

## The Economics of Cultural Transmission and the Dynamics of Preferences<sup>1</sup>

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This paper studies the population dynamics of preference traits in a model of intergenerational cultural transmission. Parents socialize and transmit their preferences to their offspring, motivated by a form of paternalistic altruism ("imperfect empathy"). In such a setting we study the long run stationary state pattern of preferences in the population, according to various socialization mechanisms and institutions, and identify sufficient conditions for the global stability of an heterogenous stationary distribution of the preference traits.

We show that cultural transmission mechanisms have very different implications than evolutionary selection mechanisms with respect to the dynamics of the distribution of the traits in the population, and we study mechanisms which interact evolutionary selection and cultural transmission. Journal of Economic Literature Classification numbers: D10, I20, J13. © 2001 Academic Press

#### 1. INTRODUCTION

This paper studies the population dynamics of the distribution of preferences in a model in which preference traits are endogenously determined by a process of intergenerational transmission of traits.

The view that preferences, norms, and, more generally, cultural attitudes should be considered as endogenous with respect to socioeconomic systems

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has been now extensively motivated. In particular, various pieces of empirical evidence have been interpreted to suggest the relevance of the endogeneity of various elements of preferences, as, for example, the discount factor, the perceived importance of education, the interdependence of agents' consumption or production patterns, and the relevance of ethnic and religious values (see Borjas [12]; Duesenberry [18]; Kapteyn *et al.* [26]; Iannaccone [25]; Leibenstein [28]; Pollak [29]). Also, Cavalli-Sforza and Feldman [15], among others, document the dependence of children's preferences on those of their parents.

Most analyses of the dynamics of preferences concentrate on evolutionary selection mechanisms. In such environments, preferences are either inherited by genetic transmission, or else are determined by an imitation process. In either case the resulting mechanism for the transmission of preferences is monotonically increasing in the material economic payoff associated to each preference trait (as the reproductive success of each trait is naturally assumed increasing in material payoffs—see, for instance, Bester and Guth [8]; Eshel *et al.* [19]; Fershtman and Weiss [20]; Kockesen *et al.* [27]; and Robson [30].<sup>2</sup>

We instead study models in which preferences of children are acquired through an adaptation and imitation process which depends on their parents' socialization actions, and on the cultural and social environment in which children live. We assume that the parents' socialization decision is motivated by their evaluation of their children's actions. Such evaluation is constructed on the basis of a form of paternalistic altruism in which parents evaluate their children's actions with their own (the parents') preferences. As a consequence, each parent always attempts to socialize his/her children to his/her own preference trait.

Since, in our analysis, the intergenerational transmission of cultural traits involves economic decisions of rational agents (the cultural parents), the transmission mechanism is not necessarily monotonic in material payoffs, as it depends on the parents' altruistic evaluation of their children's actions.<sup>3</sup>

<sup>&</sup>lt;sup>2</sup> A different approach to the endogeneity of preferences concentrates on the introspective choice of one's own preferences, as in most of Becker's work (e.g., Becker [5], and Becker and Mulligan [6]).

A large and established literature, of course, centers on the evolutionary selection of behavior, as opposed to preferences, in strategic environments (see Samuelson [33]; Vega Redondo [37]; and Weibull [38] for book-length presentations).

<sup>&</sup>lt;sup>3</sup> An explicit analysis of the transmission and adoption of cultural traits in which the transmission mechanism is exogenously specified (i.e., independently of any of the parents' decision) has been developed by Cavalli-Sforza and Feldman [15], and Boyd and Richerson [13] (see also Bandura and Walters [2] and Baumrind [3]. This literature has roots in evolutionary sociobiology (see Hamilton [22]; Dawkins [17]; Wilson [39]; and, for economic applications, Hirshleifer [23], [24]; Frank [21]; Becker [4]; Stark [35]; Bergstrom [7]; and Rubin and Paul [32].

We study the population dynamics of the distribution of preferences induced by such cultural transmission mechanisms. The main question we address is: What are the conditions on the transmission mechanisms which induce heterogeneity in the long run stationary distribution of preferences in the population? Our main result, in this respect, is the following. If the direct socialization of children inside the family and their cultural adaptation and imitation from society at large operate as substitutes in the whole cultural transmission mechanism, then there exists an heterogeneous distribution of preferences in the population, which is globally stable. When family and society are substitutes in the transmission mechanism, in fact, families will socialize children more intensely whenever the set of cultural traits they wish to transmit is common only to a minority of the population; and, on the contrary, families which belong to a cultural majority will not spend much resources directly socializing their children, since their children will adopt or imitate with high probability the cultural trait most predominant in society at large, which is the one their parents desire for them.

Since the transmission mechanisms we analyze involve agents' socialization decisions explicitly, it is of interest to study their welfare properties. We show that the paths of cultural evolution induced by such endogenous socialization and transmission mechanisms are inefficient, in the sense that too many resources are individually invested by parents to affect the preferences of their children.

The dynamics of the pattern of preferences obtained through endogenous cultural transmission mechanisms contrasts, in general, with those which would be obtained in evolutionary selection models. In simple evolutionary environments in which agents do not interact (strategically or otherwise), material payoffs, and hence fitness, can only be exogenously associated to each preference trait. Trivially, evolutionary selection models, in such environments, induce population dynamics which converge to an homogeneous distribution of preferences. Such models, in fact, lack the endogenous differential socialization success of cultural minorities, which drives the cultural heterogeneity of the stationary state distribution induced by cultural transmission mechanisms. But even if we allow the agents some choice of reproductive pattern (endogenous fitness), families in a cultural majority will choose relatively high fertility rates, since in this case their children will inherit their trait with high probability (children are of high expected "quality"). The choice of reproduction patterns, as a consequence, differently from the socialization choice, induces a culturally homogeneous configuration of the distribution of traits in the population (which configuration depends on the parameters of the transmission process, including the initial distribution of traits).

Given these differences, it is then interesting to study what happens when we extend our framework to the case in which cultural transmission interacts with evolutionary selection mechanisms. We show that our main implication that cultural substitutability generates long run cultural heterogeneity is robust to the simultaneous introduction of differential (exogenous and endogenous) fertility dynamics.

Our paper is related to a small literature investigating the issue of cultural transmission in dynamic models of preference evolution. Cavalli-Sforza and Feldman [16] and Boyd and Richerson [13], in their seminal work in evolutionary anthropology, were the first to propose models of cultural transmission with exogenous socialization efforts. Stark [35] has applied their framework to the evolution of altruism. Bisin and Verdier [10] investigate the interaction between cultural transmission and marital segregation, where marital segregation operates as a special form of a socialization mechanism, and show that such a mechanism favors, in general, heterogeneity of the distribution of the population with respect to the cultural traits. The analysis is focused on providing and discussing empirical implications for the dynamics of marriage homogamy rates and cultural heterogeneity, with specific reference to religious and ethnic traits. The present paper is broader in its focus, aiming at identifying general properties of socialization mechanisms which sustain heterogeneous limit distributions of preference traits in the population, even in environments in which socialization mechanisms interact with evolutionary selection in the process of preference formation.

The structure of the paper is as follows. Section 2 develops and analyzes a simple economic model of cultural transmission in a two-preference trait population. It studies the induced population dynamics of the distribution of traits. The crucial role of substitutability and complementarity between direct socialization and adoption from society at large as cultural transmission mechanisms is identified and analyzed. We also illustrate the analysis with various examples of interacting socialization technologies (Section 2.2.2-3). Section 3 presents a normative discussion of cultural transmission mechanisms. Section 4 compares our approach to cultural transmission to various evolutionary selection mechanisms and interacts cultural and evolutionary mechanisms. Finally, Section 5 concludes.

# 2. A SIMPLE MODEL OF CULTURAL TANSMISSION AND PREFERENCE EVOLUTION

In this section we study a simple, economic model of cultural evolution in a two-cultural trait population of individuals.<sup>4</sup> We start first by describing

 $<sup>^4</sup>$  The gist of the analysis is preserved in environments with N-traits. An Appendix available from the authors develops this point.

the class of socialization mechanisms which underlie our process of preference formation.

## 2.1. A Class of Socialization Mechanisms

We model the transmission of cultural traits as a mechanism which interacts socialization inside the family and socialization outside the family, in society at large, via imitation and learning from particular role models like teachers, peers etc. (Socialization inside the family is also called "direct vertical" socialization, while socialization by society, is also called "oblique" socialization<sup>5</sup>). Suppose there are two possible types of cultural traits in the population,  $\{a, b\}$ . The fraction of individuals with trait  $i \in \{a, b\}$  is denoted  $q^i$ . Different traits might correspond, for instance, to different perceptions of risk, different time preferences, or more generally different forms of utility functions. Families are composed of one parent and a child, and hence reproduction is asexual.<sup>6</sup> All children are born without defined preferences or cultural traits and are first exposed to their parent's trait. "Direct vertical" socialization to the parent's trait, say i, occurs with probability  $d^{i}(q^{i})$ . If a child from a family with trait i is not directly socialized, which occurs with probability  $1-d^i(q^i)$ , he or she picks the trait of a role model chosen randomly in the population (i.e., he or she picks trait i with probability  $q^i$  and trait  $j \neq i$  with probability  $q^j = 1 - q^i$ ).

Let  $P^{ij}$  denote the probability that a child from a family with trait i is socialized to trait j. By the Law of Large Numbers  $P^{ij}$  will also denote the fraction of children with a type i parent who have preferences of type j. The socialization mechanism just introduced is then characterized by the following transition probabilities, for all  $i, j \in \{a, b\}$ :

$$P^{ii} = d^{i}(q^{i}) + (1 - d^{i}(q^{i})) q^{i}$$
(1)

$$P^{ij} = (1 - d^{i}(q^{i}))(1 - q^{i}). \tag{2}$$

<sup>&</sup>lt;sup>5</sup> This terminology is taken by Cavalli-Sforza and Feldman [16].

<sup>&</sup>lt;sup>6</sup> See Section 2 for the extension to endogenous fertility.

<sup>&</sup>lt;sup>7</sup> The probability of direct socialization is allowed to depend on the fraction of individuals with trait i in the population,  $q^i$ , to capture a possible interaction between the direct vertical transmission of traits by the family and the distribution of traits in the population. Such interactions will be crucial in the models of the next section in which direct socialization constitutes an endogenous choice of the parent.

<sup>&</sup>lt;sup>8</sup> See Al-Najjar [1] and Sun [36] for formal constructions of the Law of Large Numbers with a continuum of independent agents.

The dynamics of the fraction of the population with trait i, in the continuous time limit,  $^9$  is then characterized by:

$$\dot{q}^{i} = q^{i}(1 - q^{i})[d^{i}(q^{i}) - d^{j}(1 - q^{i})]. \tag{3}$$

This is the basic equation for the dynamics of cultural traits analyzed in the paper. It is easy to derive conditions which guarantee that the distribution of cultural traits in the population will converge to a non-degenerate distribution, i.e., such that some heterogeneity of traits is maintained in the limit population. We turn to this point next.

An important determinant of the process of cultural transmission in the analysis of the population dynamics of the different cultural traits consists in how the external environment, reflected by  $q^i$ , affects direct vertical socialization. More precisely, situations where the social environment acts as a substitute or as a complement to family cultural transmission technologies have very different implications for the dynamics of cultural traits. A precise definition of "substitution" is useful.

DEFINITION (Cultural substitution). Vertical cultural transmission and oblique cultural transmission are cultural substitutes for agent i (or, equivalently,  $d^i(q^i)$  satisfies the cultural substitution property) if

 $d^{i}(q^{i})$  is a continuous, strictly decreasing function in  $q^{i}$ , and, moreover,  $d^{i}(1) = 0$ .

Intuitively, we say that direct vertical transmission acts as a *cultural* substitute to oblique transmission whenever parents have less incentives to socialize their children the more widely dominant are their values in the population (in the limit of a perfectly homogenous populations of type i, whenever parents of type i do not directly socialize their children). We now characterize the dynamics of (3) under cultural substitution (see also Fig. 1). Let  $q^i(t, q^i_0)$  denote the path of  $q^i$  solving (3) under initial condition  $q^i(0) = q^i_0$ .

<sup>9</sup> The continuous time approximation can be derived, for instance, from an economy with overlapping generations of agents living  $\Delta$  units of time and having children 1-h units of time after birth, by taking the limit for  $\Delta$ ,  $h \to 0$ , with  $h/\Delta \to 0$ . For a related continuous time approximation in evolutionary models, see, for instance, Cabrales–Sobel [14].

It is easy to see that our results hold in the discrete time dynamics in the following sense: The global stability results of the form  $q^i(t,q^i_0) \rightarrow q^{i*}$  for any initial population share  $q^i_0 \in (0,1)$ , in the continuous time dynamics (in Proposition 1, 3, 4, and 5), correspond, in the discrete time dynamics, to the existence of a neighborhood of  $q^{i*}$ , strictly contained in (0,1), which is globally attracting, for any initial population share  $q^i_0 \in (0,1)$ .

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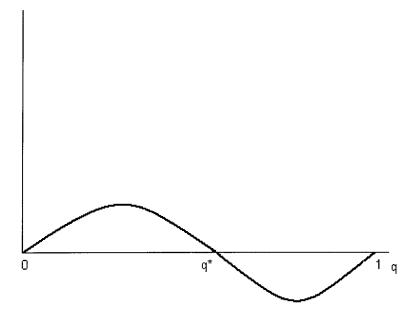


FIG. 1. The dynamics under cultural substitution.

PROPOSITION 1. Suppose vertical and oblique cultural transmission are cultural substitutes for both groups a and b. Then,  $(0, 1, q^{i*})$  are the stationary states of (3), and  $0 < q^{i*} < 1$ . Moreover,  $q^i(t, q^i_0) \rightarrow q^{i*}$ , globally, for any  $q^i_0 \in (0, 1)$ .

*Proof.* Obviously, (0, 1) are stationary states of (3), as well as all  $q^i$  which solve  $d^i(q^i) - d^j(1 - q^i) = 0$ . The equation  $d^i(q^i) - d^j(1 - q^i) = 0$  has a unique solution,  $q^i*$ , since, by cultural substitution, (i)  $d^i$  is decreasing and  $d^j$  increasing in  $q^i$ , and, moreover, (ii)  $d^i(0) > d^j(1)$ ,  $d^j(0) > d^i(1)$ . Also, (ii) implies that  $0 < q^i* < 1$ .

To show that  $q^i(t, q^i_0) \rightarrow q^i *$ , for any  $q^i_0 \in (0, 1)$ , note that  $\partial \dot{q}^i/\partial q^i|_{q^i=0} = d^i(0) - d^j(1)$  is >0, and  $\partial \dot{q}^i/\partial q^i|_{q^i=1} = d^j(0) - d^i(1)$  is >0, if  $d^i$  satisfies cultural substitution. Since  $0 < q^i * < 1$  is unique, continuity of the map (3), from  $q^i$  into  $\dot{q}^i$ , implies that the basin of attraction of  $q^i *$  is (0, 1).

If vertical and oblique transmission are cultural substitutes, the direct family socialization efforts of type i agents are decreasing in the fraction of these individuals in society, and moreover the direct socialization effort of small enough a minority is larger than that of the corresponding majority, e.g.,  $d^i(0) - d^i(1) > 0$ . As a consequence the socialization pattern moves the

system away from full homogeneity:  $q^i = 0$  and  $q^i = 1$  are locally unstable stationary states of (3), and the basin of attraction of the unique steady state associated to heterogeneous population,  $q^{i*}$ , is the whole (0, 1).

Explicit models of cultural transmission are needed to impose restrictions on the form of the function  $d^i(q^i)$ . We will next show that several "natural" models of endogenous cultural transmission satisfy cultural substitution, and hence are characterized by dynamics of the distribution of cultural traits in the population which converge to an heterogeneous population.

## 2.2. Endogenous Cultural Transmission Mechanisms

We now study cultural transmission mechanisms in which parents take actions to socialize their children. Such mechanisms determine endogenously the direct socialization maps,  $d^i(q^i)$ ,  $i \in \{a, b\}$ .

Every agents in the adult part of his lifetime makes economic and social decisions. We capture these decision, abstractly, with the choice of  $x \in X$  to maximize preferences  $u^i : X \to \Re$ , for  $i \in \{a, b\}$ . Different cultural traits are then associated to different sets of preferences over x.

Parents are altruistic towards their children and hence might want to socialize them to a specific cultural model if they think this will increase their children's welfare. Let  $V^{ij}$  denote the utility to a type i parent of a type j child,  $i, j \in \{a, b\}$ . The expected lifetime gains (abstracting from socialization costs) of a family of type i at time t are then:<sup>11</sup>

$$u^{i}(x) + (P^{ii} V^{ii} + P^{ij} V^{ij}).$$

Regarding parents' introspective view of their children's preferences (the determination of  $V^{ii}$ ,  $V^{ij}$ ), we assume that parents are only able to evaluate their children's actions with their own preferences. A type i parent, for instance, is altruistic in the sense that he or she receives utility from his or her child's future socio-economic action, but the utility he or she receives stems from the evaluation of his or her child's action with type i preferences, even if the child turns out to have type j preferences. Formally stated, the assumption has the following form.

<sup>11</sup> Separability of preferences between socio-economic activities and socialization substantially and crucially simplifies the analysis. Another important simplification we introduce is the fact that parents do not care about the whole dynasty and do not take into account socialization costs which their children bear.

<sup>&</sup>lt;sup>10</sup> Cultural substitution of direct vertical and oblique transmission is clearly much stronger than necessary for the stability results of Proposition 1. (For instance,  $d^i(1) = d^j(1) = 0$  can be substituted with  $d^i(0) > d^j(1)$ ; and the monotonicity requirement can be substantially relaxed.) A weaker definition would not expand in a substantial sense though the class of endogenous transmission mechanisms which turn out to possess the substitution property.

Assumption 1 (Imperfect empathy). For all  $i, j \in \{a, b\}$ ,  $V^{ij} = u^i(x^j)$ , where  $x^j = \arg\max_{x \in X} u^j(x)$ , and  $x^a \neq x^b$ .

We interpret such assumptions as a form of myopic or paternalistic altruism (hence the name, "imperfect empathy"). Parents are aware of the different preference traits children can adopt, and are able to anticipate the socio-economic choice a child with preference trait  $i \in \{a, b\}$  will (optimally) make. Parents are not able, though, to altruistically evaluate their children's actions with the children's utility function (to "perfectly empathize" with the children), but they are biased by their own (the parents') preference evaluations.

As a consequence of imperfect empathy, parents, while altruistic, prefer children with their own cultural trait and hence attempt at socializing them to this trait (children with a different cultural trait will not choose the same socio-economic action their parents would choose in their position):<sup>12</sup>

for all 
$$i$$
,  $j$  with  $i \neq j$ ,  $V^{ii} > V^{ij}$ . (4)

Some justifications of imperfect empathy, from an evolutionary perspective, are identified by Bisin and Verdier [9].<sup>13</sup>

To analyze the socialization decisions of parents, we then need not introduce the notation for parent's i socialization to cultural trait  $j \neq i$ ; and we denote with  $d^i$  the probability of direct socialization of parents with trait i to the i trait.

Each parent of type i can affect the probability of direct socialization of his child, by controlling an n-dimensional vector  $\tau^i$  of "inputs". Examples of the elements of the inputs vector contain the time spent with the child, the cultural homogeneity of the neighborhood in which the family locates, and of the school to which the child is sent. A map  $D: \mathfrak{R}^n_+ \times [0,1] \to [0,1]$  represents the "production function" of direct socialization:

$$d^i = D(\tau^i, q^i). \tag{5}$$

The production function D is allowed to depend on  $q^i$  to capture the possible dependence direct transmission on the distribution of the traits in the population.

<sup>&</sup>lt;sup>12</sup> If the set X depends on the agent's trait, or else if agents interact strategically and hence, for instance, agents preferences depend on  $q^i$ , it is not in general true that imperfect empathy implies (4). In this paper we do restrict ourselves, however, to environments in which  $V^{ii} > V^{ij}$ .

<sup>&</sup>lt;sup>13</sup> Such an imperfect notion of altruism is also at the core of Adam Smith's *Theory of Moral Sentiments*, where it is introduced as follows: "As we have no immediate experience of what other men feel, we can form no idea of the manner in which they are affected, but by conceiving what we ourselves should feel in the like situation. Though our brother is upon the rack, .... our senses will never inform us of what he suffers. .... By the imagination we place ourselves in his situation ....", Part I, Section I, Chap. I.

Socialization is costly. Let  $C(\tau^i)$  denote socialization costs.

Assumption 2 (Socialization). For any  $i \in \{a, b\}$ :

- (i) the utility function  $u^i(x)$  is  $\mathscr{C}^2$ , monotonic increasing and strictly concave, and the choice set X is convex and compact;
- (ii) the map D is  $\mathscr{C}^2$ , strictly increasing and strictly quasi-concave in  $\tau^i$ ; Moreover,  $D(0, q^i) = 0$ ,  $\forall q^i \in [0, 1]$ ;
- (iii) the map C is  $\mathscr{C}^2$ , strictly increasing and strictly quasi-convex; moreover, C(0) = 0, and  $\partial C(0)/\partial \tau^i = 0$ .

Parents of type *i* choose  $\tau^i \in \mathfrak{R}^n_+$ , and  $x^i \in X$  to maximize:

$$u^{i}(x^{i}) - C(\tau^{i}) + (P^{ii}V^{ii} + P^{ij}V^{ij})$$
 s.t. 1, 2, and 5. (6)

Under Assumptions 1 and 2, the argmax of the socialization problem of a type i parent, problem (6), is represented by a continuous map which we denote  $d^i = d(q^i, \Delta V^i)$ , where  $\Delta V^i = V^{ii} - V^{ij}$  is the subjective utility gain of having a child with trait i; it reflects the degree of "cultural intolerance" of type i's parents with respect to cultural deviations from their own trait. Given imperfect empathy on the part of parents,  $\Delta V^i > 0$ , by Eq. (4).

The dynamics of the fraction of the population with cultural trait i is then determined by Eq. (3) evaluated at  $d^i(q^i) = d(q^i, \Delta V^i)$ .<sup>14</sup>

The next simple Proposition develops sufficient conditions for the map d to possess the cultural substitution property.

Proposition 2. Under Assumptions 1 and 2,  $d(q^i, \Delta V^i)$  satisfies the cultural substitution property if C and D are homothetic in  $\tau^i$ , and

$$\frac{\partial D(\tau^i, q^i)}{\partial q^i} \leqslant 0. \tag{7}$$

In particular, the cultural substitution property holds if direct socialization is independent of  $q^i$ .

*Proof.* We construct first the indirect cost function of direct socialization  $d^i$ ,  $i \in \{a, b\}$ , which we denote  $H(d^i, q^i)$ :

$$H(d^{i}, q^{i}) := \min_{\tau^{i} \in \mathfrak{R}_{+}^{n}} C(\tau^{i}), \quad \text{s.t.} \quad d^{i} = D(\tau^{i}, q^{i}).$$
 (8)

Assumptions 2(ii) and 2(iii) imply that the minimization problem in (8) is convex, and hence:  $H(d^i, q^i)$  is a continuous function in  $d^i$  and  $q^i$ , the

<sup>&</sup>lt;sup>14</sup> Note that  $\Delta V^i$  must be chosen so that  $0 < d(q^i, \Delta V^i) < 1$ , for any  $q^i \in [0, 1]$ , since  $d(q^i, \Delta V^i)$  is a probability.

argmin  $\tau^i$  is a continuous mapping from  $[0,1]^2$  into  $\Re^N$ ,  $\tau(d^i,q^i)$ . As a consequence then,  $H(d^i,q^i)$  is convex in  $d^i$ , and satisfies:  $H(0,q^i)=0$ ,  $\forall q^i \in [0,1]$ , and  $\partial H/\partial d^i|_{d^i=0}=0$ .

Let  $\lambda^i$  denote the Lagrange multiplier of the constraint in Problem (8). The Implicit Function Theorem on the first order conditions of Problem (8) implies that  $\tau^i(d^i)$  is differentiable increasing in  $d^i$ , and sign  $(\partial \lambda^i/\partial q^i) = -\text{sign }(\partial D/\partial q^i)$ . But, by the Envelope Theorem,  $(\partial D/\partial q^i) = \lambda^i$ , and hence

$$\mathrm{sign}\left(\frac{\partial^2 H}{\partial q^i\,\partial d^i}\right) = -\,\mathrm{sign}\left(\frac{\partial D}{\partial q^i}\right).$$

Problem (6) can now be written as a choice problem in  $d^i$ :

$$\max_{(x^i, \, d^i) \in \, X \times \, [0, \, 1]} u^i(x^i) - H(d^i, \, q^i) + V^{ij} + \left[ \left( d^i + (1 - d^i) \, q^i \right] \varDelta V^i. \right. \tag{9}$$

The analysis above of the properties of  $H(d^i,q^i)$  guarantees that Problem (9) is convex. The Implicit Function Theorem on the first order conditions of (9) implies then that  $(\partial d^i/\partial q^i) < 0$  if  $\partial^2 H/\partial q^i \partial d^i \ge 0$ , which, we have shown above, is satisfied if  $\partial D/\partial q^i \le 0$ . Moreover, clearly d(1) = 0, since the term  $[(d^i + (1 - d^i) q^i] \Delta V^i]$  in the objective of Problem (9) is independent of  $d^i$  at  $q^i = 1$ .

For the class of endogenous socialization mechanisms introduced in this section, then, (strong but) simple conditions on the technology of direct socialization guarantee cultural substitution, and hence the heterogeneity of the long run distribution of the cultural traits in the population. In particular, besides convexity and homotheticity of C and D, it suffices that  $\partial D(\tau^i, q^i)/\partial q^i \leq 0$ . If, on the other hand, in fact  $D(\tau^i, q^i)$  is increasing in  $q^i$ , given  $\tau^i$ , a form of (cultural) complementarity between direct and oblique transmission is introduced: direct socialization is in this case more efficient, other things equal, when the trait to be transmitted is held by a majority of the population, and hence when oblique transmission is more efficient.

While  $\partial D(\tau^i, q^i)/\partial q^i \leq 0$  is clearly not a necessary condition for cultural substitution, and, *a fortiori*, it is not necessary for the stability of heterogeneous limit distribution of the population with respect to the preference traits, it is easy to show that strong enough forms of cultural complementarity can drive the dynamics of the distribution of the traits in the population towards homogeneity (see the example in Section 2.2.3).

The relevance of cultural substitution and complementarity in the socialization technology is further illustrated by the examples of socialization mechanisms that we proceed to analyze.

2.2.1. "It's the family,...!" Consider the simplest (and hence benchmark) cultural transmission technology, in which family models are the first

crucial cultural models the child is exposed to. Direct socialization is driven only by an effort variable,  $\tau^i$ , and there are no interactions between society at large and direct vertical socialization:  $d^i = D(\tau^i) = \tau^i$ . The map D in this case then trivially satisfies Assumption 2(ii) and Condition (7). If preferences and socialization costs satisfy Assumption 2(i) and 2(iii), then,  $d(q^i, \Delta V^i)$  is decreasing in  $q^i$ , and  $d^i(1) = 0$ , and, by Proposition 2, oblique and direct vertical cultural transmission are cultural substitutes. By Proposition 1, then, the dynamics of the distribution of cultural traits converges to an heterogeneous limit distribution.

Also note that  $d(q^i, \Delta V^i)$  is increasing in  $\Delta V^i = V^{ii} - V^{ij}$ , the "degree of intolerance" of family i for children with cultural trait different than the parents' own. Naturally, the more 'intolerant' a parent is, the larger are his or her incentives to socialize his or her child to his or her own trait.

2.2.2. "Do not talk to strangers." Consider the following socialization mechanism. Children are exposed simultaneously to their parent's trait, say i, and to the trait of an individual picked at random from a restricted population, composed of a fraction  $\tau_2^i$  of agents with trait i, which can be interpreted, for example, as the population of neighbors, peers and teachers. The parents direct socialization effort is denoted  $\tau_1^i \in [0, 1]$ , and controls the children's internalization of the parent's trait. If the two traits match (i.e., if the child internalizes his parent trait, i, and the trait of the individual in the restricted population is also i), then the child is socialized to trait i. Otherwise, with probability  $(1 - \tau_1^i \tau_2^i)$ , the child picks a trait from the population as a whole. The probability that a child of a type i father is directly socialized (by exposure to the parent and to the restricted pool) is then:

$$d^i = D(\tau^i) = \tau_1^i \tau_2^i$$
.

Both the direct socialization effort  $\tau_1^i \in [0, 1]$ , and the segregation effort,  $\tau_2^i \in [0, 1]$ , are chosen by parents, and

$$\tau^i = \begin{bmatrix} \tau_1^i \\ \tau_2^i \end{bmatrix}.^{16}$$

The map D, in this case also, satisfies Assumption 2(ii) and condition (7). If preferences and socialization costs satisfy Assumption 2(i) and 2(iii),

<sup>15</sup> Another interesting specification of segregation costs consists in having them depend on the distribution of traits in the population:  $C(\tau^i, q^i)$ , with  $\partial C(\tau^i, q^i)/\partial q^i > 0$ , to capture the idea that effort and segregation are more costly for minority groups than for majority groups. It can then be shown that the results described in this section still hold provided that  $\partial^2 C(\tau^i, q^i)/\partial \tau^i_s \partial q^i$ , for  $s \in \{1, 2\}$ , is not too large.

<sup>16</sup> Another interesting socialization mechanism has  $d^i = D(\tau^i) = \tau_1^i + (1 - \tau_1^i) \ \tau_2^i$ , which corresponds to the case in which children pick the trait of their parents from a restricted pool, with probability  $\tau_2^i$ , only if a first direct socialization effort,  $\tau_1^i$ , has not been successful. While such a direct socialization mapping D does not satisfy Assumption 2(ii), it still delivers cultural substitution in  $d^i(q^i)$ .

then, by Proposition 2, direct vertical and oblique transmission are substitutes for such transmission mechanisms, and, by Proposition 1, the dynamics of the distribution of cultural traits converges to an heterogeneous limit distribution.

The result is driven by the fact that socialization effort  $\tau_1^i$  and the degree of segregation  $\tau_2^i$  are good substitutes in the technology of direct vertical socialization. Because of the substitutability of effort and segregation choices, allowing for endogenous segregation choice in fact reinforces the substitutability of vertical and oblique transmission in the benchmark model in Section 2.2.1.

2.2.3. "It takes a village to raise a child." As an illustration of cultural transmission mechanisms in which complementarities arise in vertical and oblique transmission, consider the following example. Any child is first exposed simultaneously to the parent's trait and to the trait of a role model from the population with which he or she is matched randomly. If the parent and the role model are culturally homogeneous, the child is directly socialized to their common trait, otherwise the child is matched a second time randomly with a role model from the population, and adopts his or her trait. Parents of type i choose an effort variable  $\tau^i$ . The probability that a child is directly socialized is then  $d^i = D(\tau^i) = \tau^i q^i$ , which does not satisfy condition (7). Vertical and oblique transmission are not cultural substitutes in this example. As a consequence, we can easily identify classes of environments for which the dynamics of the distribution of traits in the population is such that an homogeneous population in general arises in the limit.

Suppose for instance that the cost function is quadratic,  $C(\tau^i) = \frac{1}{2} (\tau^i)^2$ . In this case then,  $d(q^i, \Delta V^i) = (q^i)^2 (1 - q^i) \Delta V^i$ ; and a simple analysis of the dynamics implies that for  $0 < q^{i*} < 1$ 

$$\begin{split} q^{i}(t,q_{0}^{i}) &\to 0, & \text{for any} \quad q_{0}^{i} \in [0,q^{i}*); \\ q^{i}(t,q_{0}^{i}) &\to 1, & \text{for any} \quad q_{0}^{i} \in (q^{i}*,1]; \end{split} \tag{10}$$

see Fig. 2.17

#### 3. WELFARE IMPLICATIONS

Without starting from some a priori ethical point of view, it is in general difficult to provide normative statements on economic situations with

<sup>17</sup> Note that  $d^i(q^i)$  is not always decreasing in the fraction of individuals of type i (it is increasing for  $q^i < 2/3$ ). This reflects the conflict between complementarities and substitutabilities of vertical and oblique transmission: for small  $q^i$  oblique transmission is complementary. Vertical transmission and socialization effort by parents of type i is increasing with the fraction of individuals of the same type.

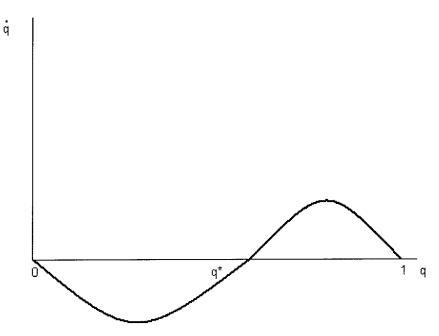


FIG. 2. The dynamics under cultural complementarity.

evolving preferences. In the present context of endogenous socialization though, we can characterize necessary and sufficient conditions for the efficiency of socialization patterns,  $d^i(q^i)$ , for  $i \in \{a, b\}$ , and the associated paths of the distribution of traits in the population.

DEFINITION (Efficiency). A pattern of direct socialization,  $(d^i(q^i), d^j(1-q^i))$ , and the induced path for the distribution of cultural traits,  $q^i(t, q_0^i)$ , are efficient if there does not exist a direct socialization pattern  $(d^{\prime i}(q^i), d^{\prime j}(1-q^i))$ , and an induced path  $q^{\prime i}(t, q_0^i)$ , which are preferred to  $(d^i(q^i), d^j(1-q^i), q^i(t, q_0^i))$ , by both types of agents,  $\{a, b\}$ , at each time t.

We are now ready to evaluate the efficiency of direct socialization patterns which derive from the agents' socialization problem, Problem (6), i.e., the efficiency of  $(d(q^i, \Delta V^i), d(1-q^i, \Delta V^j))$ .

PROPOSITION 3. Suppose the direct socialization patterns,  $(d(q^i, \Delta V^i), d(1-q^i, \Delta V^i))$ , and the induced paths of the distribution of cultural traits,  $q^i(t, q_0^i)$ , are such that, given  $q_0^i$  and for some t,

$$(d(q^{i}(t, q_{0}^{i}), \Delta V^{i}), d(1 - q^{i}(t, q_{0}^{i}), \Delta V^{j})) > 0.$$
(11)

Then  $(d(q^i, \Delta V^i), d(1-q^i, \Delta V^j))$  and  $q^i(t, q^i_0)$  are inefficient.

*Proof.* Consider the following direct socialization pattern, alternative to  $(d(q^i, \Delta V^i), d(1-q^i, \Delta V^j),$  from time 0 on:  $d'(q^i, \Delta V^i) = \max\{0, d(q^i, \Delta V^i) - d(1-q^i, \Delta V^j)\}$  and  $d'(1-q^i, \Delta V^j) = \max\{0, d(1-q^i, \Delta V^j) - d(q^i, \Delta V^i)\}$ . The same path of the distribution of traits for  $t \ge 0$  is induced by the alternative socialization pattern,  $q^i(t, q^i_0) = q^{i'}(t, q^i_0)$ . But, under the alternative socialization patterns, costs are strictly reduced for all times t such that  $d(q^i(t, q^i_0), \Delta V^i), d(1-q^i(t, q^i_0), \Delta V^j) > 0$ , and for both groups, thereby proving the statement. ▮

Direct socialization patterns are inefficient as long as at some time t the members of both cultural groups actively socialize their children (11). Conversely, a necessary and sufficient condition for the efficiency of direct socialization patterns is that, for all t, the members of at most one of the cultural groups actively socialize their children. Under Assumptions 1, 2 and cultural substitution, in particular, (11) is satisfied for all initial conditions  $q_0^i$  which do not coincide with a stationary state of the system, and hence direct socialization patterns are inefficient.

It is easy to show that the direct socialization pattern  $(d'(q^i, \Delta V^i), d'(1-q^i, \Delta V^j))$ , which we constructed in the proof of Proposition 3, not only dominates  $(d(q^i, \Delta V^i), d(1-q^i, \Delta V^j))$ , but is also efficient. For environments in which direct and oblique transmission are cultural substitutes, in particular, the optimal socialization policy at any time  $t \ge 0$  consists in not allowing direct vertical socialization for the majority cultural group, more precisely the group i such that  $q^i(t, q^i_0) \ge q^{i*}$ , and in reducing accordingly the direct socialization effort of minorities, so as to reproduce the same path of distribution of traits which would have been followed from t=0 were the socialization policy not introduced,  $q^i(t, q^i_t), t \ge 0$ .

## 4. CULTURAL TRANSMISSION AND EVOLUTIONARY SELECTION

The interaction between genetic and cultural transmission is at the core of a endless cross-disciplinary debate, often referred to as the "nature-nurture debate" (see Rogers [31] for an illuminating introduction), which we will not directly address. Rather, we are interested in two quite specific questions. First, is the cultural transmission model we introduced in the previous section distinguishable in terms of implications from a simple evolutionary selection mechanism? And secondly, are our results robust to the introduction of evolutionary as well as cultural transmission mechanisms? A qualified positive answer to both questions is given for cultural transmission mechanisms for which direct and oblique transmission are cultural substitutes.

To simplify notation, we restrict the analysis of this section to the benchmark transmission mechanism of Section 2.2.1, in which the probability of

direct socialization coincides with an effort variable chosen by parents:  $d^i = D(\tau^i, q^i) = \tau^i$ ; and vertical and oblique transmission are substitutes.

## 4.1. Evolutionary Selection and Endogenous Fitness

Suppose the cultural transmission mechanism is as introduced in the previous section, but the parent does not have a direct socialization choice, and, to guarantee symmetry,  $d^i = d^j = d \in (0, 1)$ . In such simple environments, material payoffs, and, hence, fitness are exogenously associated with each preference trait. Suppose that the number of children of each parent (i.e., his or her fitness) depends on the cultural group of the parent, and is denoted  $n^i$ ,  $i \in \{a, b\}$ . The dynamics of the distribution of cultural traits in the population is determined then by

$$\dot{q}^{i} = q^{i}(1 - q^{i}) d(v^{i} - v^{j})$$
(12)

for  $v^i = n^i/(n^i + n^j)$ . It is easy to see that

$$q^i(t, q_0^i) \to \begin{cases} 0 & \text{ for } q_0^i \in [0, 1) & \text{ if } n^j > n^i \\ 1 & \text{ for } q_0^i \in (0, 1] & \text{ if } n^i > n^j \\ q_0^i & \text{ for all } q_0^i \in [0, 1] & \text{ if } n^j = n^i \end{cases}$$

The dynamics of the distribution of traits in the population for the evolutionary selection mechanisms considered tend to an homogeneous distribution, except in the degenerate case in which the cultural groups are not distinguishable in terms of fitness. This conclusion sharply contrasts with the one we reached for cultural transmission models with cultural substitution (Proposition 1). In such analysis in fact, evolutionary selection mechanisms lack the endogenous differential socialization success of cultural minorities, which drives the cultural heterogeneity of the stationary state distribution induced by cultural transmission mechanisms.

It is of interest then, to compare the implications of cultural transmission mechanisms with those of a evolutionary selection mechanisms in which fitness is endogenously determined by parents.

Suppose each parent must choose the number of children to raise, and faces costs c(n) to raise n children.

Assumption 3. For any  $i \in \{a, b\}$ :

$$c: \Re_+ \to \Re_+$$
 is  $\mathscr{C}^2$ , strictly increasing and strictly convex,  $c(0) = 0$  and  $c'(0) = 0$ .

Suppose also that a parent of type i receives an utility gain  $V^{ij}$  for each child of type  $j, i, j \in \{a, b\}$ .

Parents of type i then choose  $n^i \ge 0$  and  $x^i \in X$  to maximize

$$u^{i}(x^{i}) - c(n^{i}) + n^{i}(P^{ii}V^{ii} + P^{ij}V^{ij}),$$
(13)

where  $P^{ii}$  and  $P^{ij}$  are defined by (2), and are evaluated at  $d^i(q^i) = d^j(1-q^i) = d$ . Let  $n(q^i, \Delta V^i)$ ,  $n(1-q^i, \Delta V^i)$ , denote the solutions of the maximization problem (13) for parents of type i and j.

Proposition 4. Under Assumption 1, 2(i), 2(iii) and 3,

If  $V^{ii} > dV^{ji} + (1-d) \ V^{ji}$ , for  $i, j \in \{a, b\}$ ,  $i \neq j$ :  $(0, 1, q^{i*})$  are the stationary states of (12) evaluated at  $n^i = n(q^i, \Delta V^i)$ ,  $n^j = n(1-q^i, \Delta V^i)$ , and  $0 < q^{i*} < 1$ ; moreover,  $q^i(t, q^i_0) \to 0$ , for any  $q^i_0 \in [0, q^{i*})$ , and  $q^i(t, q^i_0) \to 1$ , for any  $q^i_0 \in (q^{i*}, 1]$ .

If instead  $V^{ii} \leq dV^{jj} + (1-d) V^{ji}$  for a group  $i \in \{a, b\}$ ,  $q^i(t, q_0^i) \rightarrow 0$ , globally, for any  $q_0^i \in [0, 1)$ .<sup>18</sup>

*Proof.* Assumptions 1, 2(i), 2(iii), and 3 imply that problem (13) is convex. The first order condition of the maximization problem is

$$(d + (1-d) \ q^i) \ V^{ii} + ((1-d)(1-q^i)) \ V^{ij} = c'(n^i)$$

and hence  $n(q^i, \Delta V^i)$  is differentiably increasing in  $q^i$ . Any interior stationary state, must solve  $n(q^i, \Delta V^i) - n(1-q^i, \Delta V^j) = 0$ , and hence  $(d+(1-d) \ q^i) \ V^{ii} + ((1-d)(1-q^i)) \ V^{ij} = (d+(1-d)(1-q^i)) \ V^{ij} + ((1-d) \ q^i) \ V^{ji}$ . A unique such stationary state therefore exists if  $V^{ii} > dV^{ji} + (1-d) \ V^{ji}$ , for  $i, j \in \{a, b\}$ ,  $i \neq j$ . The analysis of the stability properties of the dynamics of Eq. (12), evaluated at  $n^i = n(q^i, \Delta V^i)$ ,  $n^i = n(1-q^i, \Delta V^j)$ , now simply follows.

If instead, for some  $i \in \{a, b\}$ ,  $V^{ii} \le dV^{ji} + (1 - d) V^{ji}$ , an interior steady state does not exist, and again the stability analysis simply follows.

Even if we allow the agents some choice of reproductive pattern (endogenous fitness), families in a cultural majority will choose relatively high fertility rates, since in this case their children will inherit their trait with high probability (children are of high expected "quality"). The choice of reproduction patterns, as a consequence, different from the socialization choice in an environment, characterized by cultural substitution (Proposition 1), induces a culturally homogeneous configuration of the distribution of traits

 $<sup>^{18}</sup>$  The case in which  $V^{ii}\!\leqslant\! dV^{jj}+(1-d)\;V^{ji}$  and  $V^{jj}\!>\! dV^{ii}+(1-d)\;V^{ij}$  contradicts imperfect empath, Assumption 1.

in the population (which configuration depends on the parameters of the transmission process, including the initial distribution of traits).

#### 4.2. Co-Evolution

We now study the interaction between evolutionary selection and cultural transmission. First of all, it is easy to see that the class of transmission mechanisms which interact cultural transmission and exogenous fitness maintain the qualitative properties of the underlying cultural transmission mechanism. In particular, with cultural substitution, the dynamics of the distribution of traits converge to an heterogeneous distribution, for any initial condition  $q_0^i \in (0, 1)$ . When an exogenous fitness is associated to each trait, the stable limit stationary distribution depends on the fitness ratio,  $R^i := n^i/n^j$ , in the sense that  $q^{i*}$  increases with  $R^i$ .

A more interesting class of transmission mechanisms has cultural transmission interacting with endogenous fitness. Parents of type i choose  $x^i \in X$ ,  $d^i \in [0, 1]$ , and  $n^i \ge 0$  to maximize:

$$u^{i}(x^{i}) - c(n^{i}) - n^{i}H(d^{i}) + n^{i}(P^{ii}V^{ii} + P^{ij}V^{ij})$$
(14)

where  $P^{ii}$  and  $P^{ij}$  are as in (2).<sup>19</sup> Let  $d(q^i, \Delta V^i)$ ,  $d(1-q^i, \Delta V^j)$ ,  $n(q^i, \Delta V^i)$ ,  $n(1-q^i, \Delta V^j)$ , denote the solutions to the maximization problem (14) for parents of type i and j. The dynamics of the distribution of traits in the population is then determined by

$$\dot{q}^{i} = q^{i}(1 - q^{i})(d(q^{i}, \Delta V^{i}) \ v(q^{i}, \Delta V^{i}) - d(1 - q^{i}, \Delta V^{j}) \ v(1 - q^{i}, \Delta V^{j})), \tag{15}$$

where  $v(q^{i}, \Delta V^{i}) = n(q^{i}, \Delta V^{i})/(n(q^{i}, \Delta V^{i}) + n(1 - q^{i}, \Delta V^{j})).$ 

We will next show that, for such an environment, there exists an heterogeneous limit distribution of the population with respect to the preference trait which is locally stable. To this end we require some regularity conditions: (i) that parents derive enough utility from parenthood, in particular, that they would be willing to raise a (small positive fraction of a) child with preferences different from theirs with probability one; and (ii) that the socialization gains are large enough that parents will always want to have at least a (small positive fraction of a) child even if constrained to directly socialize the offspring with probability 1. Formally:

<sup>19</sup> Following the notation introduced in the proof of Proposition 2, the choice problem of each parent of type i is considered as the choice of  $d^i$ , with socialization costs  $H(d^i, q^i)$ . In the simple case considered here in which  $d^i = \tau^i$ , this reduces to  $H(d^i) = C(d^i)$ . As a consequence the function H(.) has the same properties as C(.), from Assumption 2. Also, abusing notation, in the following we impose additional assumptions directly on  $H(d^i)$  rather than on  $C(\tau^i)$ .

Assumption 4. For any  $i \in \{a, b\}$ :

- (i)  $V^{ij} > 0$ ,
- (ii)  $V^{ii} > H(1)$ .

We can then prove:

PROPOSITION 5. Under Assumptions 1, 2, 3, and 4, (0,1) are stationary states of (15). There also exists at least one stationary state  $q^{i*}$ , such that  $0 < q^{i*} < 1$ , and  $q^{i}(t, q_0^{i}) \rightarrow q^{i*}$ , locally, for  $q_0^{i}$  in an open neighborhood of  $q^{i*}$ .

*Proof.* The first order conditions of problem (14) are:

$$V^{ij} + \lceil d^i + (1 - d^i) \ q^i \rceil \ \Delta V^i = c'(n^i) + H(d^i)$$
 (16)

$$n^{i}(1-q^{i}) \Delta V^{i} = n^{i}H'(d^{i}).$$
 (17)

It is easy to see that the second order conditions for a maximtim also are satisfied. Given Assumption 2, the function  $\Theta(d^i) = V^{ij} + d^i \Delta V^i - H(d^i)$  is strictly concave and  $\Theta'(0) = \Delta V^i$  (>0 by Assumption 1). Moreover  $\Theta(0) = V^{ij} > 0$  (by Assumption 4(i)), and  $\Theta(1) = V^{ii} - H(1) > 0$  (by Assumption 4(ii)). Thus one gets:

$$V^{ij} + \left[ d^i + (1 - (d^i) \ q^i \right] \ \Delta V^i - H(d^i) \geqslant V^{ij} + d^i \ \Delta V^i - H(d^i) = \Theta(d^i) > 0$$

for any  $d^i \in [0, 1]$  and  $q^i \in [0, 1]$ . Hence, as c'(0) = 0, it follows from (16) that  $n(q^i, \Delta V^i) > 0$ , for any  $q^i \in [0, 1]$ . Then (17) can be solved for  $d(q^i, \Delta V^i)$ , implying that  $d^i$  is differentiably decreasing in  $q^i$ , and  $d^i(1) = 0$ . As a consequence,  $\partial \dot{q}^i/\partial q^i|_{q^i=0} = n(0, \Delta V^i) \ d(0, \Delta V^i) - n(1, \Delta V^j) \ d(1, \Delta V^j) > 0$ , and  $\partial \dot{q}^i/\partial q^i|_{q^i=1} = n^j(0) \ d^j(0) - n^i(1) \ d^i(1) > 0$ , and (0, 1) are locally unstable stationary states. Continuity of the map (15), implies that at least one interior stationary state of the system exists, solving  $n(q^i, \Delta V) \ d(q^i, \Delta V^i) \ d(q^i, \Delta V^i) - d(1-q^i, \Delta V^j) \ n(1-q^i, \Delta V^j) = 0$ , which is locally stable.

While it is in general not possible to guarantee uniqueness of the interior stationary state,  $q^{i*}$ , for transition mechanisms which interact cultural transmission and evolutionary selection with endogenous fitness determination, we are able to provide a robust example here. (Note that, uniqueness of  $q^{i*}$  implies its global stability:  $q^{i}(t, q_0^{i}) \rightarrow q^{i*}$ , for  $q_0^{i} \in (0, 1)$ .)

Assume that the costs of socialization,  $H(d^i)$ , and the cost to raise children,  $c(n^i)$ , are quadratic and given by:  $H(d^i) = \frac{1}{2}(d^i)^2$ ,  $c(n^i) = \frac{1}{2}(n^i)^2$ .

Moreover, assume  $V^{ii} > \frac{2}{3}$ . In this case the interior steady state,  $q^{i*}$ , is unique.

*Proof.* The first order conditions for the maximization problem (14), in the case of the example, can be written:

$$\begin{split} n^i &= V^{ii} - (1-d^i)(1-q^i) \, \varDelta V^i - \tfrac{1}{2} \, (d^i)^2 \\ d^i &= (1-q^i) \, \varDelta V^i \end{split}$$

or

$$\begin{split} n^i &= V^{ii} - d^i + \tfrac{1}{2} \, (d^i)^2 \\ d^i &= (1-q^i) \, \varDelta V^i. \end{split}$$

Let  $X^i := (1 - q^i) \Delta V^i$ . The first order conditions can then be written:  $d^i = d^i(X^i) = X^i$ , and  $n^i = n^i(X^i) = V^{ii} - X^i + \frac{1}{2}(X^i)^2$ ; and

$$\frac{\partial d^i(X^i) \, n^i(X^i)}{\partial X^i} = V^{ii} - 2X^i + \frac{3}{2} \, (X^i)^2.$$

Since  $X^i$  is decreasing in  $q^i$ , at most one interior stationary state exists if  $\partial d^i(X^i) \, n^i(X^i)/\partial X^i > 0$ . But  $2X^i - \frac{3}{2} \, (X^i)^2$  has a maximum in  $X^i = \frac{2}{3}$ , with value  $\frac{2}{3}$ .  $V^{ii} > \frac{2}{3}$  then is sufficient to guarantee that  $\partial d^i(X^i) \, n^i(X^i)/\partial X^i > 0$ , and hence that at most one interior stationary state exists. Finally, Proposition 5 implies that at least one interior stationary state exists.

#### 5. CONCLUSIONS

This paper presented a simple model of preference evolution based on endogenous cultural transmission. We identified conditions under which the long run distribution of preference traits in the population is heterogenous. We also showed that because of natural externalities associated with social learning and influences between cultural groups, the path of evolution of preferences is inefficient in the sense that, for a given dynamic profile of preferences, too much resources are devoted to family socialization. Finally, we have shown the robustness of our results when one also

<sup>&</sup>lt;sup>20</sup> The parameters of the parents' socialization choice are normalized so that  $V^{ii}$  measures the number of children of a family with trait i, in equilibrium, when  $d^i = 0$ , i.e., the maximal number of children a family of type i would ever want to have (see the first order conditions of the fertility choice, reported in the following Proof). As a consequence,  $V^{ii} > \frac{2}{3}$  is effectively no restriction.

incorporates in the analysis evolutionary selection mechanisms with exogenous or endogenous fitness.

Clearly, however, many aspects related to cultural transmission have been left out of the analysis. In particular, our analysis did not consider situations in which agents interact in socio-economic environments, nor traits which affect, in a relevant manner, the economic environment the agents face. Such extensions would be particularly fruitful to understand the effects of market or public institutions on cultural evolution.<sup>21</sup> Also, many important aspects of the interaction of cultural transmission and evolutionary selection mechanisms have been left out by our analysis. Studying such interactions could be useful to provide micro foundations for social selection mechanisms.

#### REFERENCES

- 1. N. Al-Najjar, Decomposition and characterization of risk with a continuum of random variables, *Econometrica* **63** (1995), 119–224.
- A. Bandura and R. Walters, "Social Learning and Personality Development," Holt, Rinehart & Winston, New York, 1965.
- 3. D. Baumrind, Child care practices anteceding three pattern of preschool behavior, *Genetic Psych. Monographs* **75** (1967), 43–83.
- G. Becker, Altruism, egoism, and genetic fitness: economics and sociobiology, *J. Econ. Lit.* 14 (1970), 817–826.
- 5. G. Becker, "Accounting for Taste," Harvard University Press, Cambridge, MA, 1996.
- G. Becker and C. Mulligan, The endogenous determination of time preference, Quart. J. Econ. 3 (1993), 729–758.
- 7. T. Bergstrom, On the evolution of altruistic ethical rules for siblings, *Amer. Econ. Rev.* **85** (1995), 58–81.
- H. Bester and W. Guth, Is altruism evolutionary stable?, J. Econ. Behav. Organ. 34 (1998), 193–209.
- A. Bisin and T. Verdier, Agents with imperfect empathy might survive natural selection, mimeo, New York University, 1999.
- 10. A. Bisin and T. Verdier, Beyond the melting pot: cultural transmission, marriage and the evolution of ethnic and religious traits, *Quart. J. Econ.*, in press.
- S. Bowles, Endogenous preferences: the cultural consequence of markets and other economic institutions, J. Econ. Lit. 36 (1998), 75–111.
- G. Borjas, Ethnic capital and intergenerational income mobility, Quart. J. Econ. 57 (1992), 123–150.
- R. Boyd and P. Richerson, "Culture and the Evolutionary Process," University of Chicago Press, Chicago, 1985.
- A. Cabrales and J. Sobel, On the limit points of discrete selection dynamics, J. Econ. Theory 57 (1992), 407–419.
- L. L. Cavalli-Sforza and M. Feldman, Cultural versus biological inheritance: phenotypic transmission from parent to children, Amer. J. Human Genetics 25 (1973), 618–637.

<sup>&</sup>lt;sup>21</sup> See Bowles [11] for a motivation of such a line of research.

- L. L. Cavalli-Sforza and M. Feldman, "Cultural Transmission and Evolution: A Quantitative Approach," Princeton University Press, Princeton, 1981.
- 17. R. Dawkins, "The Selfish Gene," Oxford University Press, Oxford, 1976.
- J. Duesenberry, "Income, Saving and the Theory of Consumer Behavior," Harvard University Press, Cambridge, MA, 1949.
- I. Eshel, L. Samuelson, and A. Shaked, Altruists, egoists, and hooligans in a local interaction model, Amer. Econ. Rev. 88 (1998), 157–179.
- C. Fershtman and Y. Weiss, Social rewards, externalities and stable preferences, J. Pub. Econ. 70 (1998), 53–74.
- R. Frank, If homo economicus could choose its own utility function: would he want one with a conscience?, Amer. Econ. Rev. 77 (1987), 593–604.
- 22. W. D. Hamilton, The genetic theory of social behavior, J. Theor. Biol. 7 (1964), 1-32.
- 23. J. Hirshleifer, Economics from a biological point of view, J. Law Econ. 20 (1977), 1-52.
- J. Hirshleifer, Competition, cooperation and conflict in economics and biology, Amer. Econ. Rev. 68 (1978), 238–243.
- L. Iannaccone, Religious practice: a human capital approach, J. Sci. Study Religion 29 (1990), 297–314.
- A. Kapteyn, T. Wansbeek, and J. Buyze, The dynamics of preference formation, J. Econ. Behav. Org. 1 (1980), 123–157.
- L. Kockesen, E. Ok, and R. Sethi, The strategic advantage of negatively interdependent preferences, J. Econ. Theory 92 (2000), 274–299.
- H. Leibenstein, Bandwagon, snob and Veblen effects in the theory of consumers' demand, Quart. J. Econ. 64 (1950), 181–207.
- Quart. J. Econ. 64 (1950), 181–207.
  29. R. Pollak, Habit formation and long run utility functions, J. Econ. Theory 13 (1976),
- A. Robson, A biological basis for expected and non-expected utility, J. Econ. Theory 68 (1996), 397–424.
- 31. A. Rogers, Does biology constrain culture?, Amer. Anthro. 90 (1988), 819-831.

272-297.

- P. Rubin and C. Paul, An evolutionary model of taste for risk, Econ. Inquiry. 17 (1979), 585–596.
- L. Samuelson, "Evolutionary Games and Equilibrium Selection," MIT Press, Cambridge, MA, 1997.
- 34. A. Smith, The theory of moral sentiments, *in* "The Glasgow Edition of the Work and Correspondance of Adam Smith, 1" (D. D. Raphael and A. L. MacFie, Eds.), Liberty Fund, Glasgow, 1984. [First edition, 1759]
- 35. O. Stark, "Altruism and Beyond," Cambridge University Press, Cambridge, UK, 1995.
- Y. Sun, A theory of hyperfinite processes: the complete removal of individual uncertainty via exact LLN, J. Math. Econ. 29 (1998), 419–503.
- F. Vega Redondo, "Evolution, Games, and Economic Behavior," Oxford University Press, Oxford, 1996.
- 38. J. Weibull, "Evolutionary Game Theory," MIT Press, Cambridge, MA, 1995.
- 39. E. Wilson, "Sociobiology," Cambridge University Press, Cambridge, UK.