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# Agents with imperfect empathy may survive natural selection

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## Abstract

Cultural transmission mechanisms which favor the direct transmission of the parents' traits to their children may be adaptive to natural selection when opposed to mechanisms in which the parents choose for the offspring the highest fitness at any time. © 2001 Elsevier Science B.V. All rights reserved.

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## 1. Introduction and motivation

An example is provided to illustrate the constraints biological evolution might impose on parents-to-offspring cultural transmission mechanisms and preference evolution.<sup>1</sup>

In particular, we compare the relative evolutionary adaptiveness of cultural transmission mechanisms in which parents socialize their children to the preference trait which maximizes the children's reproductive success (mechanisms with *perfect empathy*),<sup>2</sup> with mechanisms in which parents choose for their children the cultural trait they (the parents) have, independently of fitness considerations (mechanisms with *imperfect empathy*).<sup>3</sup>

Such a comparison is of interest because the dynamic properties of the evolution of culturally

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<sup>1</sup>Rogers (1988), in a similar environment, studies the constraints imposed by natural selection on the social learning strategies of the children, rather than on the cultural transmission mechanisms of the parents.

<sup>2</sup>Models of cultural transmission motivated by perfect empathy have been forcefully introduced in the economic literature by Stigler and Becker (1977). See also Becker and Mulligan (1997).

<sup>3</sup>The study of cultural transmission mechanisms with imperfect empathy was introduced by Cavalli Sforza and Feldman (1981). See also Boyd and Richerson (1985).

transmitted preferences depend crucially on the form of altruism which motivates the transmission of preferences from parents to children: cultural transmission mechanisms characterized by perfect empathy imply dynamics of the distribution of preferences which converge to degenerate distributions concentrated on those preferences which maximize fitness, while imperfect empathy can give rise to cultural transmission mechanisms which predict heterogeneous stationary distributions of preferences (Bisin and Verdier, 2000).

If children pay a cost in terms of reproductive success to be socialized to a trait different than that of their parents, even if such cost is small enough so that tracking the most successful traits maximizes his or her own fitness at any date, and the environment is sufficiently volatile, agents with imperfect empathy may be selected by evolutionary adaptation and hence may be the only to survive natural selection.

The average reproductive disadvantage of perfect empathy is due to the fact that, in a volatile environment in which socialization is moderately costly, the reproductive success of a dynasty of agents need not coincide with that of agents at any date. In our example strategic considerations play no role in generating the adaptiveness of imperfect empathy.<sup>4</sup>

## 2. The setup

We consider here a very simple model of the evolutionary selection of different cultural transmission mechanisms, without any attempt at generality. The economy is populated by a continuum of agents. Each agent is identified by a cultural or preference trait, a parameter of his utility function, which can be of two types,  $i \in \{a, b\}$ , and is chosen by his or her parent. Agents are in fact born with a genetic ability to influence the preference trait of their children, and with different psychological abilities to evaluate their children's preferences. Agents with perfect empathy desire for their children the most reproductively successful cultural trait. Agents with imperfect empathy, on the contrary, simply want their children to have the same cultural trait they have, independently of which trait is most successful. Reproduction is asexual and each agent inherits directly from his/her parent the characteristic of the cultural transmission mechanism he/she is endowed with. It takes a cost  $c$ , in units of fitness, to socialize one's children to a trait different from one's own. We assume that parents decide which trait to socialize children to, and that children pay the fitness costs; but our analysis does not change if we assume the cost to be borne by the parents. Such cost could then both be exemplified by the time spent by parents with children to socialize them to a 'better' preference trait than their own, as well as by the learning costs children bear to adapt to the new preferences.

Time is discrete:  $t = 0, 1, \dots, \infty$ . Agents live one period, in a common environment which determines the reproductive success (fitness) associated with each cultural trait. The environment can be of two types,  $e \in \{a, b\}$ , and follows an exogenous stochastic process. More formally,  $e$  is a Markov chain on state space  $\{a, b\}$ , with transition matrix

$$\begin{bmatrix} 1 - \rho & \rho \\ \rho & 1 - \rho \end{bmatrix}^5$$

<sup>4</sup>Strategic interactions drive, for example, Banerjee and Weibull's (1991) results on the adaptiveness of imperfect learning mechanisms, as well as Kockesen et al.'s (2000) results on the adaptiveness of interdependent preferences.

<sup>5</sup>See, for instance, Bhattacharya and Waymire (1990), Chapter 2, for definitions and results on Markov chains.

In other words, each period  $t$ , the environment,  $e_t$ , switches with probability  $\rho \in [0,1]$ .

The fitness of agents with trait  $i$  is  $V$  when the environment  $e$  takes value  $i$ , and  $v$  when the environment takes value  $j \neq i$ . Without loss of generality, we let  $V > v$ , so that high fitness is achieved by the cultural type which ‘tracks’ the environment.

Agents with imperfect empathy never attempt changing their children’s cultural trait; they socialize them to their own trait at no cost. Agents with perfect empathy, on the contrary, change their children preferences, when their trait is not the most successful, if the associated costs are not too high.<sup>6</sup> To avoid the trivial case in which neither perfect nor imperfect empathy agents socialize their offspring to a trait different from their own, we in fact assume that the costs borne by children to be socialized to a different trait than their parents are low enough so that perfect empathy parents always choose to adapt the children’s trait to the environment (to socialize children to the reproductively most successful trait):

$$c \leq V - v.$$

Let  $n_t^i$  denote the number of agents in the population with imperfect empathy and trait  $i \in \{a,b\}$ , and let  $n_t^p$  be the number of agents in the population with perfect empathy (of either trait).

### 3. Imperfect empathy may be adaptive

Parents with perfect empathy always socialize their children to the most successful trait, and which trait is the most successful is assumed to switch every period with probability  $\rho$ . Parents with imperfect empathy always transmit their own trait.

The parameter  $\rho$  measures the volatility of the environment.

We are interested in the dynamic and stochastic properties of the fraction of agents with imperfect empathy:

$$\frac{n_t^a + n_t^b}{n_t^p + n_t^a + n_t^b}$$

In particular, we want to derive conditions under which the number of imperfect empathy agents dominates (is dominated by) the number of perfect empathy agents in the long run:

$$\frac{n_t^a + n_t^b}{n_t^p + n_t^a + n_t^b} \rightarrow 1 \quad (\rightarrow 0). \tag{1}$$

**Proposition 1.** *Assume  $n_0^i > 0$ ,  $i = a,b$ , and  $n_0^p > 0$ . Assume also that  $c \leq V - v$  and  $vV > (V - c)^2$ , then there exists a threshold level of volatility  $\bar{\rho}$ ,  $0 < \bar{\rho} < 1$ , such that*

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<sup>6</sup>If agents were allowed to buy fair lotteries, i.e. probabilities, of socializing their offspring, then the analysis can be somewhat carried over even if socialization is highly costly, because imperfect empathy agents would socialize with positive probability in equilibrium.

(i) for  $0 \leq \rho < \bar{\rho}$ ,  $\lim_{t \rightarrow \infty} \frac{n_t^a + n_t^b}{n_t^p + n_t^a + n_t^b} \rightarrow 0$  with probability 1;

(ii) for  $\bar{\rho} < \rho \leq 1$ ,  $\lim_{t \rightarrow \infty} \frac{n_t^a + n_t^b}{n_t^p + n_t^a + n_t^b} \rightarrow 1$  with probability 1.

**Proof.** It turns out that it is convenient to study directly the dynamics of

$$q_t^i = \frac{n_t^i}{n_t^p} \text{ for } i = a, b.$$

We will show in fact that, if  $vV > (V - c)^2$ , then there exists a  $\bar{\rho} < 1$  such that

(i) for any  $\rho \in (\bar{\rho}, 1]$

$$\lim_{t \rightarrow \infty} q_t^i = \infty, \text{ with probability 1;}$$

(ii) for any  $\rho \in [0, \bar{\rho})$

$$\lim_{t \rightarrow \infty} q_t^i = 0, \text{ with probability 1.}$$

And since

$$\frac{n_t^a + n_t^b}{n_t^p + n_t^a + n_t^b} = \frac{q_t^a + q_t^b}{1 + q_t^a + q_t^b}, \tag{2}$$

the statement of the proposition readily follows.

$q_t^i$  is a random variable, and we let  $q^i$  denote the stochastic process which drives  $q_t^i$ , for all  $t$ . It is easy to see that  $(q^i, e)$  is a Markov chain, on  $Q \times \{a, b\}$ , where  $Q$  denotes the set of rational numbers (the fitness values,  $v, V, V - c$ , are naturally integers). The transition matrix of the chain  $(q^i, e)$  (i.e., of the Markov chain which governs the joint dynamics of the environment and of the relative dimension of the population of imperfect empathy agents with trait  $i$  and of perfect empathy agents) is denoted  $P^i((q', e')|(q, e))$ . Since  $c \leq V - v$ , perfect empathy parents always choose to adapt their children's trait to the environment, and hence  $P^i((q', e')|(q, e))$  is constructed as follows:

$$P^i(q', i|q, i) = \begin{cases} 1 - \rho, & \text{if } q' = q, \\ 0, & \text{else,} \end{cases} \quad P^i(q', j|q, i) = \begin{cases} \rho, & \text{if } q' = [v/(V - c)]q, \\ 0, & \text{else,} \end{cases} \tag{3}$$

$$P^i(q', i|q, j) = \begin{cases} \rho, & \text{if } q' = [V/(V - c)]q, \\ 0, & \text{else,} \end{cases} \quad P^i(q', j|q, j) = \begin{cases} 1 - \rho, & \text{if } q' = (v/V)q, \\ 0, & \text{else.} \end{cases} \tag{4}$$

For instance, at time  $t$ , if the environment is  $e = i$ , the environment switches from  $i$  to  $j$  with probability  $\rho$ , and  $q_{t+1}^i$  then decreases by the ratio of the fitness of the two populations,  $v/(V - c)$ ; with probability  $1 - \rho$ , instead, the environment is unchanged, and  $q_{t+1}^i$  is constant. Similarly, at time

$t$ , but if the environment is  $j$ , the environment switches to  $i$  with probability  $\rho$ , and  $q_{t+1}^i$  increases by the ratio of the fitness of the two populations,  $V/(V-c)$ ; with probability  $1-\rho$ , instead, the environment is unchanged, and  $q_{t+1}^i$  decreases by  $v/V$ .

Take any  $q_0^i > 0$ , finite. Let  $f_\tau^i$  denote the ratio of the fitness of imperfect empathy agents of type  $i$  and perfect empathy agents, taking values in

$$\left\{ 1, \frac{v}{V-c}, \frac{V}{V-c}, \frac{v}{V} \right\}.$$

Note that

$$q_t^i = \prod_{\tau=1}^t f_\tau^i q_0^i. \tag{5}$$

Since  $(q^i, e)$  follows a Markov chain with transition  $P^i(q', e' | q, e)$ , as in (3) and (4),  $f_\tau^i$  also follows a Markov chain on

$$\left\{ 1, \frac{v}{V-c}, \frac{V}{V-c}, \frac{v}{V} \right\}$$

with transition matrix

$$F^i := \begin{bmatrix} 1-\rho & \rho & 0 & 0 \\ 0 & 0 & \rho & 1-\rho \\ 1-\rho & \rho & 0 & 0 \\ 0 & 0 & \rho & 1-\rho \end{bmatrix}.$$

The Markov chain for  $f_\tau^i$  is finite, and irreducible.<sup>7</sup> It is finite by construction, since

$$\left\{ 1, \frac{v}{V-c}, \frac{V}{V-c}, \frac{v}{V} \right\}$$

is a finite set. It is irreducible, since  $(F_{kl}^i)^2 > 0$  for all

$$k, l \in \left\{ 1, \frac{v}{V-c}, \frac{V}{V-c}, \frac{v}{V} \right\}.$$

As a consequence, all its states are recurrent (Bhattacharya and Waymire, 1990, Proposition 8.1, p. 136),<sup>8</sup> and it has a unique stationary distribution:

$$\mu^i := [\mu_h]_{h \in \{1, v/(V-c), V/(V-c), v/V\}},$$

<sup>7</sup>Let  $F = [F_{ij}]_{i \in S}^{j \in S}$  be the transition matrix of a Markov chain on  $S$ . Let  $F^n = F : F^{n-1}$ , with  $F^1 = F$  the  $n$ -step transition matrix. Then

- (i) A Markov chain is finite if it is defined on a finite state space,  $S$ .
- (ii) A Markov chain is irreducible if, for any  $i, j \in S$ , with  $i \neq j$ , there exists a finite  $n_{ij}$  such that  $F_{ij}^{n_{ij}} > 0$ .

<sup>8</sup>A state  $i \in S$  is recurrent if there exists a finite  $n_{ii}$  such that  $F_{ii}^{n_{ii}} > 0$ .

i.e. there exists a unique probability distribution  $\mu^i$  on

$$\left\{1, \frac{v}{V-c}, \frac{V}{V-c}, \frac{v}{V}\right\}$$

which solves  $\mu^i F^i = \mu^i$  (see Bhattacharya and Waymire, 1990, Theorem 7.1, p. 134). The stationary distribution can be calculated to be

$$\mu^i = \left(\frac{1-\rho}{2}, \frac{\rho}{2}, \frac{\rho}{2}, \frac{1-\rho}{2}\right).$$

Finally, a consequence of the (strong) Law of Large Numbers for Markov chains, e.g. Theorem 9.4 in Bhattacharya and Waymire (1990), implies that, in the limit for  $t \rightarrow \infty$ ,  $\mu_h^i$  represents the fraction of times the chain is on state  $h \in S$ ; i.e. using (5), we have that, with probability 1,

$$\lim_{t \rightarrow \infty} q_t^i = \lim_{t \rightarrow \infty} \left(\frac{v}{V-c}\right)^{(\rho/2)t} \left(\frac{V}{V-c}\right)^{(\rho/2)t} \left(\frac{v}{V}\right)^{[(1-\rho)/2]t}.$$

As a consequence,  $\lim_{t \rightarrow \infty} q_t^i = \infty$  ( $\lim_{t \rightarrow \infty} q_t^i = 0$ ) when

$$\left(\frac{v}{V-c}\right)^{\rho/2} \left(\frac{V}{V-c}\right)^{\rho/2} \left(\frac{v}{V}\right)^{(1-\rho)/2} > 1 \quad (< 1).$$

Since  $V > v$  and  $vV > (V-c)^2$ , this condition can be rewritten as  $\rho > \bar{\rho}$  ( $\rho < \bar{\rho}$ ) for

$$\bar{\rho} = \frac{\ln(V/v)}{\ln(V/v) + \ln[vV/(V-c)^2]}. \tag{6}$$

Noting finally that  $\bar{\rho} < 1$ , since  $vV > (V-c)^2$ , concludes the proof.  $\square$

One also sees immediately that the two conditions  $c \leq V-v$  and  $vV > (V-c)^2$  are necessary to drive the fraction of agents with imperfect empathy in the population to approach 1.<sup>9</sup> In other words, necessary and sufficient conditions on the set of parameters  $(\rho, v, V, c)$  which drive the fraction of agents with imperfect empathy in the population to approach 1 are

$$\rho > \bar{\rho}, \tag{7}$$

$$c < V-v, \quad vV > (V-c)^2; \tag{8}$$

that is, a sufficiently volatile environment, and socialization costs

- (i) low enough that parents with perfect empathy are willing to socialize children to track the reproductively most successful trait at any date, and

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<sup>9</sup>In fact,  $c \leq V-v$  is necessary as otherwise imperfect altruists and perfect altruists are not different in terms of cultural transmission behaviors; and the fact that  $vV > (V-c)^2$  is necessary to have  $\bar{\rho} < 1$  is a consequence of the definition of  $\bar{\rho}$  in (6).

(ii) high enough so that, for given volatility of the environment,  $\rho$ , tracking the most successful trait does not maximize the geometric average of fitness (the fitness of the whole dynasty)

$$\left(\frac{v}{V-c}\right)^{\rho/2} \left(\frac{V}{V-c}\right)^{\rho/2} \left(\frac{v}{V}\right)^{(1-\rho)/2} \text{.}^{10}$$

Fig. 1 summarizes the long-run evolution of the distribution of empathy types in the population, depending on the relative fitness cost of the two cultural traits,  $v/V$ , and on the variability of the environment,  $\rho$ . The horizontal line  $v/V = 1 - c/V$  represents the frontier over which perfect altruists do not keep track of the change of the environment for the process of cultural transmission and therefore are behaviorally not different from imperfect altruists. Below this line, differentiation between the two types of agents will lead one to dominate stochastically the other in the long run. The curve  $\rho = \bar{\rho}(v/V, V, c)$  reflects the condition expressed on the variability of the environment above which (below which) imperfect (perfect) altruists dominate. This curve crosses the horizontal line  $v/V = 1 - c/V$  at the volatility parameter  $\rho = 1/2$ . Thus imperfect altruists will dominate in the long

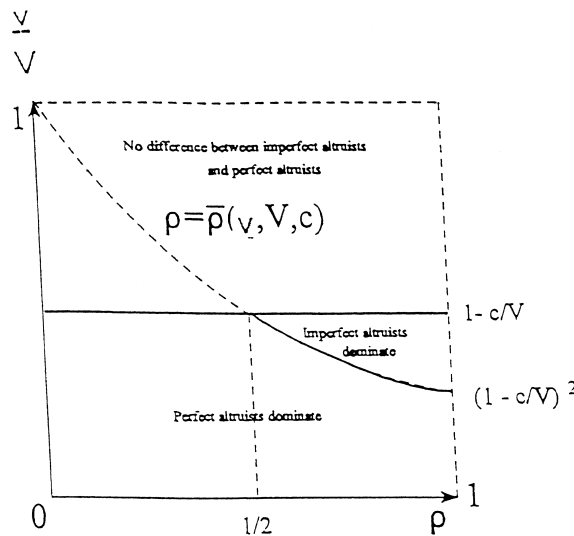


Fig. 1. Long-run distribution of types.

<sup>10</sup>Note that what determines the distribution of the population in the limit is the geometric average relative fitness

$$\left(\frac{v}{V-c}\right)^{\rho/2} \left(\frac{V}{V-c}\right)^{\rho/2} \left(\frac{v}{V}\right)^{(1-\rho)/2} \text{,}$$

which depends positively on the (arithmetic) average and negatively on the variance of relative fitness. (This is a general result in stochastic environments; see Yoshimura and Clark, 1993, for recent surveys.) It can be shown that the (arithmetic) average fitness of the population of agents with imperfect empathy and trait  $i$ , for  $\rho$  close to 1, is higher than the (arithmetic) average fitness of agents with perfect empathy (the (arithmetic) average relative fitness is  $>1$ ). This is so even if  $vV > (V - c)^2$  is not satisfied. In other words, the variance of the fitness is higher for the population of agents with imperfect empathy; this limits their adaptiveness to stochastic environments, but not enough to drive them to relative extinction, if  $vV > (V - c)^2$ .

run only for volatility parameters larger than  $1/2$ . Note also that for  $v/V < (1 - c/V)^2$  there is no environment under which imperfect altruists survive. This corresponds to the violation of the condition  $vV > (V - c)^2$ .

Finally, it is straightforward to see that for parameter values which satisfy conditions (7) and (8) the fraction of agents with imperfect empathy converges faster to 1 the higher is the volatility of the environment  $\rho$ , the higher is the socialization cost  $c$ , and the higher is the geometric average of the maximal reproduction rate in the environment,  $vV$ .

#### 4. Conclusions

Our analysis suggests that genetic evolution might impose potentially important constraints on the cultural transmission mechanisms which govern the evolution of preferences. If genetic evolution acts on the transmission mechanisms, rather than on the preference traits themselves, there is no reason to believe that the traits which maximize the instantaneous reproductive success of agents would be selected. On the contrary, and especially in environments and for traits whose fitness properties are volatile and for which socialization is moderately costly, genetic evolution might select transmission mechanisms characterized by imperfect altruism and imperfect empathy. In turn, such mechanisms will generate dynamics of preference distributions which tend to support the co-existence of preference traits associated with different rates of reproductive success.<sup>11</sup>

We just compared cultural transmission mechanisms characterized by perfect and imperfect empathy, respectively. Transmission mechanisms in which parents are motivated by a perfect form of altruism toward their whole dynasty of descendants can be constructed which internalize some appropriate average fitness over the dynasty, and hence would be selected by evolutionary adaptation. But such mechanisms would contradict the basic tenet of ‘kin selection’, that any particular form of altruism, if evolutionarily selected, can only operate among close kins rather than at the dynastic level (see Hamilton, 1964a,b, and, for a review of ‘kin selection’, Frank, 1998).

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<sup>11</sup>See Bisin and Verdier (2000) for the analysis of the dynamics of preferences in models with imperfect empathy.



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