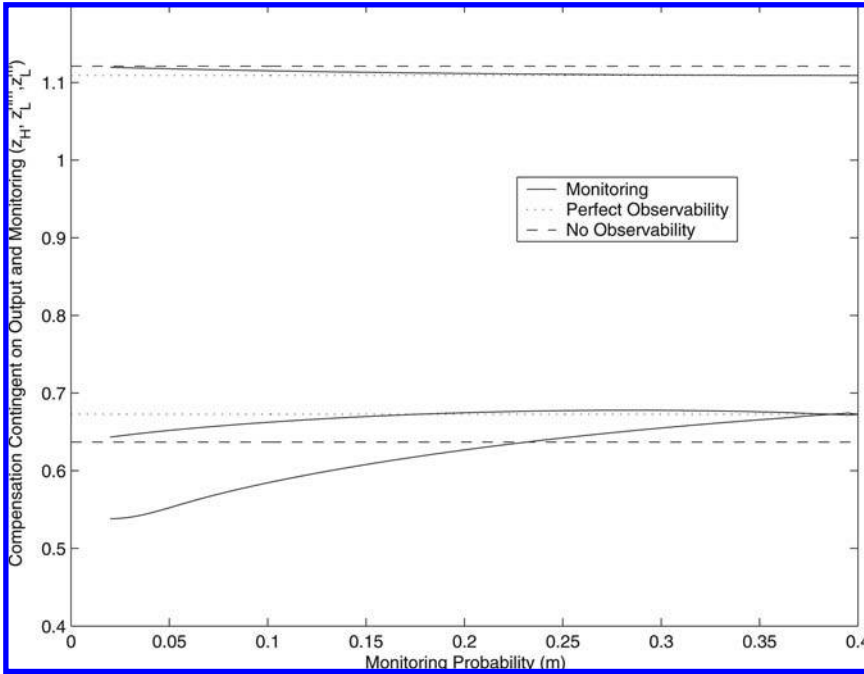


FIGURE 1. Managerial compensation with portfolio monitoring: one-period case. Compensation is shown as a function of monitoring probability  $m$ . The top three lines plot  $z_H(m)$  (solid),  $z_H^*$  (dotted), and  $z_H(0)$  (dashed). The bottom four lines plot  $z_L^m(m)$  and  $z_L^m(0)$  (both solid),  $z_L^*$  (dotted), and  $z_L^*(0)$  (dashed).

LEMMA 1.  $V(m)$  is strictly increasing in  $m$ , for  $m < m^*$ .

The optimal level of  $m$  is then obtained as the solution to the following problem:

$$\max_m V(m) - \varphi(\pi_L(a)m).$$

In fact, assuming the cost function  $\varphi(\cdot)$  is not only increasing but also sufficiently convex, the optimal level of  $m$  is uniquely determined.

### 2.4. Alternative Specification of Penalties

So far we have restricted attention to environments where the only penalty is the seizure of payoffs of side trades which the manager is due to receive. Although this specification is consistent with the limited possibilities for legal action by shareholders, as we argued in the Introduction, harsher penalties would clearly

be valuable. In this section we extend our analysis to consider an alternative specification in which a reduction in the pay to the manager can be imposed when he is monitored and caught hedging.<sup>31</sup>

Suppose, more specifically, that if managerial hedging is detected, in addition to seizing the payoffs of the hedging trades, the manager’s pay can also be reduced by a fixed amount  $k$ ; in such an event the manager’s income is then  $z_s^m - k - \max\{\tau_s, 0\}$ . It turns out that all of our results still obtain in this case, which we show in part within the set-up of the example considered earlier numerically and in part more generally.

To show that monitoring in the low state only is optimal, we take the unconditional monitoring probability, say  $\bar{m}$ , as given, and assume that the monitoring probability in the two states is chosen optimally subject to the constraint that

$$\pi_H(a)m_H + \pi_L(a)m_L \leq \bar{m}.$$

We find, within the set-up of Example 1, that it is still optimal to set  $m_H = 0$  (and hence  $m_L = \bar{m}/\pi_L(a)$ ). The intuition is as follows. Because compensation in state  $L$  is lower than in state  $H$ , the penalty  $k$  is larger in utility terms in state  $L$  and thus monitoring occurs in state  $L$  only. Moreover, because managerial hedging pays off in state  $L$  and such payoffs can be seized, this is another reason why the manager’s portfolio is monitored in state  $L$  (indeed, this is the intuition for Proposition 1).

**EXAMPLE 1 (Continued).** Consider the same environment of Example 1 but assume that monetary penalties of size  $k$  can be imposed for hedging (in addition to the seizure of all payoffs of hedging activity). When  $k = 0$  we obtain then the situation of Example 1 as a special case (thus the dotted, dash-dotted, and solid line are as in Figure 1). In the numerical computation of this example, we allow monitoring to take place in both states with  $m_H, m_L$  chosen subject to the constraint that  $\pi_H(a)m_H + \pi_L(a)m_L \leq \bar{m}$ . We find that  $m_H = 0$ , that is, that monitoring occurs in state  $L$  only and hence that  $m_H = 0$  is still optimal, even if  $k$  is positive.

In Figure 2 the optimal compensation is then again plotted as a function of  $m \equiv m_L = \bar{m}/\pi_L(a)$ , for three values of  $k$ : 0 (which, as we argued, corresponds to the case discussed previously), 0.02 (dash-dotted line), and 0.05 (bold dotted line). Consider the optimal compensation for  $k = 0.05$ , which is the bold dotted line in the figure. First, note that the compensation is only graphed for  $m$  less than approximately 14%. When  $m$  is higher than that, the compensation contract

---

31. Yet another possible specification would include penalties imposed on the investment banks offering derivative hedging contracts to managers. In practice, though, legitimate reasons for the managers to hedge might exist, and requiring investment banks to monitor the managers’ motivations for trading may then not be feasible or too costly.

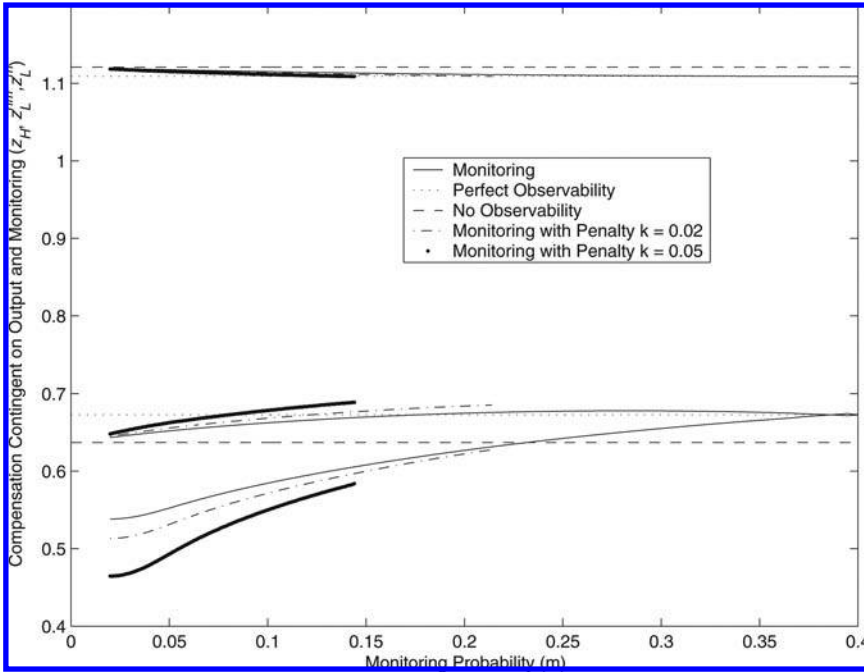


FIGURE 2. Managerial compensation with portfolio monitoring: the one-period case with alternative specification of penalties. Compensation is shown as a function of monitoring probability  $m$ . Top five lines plot  $z_H(m)$  (solid),  $z_H^*$  (dotted),  $z_H(0)$  (dashed),  $z_H(m|k = 0.02)$  (dash-dotted), and  $z_H(m|k = 0.05)$  (bold dotted). Bottom eight lines plot  $z_L^{nm}(m)$ ,  $z_L^m(m)$  (both solid),  $z_L^*$  (dotted),  $z_L^{nm}(0)$  (dashed),  $z_L^{nm}(m|k = 0.02)$  and  $z_L^m(m|k = 0.02)$  (both dash-dotted), and  $z_L^{nm}(m|k = 0.05)$  and  $z_L^m(m|k = 0.05)$  (both bold dotted).

is as in the case of perfect observability. With  $k = 0$ , this only occurs for  $m > m^* \approx 39\%$ , that is, much higher levels of monitoring were required for the compensation contract to be equivalent to perfect observability. The additional penalty imposed by  $k > 0$  clearly improves matters. When  $m$  is less than 14%, the compensation contract varies with  $m$  in a similar fashion as before (when  $k = 0$ ), but the difference between  $z_L^{nm}$  and  $z_L^m$ , which is again positive, is in fact larger: The compensation when the manager is monitored is reduced further (when  $k > 0$ ) because this gives the monetary penalty  $k$  additional bite. Note also that at  $m \approx 14\%$  there is a discontinuity in the compensation, which jumps to the perfect observability contract; this is due to the fact that there is a penalty of fixed size here.

With  $k = 0.02$ , a monitoring probability of at least 21% is required for the manager's access to hedging markets not to affect the compensation contract (i.e., for the second best contract to be implementable). Otherwise the results are comparable.

In Example 1, we have also seen that, except for the fact that the minimum level of monitoring needed to implement the second best contract is now lower and the compensation is discontinuous at that point, the optimal compensation with  $k > 0$  exhibits similar features to those found when  $k = 0$ , in particular the property  $z_L^{nm} \geq z_L^m$  is still valid. We can show that this property has general validity.

*LEMMA 2. If an additional penalty in the form of a salary reduction of size  $k$  is imposed when hedging is detected, the optimal contract is such that  $z_L^{nm} \geq z_L^m$ , with strict inequality when the optimal deviation is characterized by  $\tau_H > 0$ .*

Moreover, this result—as well as the previous findings—remains valid even if we assume that the payoffs of managerial hedging cannot be seized (in which case the only penalty for hedging is a reduction of salary of size  $k$ , so that the manager would get  $z_L^m - \tau_L - k$  in state  $L$  when monitored and hedging is detected). As already argued in Section 2.3, what is essential for the result is that there is a link between what the manager gets paid when he is monitored and did nothing wrong and what he gets paid when hedging is detected. In the presence of such link, paying the manager more when he is not monitored reduces the benefits of hedging and paying him less when he is monitored increases the penalty in utility terms if caught having traded. Furthermore, one can argue that, by continuity, even if the additional penalty can be made state dependent, say  $k_H$  and  $k_L$ , respectively, and  $k_H > k_L$ , our results hold as long as  $k_H - k_L$  is sufficiently small.

In contrast, if we were to consider the case where the penalty consists in reducing the compensation of the manager down to a minimum level  $K$ , independently of what the compensation promised to the manager in state  $L$  was (analogously to Mookherjee and Png 1989),<sup>32</sup> our results could be overturned. However, this analysis shows that the result of Mookherjee and Png (1989), which is often considered counterintuitive, does not necessarily obtain when alternative penalties are considered.

### 3. Managerial Compensation and Portfolio Monitoring: Intertemporal Case

This section extends the analysis of the contracting problem to an intertemporal framework, where there is output (and consumption) at two possible dates, date 0 and date 1. The firm produces a deterministic cash flow at date 0, given by  $y_0 > 0$ , and a random cash flow at date 1, again taking values  $y_H$  and  $y_L$  with probability dependent on the manager's effort level. The manager and shareholders have a

---

32. In their case  $K = 0$ , but their problem is still not trivial since they assume  $u(0) = 0 > -\infty$ .



common discount factor, equal to one. The manager's preferences over his income at date 0 and date 1 in every possible state are:  $u(z_0) + \sum_{s \in \{H, L\}} \pi_s(e) u(z_s) - v(e)$ .

This extension is of interest for two reasons. First, in this intertemporal framework we can distinguish between the case in which the manager can make side trades in a complete set of contingent claims, so that he is free to borrow and lend as well as to insure against any possible fluctuation in his compensation, and the case in which the manager's side trades are restricted to risk-free borrowing and lending. We examine both cases in turn. This allows us to study the effects of changes in the manager's ability to hedge his compensation due to financial innovation in the hedging markets. We find that an increase in the hedging ability implies that compensation is more distorted. This also suggests that the optimal level of portfolio monitoring is higher, the higher is the manager's ability to hedge.

Second, in this set-up, the optimal incentive contract has implications regarding the optimal distribution of the manager's compensation over time. We find that, relative to the case where the manager cannot hedge, his compensation is shifted from date 0 to date 1. In fact, as shown by Rogerson (1985), in an intertemporal agency problem with hidden action, when no side trades are possible, at the optimal contract the time profile of the compensation is distorted in favor of the initial period—that is, exhibits front loading—as this allows to improve incentives; as a consequence, the agent would want to save (if he had the option to do so). When the manager has access to hedging markets, shareholders face some limitations in the extent by which they can distort the time profile of the manager's compensation. The characterization of the optimal contract parallels otherwise the one in the case without date 0 consumption: monitoring occurs in state  $L$ , the manager bears more risk, and his compensation in state  $L$  is higher when he is not monitored than when he is monitored.<sup>33</sup>

### ***3.1. Hedging Incentive Compensation with Contingent Claims***

Suppose the manager (and shareholders) have access to financial markets where, at date 0, claims contingent on any state  $s \in S$  can be traded. As in the previous section, markets are anonymous and competitive: Agents face a given unit price, which may differ for purchases and sales, at which they are free to choose the level of their trades.

---

33. Park (2004) considers a similar environment in which the agent's date 0 consumption and savings decision is not observable and is taken prior to contracting. He concludes that only low effort is implementable.

Equilibrium prices are the same as before: For purchases of claims contingent on the  $L$  state and sales of claims contingent on the  $H$  state (corresponding to hedging trades) they are fair conditional on low effort,  $p_L^+ = \pi_L(b)$ ,  $p_H^+ = \pi_H(b)$ , whereas for sales of claims contingent on  $L$  and purchases of claims contingent on  $H$  (that correspond to betting on the firm) they are fair conditional on high effort,  $p_L^- = \pi_L(a)$ ,  $p_H^- = \pi_H(a)$ . Under Assumption 2 the optimal compensation scheme again implements high effort and we will show that, at these prices, the manager does not wish to engage in trades in the financial market.

Note that the expressions for the equilibrium prices also imply that the riskless rate at which the manager can borrow between date 0 and 1 is

$$(p_H^+ + p_L^-)^{-1} - 1 = (\pi_H(b) + \pi_L(a))^{-1} - 1 > 0,$$

whereas the riskless rate at which he can lend is

$$(p_H^- + p_L^+)^{-1} - 1 = (\pi_H(a) + \pi_L(b))^{-1} - 1 < 0.$$

Thus there is a positive spread not only for the trade of each contingent claim, but also for the trade of a claim with a riskless payoff.

In what follows, we will focus our attention on the case where monitoring only takes place at date 1, not at date 0. This is primarily for simplicity and will make the comparison with the results for the one period model easier. In this case, we are able to show (see Lemma B.1) a result analogous to Proposition 1, namely, that exerting monitoring only in state  $L$  is optimal. This obviously does not mean that if monitoring could also be exerted at date 0, this would necessarily be redundant. However, the substance of our results would not be affected if monitoring at date 0 were allowed and, moreover, monitoring in state  $L$  is most effective because, as we will show, the manager's compensation is lowest in that state and hence monetary penalties (seizing hedging payoffs or additional penalties as in Section 2.4) have the largest effect on utility.

We will show that the optimal compensation scheme for the manager in this two-period framework, when monitoring of side trades is stochastic, is obtained as the solution to the following problem:

$$\begin{aligned} & \max_{z_0(m), z_H(m), z_L^{nm}(m), z_L^m(m)} \in \mathbb{R}_+^4 \\ & u(z_0) + \pi_H(a)u(z_H) + \pi_L(a)((1-m)u(z_L^{nm}) + mu(z_L^m)) - v(a), \quad (\mathcal{P}_{\text{MON}}^0) \\ & \text{subject to} \\ & (y_0 - z_0) + \pi_H(a)(y_H - z_H) + \pi_L(a)(y_L - (mz_L^m + (1-m)z_L^{nm})) \geq 0 \quad (8) \end{aligned}$$

and

$$\begin{aligned}
 & u(z_0) + \pi_H(a)u(z_H) + p_{iL}(a)((1 - m)u(z_L^{nm}) + mu(z_L^m)) - v(a) \\
 & \geq u(z_0 - \tau_0) + \pi_H(e)u(z_H - \tau_H) \\
 & \quad + \pi_L(e)((1 - m)u(z_L^{nm} - \tau_L) + mu(z_L^m - \max\{\tau_L, 0\})) - v(e),
 \end{aligned} \tag{9}$$

for all  $e \in \{a, b\}$  and  $(\tau_0, \tau_H, \tau_L) \in \mathcal{T}(b)$ , where

$$\mathcal{T}(b) \equiv \{(\tau_0, \tau_H, \tau_L) \in \mathbb{R}^3 : \tau_H \geq 0, \tau_L \leq 0, \tau_0 + \pi_H(b)\tau_H + \pi_L(b)\tau_L = 0\}$$

is the set of trades in financial markets that are budget feasible and are restricted to be only purchases of insurance, namely, sales of  $H$  claims and purchases of  $L$  claims.<sup>34</sup>

In problem  $\mathcal{P}_{\text{MON}}^0$  we imposed two additional restrictions on the contracting problem: We required monitoring to take place only in state  $L$ , not in  $H$ , and required trades to lie in  $\mathcal{T}(b)$ . Lemma B.1 in Appendix B shows that neither of these restrictions is binding and hence that a solution to problem  $\mathcal{P}_{\text{MON}}^0$  indeed gives the optimal compensation scheme when the manager is free to choose both to sell as well as to purchase insurance in the market for contingent claims at the prices  $p^+$  and  $p^-$  as described herein, and when monitoring occurs in both states at date 1.

Let  $Z(m) \equiv [z_0(m), z_H(m), z_L^{nm}(m), z_L^m(m)]$  denote the solution to problem  $\mathcal{P}_{\text{MON}}^0$ . By the previous argument this defines the optimal compensation paid to the manager in each date and in every contingency. Whenever it is possible without generating confusion, the dependence on  $m$  will be omitted.

In what follows we will examine how different levels of ability to monitor the manager’s trades of contingent claims affect the optimal contract. The focus will be primarily on the distribution of the compensation over time (between date 0 and 1); the effects on the steepness of the compensation (its variability between the  $H$  and the  $L$  state) are—qualitatively—similar to the one found in the previous section, as we will see.

To characterize the optimal contract it is useful, as in the previous section, to begin with the two extreme cases where there is no monitoring, that is,  $m = 0$ , and where there is perfect monitoring in state  $L$ , that is,  $m = 1$ . Note that, because we ruled out by assumption the possibility of exerting monitoring at date 0, the case  $m = 1$  no longer corresponds to the second best (incentive efficient) contract, but rather to the contract obtained as the solution to the following program:

$$\max_{[z_0, z_H, z_L] \in \mathbb{R}_+^3} \quad u(z_0) + \pi_H(a)u(z_H) + \pi_L(a)u(z_L) - v(a), \quad (\mathcal{P}_{\text{SBC}}^0)$$

34. These are the trades for which prices are given by  $\pi(b)$ , that is, are fair conditional on low effort being exerted.

subject to

$$(y_0 - z_0) + \pi_H(a)(y_H - z_H) + \pi_L(a)(y_L - z_L) \geq 0$$

and

$$\begin{aligned} & u(z_0) + \pi_H(a)u(z_H) + \pi_L(a)u(z_L) - v(a) \\ & \geq (1 + \pi_H(b))u\left(\frac{1}{1 + \pi_H(b)}z_0 + \frac{\pi_H(b)}{1 + \pi_H(b)}z_H\right) + \pi_L(b)u(z_L) - v(b). \end{aligned} \tag{10}$$

Because in this case trades in state  $L$  are fully monitored, and payoffs seized, the manager will never engage in such trades:  $\tau_L \equiv 0$  (hence  $z_L^{nm} = z_L^m \equiv z_L$ ). On the other hand, the manager will now still be able to sell, unmonitored, claims contingent on  $H$ , and will then optimally use this opportunity to perfectly smooth his income between state  $H$  and date 0, as in equation (10). Let us denote a solution to problem  $\mathcal{P}_{SBc}^0$  by  $Z^+ \equiv [z_0^+, z_H^+, z_L^+]$  and the income at date 0 and in state  $H$  under the optimal deviation by

$$\bar{z}_d^+ \equiv \frac{1}{1 + \pi_H(b)}z_0^+ + \frac{\pi_H(b)}{1 + \pi_H(b)}z_H^+.$$

We can show (all results are formally stated and proven in Appendix B) that the optimal compensation with no monitoring  $Z(0)$  is characterized by perfect intertemporal smoothing ( $u(z_0(0)) = \pi_H(a)u(z_H(0)) + \pi_L(a)u(z_L^{nm}(0))$ ), and the one with full monitoring (in state  $L$ ) is distorted in favor of the initial period, that is, exhibits front loading:  $u(z_0^+) < \pi_H(a)u(z_H^+) + \pi_L(a)u(z_L^+)$ . As mentioned earlier, the latter property (i.e., the presence of front loading) was established by Rogerson (1985) for the case where no side trades are possible. Our result shows that this is also true when side trades are restricted to take place only in some markets, those for the  $H$  claims. Moreover, if  $u' > 0$ , the compensation at date 0 is lower with no monitoring (as we argued in this case there is no front loading) than with full monitoring. As in the static case, incentives are steeper and the compensation in state  $H$  higher with no monitoring than in the case of full monitoring.<sup>35</sup>

Consider then the case of intermediate levels of monitoring,  $m \in (0, 1)$ . We find again that, as long as the probability of monitoring  $m$  is sufficiently high, the optimal contract is the same as with full monitoring (in state  $L$ ). When the probability of monitoring is not sufficiently high (so that the optimal contract with

---

35. It is possible to show that exactly the same properties established in Proposition B.1 hold when the optimal compensation scheme with no monitoring,  $Z(0)$ , is compared to the optimal compensation scheme with full monitoring in all markets (also at date 0), that is, to the incentive efficient (second-best) contract  $Z^*$ . The proof is similar and is hence omitted.

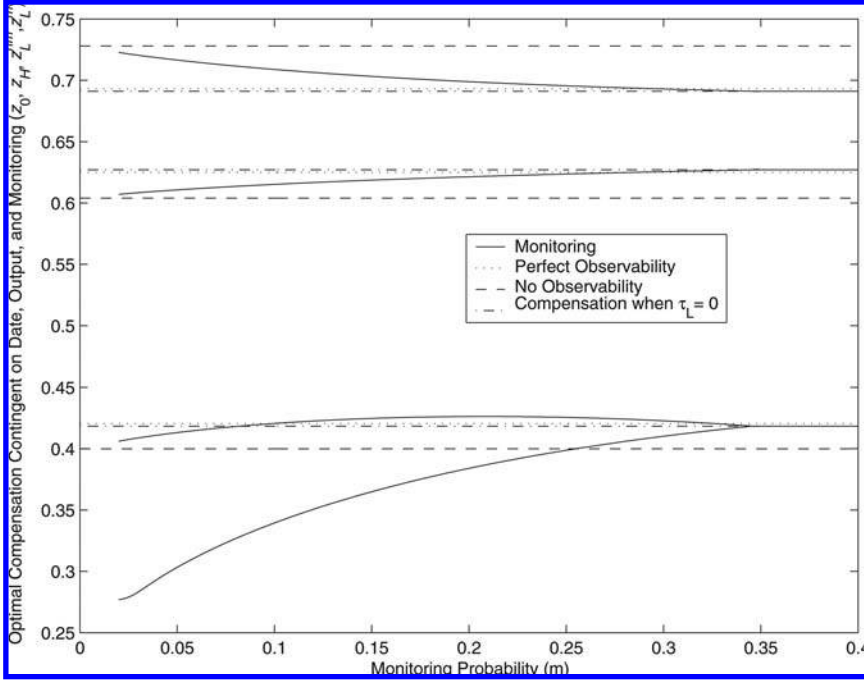


FIGURE 3. Managerial compensation with portfolio monitoring: two-period case. Compensation as a function of monitoring probability  $m$ . Top four lines plot  $z_H(m)$  (solid),  $z_H^*$  (dotted),  $z_H(0)$  (dashed), and  $z_H^+$  (dash-dotted). Middle three lines plot  $z_0(m)$  (solid),  $z_0^*$  (dotted),  $z_0(0)$  (dashed), and  $z_0^+$  (dash-dotted). Bottom four lines plot  $z_L^{nm}(m)$  and  $z_L^m(m)$  (both solid),  $z_L^*$  (dotted),  $z_L^{nm}(0)$  (dashed), and  $z_L^+$  (dash-dotted).

full monitoring is no longer implementable), the optimal compensation scheme is such that the compensation is higher in state  $L$  in the event of no monitoring than when monitoring occurs and, if the manager were to trade in the financial markets, he would choose to buy insurance,  $\tau_L < 0$ . Also, for all  $m$  we have  $z_H(m) > z_0(m) > z_L^{nm}(m)$ .

EXAMPLE 2. Modify the environment of Example 1 by introducing date 0 consumption and a date 0 endowment of  $y_0 = 1/4$ . The values of the optimal compensation in this case are reported in Panel (B) of Table 1 and in Figure 3. In this example,  $m^+ \approx 35\%$  so that this monitoring intensity alone, with no distortion in the compensation, is sufficient to get managers to refrain from hedging their compensation in state  $L$ . Furthermore, note that while the compensation with perfect monitoring in state  $L$  only,  $Z^+$ , and the compensation with perfect monitoring in both states,  $Z^*$ , do not coincide, they are almost indistinguishable; this suggests that the manager’s main concern is to insure against his low income

in state  $L$  at date 1. Once this is prevented by monitoring in that state, the compensation contract looks almost identical to the optimal compensation when there is perfect observability of trades. Also, note that the manager’s compensation at date 0,  $z_0(m)$ , increases as  $m$  increases: The higher  $m$  is, the more front loading of the compensation is possible. The other aspects of the characterization parallel the ones of Example 1.

### 3.2. Hedging Incentive Compensation with Hidden Borrowing and Lending

We turn our attention next to the case where the manager has no access to markets for contingent claims, but only to markets where a riskless asset is traded, or equivalently there can only be hidden borrowing and lending.<sup>36</sup> We interpret this as a less developed financial market. By comparing the optimal compensation contract in this case to the one obtained in the previous section, we can evaluate the consequences of a less-developed financial market for the distortions in the optimal compensation contract induced by hedging and hence for the optimal level of monitoring.

Markets are again anonymous and competitive: Agents face a given unit price at which they are free to choose the level of their trades. Because there are no informational asymmetries in this case concerning the payoff of the traded claims, their price in equilibrium will be the same for sales and purchases and equal to the common discount factor,  $p = 1$ . As in the previous section, we consider the case where monitoring takes place only at date 1. We will also assume that monitoring only takes place in state  $L$ . Indeed, numerical computations suggest that this is again optimal. The intuition is as follows: If the manager were to save using the riskless asset, these savings could be seized when he is monitored. But having the savings seized is more of a penalty when output (and hence, his compensation) is low. Hence, for any given level of monitoring it is optimal that this is concentrated in state  $L$  only.<sup>37</sup>

The optimal compensation scheme with hidden borrowing and lending (in a riskless asset) and random monitoring is then obtained as a solution to the maximization of the manager’s utility

$$\max_{z_0(m), z_H(m), z_L^{nm}(m), z_L^m(m) \in \mathbb{R}_+^4} u(z_0) + \pi_H(a)u(z_H) + \pi_L(a) \{ (1 - m)u(z_L^{nm}) + mu(z_L^m) \} - v(a), \quad (\mathcal{P}_{\text{MON}}^{0,f})$$

36. This is the case which is most studied in the literature; see, for example, Allen (1985) and Cole and Kocherlakota (2001).

37. This intuition also suggests that the same is true if additional monetary penalties (of size  $k$ ) can be imposed when hedging is detected, and numerical computations confirm that.

subject to the same participation constraint as in the previous section, equation (8), and the following new expression for the incentive compatibility constraint:

$$\begin{aligned} & u(z_0) + \pi_H(a)u(z_H) + \pi_L(a)((1 - m)u(z_L^{nm}) + mu(z_L^m)) - v(a) \\ & \geq u(z_0 - \tau_0) + \pi_H(b)u(z_H - \tau) \\ & \quad + \pi_L(b)((1 - m)u(z_L^{nm} - \tau) + mu(z_L^m - \max\{\tau, 0\})) - v(b), \end{aligned}$$

for all  $(\tau_0, \tau) \in \mathbb{R}^2$  such that  $\tau_0 + \tau = 0$ . Let  $Z^f(m)$  denote its solution.

Note that, when  $m = 0$ ,  $\mathcal{P}_{\text{MON}}^{0,f}$  is the “classic” problem yielding the optimal contract with hidden savings. On the other hand, when  $m = 1$ , its solution is given by the second best contract  $Z^*$ , that is, by the optimal contract with no side trades (with  $m = 1$  the manager can in fact only use side trades to transfer income, at a price equal to 1, from date 0 to state  $H$  at date 1 and from both states at date 1 to date 0—i.e., to borrow—and it is possible to verify that at the second best contract the manager does not wish to engage in such trades).

As mentioned in the previous section, we know from Rogerson (1985) that at the second-best contract  $Z^*$  in a two period framework the manager’s income is distorted in favor of the first period:  $u(z_0^*) < \pi_H(a)u(z_H^*) + \pi_L(a)u(z_L^*)$ .<sup>38</sup> Hence if the manager can engage in hidden trades in a riskfree asset the optimal contract would be different,  $Z^* = Z^f(0)$ , and characterized by a lower payment at the initial date,  $z_0^f(0) < z_0^*$ . In Appendix C we show that, in addition, all the properties of the optimal contract established in the previous section for the case in which the manager could hedge using a complete set of contingent claims remain valid when he is restricted to side trades in a risk-free asset.

**EXAMPLE 3.** Consider again the same set-up of Example 2. The levels of the optimal compensation for the case where side trades are restricted to risk-free borrowing and lending are reported in Panel (C) of Table 1 and in Figure 4. The results are qualitatively similar to our findings in Example 2 for the case where the manager can use contingent claims to hedge his compensation. However, because here the scope for hedging is more limited, the manager’s compensation is less distorted and the manager’s utility is reduced by less by the possibility of hedging. Indeed, we find that  $m^f \approx 30\%$ , which means that a lower monitoring probability is sufficient for the manager’s compensation to be identical to the compensation he would get with perfect observability. (In the case of hedging with contingent claims we had  $m^+ \approx 35\%$ , and even for  $m > m^+$  the optimal compensation contract was not identical—and in fact inferior—to the one under perfect observability.) Finally, note that the main distortion when  $m$  is low is that compensation is shifted from date 0 to state  $H$ .

38. In the previous section (see Lemma B.3) we established the same property for the optimal contract  $Z^+$  when there is full monitoring, but only in state  $L$ , of trades in contingent claims.

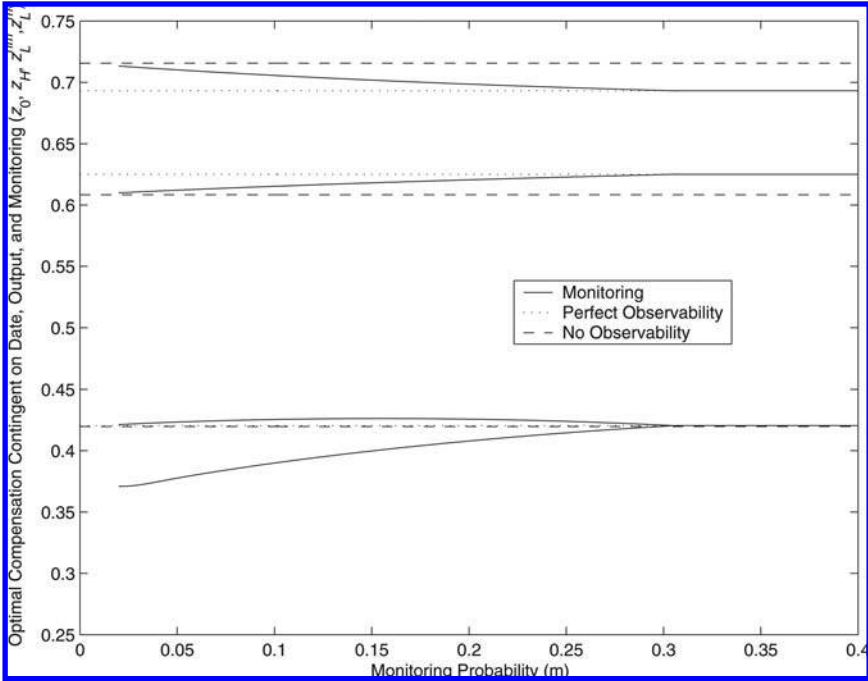


FIGURE 4. Managerial compensation with portfolio monitoring: hidden risk = free borrowing and lending only. Compensation is shown as a function of monitoring probability  $m$ . Top three lines plot  $z_H(m)$  (solid),  $z_H^*(m)$  (dotted), and  $z_H(0)$  (dashed). Middle three lines plot  $z_0(m)$  (solid),  $z_0^*(m)$  (dotted), and  $z_0(0)$  (dashed). Bottom four lines plot  $z_L^{nm}(m)$  and  $z_L^m(m)$  (both solid),  $z_L^*(m)$  (dotted), and  $z_L^{nm}(0)$  (dashed).

Hence, our results suggest that although the opportunity to trade in claims which are not contingent on the firm’s performance still poses some problems and affects the design of the optimal compensation, its quantitative effects may be smaller. They also imply that when financial markets are more developed and the scope for hedging hence larger, managerial compensation is more distorted (for a given level of monitoring) and hence the optimal level of monitoring should be higher.

**4. Discussion**

At least since the 1990s managers have had access to financial instruments which allow them to hedge the firm-specific risk in their compensation packages. Until recently, regulation has been ineffective in requiring managers to promptly disclose these financial transactions to shareholders and other investors, and executive pay contracts have rarely stated explicitly the form and amount of hedging



that managers are allowed to engage in. The adverse effects of managerial hedging on incentives in executive compensation and hence on firm performance have been pointed out by many legal and financial commentators.

We argue that as boards and shareholders recognize managers' ability to hedge their incentive compensation packages, they respond by designing the managers' incentive schemes accordingly. We show in this paper that as managers' ability to hedge increases, one should expect shareholders to monitor managers' portfolios more intensely, scrutinize their financial transactions, and possibly bring derivative suits for violation of fiduciary duty when they observe transactions by the managers which hedge the risk of incentive compensation beyond the amount mutually understood to be acceptable. Moreover, one should expect such monitoring activities in particular for managers with the easiest access to hedging markets—that is, managers of larger firms and managers in countries with well-developed financial markets. Indeed, these seem to be the firms where managerial hedging and trading activity has started to be scrutinized more carefully.

An alternative view is that corporate governance is severely ineffective and boards collude with executives to extract rents at the expense of shareholders, as argued by Bebchuck and Fried (2003). In this case incentive pay schemes should not be expected to restrict the managers' hedging ability in financial markets. The fact, however, that shareholders keep giving stocks and options to managers in their compensation package poses a challenge to this view in our opinion because it would imply that shareholders are repeatedly fooled.

It is also important to note that not all managerial hedging activity is problematic or even undesirable. For example, managers' hedging transactions may be allowed to a limited extent when firms have incentives to design excessively risky compensation packages, for example, provide compensation largely in the form of stock options, due to their advantageous tax treatment. In this case, at the optimal contract, some managerial hedging should be observed and does not constitute a violation of fiduciary duty by managers that would require legal action.

Throughout the paper we interpret managerial hedging as trades in contingent claims. But we could alternatively interpret such activity as the manager borrowing from the firm in an unobserved way and purchasing assets, such as houses. If the manager plans on repaying these loans using his bonus when the firm performance is good, but defaults on them when firm performance is bad while keeping the assets, such loans provide insurance and are a way to hedge incentive compensation. In our model, then, managers' portfolios are optimally monitored following poor performance and the extra assets bought with the loans forgiven by the firm are seized.<sup>39</sup>

---

39. Such transactions are now explicitly prohibited by Section 402 of the Sarbanes–Oxley Act of 2002.

Our model also suggests that there may be advantages to providing incentives for managers, rather than with stocks and options, with bonuses which are related to firm performance but are not a simple function of the firm’s share price. Stock and option grants are relatively easy to hedge whereas investment banks are presumably more reluctant to hedge bonuses, which are not an explicitly specified function of the price at which the shares of the firm trade.

**5. Conclusion**

Our analysis of the optimal compensation contract when managers can hedge the risk in their compensation and monitoring these hedging trades is costly, and hence does not always occur, shows that monitoring of managers’ portfolios optimally takes place when firm performance is poor. Increased scrutiny of managers’ affairs when a firm does poorly may hence be optimal rather than being an attempt by shareholders to expropriate managers ex post. Moreover, we find that, conditional on the firm’s performance, the manager’s compensation is lower when his portfolio is monitored, even if no hedging is revealed by monitoring; hence managers may be worse off, that is, their pay reduced, when their affairs are scrutinized even if they have done nothing wrong. In addition, we show that when monitoring is costly and hedging markets easily accessible or better developed, shareholders provide managers with steeper incentives. Thus, firms may respond to an increased hedging ability of managers by providing steeper incentives or by monitoring managers’ portfolios more intensely. This may explain the increased scrutiny of managerial trading over the last decade or so, in particular in the US and the UK, where hedging markets are more developed. Moreover, monitoring of managerial hedging is more important for managers for whom hedging is easier, for example, managers of large firms with liquid markets for firm-contingent claims. An additional empirical prediction of our model is that the recent increase in disclosure requirements may result in a reduction of incentive compensation (and hence of payments in the form of stocks and options).

**Appendix A: Proofs for Section 2**

*Proof of Proposition 1.* Let  $(\mathbf{z}^m, \mathbf{z}^{nm})$  be the optimal contract (i.e., a solution to  $\mathcal{P}_{\text{MON}}$  when monitoring is exerted both in  $H$  and  $L$ ). Such a contract as we said always implements the high effort level, hence we must have

$$\begin{aligned}
 &(\pi_H(a) - \pi_H(b))((1 - m_H)u(z_H^{nm}) + m_H u(z_H^m) - (1 - m_L)u(z_L^{nm}) \\
 &\quad - m_L u(z_L^m)) \geq v(a) - v(b).
 \end{aligned}$$

Any transaction in the financial market such that  $\tau_H < 0$  and  $\tau_L > 0$  (i.e., a sale of insurance) increases the manager’s income in state  $H$  (when no monitoring occurs) and lowers it in state  $L$  (whether or not monitoring occurs); as a consequence, the inequality remains valid, so that the agent still prefers to exert a high-effort level.

The optimality of  $(\mathbf{z}^m, \mathbf{z}^{nm})$  then implies that the manager cannot attain a higher level of utility by engaging in such trades. Because the manager would keep exerting high effort, his trades would have no adverse effect on the shareholders’ utility; therefore, if such trades increase the manager’s utility we would have a contradiction to the optimality of  $(\mathbf{z}^m, \mathbf{z}^{nm})$ .

We have thus shown that, if  $(\mathbf{z}^m, \mathbf{z}^{nm})$  is the solution to  $\mathcal{P}_{\text{MON}}$  (when monitoring is exerted both in  $H$  and  $L$ ), the manager never wants to engage in trades in the financial market that entail a sale of insurance, or the incentive compatibility constraint (2) never binds with  $(\tau_H, \tau_L) \in \mathbb{R}^2$  such that  $\tau_H \leq 0$ ,  $\tau_L \geq 0$ , and  $\sum_{s \in \{H, L\}} \pi_s(a) \tau_s = 0$ . This implies that monitoring is not needed to discourage trades consisting in the sale of insurance. It leaves us with only one possible role for monitoring in state  $H$ , that of introducing some randomness in the manager’s compensation in state  $H$ , which may vary according to whether or not monitoring occurs:  $z_H^{nm} = z_H^m$ . However, from the concavity of  $u(z)$  it follows that a pure randomization of the manager’s compensation, that is, not motivated by incentives, is never optimal.  $\square$

*Proof of Proposition 2.* From the form of the incentive compatibility constraint given in equation (5) for the case  $m = 0$  and the strict concavity of  $u(\cdot)$  we get<sup>40</sup>

$$\pi_H(a)u(z_H) + \pi_L(a)u(z_L^{nm}) - v(a) > \pi_H(b)u(z_H) + \pi_L(b)u(z_L^{nm}) - v(b).$$

But then

$$u(z_H) - u(z_L^{nm}) > \frac{v(a) - v(b)}{\pi_H(a) - \pi_H(b)} = u(z_H^*) - u(z_L^*),$$

which implies  $z_H(0) > z_H^* > z_L^* > z_L^{nm}(0)$ , because both  $(z_H(0), z_L^{nm}(0))$  and  $(z_H^*, z_L^*)$  have the same expected value (as they both satisfy (3) as an equality).  $\square$

LEMMA A.1. *At the optimal compensation scheme the incentive constraint (2) always holds as an equality, for all  $m$ .*

*Proof.* To induce the manager to exert high effort his compensation, as we argued, cannot be flat. If constraint (2) were holding as an inequality, it would still be

---

40. Note that for equation (5) to be satisfied we must have  $z_H(0) > z_L^{nm}(0)$ , hence the strict inequality sign.

satisfied if we consider a small change in the compensation that keeps the expected value constant and brings closer the payments in the  $H$  and the  $L$  state. This would still satisfy equation (1) and increase the manager's utility. A contradiction.  $\square$

LEMMA A.2. *Suppose that the manager's preferences are such that  $u(z) = z^{1-\sigma}/(1-\sigma)$  with  $0 < \sigma < 1$  or  $u(z) = \ln(z)$ . Then, at the optimal compensation scheme, the participation constraint (1) holds as an equality, for all  $m$ .*

*Proof.* Suppose  $u(z) = z^{1-\sigma}/(1-\sigma)$  with  $0 < \sigma < 1$  and let  $\mathbf{z} = (z_H, z_L^{nm}, z_L^m)$  be the optimal compensation. In light of Proposition 1, the incentive compatibility constraint (2), evaluated at  $\lambda\mathbf{z}$ , can be written as follows:

$$\lambda^{1-\sigma} (\pi_H(a)u(z_H) + \pi_L(a)((1-m)u(z_L^{nm}) + mu(z_L^m)) - (\pi_H(b)u(z_H - \tau_H) + \pi_L(b)((1-m)u(z_L^{nm} - \tau_L) + mu(z_L^m - \max\{\tau_L, 0\})))) \geq v(a) - v(b),$$

for all budget feasible  $\tau_H, \tau_L$ . Hence, because  $\mathbf{z}$  is incentive compatible, so is  $\lambda\mathbf{z}$  for all  $\lambda > 1$ . Evidently,  $\lambda\mathbf{z}$  is preferable to  $\mathbf{z}$ , for all  $\lambda > 1$ . Because  $\mathbf{z}$  is optimal,  $\lambda\mathbf{z}$  must then violate the participation constraint (1), for all  $\lambda > 1$ , which implies that equation (1) must hold as equality for  $\mathbf{z}$ .

Proceeding similarly for  $u(z) = \ln(z)$  we find that in that case the set of incentive compatible compensation schemes is a convex cone (if  $\mathbf{z}$  satisfies constraint (2) so does  $\lambda\mathbf{z}$  for all  $\lambda > 0$ ).  $\square$

*Proof of Proposition 3.* For all  $m \geq m^*$ , by construction we have:

$$u(z_H^*) \geq (1-m)u(z_L^*).$$

Condition (7) is thus satisfied when  $z_H = z_H^*, z_L^{nm} = z_L^*$ , so that the manager does not wish to make any trade when he switches to low effort:  $\tau_H = \tau_L = 0$ . Because high effort was sustainable at  $z_H^*, z_L^*$  with  $m = 1$ , it will also be for all  $m \geq m^*$ .  $\square$

*Proof of Proposition 4.* Fix  $m$  and omit for simplicity to write the optimal compensation as a function of  $m$ . We first show that the optimal level of trades in the market (obtained from equation [6]) is characterized by  $\tau_H > 0$ .

Suppose instead that  $(z_H, z_L^{nm}, z_L^m)$  are such that  $\tau_H = \tau_L = 0$  at the optimal contract. Thus,  $z(m)$  satisfies  $u(z_H) \geq (1-m)u(z_L^{nm})$  and

$$\begin{aligned} \pi_H(a)u(z_H) + \pi_L(a)(mu(z_L^m) + (1-m)u(z_L^{nm})) - v(a) \\ \geq \pi_H(b)u(z_H) + \pi_L(b)(mu(z_L^m) + (1-m)u(z_L^{nm})) - v(b), \end{aligned}$$

as well as the participation constraint. We will first argue that  $z_L^{nm} = z_L^m$ . To see this assume the opposite and notice that there exists a perturbation  $(dz_L^{nm}, dz_L^m)$  such that

$$mu(z_L^m)dz_L^m + (1-m)u(z_L^{nm})dz_L^{nm} = 0,$$

that is, keeping the expected utility in the low state the same, which relaxes the participation constraint because  $mdz_L^m + (1-m)dz_L^{nm} < 0$ , a contradiction. Note that, by the envelope theorem, we need not consider if the manager hedges his compensation due to the marginal change. Also, notice that the value function is differentiable at the point where  $\tau$  reaches zero, because the right-hand derivative and left-hand derivative coincide at that point. We use this fact throughout the proofs. Now, because  $m < m^*$ , we know that

$$u(z_H) \geq (1-m)u(z_L^{nm}) > \frac{u(z_H^*)}{u(z_L^*)}u(z_L^{nm}),$$

or  $u(z_H)/u(z_L^{nm}) > u(z_H^*)/u(z_L^*)$ . But because the two compensation schemes have the same expected value and do not coincide, we conclude that  $z_H^* > z_H$  and  $z_L^{nm} > z_L^*$ . (For suppose otherwise, namely,  $z_H > z_H^*$  which implies  $z_L^{nm} < z_L^*$ . Then,  $u(z_H) < u(z_H^*)$ . But the previous inequality then requires that  $u(z_L^{nm}) < u(z_L^*)$ , which contradicts  $z_L^{nm} < z_L^*$ .) But this contradicts the (second best) optimality of the compensation  $(z_H^*, z_L^*)$ .

We now turn to prove the other (and main) statement of the proposition. Suppose  $z_L^{nm} \leq z_L^m$ . Consider then an infinitesimal change in the compensation  $(dz_H, dz_L^{nm}, dz_L^m)$ , with  $dz_H = 0$ ,  $dz_L^{nm} > 0 > dz_L^m$ , leaving unchanged the manager's expected utility (and hence the term on the left-hand side of incentive compatibility constraint [2]):

$$\pi_L(a)((1-m)u(z_L^{nm})dz_L^{nm} + mu(z_L^m)dz_L^m) = 0.$$

Thus

$$(1-m)dz_L^{nm} = \frac{u(z_L^m)}{u(z_L^{nm})}m(-dz_L^m) \leq m(-dz_L^m).$$

As a consequence the participation constraint (1) still holds because the effect on it of the change in  $z$  is

$$\pi_L(a)((1-m)dz_L^{nm} + mdz_L^m) \leq 0,$$

with the inequality being strict if  $z_L^{nm} < z_L^m$ .

Finally, the effect of the change on the value of the term on the right-hand side of the incentive constraint (2) is

$$\pi_L(b)((1-m)u(z_L^{nm} - \tau_L)dz_L^{nm} + mu(z_L^m)dz_L^m) < 0,$$

where the strict inequality follows from the fact, shown in (i), that  $\tau_L < 0$ . Thus, the change allows to keep the manager's utility unchanged while making the incentive constraint slack, a contradiction.  $\square$

*Proof of Lemma 1.* Fix  $m < m^*$  and drop it as an argument of the compensation for simplicity. Consider a perturbation  $dm > 0$  and  $(dz_H, dz_L^{nm}, dz_L^m)$  with  $dz_H = dz_L^{nm} = 0$  and which satisfies the participation constraint, that is,

$$\pi_L(a)(dm(z_L^m - z_L^{nm}) + mdz_L^m) \leq 0,$$

and thus  $0 < dz_L^m \leq (z_L^{nm} - z_L^m)/mdm$ .

The effect of this perturbation on the objective and the left-hand side of the incentive constraint (2) is

$$\pi_L(a)((u(z_L^m) - u(z_L^{nm}))dm + mu(z_L^m)dz_L^m).$$

If  $dz_L^m = (z_L^{nm} - z_L^m)/m dm$ , this effect is

$$\pi_L(a)(u(z_L^m)(z_L^{nm} - z_L^m) - (u(z_L^{nm}) - u(z_L^m)))dm > 0$$

due to the concavity of  $u$ . Thus, there is a  $dz_L^m$  such that  $dz_L^m < (z_L^{nm} - z_L^m)/mdm$  and such that the effect on the objective equals zero. The effect on the right-hand side of constraint (2) evaluated at such a value of  $dz_L^m$  is

$$\pi_L(b)((u(z_L^m) - u(z_L^{nm} - \tau_L))dm + mu(z_L^m)dz_L^m) < 0,$$

because  $u(z_L^{nm} - \tau_L) - u(z_L^m) > u(z_L^{nm}) - u(z_L^m)$ . By continuity, there is a  $dz_L^m$  such that the effect on the objective and the left-hand side of the incentive constraint is strictly positive and the effect on the right-hand side is strictly negative. Such a perturbation is incentive compatible and a strict improvement.  $\square$

*Proof of Lemma 2.* The proof proceeds exactly as the proof of Proposition 4 under the assumption that monitoring occurs in state  $L$  only. We will hence only sketch the proof here. Suppose, by contradiction, that  $z_L^{nm} \leq z_L^m$  and consider a change in compensation such that  $dz_L^{nm} > 0 > dz_L^m$ , leaving the manager's expected utility (and hence the term on the left-hand side of incentive compatibility constraint [2]) unchanged:

$$\pi_L(a)((1 - m)u(z_L^{nm})dz_L^{nm} + mu(z_L^m)dz_L^m) = 0.$$

Because  $u(z_L^m) \leq u(z_L^{nm})$  the participation constraint (1) still holds. The effect of the change on the value of the term on the right-hand side of the incentive constraint (2) is

$$\pi_L(b)((1 - m)u(z_L^{nm} - \tau_L)dz_L^{nm} + mu(z_L^m - k)dz_L^m) \leq 0,$$

with strict inequality when  $\tau_L < 0$ . Thus, the change allows to keep the manager's utility unchanged while making the incentive constraint slack, a contradiction.  $\square$

### Appendix B: Formal Statements and Proofs for Section 3.1

Let  $\mathcal{T}$  denote the set of all budget feasible trades in financial markets, given by  $(\tau_0, \tau_H, \tau_L) \in \mathbb{R}^3$  such that

$$\begin{aligned} \tau_0 + \pi_H(b) \max\{\tau_H, 0\} + \pi_L(b) \min\{\tau_L, 0\} + \pi_H(a) \min\{\tau_H, 0\} \\ + \pi_L(a) \max\{\tau_L, 0\} = 0. \end{aligned}$$

LEMMA B.1. *The compensation scheme obtained as solution to problem  $\mathcal{P}_{\text{MON}}^0$  is also a solution to the same problem when (i) monitoring in state  $H$  is also allowed; and (ii) the set of admissible trades  $\mathcal{T}(b)$  is replaced by the larger set  $\mathcal{T}$ .*

*Proof.* We will rely on the characterization of  $\mathcal{P}_{\text{MON}}^0$  obtained in this appendix. In Proposition B.3, we concluded that  $z_L^m \leq z_L^{nm} < z_0 < z_H$  no matter what  $m$  is.

Let us consider purchases of claims on  $H$  ( $d\tau_H < 0$  and  $d\tau_0 > 0$ ) at price  $\pi_H(a)$  such that  $d\tau_0 + \pi_H(a)d\tau_H = 0$  starting from a zero deviation. Suppose the manager puts in high effort ( $a$ ). The benefit of the purchase would be  $-\pi_H(a)u(z_H)d\tau_H$  and the cost  $-u(z_0)d\tau_0$  and thus the net benefit

$$-\pi_H(a)(u(z_H) - u(z_0))d\tau_H < 0.$$

Suppose the manager puts in low effort ( $b$ ). The benefit would then be  $-\pi_H(b)u(z_H)d\tau_H$  and the cost would be unchanged. Thus, the net benefit would be

$$-(\pi_H(b)u(z_H) - \pi_H(a)u(z_0))d\tau_H < 0.$$

Hence, the manager would never buy claims on state  $H$  at price  $\pi_H(a)$ .

Let us consider sales of claims on  $L$  ( $d\tau_L > 0$  and  $d\tau_0 < 0$ ) at price  $\pi_L(a)$  such that  $d\tau_0 + \pi_L(a)d\tau_L = 0$  starting from a zero deviation. Suppose the manager puts in high effort ( $a$ ). The benefit of the purchase would be  $-u(z_0)d\tau_0$  and the cost would be  $-\pi_L(a)((1-m)u(z_L^{nm}) + mu(z_L^m))d\tau_L$ . Thus the net benefit would be

$$\pi_L(a)(u(z_0) - ((1-m)u(z_L^{nm}) + mu(z_L^m)))d\tau_L < 0.$$

Suppose the manager puts in low effort ( $b$ ). The benefit would then be unchanged and the cost would be  $-\pi_L(b)((1-m)u(z_L^{nm}) + mu(z_L^m))d\tau_L$ . Thus, the net benefit would be

$$(\pi_L(a)u(z_0) - \pi_L(b)((1-m)u(z_L^{nm}) + mu(z_L^m)))d\tau_L < 0.$$

Hence, the manager would never sell claims on state  $L$  at price  $\pi_L(a)$  either.

Notice that any trade involving either purchases of  $H$  claims ( $d\tau_H < 0$ ) or sales of  $L$  claims ( $d\tau_L > 0$ ) or both (with  $d\tau_0 \leq 0$ ) could hence be improved upon and thus the manager would never consider such trades.

In sum, given these prices, the manager would not want to sell claims on state  $L$  or purchase claims on state  $H$ . But then, given our assumption about the form of penalties, monitoring in state  $H$  is irrelevant.  $\square$

We establish first a preliminary result on the properties of the solutions of problem  $\mathcal{P}_{\text{MON}}^0$ , analogous to what we found in the previous section (Lemmas A.1 and A.2):

**LEMMA B.2.** *At an optimal compensation scheme,  $u(z_H) > (1 - m)u(z_L^{nm}) + mu(z_L^m)$  and the incentive compatibility constraint (9) always holds as equality, for all  $m$ . Moreover, a sufficient condition for the participation constraint (8) to also hold as equality is that  $u(z) = z^{1-\sigma} / (1-\sigma)$  with  $0 < \sigma < 1$  or  $u(z) = \ln(z)$ .*

*Proof.* The inequality  $u(z_H) > (1 - m)u(z_L^{nm}) + mu(z_L^m)$  is clearly needed to support high effort with a zero level of side trades; with non-zero trades in financial markets it must also hold, a fortiori. Suppose next that constraint (9) were not binding. Then the manager's utility could be increased by lowering the utility of the payment in state  $H$  and increasing the one in state  $L$ , while keeping unchanged the total expected payment, a contradiction.

The proof of the second claim follows the proof of Lemma A.2 quite closely and is hence omitted.  $\square$

Next we provide a comparison of the case with no monitoring, that is,  $m = 0$ , and with perfect monitoring in state  $L$ , that is,  $m = 1$ , which is the problem denoted  $\mathcal{P}_{\text{SBc}}^0$  in the text.

**LEMMA B.3.**

- (i) *The optimal contract with zero monitoring,  $Z(0)$  is such that  $z_H(0) > z_0(0) > z_L^{nm}(0)$  and*

$$u(z_0(0)) = \pi_H(a)u(z_H(0)) + \pi_L(a)u(z_L^{nm}(0)).$$

- (ii) *The optimal contract with perfect monitoring,  $m = 1$ , is given by the compensation scheme  $Z^+$  solving problem  $\mathcal{P}_{\text{SBc}}^0$ , and is such that  $z_H^+ > z_0^+ > z_L^+$  and  $u(z_0^+) < \pi_H(a)u(z_H^+) + \pi_L(a)u(z_L^+)$ .*

*Proof.* (i) When  $m = 0$ , (9) can be written as:

$$\begin{aligned} & u(z_0) + \pi_H(a)u(z_H) + \pi_L(a)u(z_L^{nm}) - v(a) \\ & \geq 2u\left(\frac{1}{2}z_0 + \frac{\pi_H(b)}{2}z_H + \frac{\pi_L(b)}{2}z_L\right) - v(b), \end{aligned} \tag{B.1}$$



where the term on the right-hand side reflects the fact that, with no monitoring, the best the manager can do by trading in the market is to perfectly smooth his income across time and the two states.<sup>41</sup> The first-order conditions for problem  $\mathcal{P}_{\text{MON}}^0$  when  $m = 0$ , can then be written as

$$\begin{aligned} u(z_0) &= \frac{\mu}{1+\lambda} + \frac{\lambda}{1+\lambda} u(\bar{z}_d), \\ u(z_H) &= \frac{\mu}{1+\lambda} + \frac{\lambda}{1+\lambda} \frac{\pi_H(b)}{\pi_H(a)} u(\bar{z}_d), \\ u(z_L^{nm}) &= \frac{\mu}{1+\lambda} + \frac{\lambda}{1+\lambda} \frac{\pi_L(b)}{\pi_L(a)} u(\bar{z}_d), \end{aligned} \quad (\text{B.2})$$

where  $\mu$  and  $\lambda$  are the Lagrange multipliers associated with the constraints (8), and (9), and

$$\bar{z}_d \equiv \frac{1}{2}z_0(0) + \frac{\pi_H(b)}{2}z_H(0) + \frac{\pi_L(b)}{2}z_L(0).$$

Because  $\pi_L(b)/\pi_L(a) > 1 > \pi_H(b)/\pi_H(a)$ , from the equations in (B.2) we get  $z_H(0) > z_0(0) > z_L^{nm}(0)$ . Furthermore,  $u(z_0(0)) = \pi_H(a)u(z_H(0)) + \pi_L(a)u(z_L^{nm}(0))$ .

(ii) Consider the first-order conditions for problem  $\mathcal{P}_{\text{SBc}}^0$ :

$$\begin{aligned} u(z_0) &= \frac{\mu}{1+\lambda} + \frac{\lambda}{1+\lambda} u(\bar{z}_d^+), \\ u(z_H) &= \frac{\mu}{1+\lambda} + \frac{\lambda}{1+\lambda} \frac{\pi_H(b)}{\pi_H(a)} u(\bar{z}_d^+), \\ u(z_L) &= \frac{\mu}{1+\lambda} + \frac{\lambda}{1+\lambda} \frac{\pi_L(b)}{\pi_L(a)} u(z_L), \end{aligned} \quad (\text{B.3})$$

where  $\mu$  and  $\lambda$  are the multipliers associated with the two constraints of  $\mathcal{P}_{\text{SBc}}^0$  and  $\bar{z}_d^+$  is as defined earlier. Hence we have  $z_H^+ > z_0^+$  and, because by construction  $\bar{z}_d^+ \in (z_H^+, z_0^+)$ ,  $z_H^+ > \bar{z}_d^+ > z_0^+$ . Furthermore, from the first equation in (B.3) we obtain

$$\mu = u(z_0^+) + \lambda(u(z_0^+) - u(\bar{z}_d^+)) > u(z_0^+),$$

and from the third one

$$\mu = u(z_L^+) + \lambda u(z_L^+)(1 - \pi_L(b)/\pi_L(a)) < u(z_L^+);$$

41. Because, as we show subsequently,  $z_H > z_0 > z_L^{nm}$ , the smoothing of income requires selling  $H$  claims and buying  $L$  claims; it will then take place at prices  $\pi(b)$ .

thus  $z_0^+ > z_L^+$ . Finally, summing the last two equations in (B.3), multiplied by  $\pi_H(a)$  and  $\pi_L(a)$ , and using the first equation, we obtain

$$\begin{aligned} \pi_H(a)u(z_H^+) + \pi_L(a)u(z_L^+) &= u(z_0^+) + \frac{\lambda}{1+\lambda}\pi_L(b)(u(z_L^+) - u(\bar{z}_d^+)) \\ &> u(z_0^+), \end{aligned}$$

where the last inequality follows from the fact that  $u(z_L^+) > u(\bar{z}_d^+)$ .  $\square$

**PROPOSITION B.1.** *Comparing the optimal compensation schemes in an intertemporal framework with full and with no monitoring, if the participation constraint binds in both cases, we have  $z_H(0) - z_L^{nm}(0) > z_H^+ - z_L^+$ ,  $z_H(0) > z_H^+$ , and, if  $u' > 0$ , then  $z_0^+ > z_0(0)$ .*

*Proof.* Comparing equations (B.1) and (10), and noting that for all  $z_0, z_H, z_L^{nm}$  we have

$$\begin{aligned} 2u\left(\frac{1}{2}z_0 + \frac{\pi_H(b)}{2}z_H + \frac{\pi_L(b)}{2}z_L^{nm}\right) - v(b) \\ \geq (1 + \pi_H(b))u\left(\frac{1}{1 + \pi_H(b)}z_0 + \frac{\pi_H(b)}{1 + \pi_H(b)}z_H\right) + \pi_L(b)u(z_L^{nm}) - v(b); \end{aligned} \quad (\text{B.4})$$

we see that the feasible set of problem  $\mathcal{P}_{\text{MON}}^0$  when  $m = 0$  is clearly contained in the feasible set of problem  $\mathcal{P}_{\text{SBc}}^0$ . As a consequence, the solution  $Z(0)$  of the first problem is also an admissible solution to the second,  $\mathcal{P}_{\text{SBc}}^0$ . However, it is not the optimal solution to such problem because, as we saw in Lemma B.3,  $z_L^{nm}(0)$  is strictly smaller than both  $z_H(0)$  and  $z_0(0)$ . So the inequality in (B.4) is strict, or the incentive compatibility constraint of  $\mathcal{P}_{\text{SBc}}^0$  is slack at  $Z(0)$ . Hence the manager, by choosing the optimal deviation when  $m = 0$ , must get a higher utility when his compensation is given by  $z^+$  rather than by  $Z(0)$ :

$$\begin{aligned} (1 + \pi_H(b))u\left(\frac{1}{1 + \pi_H(b)}z_0^+ + \frac{\pi_H(b)}{1 + \pi_H(b)}z_H^+\right) + \pi_L(b)u(z_L^+) \\ > (1 + \pi_H(b))u\left(\frac{1}{1 + \pi_H(b)}z_0(0) + \frac{\pi_H(b)}{1 + \pi_H(b)}z_H(0)\right) + \pi_L(b)u(z_L^{nm}(0)). \end{aligned} \quad (\text{B.5})$$

Define the expected cost of the manager's compensation  $z = (z_0, z_H, z_L)$ , when he exerts effort  $e$ , as

$$PV^e(z) = z_0 + \pi_H(e)z_H + \pi_L(e)z_L.$$

Notice that

$$PV^b(z) = PV^a(z) - (\pi_H(a) - \pi_H(b))(z_H - z_L).$$

Under the assumption that the participation constraint is binding both at the solution to  $\mathcal{P}_{SBc}^0$  and of  $\mathcal{P}_{MON}^0$ , the expected cost under effort  $a$  is the same at the solutions of the two problems:  $PV^a(z^+) = PV^a(Z(0))$ . Suppose the first claim in the proposition does not hold, that is,  $z_H^+ - z_L^+ \geq z_H(0) - z_L^{nm}(0)$ . Then from the previous expressions we must have  $PV^b(Z(0)) \geq PV^b(z^+)$  and the validity of (B.5) requires:

$$\begin{aligned} \frac{1}{1 + \pi_H(b)} z_0(0) + \frac{\pi_H(b)}{1 + \pi_H(b)} z_H(0) &> \frac{1}{1 + \pi_H(b)} z_0^+ + \frac{\pi_H(b)}{1 + \pi_H(b)} z_H^+ \\ &> z_L^+ > z_L^{nm}(0), \end{aligned} \quad (\text{B.6})$$

because otherwise a lottery with (weakly) lower expected value would never be preferred. The last inequality in (B.6) in turn implies, under the assumed condition  $z_H^+ - z_L^+ \geq z_H(0) - z_L^{nm}(0)$ , that  $z_H^+ > z_H(0)$ . Hence from relationship (B.6) we get  $z_0^+ < z_0(0)$ , and so, recalling the properties established in Lemma B.3,

$$\begin{aligned} \pi_H(a)u(z_H^+) + \pi_L(a)u(z_L^+) \\ > u(z_0^+) > u(z_0(0)) = \pi_H(a)u(z_H(0)) + \pi_L(a)u(z_L^{nm}(0)). \end{aligned} \quad (\text{B.7})$$

But this contradicts our previous finding that  $z_H(0) < z_H^+$  and  $z_L^{nm}(0) < z_L^+$ . Thus, we must have  $z_H^+ - z_L^+ < z_H(0) - z_L^{nm}(0)$ .

By the same argument,  $(z_H(0), z_L^{nm}(0)) \leq (z_H^+, z_L^+)$ . Suppose this was not true, that is,  $(z_H(0), z_L^{nm}(0)) \leq (z_H^+, z_L^+)$ . Because  $PV^a(z^+) = PV^a(Z(0))$ , we have  $z_0^+ \leq z_0(0)$ . Thus again  $u(z_0^+) \geq u(z_0(0))$ , which together with the properties established in Lemma B.3 leads to a contradiction. Thus,  $(z_H(0), z_L^{nm}(0)) \leq (z_H^+, z_L^+)$ .

Combining this property with the fact that, as shown herein,  $z_H^+ - z_L^+ < z_H(0) - z_L^{nm}$ , we must have  $z_H(0) > z_H^+$ .

To prove the last claim of the proposition we also proceed by contradiction: Suppose  $u' > 0$  and  $z_0^+ \leq z_0(0)$ . From the property  $u(z_0(0)) = \pi_H(a)u(z_H(0)) + \pi_L(a)u(z_L^{nm}(0))$  established in Lemma B.3, we get  $z_0(0) < \pi_H(a)z_H(0) + \pi_L(a)z_L^{nm}(0)$ . Moreover, given the properties  $z_H(0) > z_H^+$  and  $PV^a(z^+) = PV^a(Z(0))$  shown previously, if  $z_0^+ \leq z_0(0)$  the following must hold:  $z_L^+ > z_L^{nm}(0)$  and  $\pi_H(a)z_H^+ + \pi_L(a)z_L^+ \geq \pi_H(a)z_H(0) + \pi_L(a)z_L^{nm}(0)$ . As a consequence, because  $u$  is decreasing and convex, and the lottery  $(z_H(0), z_L^{nm}(0))$  has higher variance and lower mean than the lottery  $(z_H^+, z_L^+)$ , we must have

$$\pi_H(a)u(z_H(0)) + \pi_L(a)u(z_L^{nm}(0)) > \pi_H(a)u(z_H^+) + \pi_L(a)u(z_L^+).$$

This inequality in turn implies, using the relationships established in Lemma B.3, that  $z_0^+ > z_0(0)$ , that is, a contradiction.  $\square$

**REMARK B.1.** It is possible to show that exactly the same properties as those established in Proposition B.1 hold when the optimal compensation scheme with no monitoring,  $Z(0)$ , is compared to the optimal compensation scheme with full monitoring in all markets (i.e., also at date 0), given by the incentive efficient contract  $Z^*$ .

We consider then the case of intermediate levels of monitoring:  $m \in (0, 1)$ .

**PROPOSITION B.2.** *Let  $m^+ \equiv 1 - u(\bar{z}_d^+)/u(z_L^+)$ . Then,  $m^+ < 1$  and, for any  $m \geq m^+$ , the optimal contract with perfect monitoring,  $Z^+$ , solves  $\mathcal{P}_{\text{MON}}^0$ .*

*Proof.* First, as shown in Lemma B.3,  $z_H^+ > z_0^+ > z_L^+$ . By construction we have then  $z_H^+ > \bar{z}_d^+ > z_0^+$ , so that  $m^+ < 1$ .

Consider then the optimal deviation in problem  $\mathcal{P}_{\text{MON}}^0$  (i.e., the best trades the manager can do in the financial market when switching to low effort), for a given  $m$ :

$$\begin{aligned} \max_{\tau \in T(b)} \quad & u(z_0 - \tau_0) + \pi_H(b)u(z_H - \tau_H) \\ & + \pi_L(b)((1 - m)u(z_L^{nm} - \tau_L) + mu(z_L^m - \max\{\tau_L, 0\})) - v(b). \end{aligned}$$

The first-order conditions for this problem are

$$\begin{aligned} u(z_0 - \tau_0) &\leq u(z_H - \tau_H), \\ u(z_0 - \tau_0) &\geq (1 - m)u(z_L^{nm} - \tau_L), \end{aligned} \tag{B.8}$$

with equalities if, respectively  $\tau_H > 0, \tau_L < 0$ . We will show that, when  $m \geq m^+$  these conditions are satisfied at  $[z_0^+, z_H^+, z_L^+, z_L^+]$  with  $\tau_L = 0$ . Because, as we already noticed,  $z_H^+ > z_0^+$ , when  $\tau_L = 0$  the optimal choice of the trades in the other markets  $\tau_0, \tau_H$  is at a level such that  $z_0 - \tau_0 = z_H - \tau_H = \bar{z}_d^+$ . Substituting these values in the first-order conditions, the first one is trivially satisfied and the second one has the following expression:

$$u(\bar{z}_d^+) \geq (1 - m)u(z_L^+),$$

which is always satisfied for  $m^+ \leq m$ .

Thus, when  $m \geq m^+$  the manager does not wish to trade in the market for  $L$  claims. As a consequence, because  $z^+$  constitutes the optimal contract when the manager cannot engage in such trades in the  $L$  market ( $m = 1$ ), it is also the optimal choice when  $m \geq m^+$ .  $\square$

**PROPOSITION B.3.** *For any  $m < m^+$  the optimal compensation scheme  $Z(m)$  is such that (i) if the manager were to deviate, he would choose  $\tau_L < 0$ , and (ii)  $z_H(m) > z_0(m) > z_L^{nm}(m) > z_L^m(m)$ . For  $m \geq m^+$ ,  $z_H(m) > z_0(m) > z_L^{nm}(m) = z_L^m(m)$ .*

*Proof.*

(i) Notice that if the manager were to choose  $\tau_L = 0$ , then we know from the first-order conditions of  $\mathcal{P}_{MON}^0$  that  $z_L^{nm}(m) = z_L^m(m)$ . Moreover, the first-order conditions of  $\mathcal{P}_{MON}^0$  and  $\mathcal{P}_{SBc}^0$  would coincide except for the additional constraint in  $\mathcal{P}_{MON}^0$  that  $u(z_0 - \tau_0) \geq (1 - m)u(z_L^{nm})$ . But, generically,  $Z^+$  does not satisfy this additional constraint, a contradiction.

(ii) We first show that  $z_L^{nm}(m) > z_L^m(m)$  for  $m < m^+$ . The proof follows very similar lines to that of Proposition 4. Suppose  $z_L^{nm} \leq z_L^m$ . Consider the perturbation  $dz = (dz_0, dz_H, dz_L^{nm}, dz_L^m)$  with  $dz_0 = dz_H = 0$  and  $dz_L^{nm} > 0 > dz_L^m$  such that its effect on the objective and the left hand side of the incentive constraint is

$$\pi_L(a)((1 - m)u(z_L^{nm})dz_L^{nm} + mu(z_L^m)dz_L^m) = 0.$$

This perturbation satisfies the participation constraint since  $\pi_L(a)\{(1 - m)dz_L^{nm} + mdz_L^m\} \leq 0$ . The effect on the right-hand side of the incentive constraint is

$$\pi_L(b)((1 - m)u(z_L^{nm} - \tau_L)dz_L^{nm} + mu(z_L^m)dz_L^m) \leq 0,$$

with strict inequality, by claim (i) of this proposition, if  $m < m^+$ . Thus, the perturbation renders the incentive constraint slack, and the manager's utility is unchanged, a contradiction.

Next we show that  $z_H > z_0 > z_L^{nm}$  for  $m < m^+$ . By claim (i) of this proposition  $\tau_L < 0$ . The first-order condition of the optimal deviation then implies  $(1 - m)u(z_L^{nm} - \tau_L) = u(z_0 - \tau_0)$ . But then, using the envelope theorem and the first-order conditions of the maximization problem, we have

$$\begin{aligned} u(z_L^{nm}) &= \frac{\mu}{1 + \lambda} + \frac{\lambda}{1 + \lambda} \frac{\pi_L(b)}{\pi_L(a)} u(z_L^{nm} - \tau_L) \\ &= \frac{\mu}{1 + \lambda} + \frac{\lambda}{1 + \lambda} \frac{\pi_L(b)}{\pi_L(a)} \frac{1}{1 - m} u(z_0 - \tau_0) \\ &> \frac{\mu}{1 + \lambda} + \frac{\lambda}{1 + \lambda} u(z_0 - \tau_0) = u(z_0). \end{aligned}$$

Hence,  $z_L^{nm} < z_0$ . Moreover, again using the first-order condition of the optimal deviation  $u(z_0 - \tau_0) \leq u(z_H - \tau_H)$ . If the inequality is strict,  $\tau_H = 0$  which implies that  $\tau_0 > 0$  and, in turn,  $z_0 > z_H$ . But this is not possible because otherwise the perturbation  $dz_H > 0 > dz_0$  such that  $u(z_0)dz_0 + \pi_H(a)u(z_H)dz_H = 0$  would be feasible ( $dz_0 + \pi_H(a)dz_H = 0$ ) and would

relax the incentive constraint (the effect on the right-hand side, again using the envelope theorem, is  $u(z_0 - \tau_0)dz_0 + \pi_H(b)u(z_H)dz_H < 0$ ). Thus, the first-order condition holds with equality, and we can use the first-order conditions of the maximization problem to conclude that

$$\begin{aligned} u(z_H) &= \frac{\mu}{1 + \lambda} + \frac{\lambda}{1 + \lambda} \frac{\pi_H(b)}{\pi_H(a)} u(z_H - \tau_H) \\ &< \frac{\mu}{1 + \lambda} + \frac{\lambda}{1 + \lambda} u(z_0 - \tau_0) = u(z_0). \end{aligned}$$

Thus,  $z_H > z_0 > z_L^{nm} > z_L^m$  for  $m < m^+$ .

Finally, if  $m \geq m^+$ , then by Proposition B.2,  $Z(m)$  satisfies  $z_0 = z_0^+$ ,  $z_H = z_H^+$ ,  $z_L^{nm} = z_L^+$ , and  $z_L^m = z_L^+$  and, from Lemma B.3,  $z_H^+ > z_0^+ > z_L^+$ .  $\square$

### Appendix C: Formal Statements and Proofs for Section 3.2

PROPOSITION C.1. *If the participation constraint binds then we have  $z_H^f(0) - z_L^{nm,f}(0) > z_H^* - z_L^*$  and  $z_H^f(0) > z_H^*$ . If also  $u' > 0$  then  $z_0^* > z_0^f(0)$ .*

*Proof.* Consider the optimal deviation in problem  $\mathcal{P}_{MON}^{0,f}$  when  $m = 0$ ,

$$\begin{aligned} &u(z_0) + \pi_H(a)u(z_H) + \pi_L(a)u(z_L^{nm}) - v(a) \\ &\geq \max_{(\tau_0, \tau) \in \mathbb{R}^2: \tau_0 + \tau = 0} u(z_0 - \tau_0) + \pi_H(b)u(z_H - \tau) + \pi_L(b)u(z_L^{nm} - \tau) - v(b) \\ &\geq u(z_0) + \pi_H(b)u(z_H) + \pi_L(b)u(z_L^{nm}) - v(b). \end{aligned}$$

This implies that

$$u(z_H) - u(z_L^{nm}) \geq \frac{v(a) - v(b)}{\pi_H(a) - \pi_H(b)}.$$

At the second-best contract  $Z^*$ , as already mentioned in Section 3.2, Rogerson (1985) showed that  $u(z_0^*) < \pi_H(a)u(z_H^*) + \pi_L(a)u(z_L^*)$ ; moreover, the incentive compatibility constraint holds as equality, so that

$$\frac{v(a) - v(b)}{\pi_H(a) - \pi_H(b)} = u(z_H^*) - u(z_L^*),$$

and hence  $z_H^* > z_L^*$ . Therefore, we also have

$$\begin{aligned} u(z_0^*) &< \pi_H(a)u(z_H^*) + \pi_L(a)u(z_L^*) \\ &< \pi_H(b)u(z_H^*) + \pi_L(b)u(z_L^*), \end{aligned} \tag{C.1}$$

which implies that  $Z^*$  is not an admissible solution to  $\mathcal{P}_{MON}^{0,f}$ , because at that compensation contract the agent would like to save and would then be able to achieve a higher utility by engaging in side trades. Thus  $Z^f(0) = Z^*$ . Furthermore, we have

$$u(z_H^f) - u(z_L^{nm,f}) \geq \frac{v(a) - v(b)}{\pi_H(a) - \pi_H(b)} = u(z_H^*) - u(z_L^*). \tag{C.2}$$

Suppose  $(z_H^f(0), z_L^{nm,f}(0)) \leq (z_H^*, z_L^*)$ . From the participation constraint we get then  $z_0^f(0) \geq z_0^*$  and, using (C.1),  $u(z_0^f(0)) < \pi_H(b)u(z_H^f(0)) + \pi_L(b)u(z_L^{nm,f}(0))$ , which implies  $\tau_0 > 0 > \tau$ . Consider  $dz = (dz_0, dz_H, dz_L^{nm})$  with  $dz_0 < 0 < dz_H = dz_L^{nm}$  such that the change in the value of the objective function of  $\mathcal{P}_{MON}^{0,f}$  (and hence of the term on the left-hand side of the incentive constraint) is

$$u(z_0^f(0))dz_0 + (\pi_H(a)u(z_H^f(0)) + \pi_L(a)u(z_L^{nm,f}(0)))dz_H = 0.$$

Because  $u(z_0^f(0)) < \pi_H(a)u(z_H^f(0)) + \pi_L(a)u(z_L^{nm,f}(0))$  (which again follows from (C.1)), we have  $dz_0 + dz_H < 0$ , that is, the participation constraint is still satisfied. Using the first-order conditions for the optimal level of side trades from  $Z^f(0)$  in  $\mathcal{P}_{MON}^{0,f}$ ,

$$u(z_0^f(0) - \tau_0) = \pi_H(b)u(z_H^f(0) - \tau) + \pi_L(b)u(z_L^{nm,f}(0) - \tau),$$

we find

$$u(z_0^f(0) - \tau_0)dz_0 + (\pi_H(b)u(z_H^f(0) - \tau) + \pi_L(b)u(z_L^{nm,f}(0) - \tau))dz_H < 0,$$

namely, the perturbation  $dz$  also allows to relax the incentive compatibility constraint, which contradicts the optimality of  $Z^f(0)$ . Thus we must have  $(z_H^f(0), z_L^{nm,f}(0)) \leq (z_H^*, z_L^*)$ .

Suppose  $z_H^f(0) \leq z_H^*$ , and hence  $z_L^{nm,f}(0) > z_L^*$ . But this contradicts equation (C.2). As a consequence we must have  $z_H^f(0) > z_H^*$  and, using (C.2) and the concavity of  $u(\cdot)$ ,  $z_H^f(0) - z_L^{nm,f}(0) > z_H^* - z_L^*$ , as stated in the proposition.

It remains then to show that, if  $u > 0$ , then  $z_0^f(0) < z_0^*$ . Suppose not, that is,  $z_0^f(0) \geq z_0^*$ . This implies, using the participation constraint, that  $z_L^{nm,f}(0) < z_L^*$  and

$$\pi_H(a)z_H^f(0) + \pi_L(a)z_L^{nm,f}(0) \leq \pi_H(a)z_H^* + \pi_L(a)z_L^*.$$

But then, noting that the previous inequality can also be written as  $z_H^f(0) - z_H^* > z_L^{nm,f}(0) - z_L^*$ , we also have

$$\pi_H(b)z_H^f(0) + \pi_L(b)z_L^{nm,f}(0) \leq \pi_H(b)z_H^* + \pi_L(b)z_L^*.$$

If  $u' > 0$ , so that  $u$  is decreasing and convex, it follows that

$$\begin{aligned} & \pi_H(e)u(z_H^f(0)) + \pi_L(e)u(z_L^{nm,f}(0)) \\ & > \pi_H(e)u(z_H^*) + \pi_L(e)u(z_L^*) > u(z_0^*) \geq u(z_0^f(0)) \end{aligned}$$

for  $e \in \{a, b\}$  (where we again used (C.1)). This inequality again implies that the same perturbation  $dz$  considered earlier, which does not affect the value of the objective function, also satisfies the participation constraint:  $dz_0 + dz_H < 0$ . By the same argument, using the first-order conditions for the optimal level of side trades we find that such perturbation decreases the value of the term on the right-hand side of the incentive compatibility constraint:

$$u(z_0^f(0) - \tau_0)dz_0 + (\pi_H(b)u(z_H^f(0) - \tau) + \pi_L(b)u(z_L^{nm,f}(0) - \tau))dz_H < 0,$$

which is a contradiction. Thus,  $z_0^f(0) < z_0^*$ .  $\square$

**PROPOSITION C.2.** *There exists  $m^f \in (0, 1)$  such that for all  $m \geq m^f$ , the optimal compensation scheme obtained from problem  $\mathcal{P}_{\text{MON}}^{0,f}$  is given by the second best contract,  $Z^*$ , and at this contract the optimal deviation is characterized by  $\tau_0 = \tau = 0$ .*

*Proof.* Consider the first-order conditions for the optimal level of side trades at a solution  $Z^f(m)$  of problem  $\mathcal{P}_{\text{MON}}^{0,f}(m)$ . If  $\tau \leq 0$  we have

$$u(z_0^f(m) - \tau_0) \geq \pi_H(b)u(z_H^f(m) - \tau) + \pi_L(b)(1 - m)u(z_L^{nm,f}(m) - \tau)$$

whereas, if  $\tau > 0$ ,

$$\begin{aligned} u(z_0^f(m) - \tau_0) &= \pi_H(b)u(z_H^f(m) - \tau) \\ &+ \pi_L(b)(1 - m)u(z_L^{nm,f}(m) - \tau) + mu(z_L^{m,f}(m) - \tau). \end{aligned}$$

Evaluating these conditions at  $z = [z_0^*, z_H^*, z_L^*, z_L^*]$ , when  $\tau > 0$  we have

$$u(z_0^* - \tau_0) = \pi_H(b)u(z_H^* - \tau) + \pi_L(b)u(z_L^* - \tau),$$

which, because  $\tau > 0$  implies  $\tau_0 < 0$ , contradicts (C.1). Thus, we must have  $\tau \leq 0$ . Let  $m^f$  be such that

$$u(z_0^*) = \pi_H(b)u(z_H^*) + \pi_L(b)(1 - m^f)u(z_L^*);$$

note that, because from (C.1) it follows that

$$\pi_H(b)u(z_H^*) < u(z_0^*) < \pi_H(b)u(z_H^*) + \pi_L(b)u(z_L^*),$$



we have  $0 < m^f < 1$ . For all  $m \geq m^f$ , by construction the first-order conditions for the optimal level of side trades hold at  $Z^*$  with  $\tau = 0$ , hence  $Z^*$  is an admissible solution and hence the optimal solution to  $\mathcal{P}_{\text{MON}}^{0,f}(m)$ .  $\square$

**PROPOSITION C.3.** *For  $m < m^f$ , the optimal compensation contract  $Z^f(m)$  is different from the second best,  $Z^*$ , and such that  $z_L^{nm}(m) \geq z_L^m(m)$ ; at such contract, the optimal deviation is characterized by  $\tau < 0$ .*

*Proof.* To prove the first claim, suppose the optimal level of side trades is such that  $\tau > 0$ . Then the first-order conditions are

$$u(z_0^f - \tau_0) = \pi_H(b)u(z_H^f - \tau) + \pi_L(b)(1 - m)u(z_L^{nm,f} - \tau) + mu(z_L^{m,f} - \tau) \tag{C.3}$$

and, because  $\tau > 0$  implies  $\tau_0 < 0$ ,

$$u(z_0^f) > \pi_H(b)u(z_H^f) + \pi_L(b)(1 - m)u(z_L^{nm,f}) + mu(z_L^{m,f}).$$

Consider the perturbation  $dz_0 > 0 > dz_H = dz_L^{nm} = dz_L^m \equiv dz_1$  such that  $dz_0 + dz_1 = 0$ . Notice that the first-order conditions of  $\mathcal{P}_{\text{MON}}^{0,f}(m)$  imply that

$$\begin{aligned} & \pi_H(a)u(z_H^f) + \pi_L(a)(1 - m)u(z_L^{nm,f}) + mu(z_L^{m,f}) \\ &= \frac{\mu}{1 + \lambda} + \frac{\lambda}{1 + \lambda}(\pi_H(b)u(z_H^f - \tau) + \pi_L(b)(1 - m)u(z_L^{nm,f} - \tau) + mu(z_L^{m,f} - \tau)) = u(z_0^f), \end{aligned}$$

where  $\mu$  and  $\lambda$  are the Lagrange multipliers associated with the constraints of problem  $\mathcal{P}_{\text{MON}}^{0,f}(m)$ , and the last equality follows from (C.3) together with the first-order condition with respect to  $z_0^f$ . As a consequence, the effect of the perturbation  $dz_0, dz_1$  on the value of the objective function of  $\mathcal{P}_{\text{MON}}^{0,f}$  and of the term on the left-hand side of the incentive constraint is

$$u(z_0^f)dz_0 + \pi_H(a)u(z_H^f) + \pi_L(a)(1 - m)u(z_L^{nm,f}) + mu(z_L^{m,f})]dz_1 = 0,$$

Also, its effect on the value of the term on the right-hand side of the incentive compatibility constraint is

$$u(z_0 - \tau_0)dz_0 + (\pi_H(b)u(z_H - \tau) + \pi_L(b)(1 - m)u(z_L^{nm} - \tau) + mu(z_L^m - \tau))dz_1 = u(z_0 - \tau_0)(dz_0 + dz_1) = 0.$$

Thus, the perturbation is admissible and does not decrease the value of the objective function. Hence, whenever the optimal deviation is characterized by  $\tau > 0$  we can always find an alternative solution, with higher  $z_0$  and lower  $z_H, z_L^{nm}, z_L^m$  at which the optimal deviation is  $\tau \leq 0$ .

Next, suppose that  $m < m^f$  but  $\tau = 0$ . First, note that when  $m < m^f$ ,  $Z^f(m) = Z^*$  because  $u(z_0^*) < \pi_H(b)u(z_H^*) + \pi_L(b)(1-m)u(z_L^*)$ , so that the manager would save at  $Z^*$ . Moreover, using the first-order conditions of problem  $\mathcal{P}_{\text{MON}}^{0,f}$  at  $\tau = 0$ , we have  $z_L^{nm} = z_L^m \equiv z_L$ . Next, note that given  $\tau = 0$ , the incentive compatibility constraint implies

$$u(z_0) + \pi_H(a)u(z_H) + \pi_L(a)u(z_L) - v(a) = u(z_0) + \pi_H(b)u(z_H) + \pi_L(b)u(z_L) - v(b)$$

and hence

$$u(z_H) - u(z_L) = \frac{v(a) - v(b)}{\pi_H(a) - \pi_H(b)} = u(z_H^*) - u(z_L^*), \quad (\text{C.4})$$

where the second equality uses the incentive compatibility constraint of the second best problem. Now, there are two cases to consider: On the one hand, if  $z_H > z_H^*$ , then using (C.4)  $z_L > z_L^*$  and, using the participation constraint,  $z_0 < z_0^*$ ; on the other hand, if  $z_H < z_H^*$ , then  $z_L < z_L^*$  and  $z_0 > z_0^*$ . The first-order conditions of  $\mathcal{P}_{\text{MON}}^{0,f}$  imply

$$\pi_H(a) \frac{1}{u(z_H)} + \pi_L(a) \frac{1}{u(z_L)} = \frac{1}{u(z_0)},$$

and  $Z^*$  satisfies an equivalent equation. But then  $(z_H, z_L) > (z_H^*, z_L^*)$  and  $z_0 < z_0^*$  would imply

$$\begin{aligned} \pi_H(a) \frac{1}{u(z_H)} + \pi_L(a) \frac{1}{u(z_L)} &> \pi_H(a) \frac{1}{u(z_H^*)} + \pi_L(a) \frac{1}{u(z_L^*)} \\ &= \frac{1}{u(z_0^*)} > \frac{1}{u(z_0)}, \end{aligned}$$

which is a contradiction. When  $(z_H, z_L) < (z_H^*, z_L^*)$  and  $z_0 > z_0^*$ , both inequalities are reversed, again a contradiction. We conclude that  $\tau < 0$ .

The proof that  $z_L^{nm}(m) \geq z_L^m(m)$  is identical to the proof of the corresponding claim in Proposition B.3.  $\square$

## References

- Acharya, Viral V., and Alberto Bisin (2005). "Managerial Hedging, Equity Ownership, and Firm Value." Working paper, New York University.
- Admati, Anat R., Paul Pfleiderer, and Josef Zechner (1994). "Large Shareholder Activism, Risk Sharing, and Financial Market Equilibrium." *Journal of Political Economy*, 102, 1097–1130.
- Allen, Franklin (1985). "Repeated Principal–Agent Relationships with Lending and Borrowing." *Economics Letters*, 17, 27–31.

- Arnott, Richard, and Joseph Stiglitz (1991). "Equilibrium in Competitive Insurance Markets with Moral Hazard." NBER Working Paper No. 3588.
- Baiman, Stanley, and Joel S. Demski (1980). "Economically Optimal Performance Evaluation and Control Systems." *Journal of Accounting Research*, 18 (Supplement), 184–220.
- Bank, Steven A. (1995). "Devaluing Reform: The Derivatives Markets and Executive Compensation." *DePaul Business Law Journal*, 7, 301–332.
- Bebchuk, Lucian Arye, and Jesse M. Fried (2003). "Executive Compensation as an Agency Problem." *Journal of Economic Perspectives*, 17, 71–92.
- Bebchuk, Lucian Arye, Jesse M. Fried, and David I. Walker (2002). "Managerial Power and Rent Extraction in the Design of Executive Compensation." *University of Chicago Law Review*, 69, 751–846.
- Bennardo, Alberto, and Pierre-André Chiappori (2003). "Bertrand and Walras Equilibria under Moral Hazard." *Journal of Political Economy*, 111, 785–817.
- Bettis, J. Carr, John M. Bizjak, and Michael L. Lemmon (2001). "Managerial Ownership, Incentive Contracting, and the Use of Zero-Cost Collars and Equity Swaps by Corporate Insiders." *Journal of Financial and Quantitative Analysis*, 36, 345–370.
- Bettis, J. Carr, Jeffrey L. Coles, and Michael L. Lemmon (2000). "Corporate Policies Restricting Trading by Insiders." *Journal of Financial Economics*, 57, 191–220.
- Bisin, Alberto, and Piero Gottardi (1999). "General Competitive Analysis with Asymmetric Information." *Journal of Economic Theory*, 87, 1–48.
- Bisin, Alberto, and Piero Gottardi (2006). "Efficient Competitive Equilibria with Adverse Selection." *Journal of Political Economy*, 114, 485–516.
- Bisin, Alberto, and Danilo Guitoli (2004). "Moral Hazard and Non-exclusive Contracts." *RAND Journal of Economics*, 35, 306–328.
- Bisin, Alberto, and Adriano A. Rampini (2006). "Exclusive Contracts and the Institution of Bankruptcy." *Economic Theory*, 27, 277–304.
- Bizer, David S., and Peter M. DeMarzo (1992). "Sequential Banking." *Journal of Political Economy*, 100, 41–61.
- Bizer, David S., and Peter M. DeMarzo (1999). "Optimal Incentive Contracts when Agents Can Save, Borrow, and Default." *Journal of Financial Intermediation*, 8, 241–269.
- Bolster, Paul, Don Chance, and Don Rich (1996). "Executive Equity Swaps and Corporate Insider Holdings." *Financial Management*, 25, 14–24.
- Cole, Harold L., and Narayana R. Kocherlakota (2001). "Efficient Allocations with Hidden Income and Hidden Storage." *Review of Economic Studies*, 68, 523–542.
- Dye, Ronald A. (1986). "Optimal Monitoring Policies in Agencies." *RAND Journal of Economics*, 17, 339–350.
- Easterbrook, Frank H. (2002). "Derivative Securities and Corporate Governance." *University of Chicago Law Review*, 69, 733–747.
- Easterbrook, Frank H., and Daniel R. Fischel (1991). *The Economic Structure of Corporate Law*. Harvard University Press.
- Economist* (1999a). "Executive Relief." 3 April, 351, 64–66.
- Economist* (1999b). "The Trouble with Stock Options." 7 August, 352, 13–14.
- Economist* (1999c). "Share and Share Unalike." 7 August, 352, 18–20.
- Economist* (2002). "Taken for a Ride." 13 July, 364, 64.
- Fox, Merritt B. (1999). "Required Disclosure and Corporate Governance." *Law and Contemporary Problems*, 62, 113–124.
- Gale, Douglas, and Martin Hellwig (1985). "Incentive-Compatible Debt Contracts." *Review of Economic Studies*, 52, 647–663.
- Garvey, Gerald T. (1993). "The Principal-Agent Problem when the Agent Has Access to Outside Markets." *Economics Letters*, 43, 183–186.
- Garvey, Gerald T. (1997). "Marketable Incentive Contracts and Capital Structure Relevance." *Journal of Finance*, 52, 353–378.

- Garvey, Gerald, and Todd Milbourn (2003). "Incentive Compensation when Executives Can Hedge the Market: Evidence of Relative Performance Evaluation in the Cross-Section." *Journal of Finance*, 58, 1557–1582.
- Glosten, Lawrence R., and Paul R. Milgrom (1985). "Bid, Ask and Transaction Prices in a Specialist Market with Heterogeneously Informed Traders." *Journal of Financial Economics*, 14, 71–100.
- Grossman, Sanford J., and Oliver D. Hart (1983). "An Analysis of the Principal Agent Problem." *Econometrica*, 60, 7–45.
- Ip, Greg (1997). "Collars Give Insiders Way to Cut Risk." *Wall Street Journal*, 17 September, C1.
- Jensen, Michael C., and Murphy, Kevin J. (1990). "Performance-Pay and Top-Management Incentives." *Journal of Political Economy*, 98, 225–264.
- Jin, Li (2002). "CEO Compensation, Diversification, and Incentives." *Journal of Financial Economics*, 66, 29–63.
- Kahn, Charles M., and Dilip Mookherjee (1998). "Competition and Incentives with Nonexclusive Contracts." *RAND Journal of Economics*, 29, 443–465.
- Klausner, Michael D., and Kate Litvak (2000). "A Conceptual Analysis of Derivative Suits." Working paper, Stanford University.
- Krasa, Stefan, and Anne P. Villamil (2000). "Optimal Contracts when Enforcement Is a Decision Variable." *Econometrica*, 68, 119–134.
- Kyle, Albert S. (1985). "Continuous Auctions and Insider Trading." *Econometrica*, 53, 1315–1335.
- Lavelle, Louis (2001). "Undermining Pay for Performance." *Business Week*, 15 January, 70–71.
- Mookherjee, Dilip, and Ivan Png (1989). "Optimal Auditing, Insurance, and Redistribution." *Quarterly Journal of Economics*, 104, 399–415.
- Morck, Randall, Andrei Shleifer, and Robert W. Vishny (1988). "Management Ownership and Market Valuation: An Empirical Analysis." *Journal of Financial Economics*, 20, 293–315.
- Murphy, Kevin J. (1999). "Executive Compensation." In *Handbook of Labor Economics*, Vol. 3, edited by O. Ashenfelter and D. Card. North Holland.
- Ozerturk, Saltuk (2006). "Financial Innovations and Managerial Incentive Contracting." *Canadian Journal of Economics*, 39, 434–454.
- Park, In-Uck (2004). "Moral Hazard Contracting and Private Credit Markets." *Econometrica*, 72, 701–746.
- Pauly, Mark V. (1974). "Overinsurance and Public Provision of Insurance: The Roles of Moral Hazard and Adverse Selection." *Quarterly Journal of Economics*, 88, 44–62.
- Parlour, Christine A., and Uday Rajan (2001). "Competition in Loan Contracts." *American Economic Review*, 91, 1311–1328.
- Prescott, Edward C., and Robert M. Townsend (1984). "Pareto Optima and Competitive Equilibria with Adverse Selection and Moral Hazard." *Econometrica*, 52, 21–46.
- Puri, Shaifali (1997). "New Tools for the Options Crowd." *Fortune*, 10 November, 136, 308.
- Rogerson, William P. (1985). "Repeated Moral Hazard." *Econometrica*, 53, 69–76.
- Shleifer, Andrei, and Robert W. Vishny (1996). "A Survey of Corporate Governance." *Journal of Finance*, 52, 737–783.
- Schizer, David M. (2000). "Executives and Hedging: The Fragile Foundation of Incentive Compatibility." *Columbia Law Review*, 100, 440–504.
- Smith, Randall (1999). "Internet Rich: Cashing in the Chips; For Internet Glitterati Is a Matter of Timing." *Wall Street Journal*, 28 June, C1.
- Townsend, Robert M. (1979). "Optimal Contracts and Competitive Markets with Costly State Verification." *Journal of Economic Theory*, 21, 265–293.
- Winton, Andrew (1995). "Costly State Verification and Multiple Investors: The Role of Seniority." *Review of Financial Studies*, 8, 91–123.

**This article has been cited by:**

1. Viral V. Acharya, Alberto Bisin. 2009. Managerial hedging, equity ownership, and firm value. *The RAND Journal of Economics* **40**:1, 47-77. [[CrossRef](#)]