Efficient Competitive Equilibria with Adverse Selection

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Do Walrasian markets function orderly in the presence of adverse selection? In particular, is their outcome efficient when exclusive contracts are enforceable? This paper addresses these questions in the context of a Rothschild-Stiglitz insurance economy. We identify an externality associated with the presence of adverse selection as a special form of consumption externality. Consequently, we show that competitive equilibria always exist but are not typically incentive efficient. However, as markets for pollution rights can internalize environmental externalities, markets for consumption rights can be designed to internalize the consumption externality due to adverse selection. With such markets competitive equilibria exist and incentive-constrained versions of the first and second welfare theorems hold.

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I. Introduction and Motivation

We study competitive exchange economies with adverse selection. Agents have private information regarding the probability distribution of their endowments. In addition, firms offer contracts providing the agents with insurance against the realization of any shock affecting their individual endowment. Agents' trades can be fully observed, so that firms can enforce exclusive contractual relationships.[1] The agents' private information is then the only "friction" to the operation of markets. We analyze Walrasian equilibria in which both consumers and firms act as price takers.

We intend to address the following questions: Do Walrasian markets function orderly in the presence of adverse selection? What are the properties of allocations attainable as Walrasian equilibria? And in particular, are Walrasian equilibria incentive efficient?

Our analysis is motivated by the fundamental contribution of Prescott and Townsend (1984*a*, 1984*b*). They analyze Walrasian equilibria of economies with moral hazard and with adverse selection when exclusive contracts are enforceable. While for moral hazard economies they prove existence and constrained versions of the first and second theorems of welfare economics (see also Kocherlakota 1998; Kehoe, Levine, and Prescott 2002; Bennardo and Chiappori 2003), they show that their approach cannot be successfully extended to adverse selection economies. They conclude that "there do seem to be fundamental problems for the operation of competitive markets for economies or situations which suffer from adverse selection" (1984*b*, 44).

In this paper we identify first a special form of consumption externality that arises in Walrasian economies with adverse selection and full observability of trades. In such economies agents face a complete set of markets for insurance at given prices. In particular, they face different markets and prices for each different risk type. For instance, insurance might be more expensive for high-risk types than for low-risk types. The risk type of any agent is not observable, however, because of adverse selection. Nonetheless, it is implicitly revealed by the agent's trades, which are observable. Therefore, a contract that only a high-risk type will want to buy will not be offered in the market (and at the prices quoted) for low-risk types. In other words, only incentive-compatible insurance contracts will be offered and hence will be available for trade in this economy. But the trades chosen by agents of one risk type in-

[1] This is a strong assumption. It is nonetheless the benchmark case considered in contract theory as well as in general equilibrium analyses of economies with asymmetric information, e.g., in Prescott and Townsend (1984*b*). When full observability does not hold, exclusive contracts are not enforceable and agents might undo by trading one contract the incentives provided by another contract; the properties of competitive equilibria in that case are investigated in Bisin and Gottardi (1999).

fluence the set of incentive-compatible contracts, that is, the set of contracts that will be offered to agents of other types. For example, the trades that reveal that high-risk agents misrepresent themselves as low-risk agents depend on the actual level of trades of low-risk agents. This is the fundamental consumption externality that is present in economies with adverse selection.

Competitive equilibrium allocations are then in general not efficient, as we know is the case, for instance, in economies with environmental externalities.[2] In the presence of environmental externalities, however, efficient allocations can be decentralized, following the approach pioneered by Lindahl (1919) and Arrow (1969), when a competitive market for "pollution rights" is set up with a regulatory mechanism requiring that firms do not produce unless they acquire the appropriate amount of rights in the market. We show in this paper that a similar result can be obtained in adverse-selection economies as well. The decentralization of incentive-efficient allocations can be attained if competitive *markets for consumption rights* are introduced and a regulatory enforcement mechanism is imposed that requires each agent wishing to acquire a specific insurance contract to hold an appropriate amount of consumption rights.

The link between the level of consumption and the holdings of rights can be designed to induce agents to internalize the externality their own consumption imposes on the economy in the presence of adverse selection. For instance, a higher level of consumption by agents of a low-risk type might exert a negative externality on agents of a high-risk type by tightening their incentive constraint. That is, a higher consumption level by the low-risk agents raises the incentives of high-risk agents to misreport their risk type and choose to consume the same amount as the low-risk agents. It reduces, as a consequence, the set of incentive-compatible contracts for the high-risks by rendering a higher level of consumption for the high-risks necessary to avoid their misreporting their risk type. When markets for consumption rights are introduced, the low-risk types will internalize this externality because at equilibrium they will have to buy, at a positive price in a competitive market, an amount of consumption rights that appropriately increases with their own consumption level.

Evidently, the implementation of the structure of markets that guarantees the decentralization of incentive-constrained efficient allocations in adverse-selection economies requires an enforcement mechanism

[2] In contrast, in moral hazard economies, incentive-compatibility constraints do not relate the trades of different agents in the economy, but rather the trades made by the same agent under different circumstances (e.g., different effort levels). No consumption externality is therefore present, and equilibrium allocations, as shown by Prescott and Townsend (1984*b*), are incentive efficient.

that, as we noted, prevents agents from acquiring commodities for their own consumption without acquiring also the appropriate amount of consumption rights. We will argue that the implementation of markets for consumption rights can be greatly simplified by designing a regulatory mechanism that operates on insurance firms rather than directly on agents and by having consumption rights take the simple form of the "right" to trade in the market designated for the low-risk agents (the high-quality market in our setup). At equilibrium, firms offering contracts exclusively to low-risks would acquire the right to do so from high-risk agents at market-determined prices. When at equilibrium the prices of the rights are positive, an efficient allocation is decentralized at which low-risks subsidize high-risks by paying the cost of buying their rights to trade in the low-risk insurance market.

We call the Walrasian equilibria in which only insurance contracts are traded EPT, for Externality-Prescott-Townsend. We call the Walrasian equilibria in which markets for consumption rights are also present ALPT, for Arrow-Lindahl-Prescott-Townsend. EPT and ALPT are therefore equilibrium concepts that are associated with different market structures and different institutional environments. ALPT requires a regulatory intervention, not needed for EPT, to introduce and enforce the operation of markets for consumption rights.

We show that there are no fundamental problems associated with Walrasian equilibrium for either of these market structures and institutional environments: EPT and ALPT equilibria always exist. We also show that EPT provides a useful—and somewhat robust—prediction to the outcome of competitive markets in adverse-selection economies. While, as we noted, EPT equilibria are not ensured to be incentive efficient, they satisfy an appropriately defined notion of third-best efficiency, and the second welfare theorem also holds for this market structure: any incentive-efficient allocation can be decentralized as a competitive equilibrium. ALPT equilibria are instead ensured to be incentive efficient: we show that for this market structure (incentive-constrained versions of) the first and second theorems of welfare economics hold. Therefore, our results in this paper replicate for adverse-selection economies, in the presence of markets for consumption rights, the results that Prescott and Townsend (1984*a*, 1984*b*) obtain for moral hazard economies, thereby offering a solution to the problem their papers posed.

The analysis of this paper is developed for a simple insurance economy with adverse selection like the one considered by Rothschild and Stiglitz (1976). This constitutes a very important test case for any equilibrium notion of adverse-selection economies. In addition, by exploiting the simple structure of the economy, we are able to clearly illustrate the features and to provide a complete characterization of the various notions of Walrasian equilibrium that we study. In particular, the allocation we obtain as a unique EPT equilibrium corresponds to the Rothschild-Stiglitz separating candidate equilibrium (this is so even for those parameter values for which Rothschild and Stiglitz found nonexistence). On the other hand, ALPT equilibria might entail a nonzero level of cross-subsidization, that is, some degree of "pooling" among types.

The paper is organized as follows. The structure of the economy is presented in Section II; incentive-efficient allocations are then characterized in Section III. In Section IV, first EPT equilibria and then ALPT equilibria are defined, their existence established, and their efficiency properties characterized. The proofs of the main results are collected in the Appendix.[3]

II. The Economy

Consider an economy with adverse selection populated by a continuum of agents of two different types, b and g. Let ξ^b denote the fraction of agents of type b and ξ^g the fraction of agents of type g in the population. We assume $\xi^b, \xi^g > 0$.

There is a single consumption good. Uncertainty enters the economy via the level of the agents' endowment and is purely idiosyncratic. There are two possible states, H and L, for every individual, and his endowment when H (respectively L) is realized is ω_H (ω_L). Let π^i_s be the probability that individual state $s \in S \equiv \{H, L\}$ is realized for an agent of type $i \in \{g, b\}$. These random variables are independently distributed across all agents and identically distributed across agents of the same type.

Each agent is privately informed about his type. On the other hand, the realization of individual states is commonly observed.

With no loss of generality, let $\omega_L < \omega_H$ and $0 < \pi^b_H < \pi^g_H < 1$. It follows that state H corresponds to a high-endowment realization and type b is the high-risk type.

The preferences of each agent are described by a von Neumann–Morgenstern utility function with type-independent utility index u: $\mathbb{R}_+ \rightarrow \mathbb{R}_+$ defined over consumption in each idiosyncratic state $s \in S$. Then let $U^i(x^i) \equiv \sum_{s\in S} \pi^i_s u(x^i_s)$, for $i \in \{g, b\}$, where $x^i \equiv (x^i_H, x^i_L)$.

We make the following assumption.

ASSUMPTION 1. Endowments are always strictly positive for all agents: $\omega_L, \omega_H > 0$. Preferences are strictly monotonic, strictly concave, and twice continuously differentiable, and $\lim_{x\rightarrow 0} u(x) = \infty$.

Note that the economy is the insurance economy with adverse selection considered by Rothschild and Stiglitz (1976).

[3] A complete presentation of the proofs of all other results in the paper, as well as some additional discussions of our results, can be found in Bisin and Gottardi (2005).

III. Incentive-Efficient Allocations

Let $\omega \equiv \{\omega_H, \omega_L\} \in \mathbb{R}^2_+$. Then $z^i \equiv (z^i_H, z^i_L) \in \mathbb{R}^2$ denotes the *net transfers* in each state to type $i \in \{g, b\}$. The consumption level induced by such transfers is $x^i = \omega + z^i$. A (symmetric) *feasible allocation* is then described by a pair of net transfers $\{z^g, z^b\}$ satisfying the following resource feasibility constraint:

$$\sum_{s\in S} (\xi^g \pi^g_s z^g_s + \xi^b \pi^b_s z^b_s) \leq 0, \tag{1}$$

where the purely idiosyncratic nature of the uncertainty and the law of large numbers have been used to take the sum of the net transfers of the commodity contingent on each individual state, weighted by their probability.

An *incentive-compatible allocation* is a pair $\{z^g, z^b\}$ that satisfies the constraints that type g agents prefer net trade z^g to z^b and type b agents prefer z^b to z^g:

$$\sum_{s\in S} \pi^g_s u(\omega_s + z^g_s) \geq \sum_{s\in S} \pi^g_s u(\omega_s + z^b_s) \tag{2}$$

and

$$\sum_{s\in S} \pi^b_s u(\omega_s + z^b_s) \geq \sum_{s\in S} \pi^b_s u(\omega_s + z^g_s). \tag{3}$$

DEFINITION 1. An allocation $\{z^g, z^b\}$ is incentive efficient if it is feasible and incentive compatible (i.e., satisfies [1]–[3]) and if there does not exist another allocation $\{\hat{z}^g, \hat{z}^b\}$, also feasible and incentive compatible, such that $U^b(\omega + \hat{z}^b) \geq U^b(\omega + z^b)$ and $U^g(\omega + \hat{z}^g) \geq U^g(\omega + z^g)$, with at least one inequality being strict.[4]

For the simple adverse-selection economy under consideration, Prescott and Townsend (1984*b*) have provided a complete characterization of the set of incentive-efficient consumption allocations (see also Crocker and Snow 1985; Jerez 2003). We summarize its main elements below.

At any incentive-efficient allocation, at least one of the two types of agents is fully insured (has a deterministic consumption bundle). Incentive-efficient allocations can then be classified according to which type of agent is fully insured.

Consider first the incentive-efficient allocations in which the type b

[4] As shown by Prescott and Townsend (1984*b*) (see also Cole 1989), in the presence of asymmetric information, it may be desirable to expand the commodity space so as to allow for random allocations of contingent commodities or lotteries over consumption bundles. This is not the case for our simple adverse-selection economy, for which allocations involving nondegenerate lotteries are always suboptimal (see Prescott and Townsend 1984*b*). To keep the notation simpler, definitions are then stated for the case of nonrandom allocations.

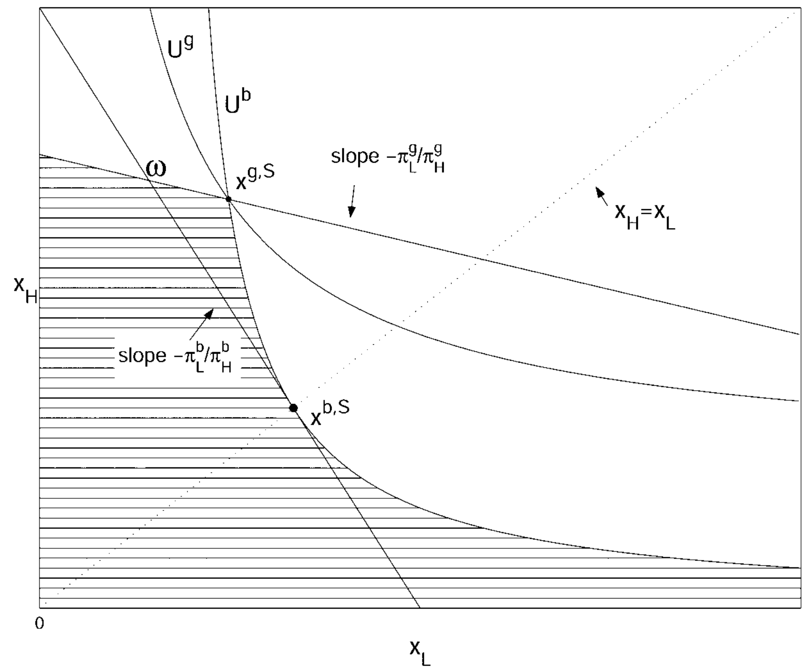

FIG. 1.—The separating allocation. ω denotes the individual endowment; $(x^{i,S})_{i=g,b}$ the consumption level at the separating allocation, i.e., $x^{i,S} = \omega + z^{i,S}$; and U^b and U^g the indifference curves of types b and g, respectively, at this allocation. Type b agents are fully insured at fair odds: $x_s^{b,S} = \sum_{s\in S}(\pi_s^b \omega_s)$ for each s. Type g agents are only partially insured, again at fair odds, so as to satisfy the type b incentive-compatibility constraint: $x^{g,S}$ lies at the intersection of U^b and g's fair odds line.

agents are fully insured. At such allocations the type g agents are only partially insured; that is, their consumption is higher in state H than in state L. The level of insurance provided to type g agents is in fact limited by the incentive constraint requiring that type b agents prefer z^b to z^g, as in (3) above. Type b agents are instead fully insured because the other incentive constraint is not binding. Allocations in this class can be parameterized by the level of consumption of type b agents or, equivalently, by the expected value of the net transfer they receive, on a per capita basis, given by $\sum_{s\in S}\pi_s^b z_s^b$. The allocation obtained when the expected net transfer to type b agents is zero is the *separating* allocation induced by the Rothschild-Stiglitz separating pair of contracts, illustrated in figure 1.

The minimum level of the expected net transfer to type b agents that is compatible with incentive efficiency may turn out to be strictly positive. This occurs in particular when the fraction ξ^g of type g agents is large

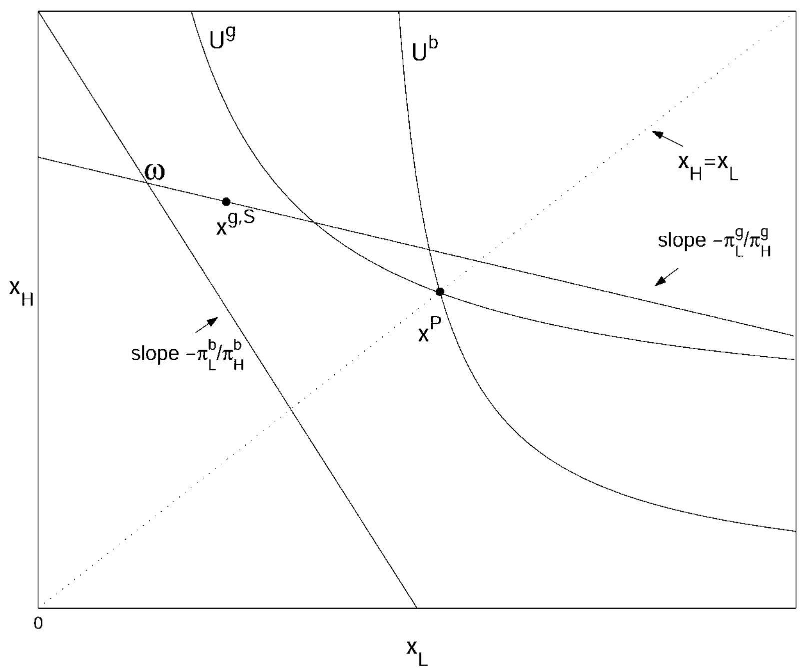

FIG. 2.—The pooling allocation. $x^P = \omega + z^P$ denotes the consumption level at the pooling allocation, and U^b and U^g the indifference curves of types b and g, respectively, at the pooling allocation. Both types of agents are fully insured and consume the aggregate per capita endowment: $x^P = \sum_{s \in S} (\xi^g \pi^g_s \omega_s + \xi^b \pi^b_s \omega_s)$.

enough. In this case the Rothschild-Stiglitz *separating* allocation is not incentive efficient: the utility of type g agents is increased if they make a positive subsidy to type b's consumption, since the subsidy relaxes their binding incentive constraint and this more than compensates the cost of the subsidy.

At incentive-efficient allocations in which the type b agents are fully insured, the higher the expected value of the net transfer to type b agents, the weaker the incentive constraint (3) and, hence, the greater the amount of insurance that can be provided to type g agents. Then there also exists a maximum level of the expected net transfer compatible with the incentive efficiency of allocations in this class, at which (3) becomes nonbinding. At the allocation corresponding to this level, both types of agents are fully insured and consume the same amount. This is the *pooling* allocation induced by the Rothschild-Stiglitz pooling contract, illustrated in figure 2. No incentive constraint binds at this allocation, and Pareto efficiency also obtains.

The second class of incentive-efficient allocations has the property that type g agents are fully insured. Type b agents are then overinsured; that is, their consumption is higher in state L. The extent by which this happens is determined by the incentive constraint requiring that type g agents prefer z^g to z^b, as in (2). The characterization of incentive-efficient allocations when type g agents are fully insured is symmetric to the characterization obtained above when type b agents are fully insured. Allocations can now be parameterized by the per capita level of the expected net transfer type g agents receive, given by $\sum_{s\in S}\pi_s^g z_s^g$.

As in Rothschild and Stiglitz, the pooling and the separating allocations will play a central role in our analysis.

IV. Walrasian Equilibria

Various competitive equilibrium concepts have been used in the analysis of adverse-selection economies with exclusive contracts. The standard strategic analysis of such economies, due to Rothschild and Stiglitz (1976), considers the Nash equilibria of a game in which insurance companies simultaneously choose the contracts they issue. In this game, the competitive aspect of the equilibrium is captured by allowing the free entry of insurance firms.[5]

Our approach consists instead in studying Walrasian equilibrium concepts in which both agents and insurance firms act as price takers in competitive markets. At equilibrium, a price is quoted for all possible contracts—where a contract is defined by a bundle of consumption claims, contingent on each individual state—and contracts are indexed by the agent's type. In other words, distinct insurance markets exist for high- and low-risk agents. Since each agent's risk type is unobservable whereas trades are observable, the viability of markets indexed by the agents' types requires the presence of appropriate restrictions on the trades agents can make in each market. In fact, firms contemplating which contracts to offer, for instance, in the market for the low-risk agents, will not want to offer contracts that a high-risk agent would prefer at the market prices to all the contracts offered in the market for high-risks. More generally, the set of contracts firms would be willing to offer restricts admissible trades to incentive-compatibility ones. Following the method used by Prescott and Townsend (1984*b*) for moral hazard economies, we directly assume that agents are restricted to trade only incentive-compatible contracts, without explicitly deriving the restriction from the firms' problem. Prices are then indexed by the agent's

[5] Different specifications of the game among insurance companies have been considered in the literature, with the aim of better capturing some aspects of the competition among firms under adverse selection; in particular, Wilson (1977), Riley (1979), and Hellwig (1987) allow for dynamic reactions to new contract offers; see also Maskin and Tirole (1992).

declared type and are linear over the restricted domain of incentive-compatible trades.[6]

We will consider Walrasian equilibrium concepts that are associated with different market structures and different institutional environments. First we will introduce EPT equilibria, where each agent can trade any incentive-compatible insurance contract. In fact, we shall allow agents to trade a complete set of contingent consumption claims, restricted only by the incentive-compatibility constraints. As already noted, the incentive constraints link the trades made by the different types of agents in the markets, therefore generating an externality. Next, we will examine ALPT equilibria in which markets for consumption rights, which induce agents to internalize the externality, are introduced and enforced by a regulatory mechanism.

We will show that, in contrast to the strategic approach by Rothschild and Stiglitz, for either of the market structures and institutional environment we study, Walrasian equilibria always exist. We characterize equilibrium allocations and study the welfare properties of both EPT and ALPT equilibria.

A. *EPT Equilibria*

We start by describing, first in words and then more formally, the set of admissible trades that agents face in the market structure associated with EPT equilibria. Each agent can trade, at linear prices, claims contingent on every realization of his individual uncertainty. There is a different market designated for each type. The agent first has to choose whether to trade in the market designated for type g agents or in the one designated for type b. The set of admissible trades in each of these markets is restricted by the incentive-compatibility constraints. In other words, agents (who declare to be) of type g can choose how many contingent claims to trade at the prices designated for type g agents. However, they are prohibited from trading in this market any amount of claims that is strictly preferred by type b agents to the amount chosen in equilibrium by the agents trading in the market designated for type b; similarly for agents (declaring to be) of type b.

[6] In this way the decision of firms is greatly simplified; firms are in fact free to offer any contract in the existing markets at the given prices. An equivalent specification in which the constraints are imposed directly on the firms rather than on consumers is possible but significantly more complex, as shown by Jerez (2003) in the context of moral hazard economies. An alternative way of modeling exclusive contractual relationships in Walrasian economies allows for nonlinear prices over the space of contingent claims while restricting admissible prices via suitable "refinements" to select among the resulting large set of equilibria. It is interesting to notice that in our setup, this method yields the same equilibrium allocations as the ones we get at EPT equilibria (see Gale 1992; Dubey, Geanakoplos, and Shubik 2005).

More precisely, the specification of the set of admissible trades for every agent is constructed as follows. Let $q_{i,s} \in \mathbb{R}_+$ be the unit price at which any agent who claims to be of type $i \in \{g, b\}$ can trade the consumption good for delivery in his individual state $s \in S$, $\boldsymbol{q} \equiv \{q_{g,H}, q_{g,L}, q_{b,H}, q_{b,L}\} \in \mathbb{R}^4_+$. To fix notation, note that here and in what follows a subscript $i \in \{g, b\}$ denotes the type declared by the agent, and a superscript denotes his actual unobservable type. The agent has to choose a vector $\boldsymbol{z} \equiv \{z_g, z_b\} \in \mathbb{R}^4$, where $z_i \equiv \{z_{i,H}, z_{i,L}\}$ denotes the net trades made in the market in which agents who declare to be of type $i \in \{g, b\}$ trade. Nonnegativity of consumption requires that $\boldsymbol{z} + (\omega, \omega)$ be nonnegative; this is the first part of the definition of the set of admissible trades (definition 2 below).

Every agent can claim to be of type g and trade in the market designated for type g's at the prices q_g. Alternatively, he can claim to be of type b and trade in the market designated for b at the prices q_b. If an agent declares to be of type g and hence trades in the market for g, that is, if he chooses $z_g \neq 0$, then he cannot trade in the market for b and he must choose $z_b = 0$; the second part of definition 2 formally states this requirement.

Moreover, the net trades of an agent in the market for g have to be incentive compatible with respect to the net trades made in the market by agents who claim to be of type b. In other words, type g agents are prohibited from choosing net trades such that type b agents strictly prefer them to the trades that can be made in the market for type b agents (and, in particular, to the amount traded in that market in equilibrium). Let $\bar{\boldsymbol{z}}_b = (0, \bar{z}_b)$ denote the net trades made in the market by agents who claim to be of type b. A trade of contingent claims z_g in the market for g is incentive compatible, and hence admissible, only if type b agents weakly prefer $\bar{z}_b$ to z_g.[7] Similarly, if the agent chooses instead to trade in the market for the b types, that is, $z_b \neq 0$, type g agents must prefer $\bar{z}_g$ to z_b, where $\bar{\boldsymbol{z}}_g \equiv (\bar{z}_g, 0)$. This condition is stated in the third part of definition 2.

Definition 2. The set of admissible net trades for each agent, $Z(\bar{\boldsymbol{z}}_g, \bar{\boldsymbol{z}}_b)$, is given by the vectors $\boldsymbol{z} \in \mathbb{R}^4$ such that (a) $\boldsymbol{z} + (\omega, \omega) \gg 0$; ($b$) for all $i \in \{g, b\}$, $z_i \neq 0 \Rightarrow z_j = 0$ for $j \neq i$; and (c)

[7] It is not explicitly required that type g agents prefer z_g to $\bar{z}_b$, as in the first of the two incentive constraints appearing in the definition of incentive-constrained allocations, (2). Such a constraint would in fact be redundant. Each agent trading in the market for g will always choose his most preferred allocation. Therefore, he will choose an allocation that he prefers to $\bar{z}_b$ if such an allocation is available. But at equilibrium this will always be the case since type g's incentive constraint (2) is imposed on the problem of agents trading in the market for b, and at equilibrium we require $\bar{z}_b$ to be the actual choice made by agents trading in market b.

$$\sum_{s\in S}\pi_s^j u(\omega_s+\bar{z}_{j,s})\geq\sum_{s\in S}\pi_s^j u(\omega_s+z_{i,s}). \tag{4}$$

The set of admissible net trades depends on the level of trades made in the market $\bar{\boldsymbol{z}}_g$, $\bar{\boldsymbol{z}}_b$ via the incentive-compatibility constraints imposed in the specification of this set. This is the formal representation of the consumption externality arising in Walrasian equilibria of economies with adverse selection and fully observable trades. Our first contribution is to show that once the presence and the nature of this externality are clearly identified, Walrasian equilibria can be defined and their properties analyzed as in other economies with externalities in consumption.

The choice problem of an agent of type $i\in\{g, b\}$ then has the following form:

$$\max_{z\in Z(\bar{z}_g,\bar{z}_b)}\sum_{s\in S}\pi_s^i u\Big(\omega_s+\sum_{j\in\{g,b\}}z_{j,s}\Big) \qquad (\mathrm{P}^{\mathrm{EPT},i})$$

subject to $\boldsymbol{q}\cdot\boldsymbol{z}\leq 0$.

Insurance firms supply insurance contracts, defined by purchases and sales of claims contingent on the realization of the individual uncertainty and the agents' declared type. Moreover, firms can construct aggregates—or "pools"—of such contracts and transform them into riskless claims by the law of large numbers. Let $\boldsymbol{y}\equiv\{y_{g,H}, y_{g,L}, y_{b,H}, y_{b,L}\}$ denote the vector describing the supply of net trades of contingent commodities, on a per capita basis. Firms are then characterized by the following constant returns to scale technology:

$$Y=\Big\{\boldsymbol{y}\in\mathbb{R}^4:\sum_{i\in\{g,b\}}\sum_{s\in S}\pi_s^i y_{i,s}\leq 0\Big\}.$$

The technology requires that firms offer individual contracts that are self-financing in the aggregate.

The firms' problem consists in the choice of a vector $\boldsymbol{y}$ lying in the set Y, so as to maximize profits:[8]

$$\max_{y\in Y}\boldsymbol{q}\cdot\boldsymbol{y}. \qquad (\mathrm{P}^{\mathrm{EPT},f})$$

We restrict our attention here, and in what follows, to symmetric equilibria, where all agents of the same type make the same choice.

Definition 3. An EPT equilibrium is given by a collection of net trades for each type of consumer, $(\boldsymbol{q}, \boldsymbol{z}^g, \boldsymbol{z}^b)$, a production vector $\boldsymbol{y}$, a price vector $\boldsymbol{q}$, and a pair $(\bar{\boldsymbol{z}}_g, \bar{\boldsymbol{z}}_b)$ such that

[8] As we noted, the presence of incentive compatibility as a restriction on consumers' admissible trades can be justified as a restriction on the set of contracts firms can offer to consumers (and such a restriction could be explicitly imposed on the set Y).

a. for each $i \in \{g, b\}$, $\boldsymbol{z}^i$ solves the optimization problem ($\mathrm{P}^{\mathrm{EPT},i}$) of consumers of type *i*, given ($\boldsymbol{q}$, $\bar{x}_g$, $\bar{x}_i$);

b. $\boldsymbol{y}$ solves the firms' profit maximization problem ($\mathrm{P}^{\mathrm{EPT},f}$), given $\boldsymbol{q}$;

c. markets clear:

$$\sum_i \xi^i \boldsymbol{z}^i \leq \boldsymbol{y}; \tag{5}$$

d. the level of trades in each market, taken as given by agents, is consistent with the agents' actual choice:

$$\bar{\boldsymbol{z}}_g = \boldsymbol{z}^g,$$

$$\bar{\boldsymbol{z}}_b = \boldsymbol{z}^b.$$

Note that condition *d* requires that at equilibrium each agent chooses to declare his true type: type *g* agents choose to trade in the market designated for *g* and type *b* agents prefer to trade in the market designated for *b*.

The formulation of the agents' set of admissible trades in ($\mathrm{P}^{\mathrm{EPT},i}$), together with the consistency condition *d*, ensures that the equilibrium allocation (z_g^g, z_b^b) is mutually incentive compatible; that is, it satisfies (2)–(3).

We are able to completely characterize the EPT equilibria of the economy under consideration. First of all, it is easy to verify that the specification of the set of admissible trades in definition 2 ensures that each agent will choose to trade in the market designated for his own type. Moreover, the constant returns to scale property of the firm's technology, which characterizes the set *Y* of technologically feasible contracts firms can offer, implies that a solution of the firms' maximization problem ($\mathrm{P}^{\mathrm{EPT},f}$) requires prices $\boldsymbol{q}$ to be *fair*.

LEMMA 1. At an EPT equilibrium, prices of contingent commodities have to be "fair":

$$q_{i,s} = \pi_s^i, \quad i \in \{g, b\},\ s \in S. \tag{6}$$

If in fact prices were not fair, for example, $\pi_L^b > q_{b,L}$, $\pi_H^b \leq q_{b,H}$, firms could achieve unboundedly large positive profits by selling commodity (*b*, *H*) and buying (*b*, *L*).

At an EPT equilibrium, therefore, type *b* agents face fair prices, and we can show that they face no binding incentive constraint. They will then fully insure and choose a level of net trades corresponding to *b*'s component of the separating allocation, $z^{b,S} = x^{b,S} - \omega$.

The set of budget-feasible and incentive-compatible consumption levels of type *g* agents at the equilibrium prices is illustrated in figure 1. It corresponds to the shaded area lying below the lower contour of the indifference curve U^b passing through $x^{b,S}$ and the fair odds line for type g, which by lemma 1 is the budget constraint in the market for g. The preferred point by type g agents in this set, $x^{g,S}$, is the consumption level induced by type g's net trades at the separating allocation. We conclude that the only EPT equilibrium of the economy is the Rothschild-Stiglitz separating allocation.

Theorem 1. Under assumption 1, a unique EPT equilibrium allocation always exists and coincides with the Rothschild-Stiglitz separating allocation.

Thus the Rothschild-Stiglitz pooling allocation is never an EPT equilibrium. To understand why, it is important to note at the outset that this allocation is incentive compatible, and hence it is included in the commodity space. Nonetheless, no vector of prices supports the pooling allocation as an EPT equilibrium. As shown in lemma 1, in equilibrium, prices have to be fair. At fair prices, though, the pooling allocation is not budget-feasible for the type b agents, and hence it cannot be an equilibrium. This is just a consequence of the fact that the pooling allocation requires cross-subsidization from type g's to type b's, and cross-subsidization requires prices not to be fair.[9]

We have shown that the separating allocation is always an EPT equilibrium. For the same economy, Rothschild and Stiglitz (1976) find instead robust instances in which the separating allocation is not an equilibrium (and no other allocation is). When all agents prefer the pooling allocation to the separating one, their argument goes, the latter is not an equilibrium because the introduction of a pooling contract constitutes a profitable deviation, since all agents will acquire it. To understand why this argument does not apply in our context, we should point out that in a Walrasian equilibrium, *all* technologically feasible contracts $y \in Y$ are simultaneously available for trade in a competitive market. This includes the pooling contract, the separating pair of contracts, and the contracts that type g agents prefer to the pooling contracts. It is indeed the availability of contracts that "skim" type g agents from the pooling contract, which implies that the pooling contract is never a profitable deviation from the separating allocation at a Walrasian equilibrium.[10]

Welfare properties of EPT equilibria.—As we noticed earlier, when the fraction ξ^b of type b agents is sufficiently small, the separating allocation

[9] The pooling allocation would be budget feasible for both types if $q_g = q_b$. But at these prices, as argued for lemma 1, firms could achieve unboundedly large positive profits by selling commodity (g, L) and buying (b, L), i.e., by selling only insurance to the g types. In other words, firms could make positive profits by "breaking" the pooling and introducing some separation, as in the argument used by Rothschild and Stiglitz (1976) to explain why the pooling allocation is not supported as an equilibrium.

[10] This is not unrelated to Wilson's (1977) and Riley's (1979) critique of the strategic notion of equilibrium of Rothschild and Stiglitz (1976); see Bisin and Gottardi (2005) for an extended discussion of this point.

is not incentive efficient. The characterization of EPT equilibria obtained in theorem 1 therefore reveals that the first welfare theorem does not hold: that EPT equilibria may not be incentive efficient. This should not come as a surprise: our formulation of Walrasian equilibria in adverse-selection economies, EPT, clearly identifies an externality, not internalized by the structure of markets considered, that may preclude the incentive efficiency of equilibrium allocations.

It is useful, however, to examine more closely what the precise source of the inefficiency is. As shown by lemma 1, at an EPT equilibrium, the prices of contracts traded by each type are always fair. Thus at equilibrium, there is never cross-subsidization across types. We show next that such lack of cross-subsidization is clearly responsible for the possible inefficiency of EPT equilibria; moreover, it is in fact the only source of inefficiency. EPT equilibrium allocations are efficient within the restricted subset of allocations that are incentive efficient and satisfy an additional condition requiring that there is no cross-subsidization across types.[11] Thus the following third-best version of the first welfare theorem holds.

PROPOSITION 1. Under assumption 1, all EPT equilibrium allocations are efficient within the restricted set of feasible allocations that are incentive compatible and, in addition, satisfy the condition

$$\sum_{s\in S} \pi_s^i z_s^i \leq 0, \quad i \in \{g, b\}. \tag{7}$$

On the other hand, the second welfare theorem holds for the present structure of markets: any incentive-efficient consumption allocation can be decentralized as an EPT equilibrium *with transfers*. Let $(t_H, t_L) \in \mathbb{R}^2$ denote the vector of state-contingent transfers to the agents. Note that transfers are the same for both types since types are private information. Thus the budget constraint of an agent who chooses to trade in the market for i, in the presence of the transfers (t_H, t_L), is

$$\sum_{s\in S} q_{i,s}(z_{i,s} - t_s) \leq 0, \tag{8}$$

for $i \in \{g, b\}$. Finally, we say that a vector of transfers (t_H, t_L) is *feasible* if

$$\sum_{s\in S} (\xi^g \pi_s^g t_s + \xi^b \pi_s^b t_s) \leq 0.$$

PROPOSITION 2. Let (z^g, z^b) be an arbitrary incentive-efficient allo-

[11] See Gale (1996) for a similar result for competitive equilibria of adverse-selection economies with fully nonlinear price schedules.

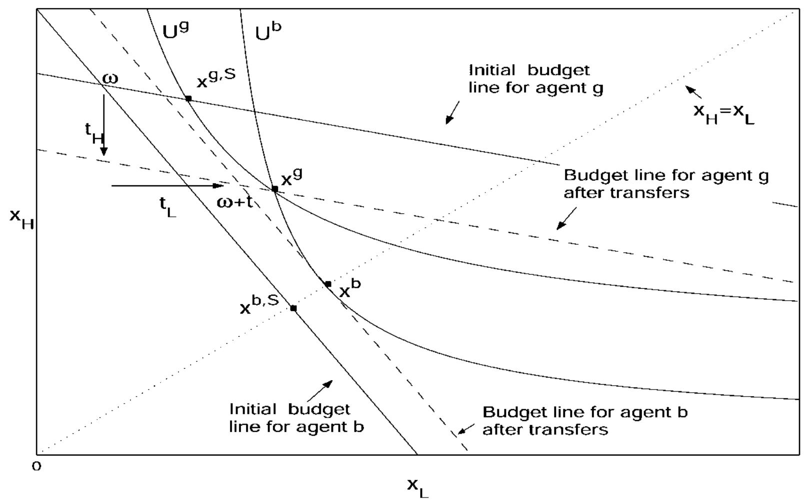

FIG. 3.—The second welfare theorem. $t = \{t_H, t_L\}$ denotes the transfer to the agents, and U^b and U^g the indifference curves of types b and g at the EPT equilibrium after the transfer. The consumption level at the initial EPT equilibrium is $(x^{i,S})_{i=g,b} = (z^{i,S})_{i=g,b} + \omega$. The consumption at the EPT equilibrium with transfers t is $(x^i)_{i=g,b} = (z^i)_{i=g,b} + \omega$.

cation. Under assumption 1 there exists a set of feasible transfers $(t_H, t_L) \in \mathbb{R}^2$ such that (z^g, z^b) is the allocation obtained at the EPT equilibrium of the economy under consideration when each agent receives a transfer (t_H, t_L), that is, when each agent's budget constraint is as in (8).

Figure 3 illustrates this result. For the given initial endowment ω, the EPT equilibrium is the corresponding separating allocation $(z^{i,S})_{i=g,b}$. Consider now the class of incentive-efficient allocations characterized by full insurance of type b agents and partial insurance of type g agents. They have the property that the consumption level of type g satisfies the incentive constraint of type b with equality; that is, the consumption level of type g lies on type b's indifference curve U^b. Take an arbitrary incentive-efficient allocation $(z^i)_{i=g,b}$ in this class. Transfers $t = (t_H, t_L)$ can be constructed as in the figure so that the EPT equilibrium (the separating allocation) associated with the endowment $\omega + t$ is in fact $(z^i)_{i=g,b}$. The formal proof is contained in the Appendix.

B. *ALPT Equilibria*

In the definition of EPT equilibria the set of admissible trades of each agent is restricted by incentive-compatibility constraints that relate the level of net trades an agent can make in a market to the net trades made in the other market. This fact generates, as we noticed, an externality in consumption. To decentralize incentive-efficient allocations in the adverse-selection economies under consideration, it is then necessary to introduce markets in which agents can trade the "commodity" that generates the externality (as suggested by Lindahl [1919] and Arrow [1969] for general economies with externalities and public goods).[12]

The design of a structure of markets for *consumption rights* that allows one to decentralize incentive-efficient allocations as Walrasian equilibria contains three main components: a definition of the agents' choice set, which includes holdings of consumption rights; an enforcement mechanism that specifies exactly which "rights" holding consumption rights provides the agent with; and a specification of the agents' initial endowments of consumption rights as well as of the technology to produce them. The design of the markets for consumption rights requires some regulatory provisions set by, for example, a government agency, as in the case of markets of pollution rights in which a government agency distributes the rights and designs and enforces all regulations specifying the amounts of pollution rights to be acquired per unit produced or polluted (see Ledyard and Szakaly-Moore 1994).

We discuss the components of the decentralization mechanism we propose for our economy with adverse selection below, first informally and then more formally.

Consider an agent, say of type g. As in the market structure considered in the previous section, first he has to choose in which market to trade, either the market designated for type g agents or the market for type b agents. The set of admissible net trades he will face in each market will be such as to induce him to trade in the market designated for his own type, type g. However, now in this market he will trade consumption rights as well as contingent claims. To be able to realize any desired level of consumption by trading contingent claims in the market for the type g's, the agent is required in fact to hold an appropriate amount of *consumption rights for market* g. This is the enforcement mechanism component of the market design. It provides a link between consumption and holdings of consumption rights, with the objective of making the agent internalize the effect of his consumption on the incentive-compatibility constraints of the other type: the higher the level of consumption by type g agents, the tighter the incentive constraint that has to be satisfied to prevent type b agents from pretending to be of type g, and hence the higher the amount of consumption rights for market

[12] Arrow-Lindahl equilibrium concepts are sometimes criticized on the basis of the fact that the price-taking assumption is inconsistent with individualized prices (since markets are too "thin"); see, e.g., Chari and Jones (2000). But in the economy considered here, there is a continuum of agents of each type; hence the presence of markets for consumption rights for each type is still consistent with price-taking behavior.

g a type g agent needs to hold.[13] Only when the price of consumption rights for market g is zero will the agent be free to choose his trades of contingent claims in this market. In this case the enforcement mechanism does not constrain his choice. Furthermore, the units of denomination of the consumption rights are specified so that, in equilibrium, each agent trading in the market for g holds an amount of rights equal to the consumption level of the agents trading in the market for b (and vice versa).

Finally, the supply of consumption rights needs to be specified. Consider the rights for market g (the supply of rights for market b is symmetric). Both agents' types are provided by a regulatory mechanism with a certain initial amount of rights for market g (which we call "endowment"). In addition, the agents who choose to trade in the market designated for b are given by the same regulatory mechanism (we say "produce") an additional amount of consumption rights for market g directly proportional to their net trades of contingent claims. A higher level of net trades in market b relaxes the type b's incentive constraint and hence makes the participation in the market for type g agents easier. The production of consumption rights for market g by agents trading in market b therefore allows agents operating in market b to internalize the positive externality their own trades exert on the agents trading in the other market.

We turn now to a formal description of the market design. Let $\mathbf{z} \equiv \{z_g, z_b\} \in \mathbb{R}^4$ be, as before, the vector of net trades of contingent claims, where $z_i \equiv \{z_{i,H}, z_{i,L}\}$ denotes the trades of an agent in the market designated for type $i \in \{g, b\}$. Nonnegativity of consumption again requires that $\mathbf{z} + (\omega, \omega)$ be nonnegative, as stated in the first part of the specification of the set of admissible trades (definition 4 below).

Also as before, an agent who chooses net trades $z_g \neq 0$ in the market for g, effectively declaring to be of type g, cannot trade in the market designated for type b agents and must therefore choose $z_b = 0$. This is summarized by the condition that

$$\forall i \in \{g, b\}, \quad z_i \neq 0 \Rightarrow z_j = 0 \quad \text{for } j \neq i,$$

which is the second part of definition 4.

The next conditions describing the set of admissible trades differ instead from those in the previous section. To be able to trade z_g units in the contingent-claims market for g and hence to consume $x_g =$

[13] The enforcement mechanism might be designed to operate on net trades rather than on consumption, to allow agents to consume their own endowment without holding any rights. We avoid making this distinction to keep simpler the presentation and the interpretation of the market mechanism.

$\omega + z_g$ units, the agent must hold an amount $\chi(g) \equiv \{\chi_H(g), \chi_L(g)\} \in \mathbb{R}^2_+$ of consumption rights for market g, which satisfies[14]

$$\sum_{s\in S} \pi_s^b u(\chi_s(g)) \geq \sum_{s\in S} \pi_s^b u(\omega_s + z_{g,s}). \tag{9}$$

This specification of the enforced link between the contingent trades in the market for g and the holdings of consumption rights for market g guarantees that type b's incentive constraint is satisfied for the amount of rights held by the agent. Therefore, type b's incentive constraint is internalized by agents trading in the market designated for type g agents. Symmetrically, agents trading in the market for b must hold an amount $(\chi_H(b), \chi_L(b))$ of consumption rights so that type g's incentive constraint is satisfied.

The total initial endowment of consumption rights for market g is $\omega\xi^g$. A fraction $1 - \alpha$ of this endowment is equally distributed among the agents trading in the market for g, and the remaining fraction α is distributed to the agents trading in the market for b. Each agent trading in the market for g therefore receives $(1 - \alpha)\omega$ units of endowment of rights for consumption in market g, whereas $(\xi^g/\xi^b)\alpha\omega$ units of the same rights go to each agent choosing to trade in the market for b. The distribution of consumption rights for market b is symmetric. Therefore, each agent who trades in the market for g also has an endowment of $(\xi^b/\xi^g)\alpha\omega$ units of consumption rights for market b.[15] The constant α conveniently parameterizes the distribution of the initial endowment of consumption rights across agents. The higher α is, the lower the initial endowment an agent has of consumption rights for the market he chooses, and hence the higher the amount of consumption rights an agent has to buy to be able to satisfy the constraints required to consume and trade in this market (i.e., the greater the payment needed to internalize the externality). We restrict α to be strictly positive: $\alpha \in (0, 1]$.[16]

The outstanding amount of consumption rights is not limited to the total initial endowment distributed across the agents. Any trade of contingent claims to the consumption good in one market, in fact, produces consumption rights for the other market. More specifically, an agent choosing an amount z_g of net trades in the market for g, for instance,

[14] Note that consumption rights for market g are thus denominated in units of type b's consumption.

[15] Note that in our formulation the endowment of consumption rights of each agent depends on his choice of the market, i.e., on his implicit declaration of his type.

[16] This is needed to ensure that agents can find some trades in the interior of their budget set at all prices and hence to ensure the existence of an equilibrium.

produces an additional amount $(\xi^b/\xi^g)z_g$ of consumption rights for market b that he can trade in the market.

Let $\zeta(i) \equiv \{\zeta_H(i), \zeta_L(i)\} \in \mathbb{R}^2$ denote the net trades of consumption rights for market $i \in \{g, b\}$. The total amount of rights held by an agent then equals his initial endowment plus the amount that is possibly produced and the one traded. In particular, an agent who chooses to trade in the market for g then holds a total amount of rights for consumption in the market for g given by $\chi(g) = (1-\alpha)\omega + \zeta(g)$. The total amount of consumption rights for market b held by this agent is then $\chi(b) = (\xi^b/\xi^g)(\alpha\omega + z_g) + \zeta(b)$. The amount held of consumption rights, for both the market that is chosen (g in this case) and the other market, must be nonnegative:

$$(1-\alpha)\omega + \zeta(g) \geq 0,$$

$$\zeta(b) + \frac{\xi^b}{\xi^g}(\alpha\omega + z_g) \geq 0.$$

In addition, the amount of rights for market g must satisfy the incentive-compatibility condition (9). Symmetric conditions must hold for agents choosing to trade in the market for b. These are, respectively, conditions c and d in the definition of the set of admissible trades (definition 4 below).

Note that agents have no benefit from holding rights for the market in which they choose not to trade. They will therefore sell in the market their entire endowment as well as their production of such rights, as long as the price is positive (and free disposal guarantees that the equilibrium prices of consumption rights are nonnegative). Therefore, the net sales of consumption rights for market b, $-\zeta(b)$, by an agent trading in the market for g typically equals $(\xi^b/\xi^g)(\alpha\omega + z_g)$.

The presence of different markets designated for each type requires, as already noted, that the set of admissible trades be restricted so as to ensure that agents self-select in the market for their own private type. The imposition of incentive-compatibility constraints (9) ensures, as we shall see, that at equilibrium type b agents prefer to trade in the market for b rather than choosing the same trades as the agents who selected to trade in the market for g. However, this is not enough to guarantee that they also do not prefer other trades available in the market for g. To rule out such a possibility, an additional constraint needs to be imposed.[17] More specifically, we require that an agent cannot make trades

[17] Such a constraint is not needed, as we saw, in the case of EPT. On the other hand, in ALPT, where the incentive-compatibility constraint faced by an agent in a market is required to hold with respect to the amount of consumption rights chosen by the same agent rather than to the trades made by agents in the other market, the set of admissible trades is larger and self-selection might not be induced in the absence of this constraint. Rustichini and Siconolfi (2004) do not impose this constraint and find that equilibria may not exist.

in the market for g that only a type b agent would choose in that market, since these trades would clearly signal that the agent has untruthfully declared his type to be g. We exclude then as nonadmissible all those trades that only a type b agent would prefer to the trades that are made in this market, $\bar{z}_g$. This is formally stated as condition e in definition 4 below. As we will see, this guarantees that at equilibrium agents always declare their true type.

DEFINITION 4. The set of admissible trades of every agent, $\mathcal{Z}$, is given by the vectors $\boldsymbol{z} \in \mathbb{R}^4$ and $\boldsymbol{\zeta} \in \mathbb{R}^4$ such that

a. $\boldsymbol{z} + (\omega, \omega) \gg 0$;
b. for all $i \in \{g, b\}$, $z_i \neq 0 \Rightarrow z_j = 0$ for $j \neq i$; and the following conditions hold:
c. $(1 - \alpha)\omega + \zeta(i) \geq 0,\ \zeta(j) + (\xi^j/\xi^i)(\alpha\omega + z_i) \geq 0$;
d. $\sum_{s\in S}\pi_s^j u((1 - \alpha)\omega_s + \zeta_s(i)) \geq \sum_{s\in S}\pi_s^j u(\omega_s + z_{i,s})$;
e. if $\sum_{s\in S}\pi_s^i u(\omega_s + z_{i,s}) < \sum_{s\in S}\pi_s^i u(\omega_s + \bar{z}_{i,s})$, then $\sum_{s\in S}\pi_s^j u(\omega_s + z_{i,s}) < \sum_{s\in S}\pi_s^j u(\omega_s + \bar{z}_{i,s})$.

As we already mentioned, condition c requires the nonnegativity of the amount held of consumption rights, and conditions d and e are the incentive constraints restricting admissible trades. Condition d requires each agent choosing to trade in the market for type i to internalize the incentive-compatibility constraint of type j agents by means of his holdings of consumption rights for market i; condition e together with condition d guarantees, as we argued, that agents declaring to be of type i, and hence trading in the market designated for i, are in fact type i agents.

As we noted, in EPT the presence of the incentive-compatibility constraints in the specification of the set of admissible trades generates an externality. However, in ALPT, where markets for consumption rights are introduced, no externality is induced by the presence of those constraints, as in condition d. At the same time, condition e in $\mathcal{Z}$, where $\bar{z}_i$ appears, might seem to introduce once again an externality in the specification of the trading set and hence in the equilibrium notion. However, this is not the case (and this is why, somewhat abusing notation, we have not indexed the set of admissible net trades $\mathcal{Z}$ by $\bar{z}$). Condition e in fact guarantees that agents self-select in the market designated for their own type but *does not affect the equilibrium allocation and prices* in any other way: the condition rules in fact as nonadmissible in market i the trades that only type j but not type i would make in this market. Formally, we show (see lemma A.1 in the proof of theorem 2 in Bisin and Gottardi [2005]) that the set of equilibria of the economy under consideration, where the choice of any agent is restricted to lie in $\mathcal{Z}$, coincides with the set of equilibria of an artificial economy in which the agents' choice is restricted only by conditions a–d of the definition of $\mathcal{Z}$, but type i

agents are required to trade in the market designated for type i.[18] As a consequence, even though formally the set $\mathcal{Z}$ of feasible trades of every agent still depends on the level of trades made in the market, we can argue that in this case the externality does not matter,[19] since the market for consumption rights allows us to properly internalize it.

The vector of prices at which consumption rights can be traded is $\boldsymbol{p} \equiv \{p_H(g),\ p_L(g),\ p_H(b),\ p_L(b)\} \in \mathbb{R}^4_+$, and the vector of prices of state-contingent commodities is again denoted as $\boldsymbol{q} \equiv \{q_{g,H},\ q_{g,L},\ q_{b,H},\ q_{b,L}\} \in \mathbb{R}^4_+$. The choice problem of an agent of type $i \in \{g,\ b\}$ now has the following form:

$$\max_{(\boldsymbol{z},\boldsymbol{\zeta}) \in \mathcal{Z}} \sum_{s \in S} \pi^i_s u\Big(\omega_s + \sum_{j \in \{g,b\}} z_{j,s}\Big) \qquad (\mathrm{P}^{\mathrm{ALPT},i})$$

subject to

$$\boldsymbol{q} \cdot \boldsymbol{z} + \boldsymbol{p} \cdot \boldsymbol{\zeta} \leq 0.$$

Firms are characterized by the same technology Y as in EPT, and their choice problem is also the same:

$$\max_{\boldsymbol{y} \in Y} \boldsymbol{q} \cdot \boldsymbol{y}. \qquad (\mathrm{P}^{\mathrm{ALPT},f})$$

We then have the following definition.

Definition 5. An ALPT equilibrium is a collection of net trades of contingent commodities and consumption rights for each type of consumer $(\boldsymbol{z}^i,\ \boldsymbol{\zeta}^i)_{i \in \{g,b\}}$, a production vector $\boldsymbol{y}$, a price vector $(\boldsymbol{p},\ \boldsymbol{q})$, and a pair $(\bar{z}_g,\ \bar{z}_b)$ such that

a. for each $i \in \{g,\ b\}$, $(\boldsymbol{z}^i,\ \boldsymbol{\zeta}^i)$ is a solution of the optimization problem $(\mathrm{P}^{\mathrm{ALPT},i})$ of type i consumers, given $(\boldsymbol{p},\ \boldsymbol{q})$ and $\bar{z}_j$, $j \in \{g,\ b\}$;
b. $\boldsymbol{y}$ solves the firms' profit maximization problem $(\mathrm{P}^{\mathrm{ALPT},f})$, given $\boldsymbol{q}$;
c. markets clear, both for contingent commodities, $\sum_i \xi^i \boldsymbol{z}^i \leq \boldsymbol{y}$, and for consumption rights, $\sum_i \xi^i \boldsymbol{\zeta}^i \leq 0$;
d. the level of trades in each market, taken as given by agents, is consistent with the agents' actual choice: $\bar{z}_j = z^j_j$, $j \in \{g,\ b\}$.

As the analogous condition in EPT, the consistency condition d requires that at equilibrium agents choose to declare their type truthfully. Also, the specification of the set of admissible trades $\mathcal{Z}$, together with

[18] In this sense we can say that condition e does not bind locally at equilibrium allocations. This is not to say that it is redundant. We have in fact already noted (see n. 17) that it is not. To understand this, we should observe that the maximization problem of each agent in our economy is not convex and includes in particular a binary choice regarding which market to trade in.

[19] This can be done formally. An alternative equivalent specification of condition e, where no reference to $\bar{z}_i$ is made, is in fact possible, but at the cost of some nontransparent notation; hence we opted here for the specification in definition 4.

the consistency and the market-clearing conditions c and d, implies that the level of consumption of type b agents, $\omega + z_b$, equals the amount of consumption rights for market g held by type g, $(1 - \alpha)\omega + \zeta(g)$. As a consequence, again, the equilibrium consumption allocation (z_b^b, z_g^g) is mutually incentive compatible.

We can show[20] that, for the economy under consideration, an ALPT equilibrium always exists.[21]

THEOREM 2. Under assumption 1, an ALPT equilibrium always exists.

We derive next a characterization of ALPT equilibria. This allows us to better understand how the markets for consumption rights we designed work and will also be of use in our analysis of the welfare properties of ALPT.

First, in the proof of theorem 2 in Bisin and Gottardi (2005), we show that there is always an ALPT equilibrium in which prices are as follows:

$$q_{i,s} = \pi_s^i, \quad s \in S, \; i \in \{g, b\}, \tag{10}$$

and

$$p_s(g) = \beta\pi_s^b, \quad p_s(b) = 0, \; s \in S, \tag{11}$$

for some $\beta \in (0, \xi^b/\xi^g)$. The form of these equilibrium prices clearly illustrates how incentive-efficient allocations are sustained as ALPT equilibria. The prices $\boldsymbol{q}$ at which agents can trade the contingent claims for the consumption good are fair, as in EPT. But then the type g agents have to pay a positive price to acquire the consumption rights for market g needed to satisfy the required incentive-compatibility constraint. Type b agents, on the other hand, receive a positive revenue from the sale of these consumption rights. Furthermore, the consumption rights for market b trade at a zero price. As a consequence, type b agents do not have to pay any cost to ensure the incentive compatibility of their consumption, and type g agents receive nothing from the sale of these rights. At the equilibrium allocation obtained at the above prices, type b's incentive constraint (3) is binding, the consumption of the g agents

[20] Standard arguments cannot be directly applied here. We refer to Bisin and Gottardi (2005) for a discussion of the methods used to establish existence and for the proof of the result.

[21] In their working paper (Prescott and Townsend [1982], which is an extended version of their 1984b article), the authors briefly discuss a notion of competitive equilibrium for adverse-selection economies that is related to ALPT. In their formulation, however, each type of agent chooses directly the consumption level of the other type rather than the amount of consumption rights. In our context this is equivalent to the requirement that, e.g., the price agents face for consumption rights for market g, $p(g)$, is the same as the price that agents face for consumption claims in the market for type b, q_b. At these prices, the choices made by the two types of agents in the two markets will typically not be consistent; i.e., in our notation, $\zeta(g) + (1 - \alpha)\omega \neq z_b + \omega$. As a consequence, as Prescott and Townsend notice, an equilibrium will generally fail to exist.

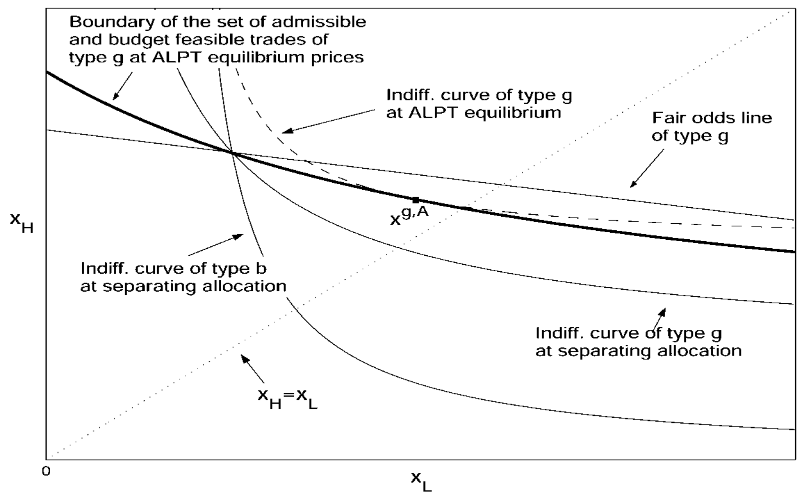

FIG. 4.—ALPT equilibrium. $x^{g,A}$ denotes the ALPT equilibrium consumption level for type g, and the solid curve depicts the boundary of the set of admissible and budget-feasible trades of contingent claims in the market for type g at the equilibrium prices. To find the boundary of this set, we proceed as follows. At the prices (10) and (11), it is immediate to see that the optimal choice of consumption rights for market g is at a full-insurance level: $\zeta_H(g) + (1-\alpha)\omega_H = \zeta_L(g) + (1-\alpha)\omega_L$. We can then solve this equation together with the incentive-compatibility constraint d in $\mathcal{Z}$, $\sum_{s\in S}\pi_s^b[\zeta_s(g) + (1+\alpha)\omega_s] = \sum_{s\in S}\pi_s^b u(z_{g,s}+\omega_s)$ for $\zeta(g)$ in terms of z_g. Substituting the resulting expression into the budget constraint in problem ($P^{\mathrm{ALPT},g}$) and noting that $z_b = 0$, for the type g agents, and $p(b) = 0$, we obtain an equation whose only variable is z_g, describing the boundary of the set required.

then exerts a negative externality on type b agents, and the positive equilibrium price of consumption rights for market g induces a subsidy from type g agents to type b agents. Consumption rights for market b trade at a zero price, however, because at equilibrium type g's incentive constraint (2) is not binding, and hence the consumption of type b agents generates no externality on type g agents.

Figure 4 illustrates such an ALPT equilibrium. In particular, the bold, solid curve describes the boundary of the set of admissible and budget-feasible trades of contingent claims in the market designated for type g, at an ALPT equilibrium. This set is typically larger than the corresponding set in EPT, depicted by the shaded area in figure 1. The bold, solid curve in figure 4, representing as we said the boundary of such a set, is obtained using the characterization of the equilibrium prices in (10) and (11) (details are in the figure legend). The equilibrium level of consumption for type g agents, given by the point $x^{g,A}$, corresponds to their preferred bundle on this curve. The equilibrium consumption level of type b agents (not in the figure) is then given by the highest full-insurance point in their budget set, where their income includes the revenue from the sale of consumption rights.

Is the presence of some level of cross-subsidization an intrinsic feature of ALPT equilibria? To answer this question, we examine how the equilibrium varies with the initial distribution of consumption rights, as parameterized by α. Let us denote by $\beta(\alpha)$ the equilibrium value of β associated with a given level of α in expression (11) of equilibrium prices; analogously, let $x(\alpha)$ be the corresponding equilibrium consumption level. In the next proposition we show that, as $\alpha \to 0$, the sequence of ALPT equilibrium consumption levels $\{x^g(\alpha),\ x^b(\alpha)\}$ converges to $(\underline{x}^i \equiv \omega + \underline{z}^i)_{i\in\{g,b\}}$, where $(\underline{z}^g,\ \underline{z}^b)$ is the incentive-efficient allocation characterized by the *minimum level of subsidization from type* g *agents to type* b *agents*. More formally, $\{\underline{z}^g,\ \underline{z}^b\}$ is obtained as a solution of the problem of maximizing the expected utility of type g agents, $U^g(\omega + z^g)$, subject to the resource feasibility and incentive-compatibility constraints, and the additional constraint that $\sum_s \pi_s^b z_s^b \geq 0$, that is, that b is not subsidizing g. It is immediate to see, given the characterization provided of incentive-efficient allocations, that $(\underline{z}^g,\ \underline{z}^b)$ coincides with the Rothschild-Stiglitz separating allocation whenever this is incentive efficient, and is otherwise (when the constraint $\sum_s \pi_s^b(z_s^b) \geq 0$ is not binding) given by the incentive-efficient allocation at which b's welfare is minimal.[22]

PROPOSITION 3. Under assumption 1, $\lim_{\alpha\to 0} x^i(\alpha) = \underline{x}^i$, for $i \in \{g, b\}$.

As shown in the proof of the proposition in the Appendix, for each $\alpha > 0$, the ALPT equilibrium characterized by prices of the form (10)–(11) exhibits a positive subsidy from the g types to the b types. Type b agents receive a subsidy through the sale of consumption rights for market g. When $\alpha \to 0$, the initial distribution of consumption rights is the most favorable to the type g agents, since they receive in the limit the entire amount of the total initial endowment of consumption rights for market g; hence the sale of such rights by the b agents will be the lowest, and the equilibrium will be the incentive-efficient allocation $\{\underline{z}^g,\ \underline{z}^b\}$ in which the subsidy to the b agents is minimal. We show in the proof of proposition 3 that, for the economies in which the Rothschild-Stiglitz separating allocation is incentive efficient, this allocation is decentralized as an ALPT equilibrium with $\alpha = 0$, and the equilibrium prices satisfy (11) with $\beta = \lim_{\alpha\to 0}\beta(\alpha) < \xi^b/\xi^g$. On the other hand, for the economies in which the Rothschild-Stiglitz separating allocation is not incentive efficient, an ALPT equilibrium does not exist when $\alpha = 0$ (see n. 16), but the sequence of ALPT equilibria allocations

[22] Note that $\{\underline{z}^g,\ \underline{z}^b\}$ is also the allocation induced by what is sometimes referred to as the Wilson-Miyazaki pair of contracts.

converges, as $\alpha \to 0$, to $\{\underline{z}^g, \underline{z}^b\}$, the allocation with the minimum level of subsidization from type g agents to type b agents, and equilibrium prices in the limit satisfy (11) with $\lim_{\alpha \to 0} \beta(\alpha) = \xi^b/\xi^g$.

1. Welfare Properties of ALPT Equilibria

First we show that the structure of markets for consumption rights we have designed does indeed solve the problem of decentralizing incentive-efficient allocations. Proposition 4 represents (an incentive-constrained version of) the first theorem of welfare economics for economies with adverse selection.

PROPOSITION 4. All ALPT equilibria are incentive efficient.

Moreover, when one uses the characterization of ALPT equilibria with $\alpha = 0$ provided by proposition 3, it is straightforward to extend the second welfare theorem obtained for EPT equilibria, proposition 2, to ALPT equilibria. Any incentive-efficient consumption allocation can be decentralized as an ALPT equilibrium with an initial distribution of rights given by $\alpha = 0$ and with transfers that are possibly dependent on the state but not on the agents' type.

PROPOSITION 5. Under assumption 1, for any incentive-efficient consumption allocation (z^g, z^b), there exists a set of feasible transfers (t_H, t_L) such that (z^g, z^b) is an ALPT equilibrium allocation when each agent receives the transfer (t_H, t_L) and $\alpha = 0$.

2. The Implementation of ALPT Equilibria

While we have shown that when markets for consumption rights are appropriately designed, incentive Pareto-optimal allocations can be decentralized as Walrasian equilibria, such markets might appear difficult to implement in actual economies. The objective of this subsection is to argue that, on the contrary, markets for consumption rights can be more simply implemented when designed along the lines, for example, of markets for pollution rights or of markets for the access to clubs. More specifically, this is the case if the enforcement mechanism operates on insurance firms rather than on consumers, linking the contracts they may offer to their holdings of appropriate amounts of *trading rights*. Moreover, the implementation is simplified if holding such rights is required only to offer contracts in the market for type g agents, the high-quality market in our setup, and if units of the rights are redefined so that every consumer is endowed with a single unit of the right to trade in the market for g.[23]

[23] See Carlson et al. (1993) and Ledyard and Szakaly-Moore (1994) for a discussion of the properties of enforcement mechanisms in the market for pollution rights that make them easily and effectively implementable.

We now illustrate how such a decentralization scheme may work and give rise to the same equilibrium outcomes as the structure of markets considered in ALPT. There are again two markets, one for the agents (declaring to be) of type b and one for those (declaring to be) of type g. If a consumer chooses to trade in the market for b, he can sell the right to trade in the market for g, at a price determined in equilibrium. Such rights are purchased by insurance firms. To be able to sell a contract in the market for g, a firm in fact has to satisfy an enforcement constraint, which prescribes that the contract is backed by an appropriate amount of trading rights. More precisely, the contract has to satisfy an incentive constraint requiring that only type g agents prefer it to the consumption level that can be attained in the market for b with the sale of the amount of trading rights backing the contract. Each firm may then offer a menu of contracts in the market for g, indexed by the amount of rights backing them: the larger this amount, the better the contract, since the incentive-compatibility constraint will be looser. No enforcement constraint, however, operates on contracts offered in the market for type b, the low-quality market. In this market firms are free to choose the specification of the contract, that is, the net payment to the agent in each state, so as to maximize profits.

The consumers trading in each market then face a given menu of contracts among which they can choose, which at equilibrium coincides with the menu of contracts firms are willing to offer in the two markets. At a competitive equilibrium, every agent chooses to trade in the market designated for his type and markets clear. In particular, the number of trading rights bought by firms operating in the market for g is equal to the number of type b agents in the economy, who all sell the right to trade in the market for g they are endowed with.

It is possible to show that there is always a competitive equilibrium for this economy in which prices have the following properties: (i) the rights to trade in the market for g have a positive price, (ii) each contract offered in the market for b is priced fairly, and (iii) the price of any contract offered in the market for g equals the sum of the value of the contingent claims included in the contract, evaluated at the fair price $q_{g,s} = \pi_s^g$, $s \in S$, and the market value of the trading rights backing the contract. At such prices, the type b agents choose to fully insure in the market for b and use the proceeds from the sale of their right to trade in the market for g to attain a level of consumption that is higher than the expected value of their endowment. Type g agents choose instead a partial insurance contract in the market for g. This contract is backed by a smaller amount of rights and hence is cheaper than the full-insurance contract. In particular, we can also show that the competitive equilibrium allocation is the same as the one obtained as the limit of ALPT equilibria, for $\alpha \to 0$ (see proposition 3): the structure of markets described then decentralizes the allocation $\{\underline{z}^g, \underline{z}^b\}$, the incentive-efficient allocation that is preferred by the type g agents, that is, the Rothschild-Stiglitz separating allocation when it is incentive efficient.

With the objective of exploring some strategic foundations for our notion of ALPT equilibrium, we have also studied an extension of the definition of the core proposed by Marimon (1988) (see also Boyd, Prescott, and Smith 1988) for adverse-selection economies.[24] Such a notion is characterized by the fact that blocking coalitions cannot tax, only subsidize agents outside the coalition, but otherwise agents of one type can separate at no cost. It is fairly immediate to see that a single allocation is in the core according to this notion and is $\{\underline{z}^g, \underline{z}^b\}$. But suppose instead that the high-quality types (g in our setup) can form a coalition excluding the low-quality types (b) only if they pay a given amount C, which we can interpret as the cost of acquiring the right to separate from the low-quality types (of course, the allocation proposed by the deviating coalition must also be incentive compatible). This clearly parallels the role of the distribution of the endowment of consumption rights in ALPT. It can be shown that, by varying C, we can obtain as core allocations the set of ALPT equilibrium allocations corresponding to different values of α.

V. Conclusions

We have studied in this paper how Walrasian markets work in economies with adverse selection when exclusive contracts are available and whether their outcome is efficient. In particular, we have identified a form of externality as a potential problem for the operation of Walrasian markets in this setup. We have then shown that an enlarged structure of markets, which includes markets for consumption rights, allows us to internalize this externality and hence to decentralize incentive-efficient allocations. All Walrasian (ALPT) equilibrium allocations are incentive efficient in this case. We have also shown that when such an enlarged set of markets is not available, a Walrasian (EPT) equilibrium always exists but may fail to be incentive efficient, and inefficiency is robust.

Our results have been derived for a class of simple insurance economies with adverse selection in which agents can be of two possible types, and the privately observed type of each agent concerns only the probability structure of the idiosyncratic shocks affecting the agent. Such economies provide an important benchmark for the analysis of markets

[24] It should be pointed out, though, that various other notions have been proposed, and agreement has not yet been reached on what is an appropriate notion of the core for economies with adverse selection.

and equilibria with asymmetric information, at least since the work by Rothschild and Stiglitz (1976). However, the equilibrium concepts we introduced can be extended to more general classes of economies with adverse selection.

Appendix

We omit the proof of theorems 1 and 2, showing the existence of EPT and of ALPT equilibria. We refer the interested reader to Bisin and Gottardi (2005).

Proof of Proposition 1

If, at a solution of the problem of maximizing the utility of the two types of agents subject to (2), (3), and (7), both incentive-compatibility constraints hold as equalities, under the assumptions made on agents' preferences (in particular, the single-crossing property), we must have $z^g = z^b$. But then (7) implies that z lies on, or below, the fair odds line for type b. Since the separating allocation is always weakly preferred by both types of agents to any point on this line and strictly by at least one type, this cannot be a solution.

On the other hand, if only one of the two incentive constraints is binding, say the one for type b (given by [3]), then the optimal level of z^b is simply obtained by maximizing $U^b(\omega + z^b)$ over (7); thus it will always be at the full-insurance point $\omega_H + z^b_H = \omega_L + z^b_L$ satisfying (7). The level of z^g is then determined by maximizing $U^g(\omega + z^g)$ subject to (7) and (3), taking $\omega + z^b$ as given at the full-insurance level determined before. It is immediate to see that the pair (z^g, z^b) is the same as the separating allocation of Rothschild and Stiglitz. If we apply a symmetric argument when (2) is the only constraint binding, we find that no solution exists in this case (when z^g is at the full-insurance level satisfying [7], no value exists for z^b that also satisfies [7] and [2]). QED

Proof of Proposition 2

Let (z^g, z^b) be an arbitrary incentive-efficient allocation. By lemma 1, at an EPT equilibrium, prices of contingent commodities are necessarily fair. If we then consider the budget lines $\sum_{s\in S}\pi^b_s(\omega_s + z^b_{b,s})$ and $\sum_{s\in S}\pi^g_s(\omega_s + z^g_{g,s})$ going through, respectively, the points $\omega + z^b$ and $\omega + z^g$, they will intersect at a single point, call it $\tilde{x}$. Since $\sum_{s\in S}\pi^i_s(\omega_s + z^i_s) = \sum_{s\in S}\pi^i_s\tilde{x}_s$, $i \in \{g, b\}$ and (z^g, z^b) satisfies the resource feasibility condition (1), the allocation $(\tilde{x} - \omega, \tilde{x} - \omega)$ is also feasible:

$$\sum_{s\in S}[\xi_g\pi^g_s(\tilde{x}_s - \omega_s) + \xi_b\pi^b_s(\tilde{x}_s - \omega_s)] \leq 0,$$

which implies that a transfer $t_H = \tilde{x}_H - \omega_H$, $t_L = \tilde{x}_L - \omega_L$ to all the agents is also feasible.

We show next that (z^g, z^b) is the (unique in fact) EPT equilibrium allocation of the economy when the agents receive the transfer $t = (t_H, t_L)$. Suppose not. Then there exists another admissible choice, say $(\hat{z}^g, 0)$ for g, that also lies in $Z[(z^g, 0), (0, z^b)]$; that is, $\hat{z}^g$ is incentive compatible relative to z^b and is budget feasible $(\sum_s\pi^g_s(\hat{z}^g_s - t_s) \leq 0)$, and $\hat{z}^g$ is strictly preferred to z^g by type g. But then $\sum_{s\in S}[\xi_g\pi^g_s(\hat{z}^g_s - t_s) + \xi_b\pi^b_s(z^b_s - t_s)] \leq 0$; thus the allocation $(\hat{z}^g, z^b)$ is also feasible and incentive compatible and Pareto-dominates (z^g, z^b), a contradiction. QED

Proof of Proposition 3

At any ALPT equilibria in which prices are of the form (10)–(11), type b agents face a zero price for the rights for consumption in market b. Hence their consumption choice is not constrained by incentive compatibility and they always choose to fully insure:

$$z^b_{b,H} + \omega_H = z^b_{b,L} + \omega_L = \frac{[1 - \beta(\xi^g/\xi^b)(1-\alpha)](\sum_{s\in S}\pi^b_s\omega_s)}{1 - \beta(\xi^g/\xi^b)}. \quad \text{(A1)}$$

The term on the right-hand side is the income of a type b agent, given by the value of his endowment of contingent claims, evaluated at $q_b = \pi^b$, plus the value of the amount of consumption rights for market g the agent possesses, evaluated at $p(b) = \beta\pi^b$. The equilibrium consumption level of the type b agents, as α varies, is then

$$x^b_s(\alpha) = \frac{[1 - (\xi^g/\xi^b)\beta(\alpha)(1-\alpha)](\sum_{s\in S}\pi^b_s\omega_s)}{1 - (\xi^g/\xi^b)\beta(\alpha)}, \qquad s \in S,$$

and the subsidy received by b, $\sum_{s\in S}\pi^b_s[x^b_s(\alpha) - \omega_s]$, is equal to

$$\frac{(\xi^g/\xi^b)\alpha\beta(\alpha)}{1 - (\xi^g/\xi^b)\beta(\alpha)}\sum_{s\in S}\pi^b_s\omega_s.$$

Both sequences $x(\alpha)$ and $\beta(\alpha)$, giving the equilibrium consumption level and rights' prices for different values of α, lie in compact sets. Thus they admit convergent subsequences; let $\hat{x}$ and $\hat{\beta}$ be their limit. On the basis of the above argument, if $\hat{\beta} = \lim_{\alpha\to 0}\beta(\alpha)$ is strictly less than ξ^b/ξ^g, the value of the subsidy converges to zero as $\alpha \to 0$, so that $x^b(\alpha)$ converges to full insurance at fair odds, that is, to type b's component of the consumption level at the Rothschild-Stiglitz separating allocation, $x^{b,S} = \omega + z^{b,S}$. From the first welfare theorem (proposition 4), $z^{b,S}$ must be part of an incentive-efficient allocation; hence $x^g(\alpha)$ must converge to $x^{g,S} = \omega + z^{g,S}$, and the separating allocation $z^{g,S}$, $z^{b,S}$ coincides with $\{\underline{z}^g, \underline{z}^b\}$ in this case.

On the other hand, if $\hat{\beta} = \xi^b/\xi^g$, the problem of type g agents, $(P^{\text{ALPT},g})$, in the limit (for $\alpha = 0$ and $\beta = \xi^b/\xi^g$), reduces to the problem of maximizing $U^g(\omega + z^g)$ (with respect to z^g, z^b) subject to the resource feasibility and incentive constraints (1)–(3) (since the budget constraint coincides with [1]). The solution of this problem is $\{\underline{z}^g, \underline{z}^b\}$. Note that the value of the subsidy to type b agents in this case converges to a level that is either positive or zero (but cannot be negative since it is strictly positive for all $\alpha > 0$). QED

Proof of Proposition 4

The proof is quite standard. Suppose not; that is, there exists a feasible, incentive-compatible allocation $(\hat{z}^g, \hat{z}^b)$ that Pareto-dominates the equilibrium allocation (z^g_g, z^b_b). Then, note that the following constitutes an admissible choice (i.e., that lies in $\mathcal{Z}$) for agent $i \in \{g, b\}$: $\hat{z}^i_i = \hat{z}^i$, $\hat{z}^i_j = 0$, $\hat{\zeta}^i(i) = \alpha\omega + \hat{z}^j$, and $\hat{\zeta}^i(j) = -(\xi^j/\xi^i)(\alpha\omega + \hat{z}^i)$, for $j \neq i$. Therefore, given the fact that local nonsatiation holds, as shown in the proof of theorem 2 in Bisin and Gottardi (2005), it must be

$$q_g \cdot \hat{z}^g + p(g)(\alpha\omega + \hat{z}^b) - p(b)\left(\frac{\xi^b}{\xi^g}\right)(\alpha\omega + \hat{z}^g) \geq 0$$

and

$$q_b \cdot \hat{z}^b + p(b)(\alpha\omega + \hat{z}^g) - p(g)\left(\frac{\xi^g}{\xi^b}\right)(\alpha\omega + \hat{z}^b) \geq 0,$$

one of the two inequalities being strict. Summing then the two inequalities, multiplied respectively by ξ^g and ξ^b, we get

$$\xi^g q_g \cdot \hat{z}^g + \xi^b q_b \cdot \hat{z}^b > 0.$$

Since $(\hat{z}^g, \hat{z}^b)$ is a feasible allocation, $(\xi^g\hat{z}^g, \xi^b\hat{z}^b)$ lies in the firms' production possibilities' set Y and, by the previous inequality, yields positive profits. This contradicts the fact that the production plan chosen by the firms at equilibrium, $(\xi^g z^g_g, \xi^b z^b_b)$, maximizes their profits at the prices q, since its profits equal zero. QED

Proof of Proposition 5

Let (z^g, z^b) be any incentive-efficient allocation. By proposition 2, there exists a set of feasible transfers (t_H, t_L) such that (z^g, z^b) is an EPT equilibrium allocation for the economy under consideration when each agent receives a transfer (t_H, t_L). Moreover, by theorem 1 and proposition 2, the economy in which agents' endowment equals $\omega + t$ has a unique EPT equilibrium (where the agents' consumption is given by $(\omega + z^g, \omega + z^b)$), coinciding with the Rothschild-Stiglitz separating allocation for the economy with endowment $\omega + t$. Since $(\omega + z^g, \omega + z^b)$ is incentive efficient, from proposition 3 it follows that it can be decentralized as an ALPT equilibrium with $\alpha = 0$ of the economy with endowments $\omega + t$. QED

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