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Competitive Equilibria with Asymmetric Information*

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This paper studies competitive equilibria in economies where agents trade in markets for standardized, non-exclusive financial contracts, under conditions of asymmetric information (both of the moral hazard and the adverse selection type). The problems for the existence of competitive equilibria in this framework are identified, and shown to be essentially the same under different forms of asymmetric information. We then show that a "minimal" form of non-linearity of prices (a bid-ask spread, requiring only the possibility to separate buyers and sellers), and the condition that the aggregate return on the individual positions in each contract can be perfectly hedged in the existing markets, ensure the existence of competitive equilibria in the case of both adverse selection and moral hazard. *Journal of Economic Literature* Classification Numbers: D50, D82. © 1999 Academic Press

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1. INTRODUCTION

This paper studies competitive equilibria in economic environments characterized by the presence of asymmetric information; both situations of moral hazard and adverse selection are allowed for.

Agents are assumed to trade in spot markets and markets for financial contracts. These contracts are (i) standardized and (ii) non-exclusive, in the sense that the terms of each contract (its price and its payoff) are, respectively, common across many relationships involving different agents, and independent of the level of transactions made by an agent in other markets. Under these conditions the partners of a contractual relationship have a very limited power over the terms of the contract. Also, at an equilibrium each agent will typically trade in different markets, and enter different contracts at the same time. We argue that in such a situation a general equilibrium approach is useful to analyze the interaction among trades in different markets; also that we may analyze contracts, as well as commodities, as traded in competitive markets. Our analysis builds on the earlier work by Dubey, Geanakoplos and Shubik [12], who explicitly addressed this issue in a model where asymmetric information is generated by the possibility of default.

Hence we depart from the analysis of exclusive contractual relationships, where an agent can only choose one out of a menu of contracts, or equivalently the terms of a contract depend on the position of the agent in all markets. The implementation of these contracts imposes a very strong informational requirement as all the transactions of an agent need to be observed. The non-exclusivity of contracts matches then the observation that, for instance, agents often hold various insurance policies, and get loans both from banks and from credit card companies[1]. Also, the terms of contracts very rarely depend on the agents' transactions in other markets[2]. Moreover, well-defined markets operate where standardized contracts are traded: markets for credit and insurance contracts, or mortgages, are important examples, as well as markets for any financial security which allows for a default clause. The recent diffusion of the securitization of payoffs of contracts has also enhanced the use of standardized contracts (see Kendall and Fishman [27]).

The main objective of this paper is to analyze the conditions for the "viability" of competitive markets for contracts in the presence of asymmetric information. In this context, the payoff of a contract typically

[1] Petersen and Rajan [36] provide some evidence on the composition of credit sources for small businesses in the U.S.

[2] See Smith and Warner [42] for an analysis of the terms of debt contracts in the U.S.

depends on the characteristics of the agent trading the contract (as, for instance, in insurance contracts against an individual source of risk). Hence when the same (type of, standardized) contract is entered by different agents, this is effectively a different contract. However, when the agents' characteristics are only privately observed these different contracts cannot be separated and are traded together in a single market. As a consequence a problem may arise in ensuring feasibility in such markets since there may not be "enough prices" to clear these markets.

If the quantities traded by an agent in all existing markets are a fully revealing signal of the agent's type, of his private characteristics, exclusive contracts, where the terms of a contract depend on the trades of the agent in the various markets, allow to solve this problem—by separating agents of different types—and to clear markets. This was shown by Prescott and Townsend [38] to be possible, under general conditions, when asymmetric information is of the moral hazard type, though not in the case of adverse selection. In adverse selection economies in fact the quantities traded may not fully reveal the private characteristics of the agents trading the contract, i.e. agents cannot be separated, so that what are effectively different contracts have to be traded in a single market at the same price. Furthermore, as we already argued, the possibility of implementing exclusive contracts is a very demanding requirement. When exclusive contracts are not available, the feasibility problem we identified above arises also for moral hazard economies.

In this paper we show that two prices (a bid and an ask price) for each contract are enough to guarantee the existence of a competitive equilibrium in economies where trade takes place under asymmetric information. The result requires the additional condition that the aggregate return on the individual positions in a given contract can be perfectly hedged on the existing markets (asset markets are "sufficiently" complete), or is also marketed as a distinct claim (i.e. all individual positions in the contract are pooled together and securitized in a "pool" security, whose payoff is the average total net amount due to agents who traded the contract[3]). The importance of the role of "pool" securities for economies with asymmetric information was first stressed by Dubey, Geanakoplos and Shubik [12].

When "pool" securities are either directly or indirectly traded, the simple ability to differentiate prices for buying and selling positions is sufficient for the existence of competitive equilibria. We are able to show in particular that non-trivial equilibria always exist, and to derive some properties of the equilibrium prices of contracts. We should stress that our results hold both under moral hazard and adverse selection, and no matter what is the "dimension" of the sources of asymmetric information in the economy (i.e.

[3] See also the final section for further discussions on this.

of the cardinality of the set of unobservable possible types or actions of the agents trading the contract).[4]

This form of non-linearity of prices (i.e. of dependence of the unit price of a contract on the quantity traded of that contract) is “minimal” in the sense that it only arises at one point and, more importantly, can be implemented by observing only the level (in fact the sign) of trades in each particular transaction, without even knowing the other transactions of the agent in the same contract. But it is also “minimal” in the sense that we will show that without it, i.e. if prices of contracts are linear over the whole domain, competitive equilibria may fail to exist in economies with asymmetric information: a robust example of an economy with adverse selection is presented where at all prices the total net payoff to agents trading individual contracts is positive. Evidently, our results also imply the existence of competitive equilibria for the case in which stronger forms of non-linearity of prices can be implemented (i.e. when all trades in the same market, or even possibly in other markets, can be monitored), as long as the price is allowed to be non-linear at the level of zero trades.

The identification of the “problems” for the existence of competitive equilibria with linear prices in economies with asymmetric information is one of the main contributions of this paper. Not only these problems are shown to be common both to moral hazard and adverse selection economies, but in our framework adverse selection can be seen, at an abstract level, as a reduced form of moral hazard. As a consequence of such problems, while in the case of symmetric information competitive equilibria always exist under standard assumptions, when there is asymmetric information some restriction on trades is needed. The main purpose of such restrictions is to provide a mechanism according to which gains and losses agents make on the basis of their private information are redistributed in the economy.

The analysis is developed in the framework of a two-period, pure exchange economy. A large economy is considered, with infinitely many agents, of finitely many types. There is both aggregate and idiosyncratic uncertainty. Aggregate shocks are commonly observed. On the other hand agents may have private information over the realization of their idiosyncratic shocks. Agents can trade contracts whose payoff depends on the realization of the aggregate as well as the idiosyncratic shocks affecting them. These are standardized contracts, in the sense that their specification (their payoff structure) is common across agents: all contracts of the same

[4] The only restriction we (in fact have to) impose on the specification of contracts’ payoffs is that, even in the presence of asymmetric information, there are no unbounded arbitrage opportunities, for at least some prices. Alternatively, a bound on the set of admissible trades in contracts can be imposed, without the need in such case of any restriction on the specification of the contracts’ payoff.

type are ex ante identical and sell at the same price, though the payoff of a unit of them depends on the realization of a specific idiosyncratic shock.

The structure of the paper is as follows. Section 2 describes the class of economies we consider and the case of symmetric information is examined in Section 3. Asymmetric information is introduced then in Section 4 and the problems for the existence of competitive equilibria are identified. Section 5 provides a robust example of non-existence of competitive equilibrium for a simple adverse selection economy. In Section 6 the existence of competitive equilibria is established. Section 7 concludes.

Related literature. Our analysis was significantly inspired by the work of Dubey, Geanakoplos and Shubik [12] (also Geanakoplos, Zame and Dubey [18]). These authors study the competitive equilibria of economies where standardized securities are traded and agents have the choice to default on their contractual obligations; thus a situation of asymmetric information originates and is captured by the possibility of default. The fact that both in their and our set-up agents have some control over the payoff of the securities they trade generates important similarities in the analysis. Moreover, we also use Dubey, Geanakoplos and Shubik [12]'s construction of "pool" securities to aggregate the payoff of securities traded by agents under asymmetric information.[5] In the model considered by Dubey *et al.* the possibility of default is the only source of an agent's ability to affect the payoff of the contracts he trades; since agents may only default on their short positions, a one-side constraint naturally originates in such framework. Consistently with our existence result then, no feasibility problem arises in this model and existence can be proved under standard conditions.

Extending this work, Bisin, Geanakoplos, Gottardi, Minelli and Polemarchakis [7] show that competitive equilibria for several types of asymmetric information economies (including for instance insurance economies with adverse selection and/or moral hazard, Akerlof's "lemons" economies, economies with default, monitoring, tournaments, and others) share a common structure and can be analyzed within the set-up of a common abstract model.

The pioneering work of Prescott and Townsend [38, 39] constitutes an important reference for any analysis of economies with asymmetric information in the framework of general equilibrium, competitive models. These authors consider an economy where competitive markets for all state-contingent commodities are open at the beginning of time and agents' consumption plans are subject to the additional restriction that they have to be incentive compatible. Prescott and Townsend show that, for economies

[5] This construction is also used by Minelli and Polemarchakis [35] in an analysis of Akerlof's model of the used car market.

with moral hazard, competitive equilibria always exist, and are constrained efficient, while the extension of their approach to economies with adverse selection is problematic.[6] It is easy to see that the contractual relationships which generate the equilibria considered in their analysis satisfy a very strong exclusivity condition (indirectly induced by the fact that contingent consumption plans are restricted by an overall incentive compatibility constraint).[7]

Helpman and Laffont [24] (see also (Laffont [28]) present an example of an economy with moral hazard where, with linear prices, no competitive equilibrium exists. As we argue more in detail in another paper (Bisin and Gottardi [8]), the lack of existence in that example can be imputed to the same kinds of problems as the ones discussed here.

Competitive equilibria for adverse selection economies in a general equilibrium framework are also studied by Gale [14, 15]. Though the structure of markets, and in particular the role of prices and the specification of the market clearing conditions are rather different from the ones considered here, it is important to notice that Gale looks at economies where agents can be ex-ante partitioned into buyers and sellers (which again introduces what is effectively a form of one-side constraint).

The importance and the consequences of asymmetric information in large economies are examined by Gul and Postlewaite [20], Postlewaite and McLean [37]. They study a class of economies with adverse selection for which they show that, as the economy becomes large, the consequences of the presence of asymmetric information tend to disappear (the set of constrained efficient allocations tends to coincide with the set of fully Pareto efficient allocations). This is not the case in our set-up: even though the economy is large, agents still retain some private information over the payoff of the securities they trade.

The efficiency of competitive equilibria for economies with moral hazard and exclusive contracts, after the decentralization result obtained by Prescott and Townsend, has been recently examined by Lisboa [30], Bennardo [6], Citanna and Villanacci [11], Magill and Quinzii [31]. On the other hand, the consequences for efficiency of abandoning exclusivity have been discussed (again only for the moral hazard case, and for simple economies with a single market and ex-ante identical agents) by Hellwig [23], Arnott and Stiglitz [3], Bisin and Guaitoli [9], Kahn and Mookerjee [25]. The efficiency properties of competitive equilibria in our framework will be analyzed in another paper.

Finally the literature on General Equilibrium models with Incomplete Markets should also be mentioned.[8] This has developed a general frame-

[6] See however, Hammond [21], Bennardo [6].

[7] See Lisboa [30], Bennardo [6], Citanna and Villanacci [11] for existence results under an explicit exclusivity condition when, in addition, re-trading in future spot markets is allowed.

[8] See Geanakoplos [17], and Magill and Shafer [32] for surveys of this literature.

work which extends the methodology of the Arrow-Debreu model to the analysis of competitive equilibria under uncertainty (with symmetric information), when agents are not able to fully insure against all sources of risk. In such framework the set of markets in which agents are allowed to trade (in particular of contingent markets) is taken as exogenously given. We show in this paper that the presence of asymmetric information generates some restrictions on the set of the agents' insurance opportunities arising endogenously from the agents' incentive compatibility constraints and the conditions for the viability of markets.[9]

2. THE STRUCTURE OF THE ECONOMY

We consider a two-period pure exchange economy. There are L commodities, labelled by $l \in L = \{1, ..., L\}$, available for consumption both at date 0 and at date 1; commodity 1 is the designated numeraire in every spot. The agents in the economy are of finitely many types, indexed by $h \in H = \{1, ..., H\}$, and there are countably many agents of each type. An agent is then identified by a pair (h, n), where $n \in N$ (N is the set of natural numbers); λ^h denotes the fraction of the total population made of agents of type h.

Uncertainty. Uncertainty is described by the collection of random variables $\tilde{\sigma}$, $(\tilde{s}^{h,n})_{h \in H, n \in N}$, with support, respectively Σ and $(S^h)_{h \in H}$ (the same for all n). Both Σ and S^h are assumed to be finite sets, $\Sigma = \{1, ..., \Sigma\}$ and $S^h = \{1, ..., S^h\}$, with generic element σ and s^h.

The random variable $\tilde{\sigma}$ describes the economy's aggregate uncertainty, which affects all agents in the economy, while $\tilde{s}^n \equiv (\tilde{s}^{h,n})_{h \in H}$ is an idiosyncratic shock, which only affects the (H) agents of index n. We assume that the variables $(\tilde{s}^n)_{n \in N}$ are, conditionally on σ, identically and independently distributed across n. Let π denote the common probability distribution of $(\tilde{\sigma}, \tilde{s}^{h,n})$, and $\pi(s \mid \sigma) \equiv \pi((\tilde{s}^{1,n} = s^1, ..., \tilde{s}^{H,n} = s^H) \mid \sigma)$. We have so:

Assumption 1

- $\pi(\tilde{s}^{h,n} = s^h) = \pi(\tilde{s}^{h,n'} = s^h)\ \forall n, n' \in N, s^h \in S^h$.
- $\pi(\tilde{s}^{h,n} = s^h, \tilde{s}^{h',n'} = s^{h'} \mid \sigma) = \pi(\tilde{s}^{h,n} = s^h \mid \sigma)\, \pi(\tilde{s}^{h',n'} = s^{h'} \mid \sigma)\ \forall n \neq n' \in N$; $h, h' \in H$; $s^h, s^{h'} \in S^h$.

[9] See Duffie and Rahi [13] for a survey of other approaches to endogenizing market incompleteness.

On the other hand we allow $\tilde{s}^{h,n}$ to be correlated, conditionally on σ, with $\tilde{s}^{h',n}$, for $h' \neq h$. We also allow for the possible correlation of $\tilde{s}^n$ with $\tilde{\sigma}$.[10]

A metaphor may be useful to clarify the structure of the uncertainty: we may think of n as indexing different "villages" (there are then infinitely many, ex-ante identical villages), while h indexes different professional types inside each village. The idiosyncratic shocks affecting the H different professional types in each village may be correlated among them, but are independent across villages, conditionally on the aggregate shock.

Remark 1. Both the correlation properties of the various sources of risk and the general structure of the uncertainty will play an important role when asymmetric information is introduced as they allow us to have competitive markets with various types of informational asymmetries simply in correspondence of different kinds of private information over the agents' idiosyncratic shocks.

The possible correlation among the idiosyncratic shocks affecting the agents in the same village and their correlation with the aggregate shocks ensure that a non-trivial specification of the contracts is possible for all the types of asymmetric information considered. In particular, it will allow us to have contracts exploiting such correlations to extract some of the agents' private information as for instance in situations with relative performance evaluation[11]. Moreover, the presence of aggregate uncertainty, besides allowing for greater generality, also implies that the return on the aggregate of individual positions in a given contract is a complex bundle of commodities, contingent on the realization of the aggregate uncertainty, so that the possibility of hedging, directly or indirectly, this "pool" is a non-trivial issue and requires the availability of appropriate securities.

On the other hand, the consideration of a large economy, with idiosyncratic risk, implies that with private information over this risk agents will be "small" as far as the level of their trades is concerned (so that their price-taking behavior is justified), but retain some monopolistic power with regard to their information, i.e. some specific and exclusive information. Thus agents are not "informationally small", even though the economy is large (unlike in the models considered by Gul and Postlewaite [20], Postlewaite and McLean [37]).

Endowments. We will consider, with no loss of generality, the case in which uncertainty enters the economy via the level of the agents' date 1

[10] In the following analysis the decomposition of the idiosyncratic shock $\tilde{s}^n$ into the variables $\tilde{s}^{h,n}$, $h \in H$, will be used to describe in turn the components of the idiosyncratic shock which is only affecting agent (h, n), or a signal such agent receives over the realization of his idiosyncratic uncertainty.

[11] See Remark 4 for a more extended discussion on this.

endowment. Each agent $(h, n) \in H \times N$ has an endowment w_0^h at date 0, and his date 1 endowment, $w_1^h(\tilde{\sigma}, \tilde{s}^n)$, depends upon the realization of $\tilde{\sigma}$ and $\tilde{s}^n$. Let $S \equiv \Pi_{h \in H} S^h$ and $s \equiv (s^h)_{h \in H}$. We assume:

Assumption 2. $w_0^h \in \Re_{++}^L$, $w_1^h \equiv (w_1^h(\sigma, s); \sigma \in \Sigma, s \in S) \in \Re_{++}^{L(\Sigma S)}$.

Preferences. A consumption plan for an arbitrary agent (h, n) specifies the level of consumption of the L commodities at date 0 and 1 in every state. The consumption set is the non-negative orthant of the Euclidean space. Agents are assumed to have Von Neumann–Morgenstern preferences over consumption plans. The utility index of agent (h, n) is given by a function u^h: $\Re_+^{2L} \to R$ satisfying:

Assumption 3. u^h is continuous, strictly increasing and strictly concave.

Information Structure. The structure and distribution of the uncertainty is known by all agents at the initial date 0. Throughout the analysis we will also maintain the assumption that the aggregate shock $\tilde{\sigma}$ is realized at date 1, and its realization is commonly observed by all agents. On the other hand, different cases will be considered with respect to the information agents have over their idiosyncratic shocks.

In our framework *asymmetric information economies* are characterized by the fact that either the realization of the idiosyncratic shock component $\tilde{s}^{h,n}$ or its distribution are private information of agent (h, n). In particular, we will examine the case of *adverse selection* economies, where the agents have private information at the beginning of date 0, before markets open, over the realization of their idiosyncratic shock, and of *hidden information* economies, where it is the realization of the shock at date 1 to be private information. We intend to argue that the latter have essentially the same properties as economies with the more standard form of moral hazard, hidden action.[12] Moreover, having reduced the various types of informational asymmetries to various types of information over realizations of the uncertainty provides a clear benchmark of what the consequences of private information are in terms of the extent to which insurance markets are missing. The results we present here however do not depend on this particular specification and extend to other set-ups (see also Bisin, Geanakoplos, Gottardi, Minelli, Polemarchakis [7] for a more explicit discussion on this).

[12] The main distinguishing feature both of economies with hidden information and moral hazard, as we will see in the next sections (also Bisin and Gottardi [8]), is that agents are symmetrically informed at the time markets open, but have the possibility to affect (the distribution of) the payoff of the contracts they enter.

3. SYMMETRIC INFORMATION ECONOMY

We will consider first the case of symmetric information. This provides a natural and useful benchmark for the rest of the analysis. In presenting this case we will introduce the structure of markets and the nature of the market clearing conditions in our framework, which we will maintain throughout the analysis.

In this section we suppose that

- all idiosyncratic shocks $(\tilde{s}^{h,n})_{h \in H, n \in N}$ are realized at date 1 and are commonly observed.

The same is always true, as we said, for the aggregate shocks $\tilde{\sigma}$. Agents' information is then perfectly symmetric.

Market Structure. Spot markets for the L commodities open both at date 0 and in every possible state at date 1.

At date 0 a set of markets for contingent contracts (or securities)[13] also open. More precisely, for every pair $(\tilde{s}^n, \tilde{\sigma})$ there are J securities: each security $j \in J$ pays $r_j(s, \sigma)$ units of numeraire if and only if the realization of $(\tilde{s}^n, \tilde{\sigma})$ equals (s, σ); and there is one of these securities for every n. These are standardized securities in the sense that ex-ante all securities of a given type j are identical, i.e. their payoff has the same distribution for all n; ex-post however, their payoff will be different, as it will vary with the specific realization of $(\tilde{s}^n)_{n \in N}$ across n. This is natural in insurance markets, where insurance policies are standardized, but payments depend on individual realization of shocks; similar considerations hold for standard credit contracts, mortgages,...

Altogether there are then countably many of these securities in the economy, but we will consider the case in which each agent (h, n) can only trade the J securities with payoff contingent on the idiosyncratic shock $\tilde{s}^n$ affecting him (or his "village"), and on $\tilde{\sigma}$. We will show that in the present framework this is not restrictive provided all agents can also trade J "pool" securities: these securities summarize in fact all agents could do by trading in the existing securities of index n different from their own.

The payoff of "pool" security j is defined so as to equal the opposite of the average net amount (of the numeraire commodity) due to—or owed by—all agents who traded securities of type j. Hence each "pool" security can be viewed as a claim against the (net) position of all agents in "individual" securities of a given type. By the Law of Large Numbers the payoff of "pool" security j will only depend on σ and be given by:

$$r_j^p(\sigma) = -\frac{\sum_h \lambda^h \sum_s \pi(s \mid \sigma)\, r_j(s, \sigma)\, \theta_j^h}{\sum_h \lambda^h \theta_j^h}, \quad \sigma \in \Sigma$$

[13] In what follows we will use interchangeably the terms contract and security.

where θ_j^h denotes the amount of security j held by agents of type h (independent of n as we will show). This expression clearly simplifies to $r_j^p(\sigma) = -\sum_s \pi(s \mid \sigma)\, r_j(s, \sigma)$ (and we can take this as the obvious specification of $r_j^p(\sigma)$ also when $\sum_h \lambda^h \theta_j^h = 0$). All this has a very natural interpretation: the payoff of each "pool" security is obtained from the payoff of the underlying security simply by averaging out the idiosyncratic component of its return (this is in fact diversified away when we consider the total—average—return on positions in infinitely many ex-ante identical securities).

Pricing Structure. Markets are perfectly competitive, i.e. agents act as price-takers in all markets. Moreover, all securities that are ex-ante identical (which only differ in the index n) sell at the same price. Securities offering the same type of insurance against the idiosyncratic shocks in different "villages" are in fact equivalent to the eyes of the outside investors, and hence, we argue, should sell at the same price. On this basis we can claim that there is a unified, large, competitive market where all the (standardized) securities of a given type are traded together. The level of trade of each agents will then be negligible with respect to the aggregate level of trade in the market, thus justifying the assumption of price-taking behavior on these markets too.[14]

We also consider the case where prices in both financial and spot markets are a linear function of the level of their trades, and are also independent from agents' observable characteristics (e.g. of their type h). The unit price of security j is then denoted by q_j (by the above perfect competition assumption, independent of n); $q \equiv (q_j)_{j \in J}$. The (normalized) vector of spot prices of the L commodities at date 0 and at date 1 when the aggregate shock is σ, are denoted respectively by p_0 and $p_1(\sigma)$.

With regard to "pool" securities, we impose the condition that each "pool" security j sells at the opposite price of the underlying "individual" security; $-q_j$ is then the price of "pool" security j. This can be viewed as a no arbitrage condition whenever agents are free to trade on securities with payoff contingent on other agents' (other "villages") idiosyncratic shocks or, as we will argue later, as a zero profit condition if intermediaries are explicitly modelled.

In the present framework all agents of a given type h face the same choice problem, and this problem is convex. All these agents make then the same choice, so that this will be independent of n, and will be described by a portfolio respectively of "individual" and "pool" securities, $\theta^h = (\ldots, \theta_j^h, \ldots) \in R^J$, $\theta_p^h = (\ldots, \theta_{p,j}^h, \ldots) \in R^J$, and a consumption plan $c^h = (c_0^h; c_1^h = c_1^h(s, \sigma), s \in S, \sigma \in \Sigma) \in \Re_+^{L(1+S\Sigma)}$. The consumption plan specifies the level of consumption at date 0 and at date 1, for every possible realization

[14] Since each "village" is populated by a finite number of agents, price taking would not be justified in fact in an economy with securities' prices indexed by the name of the "village".

of the aggregate uncertainty and the idiosyncratic uncertainty affecting the agent.

A *competitive equilibrium with symmetric information* is then a collection of prices $\langle p_0, (p_1(\sigma)_{\sigma \in \Sigma}), (q_j)_{j \in J} \rangle$, consumption and portfolio plans for every agent's type $\langle (c^h, (\theta^h, \theta^h_p))_{h \in H} \rangle$, and a specification of the payoff of "pool" securities $[r^p_j(\sigma)]_{\sigma, j}$ such that:

- agents optimize: for all $h \in H$ the plan $(c^h, (\theta^h, \theta^h_p))$ solves the problem

$$(c^h_0, c^h_1, \theta^h, \theta^h_p) \in \arg\max \sum_{s, \sigma} \pi(\sigma)\, \pi(s \mid \sigma)\, u^h(c^h_0, c^h_1(s, \sigma)) \qquad (P^h_{SI})$$

s.t.

$$p_0 \cdot (c^h_0 - w^h_0) + q \cdot (\theta^h - \theta^h_p) \leqslant 0$$

$$p_1(\sigma) \cdot (c^h_1(s, \sigma) - w^h_1(s, \sigma)) \leqslant \sum_j r_j(s, \sigma)\, \theta^h_j + r^p_j(\sigma)\, \theta^h_{j, p}, \quad \forall (s, \sigma) \in S \times \Sigma$$

$$(c^h_0, c^h_1) \in \Re^{L(1 + S\Sigma)}_+, \quad (\theta^h, \theta^h_p) \in \Re^{2J}$$

- markets clear:

$$\sum_h \lambda^h (c^h_0 - w^h_0) \leqslant 0 \tag{3.1}$$

$$\sum_h \lambda^h \sum_s \pi(s/\sigma)(c^h_1(s, \sigma) - w^h_1(s, \sigma)) \leqslant 0, \qquad \sigma \in \Sigma \tag{3.2}$$

$$\sum_h \lambda^h (\theta^h_j - \theta^h_{p, j}) = 0, \qquad j \in J \tag{3.3}$$

- the payoff $r^p_j(\sigma)$ of each "pool" security satisfies:

$$r^p_j(\sigma) = -\sum_s \pi(s \mid \sigma)\, r_j(s, \sigma), \quad j \in J, \quad \sigma \in \Sigma \tag{3.4}$$

Under assumption 1, we have been able to exploit the Law of Large Numbers to write the feasibility condition for date 1 in (3.2) in terms of conditional expectations. The market clearing condition for securities (3.3) is then stated as the equality of the total position in "individual" securities of a given type and the total position in the associated "pool" security. It is easy to show, by using again the Law of Large Numbers and the above specification of the payoff of "pool" securities that this ensures that the aggregate payoff of the portfolios held by agents equals 0, for all possible realizations of the uncertainty at date 1, i.e. ensures feasibility. This formulation of the equilibrium condition for securities implies that trades among agents of different index n (across different "villages") take place both by compensating long and short positions in the same type of security in different "villages" (i.e. by aggregating together their positions in this security) as well as by compensating their net total position with positions in the associated "pool" securities.

It is immediate to see that the set of securities' prices precluding arbitrage opportunities is a non-empty, open set. Moreover, both the agents' choice problem P^h_{SI} and the equilibrium problem are finite-dimensional and well-behaved problems. The following result then follows by an application of standard arguments:

THEOREM 1. *Under assumptions* 1–3, *a competitive equilibrium with symmetric information exists, such that the price of every security* $j \in J$ *is "fair", conditionally on* σ:

$$q_j = \sum_{\sigma \in \Sigma} \rho_\sigma \sum_{s \in S} \pi(s \mid \sigma)\, r_j(s, \sigma) = -\sum_{\sigma \in \Sigma} \rho_\sigma r_j^p(\sigma)$$

for some $\rho \equiv (..., \rho_\sigma, ...) \gg 0$.

Let R denote the $S\Sigma \times J$ payoff matrix, with generic element $r_j(s, \sigma)$, and $Sp[R]$ the linear space generated by the columns of R. We also have:

COROLLARY 1. *If, in addition,* $Sp[R] = \Re^{S\Sigma}$, *competitive equilibria with symmetric information and fair prices are Pareto optimal and such that consumption allocations only depend on the aggregate shock* σ (*i.e. all idiosyncratic shocks are fully insured*).

When $Sp[R] = \Re^{S\Sigma}$ we can say therefore that markets are complete and that the above market structure allows to decentralize Pareto optimal allocations via securities with exogenously given payoff. Our result complements the results of Magill and Shafer [33], Cass, Chichilniski and Wu [10] where, building on the original analysis of Malinvaud [34], Pareto optima are decentralized via a set of mutual insurance contracts. It also confirms the fact that the restriction we imposed on agents' behavior, by preventing them from trading in securities of different index n, is not binding.

Remark 2. Though the set of available securities is taken as given and financial intermediaries are not explicitly modelled, competitive intermediaries, who design and market these securities, could be introduced with no substantial change in the structure of the model or the definition of an equilibrium. In particular, the economies we study are equivalent to economies in which intermediaries take positions in individual securities, compensate them across "villages", and issue, on that basis, "pool" securities. Intermediaries maximize profits and act on the basis of competitive conjectures. The condition we imposed on the price of "pool" securities together with the specification of the market clearing condition for securities imply then that a zero-profit condition holds, at equilibrium, for all intermediaries.

This equivalence between the specification of the model with an exogenously given set of financial markets and the one with competitive, profit-maximizing intermediaries extends to all the following analysis of asymmetric information economies.

4. ASYMMETRIC INFORMATION ECONOMIES

In this section asymmetric information is introduced: different information structures, leading to different types of economies with asymmetric information are presented. We show that in these economies the existence of competitive equilibria cannot be proved under the same set of assumptions as with symmetric information (i.e. assumptions 1–3 are no longer enough to ensure that competitive equilibria exist). The nature of the existence problems is identified, and is shown to be common to economies with various kinds of informational asymmetries. This will provide the basis for the determination of additional conditions under which general existence results will be proved in Section 6.

4.1. *Adverse Selection Economy*

Consider the case in which:

- The idiosyncratic shocks $(\tilde{s}^{h,n})_{h \in H, n \in N}$ are realized at the beginning of date 0, but the realization of $\tilde{s}^{h,n}$ is privately observed by agent (h, n) and becomes commonly known only at date 1.

Let the structure of markets be the same as in the previous section. At date 0, markets for the L commodities and securities open. For every n there are J securities with payoff contingent on $((\tilde{s}^n), \tilde{\sigma})$; in addition, there are J "pool" securities. At date 1, after the realization of $((\tilde{s}^n)_{n \in N}, \tilde{\sigma})$ becomes known to all agents, securities liquidate their payoff and the commodities are again traded on spot markets. All markets are perfectly competitive, and we examine first the case in which all prices are restricted to be linear.

With the above information structure the economy will be characterized by the presence of adverse selection: at date 0 agents trade contingent securities having different information over their payoff. In particular each agent (h, n), before choosing the level of trade in "individual" securities, knows the realization of $\tilde{s}^{h,n}$, i.e. has some information over the payoff of these securities.

Since the economy is large and all "individual" securities of a given type are traded together in a single market, the private information of an agent over an idiosyncratic source of uncertainty will have a negligible impact on the total level of trades in the market. As a consequence, in the present framework date 0 prices can only reveal the information contained in aggregate trades, and this can at most be the component of the aggregate uncertainty which is correlated with the agents' signals. Thus no idiosyncratic uncertainty can be revealed at equilibrium, i.e. the equilibrium will never be fully revealing (differently from Radner [40]). For the clarity of the exposition, but clearly with no loss of generality, we will assume here that the component of the agregate uncertainty which is revealed by aggregate trades is null, i.e.

- the idiosyncratic shocks (and signals) $(\tilde{s}^{h,n})_{h \in H, n \in N}$ are independent of $\tilde{\sigma}$: $\pi(s \mid \sigma) = \pi(s)\ \forall s, \sigma$.

Thus no information is revealed at a competitive equilibrium.

A formal description of the agents' problem and a definition of competitive equilibrium for the adverse selection economy is now presented.

Let q_j be the price of securities of type j (again, by the assumption of perfect competition, the same for all n), and $-q_j$ be the price of the associated "pool" security; $q \equiv ((q_j)_{j \in J})$; p_0 and $p_1(\sigma)$ are commodity spot prices. Moreover, we will still consider the case in which agent (h, n) is restricted to trade only the J securities contingent on his own idiosyncratic shock $\tilde{s}^n$ as well as the J "pool" securities.[15]

Given the assumed information structure the agent will choose the level of trades at date 0, in securities and consumption goods, after learning the realization s^h of $\tilde{s}^{h,n}$. His portfolio and consumption plans are then contingent on s^h. At the same time his date 1 consumption plan will specify now the level of consumption for every possible realization of the remaining uncertainty, i.e. for every possible value $s^{-h} \equiv ((s^{h'})_{h' \neq h})$ of the shocks affecting the other agents, and for every σ. See Figure 1.

We will see below that all agents of the same type face the same optimization problem, that the feasible set is convex, and their objective function is strictly concave; their optimal choice therefore will be, as in the case of symmetric information, identical for all n (and the index n can then be omitted here).

[15] We should note however that in the presence af asymmetric information this restriction does not bind only if it is assumed that agents are unable to "control" for the identity (in particular the "village") of the partner of each of their transactions, i.e. of whom they are buying or selling the contract from. In that case "pool" securities summarize again all that agents could do if they were able to trade all "individual" securities, including the ones of the other "villages".

Adverse Selection

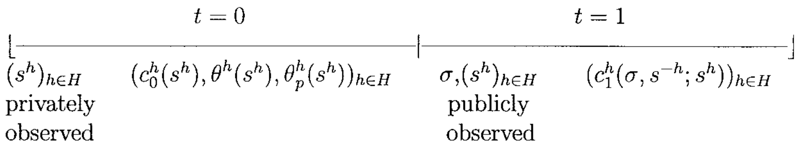

FIG. 1. Timing.

Let $S^{-h} \equiv \prod_{h' \neq h} S^h$ and $\pi(s^{-h} \mid s^h) \equiv \pi((\tilde{s}^{h', n} = s^{h'})_{h' \neq h} \mid s^h)$. The consumption and portfolio plans of agents of type h are then described by the vectors $(\theta^h(s^h); \theta_p^h(s^h)) = (..., \theta_j^h(s^h), ...; ..., \theta_{p,j}^h(s^h), ...) \in R^J \times R^J$, and $c^h(s^h) = (c_0^h(s^h); c_1^h(s^h) = c_1^h(s^{-h}, \sigma; s^h), s^{-h} \in S^{-h}, \sigma \in \Sigma)) \in \Re_+^{L(1 + S^{-h}\Sigma)}$, $s^h \in S^h$, and are obtained as solutions of the following program:

$$(c^h(s^h), \theta^h(s^h), \theta_p^h(s^h)) \in \arg\max \sum_{s^{-h}, \sigma} \pi(\sigma)\, \pi(s^{-h} \mid s^h)\, u^h(c^h(.; s^h)) \qquad (P_{AS}^h)$$

s.t.

$$p_0 \cdot (c_0^h(s^h) - w_0^h) + q \cdot (\theta^h(s^h) - \theta_p^h(s^h)) \leqslant 0$$

$$p_1(\sigma) \cdot (c_1^h(s^{-h}, \sigma; s^h) - w_1^h(s, \sigma)) \leqslant \sum_j \theta_j^h(s^h)\, r_j(s, \sigma) + \sum_j \theta_{p,j}^h(s^h)\, r_j^p(\sigma), \qquad \forall (s, \sigma) \in S \times \Sigma$$

$$(c^h(s^h)) \in \Re_+^{L(1 + \Sigma S^{-h})}; \quad (\theta^h(s^h), \theta_p^h(s^h)) \in \Re^{2J}$$

The unit payoff of "pool" security $j \in J$ is again defined by the opposite of the average total net amount (of the numeraire commodity) due to—or owed by—all agents who traded securities of type j, for all n; this is when the average is well defined, and it takes an arbitrary value otherwise:

$$r_j^p(\sigma) = \begin{cases} -\dfrac{\sum_h \lambda^h \sum_s \pi(s)\, r_j(s, \sigma)\, \theta_j^h(s^h)}{\sum_h \lambda^h \sum_{s^h} \pi(s^h)\, \theta_j^h(s^h)} & \text{if } \sum_h \lambda^h \sum_{s^h} \pi(s^h)\, \theta_j^h(s^h) \neq 0 \\ \text{arbitrary,} & \text{if } \sum_h \lambda^h \sum_{s^h} \pi(s^h)\, \theta_j^h(s^h) = 0 \end{cases}$$

for all $\sigma \in \Sigma$. (4.1)

Let R^p be the $\Sigma \times J$ matrix with generic element $r_j^p(\sigma)$.

A *competitive equilibrium with adverse selection* is defined by a specification of the "pool" securities' payoff R^p, a collection of prices $(p_0, (p_1(\sigma))_{\sigma \in \Sigma}, q)$, and of contingent consumption and portfolio plans for every agents' type $(c^h(s^h), \theta^h(s^h), \theta^h_p(s^h); s^h \in S^h)_{h \in H}$ such that:

- for all h, the plan $(c^h(s^h), \theta^h(s^h), \theta^h_p(s^h); s^h \in S^h)$ solves (P^h_{AS}) at the prices $(p_0, (p_1(\sigma))_{\sigma \in \Sigma}, q)$ and "pool" securities' payoff R^p;
- commodity markets clear:

$$\sum_h \lambda^h \sum_{s^h} \pi(s^h)(c^h_0(s^h) - w^h_0) \leqslant 0 \tag{4.2}$$

$$\sum_h \lambda^h \sum_s \pi(s)((c^h_1(s^{-h}, \sigma; s^h) - w^h_1(s, \sigma))) \leqslant 0, \quad \forall \sigma \in \Sigma \tag{4.3}$$

- security markets clear, for all $j \in J$:

$$\sum_h \lambda^h \sum_{s^h} \pi(s^h)(\theta^h_j(s^h) - \theta^h_{p,\,j}(s^h)) = 0 \tag{4.4}$$

- the payoff $r^p_j(\sigma)$ of each "pool" security j satisfies (4.3), for all σ.

4.1.1. *Why existence is a problem with adverse selection.* The presence of adverse selection, specifically the fact that each agent (h, n) trades securities (of index n) by having some private information over its payoff, poses two main problems for the analysis of this economy with respect to the case of symmetric information considered in Section 3.

1. *Feasibility*. Market clearing for the aggregate positions on "individual" and the associated "pool" securities (as in condition (4.4)) is no longer enough to ensure feasibility of trades in securities.

The problem is that now security holdings, unlike in the case of symmetric information, are not the same for all agents of the same type h as the portfolio choice of each agent (h, n) depends on the observed realization s^h of the signal $\tilde{s}^{h,n}$. Since the payoff of the securities purchased also depends on s^h, the individual portfolio choice is then correlated with the return on the portfolio. As a consequence, the aggregate return on the positions held by agents in a given contract is no longer a linear function of the total level of trades in that contract. In particular, condition (4.4), which is the direct analogue of the market clearing condition (3.3) considered for the symmetric information case, does not ensure that the aggregate payoff on securities is 0 (and this is obviously required for agents' trades in securities to be feasible):

$$\sum_h \lambda^h \sum_s \pi(s)(r_j(s, \sigma)\, \theta^h_j(s^h) + r^p_j(\sigma)\, \theta^h_{p,\,j}(s^h)) = 0 \tag{4.5}$$

To see this, suppose (4.6) holds and, furthermore, we have $\sum_h \lambda^h \sum_{s^h} \pi(s^h)\, \theta_j^h(s^h) = \sum_h \lambda^h \sum_{s^h} \pi(s^h)\, \theta_{p,\,j}^h(s^h) = 0$ (i.e. transactions in "individual" and "pool" securities clear separately).

Then, while $\sum_h \lambda^h \sum_{s^h} \pi(s^h)\, \theta_{p,\,j}^h(s^h) = 0$ implies $\sum_h \lambda^h \sum_s \pi(s)\, r_j^p(\sigma)\, \theta_{p,\,j}^h(s^h) = 0$, the equality $\sum_h \lambda^h \sum_{s^h} \pi(s^h)\, \theta_j^h(s^h) = 0$ does not imply $\sum_h \lambda^h \sum_s \pi(s)\, r_j(s, \sigma)\, \theta_j^h(s^h) = 0$, since the term $\sum_h \lambda^h \sum_{s^h} \pi(s^h)\, \theta_j^h(s^h)$ cannot be factored out of this sum when $\theta_j^h(s^h)$ depends non trivially on s^h, i.e. when adverse selection matters.[16]

The nature of the problem can be clearly seen by considering the following extreme case. Suppose signal s implies that the return on buying a certain "individual" contract will be high, while s' on the contrary implies that the return will be low. Then it may happen that agents who received signal s will buy this contract, while agents who received s' will sell. In this case, even if the aggregate position on this type of contract is 0, still in period 1 the agents who bought the contract cannot be paid out of the proceeds from agents who sold it, so that feasibility is not satisfied.

At a more general level we can view the feasibility problem as arising from the fact that each of the various contracts of the same type is now a different object not only ex-post (as the realization of the payoff depends on the village) but also ex-ante, as the level of trades by an agent depends on the specific realization of signal received over its payoff. On the other hand, with linear prices, only one price exists to clear the market for all these contracts.

2. *Arbitrage.* Agents have additional arbitrage opportunities.

With symmetric information the set of securities' prices precluding arbitrage is always non-empty and open. On the other hand, when agents have private information over the support of the payoff of securities (as in the situation we are considering) this set may well be empty.

More precisely, the set:

$$K(s^h) \equiv \left\{ q \in R^J : \exists \rho \in \Re_{++}^{S^{-h}\Sigma},\ \text{s.t. } q = \sum_{s^{-h},\,\sigma} \rho_{s^{-h},\,\sigma}\, r(s^h, s^{-h}, \sigma) \right\}$$

denotes the set of prices of the J individual securities precluding arbitrage opportunities to agents of type h when they observed state s^h. Therefore, for no agent to have any arbitrage opportunity we need:

$$\bigcap_{h \in H,\, s^h \in S^h} K(S^h) \neq \varnothing \qquad (NA)$$

[16] Even though the expression defining the payoff of the "pool" security is not defined when $\sum_h \lambda^h \sum_{s^h} \pi(s^h)\, \theta_j^h(s^h) = 0$, and hence the payoff of the "pool" can be set at an arbitrary value in this case, the statement in the text is true no matter what is the payoff of the "pool" in this case.

The greater the set of securities with payoff contingent on $(\tilde{s}^{h,n})_{h \in H, n \in N}$, i.e., the larger the insurance offered against the states over which some agents have private information, the less likely it is that condition (NA) will be satisfied. In particular it will always be empty if R has full rank, so that un-restricted trade in a complete set of markets is not feasible in the present situation.

Again the nature of the problem can be clearly seen by considering an extreme case. Agents receive different signals over the future realization of the idiosyncratic uncertainty, so it may happen that agent (h, n) knows that some shock realization s is not possible, while some other agent (h', n) gives it positive probability. Suppose there is a security paying one unit in state s and 0 in all other states. No-arbitrage for agent (h', n) requires that this security sells at a positive price, while no-arbitrage for agent (h, n) requires that the security's price is 0. Hence the no-arbitrage set is empty in this case.

4.2. *Hidden Information Economy*

Consider next the case in which:

- the idiosyncratic shocks $(\tilde{s}^{h,n})_{h \in H, n \in N}$ are realized at date 1 and may be correlated with $\tilde{\sigma}$ (as in the symmetric information case);
- the realization of $\tilde{s}^{h,n}$ is privately observed by agent (h, n)—before the realization of $\tilde{\sigma}$ is commonly observed -and never becomes known to the other agents, for all h, n.

Under these conditions contracts with payoff directly contingent on $\tilde{s}^{h,n}$ can no longer be written, as $\tilde{s}^{h,n}$ is private information and never publicly observable. Thus agents will only be able to get some insurance against their idiosyncratic shocks as long as this is compatible with their incentives. We will model this by considering securities whose payoff is contingent on what the agents say about the state, on the messages they send after learning the realization of their idiosyncratic shock.

More precisely, let M^h be the space of messages which an agent of type h can send. We will assume that M^h is finite; let M^h also be its cardinality and denote by m^h its generic element (we can have, for instance, $M^h = S^h$, i.e. each agent simply announces one of the possible states he has privately observed). For every n there are so J securities whose payoff depends on the realization of $\tilde{\sigma}$, commonly observed, and on the messages sent by agents of index n over the realization of $(\tilde{s}^{h,n})_{h \in H}$. One unit of security j, $j = 1, \ldots, J$ pays $r_j(m, \sigma)$ units of the numeraire commodity at date 1 when state σ is realized, and $m \equiv (m^1, \ldots, m^H) \in M = \prod_{h \in H} M^h$ is the collection of messages sent by the H agents of index n. In addition, there are again J "pool" securities, associated with each type of "individual" security, and the $\Sigma \times J$ matrix R^p, with generic element $r_j^p(\sigma)$, describes their payoffs.

Except for this difference in the specification of contracts, the structure of markets is unchanged. Spot markets for the commodities and securities' markets open at date 0, before any realization of the uncertainty. At the beginning of date 1 agents send their message; the realization of $\tilde{\sigma}$, as well as the messages are then commonly observed, the payoff of contracts is determined and liquidated. Spot markets subsequently open where the commodities are traded. Markets are competitive. As in the case of adverse selection we consider first the case in which prices are restricted to be linear; in particular, q_j is again the unit price of securities of type j (and $-q_j$ the price of the associated "pool" security).

To ensure that agents are able to observe their own endowment we will also assume here that:

- the endowment of agent (h, n) depends upon the realization of $\tilde{\sigma}$ and $\tilde{s}^{h,n}$ only.

Under the present characterization of the individuals' information structure and of the nature of the existing securities, agents can exploit their private information to affect the payoff of "individual" securities. Though agents have no private source of information when they trade securities at date 0, they do take into account their future ability to "choose", to a certain extent, the level of the return on these securities. The markets for such claims are then characterized by the presence of hidden information[17].

Remark 3. We can now see more precisely how the non-triviality of the choice of the message sent by agents (and hence the fact that indeed some amount of insurance against the agents' privately observed states can be achieved) is ensured by the following features of the information structure:

(a) the correlation of $(\tilde{s}^{h,n})_{h \in H}$ across h;

(b) the correlation of $\tilde{s}^{h,n}$ with $\tilde{\sigma}$.

[17] The very close similarity with the classical case of moral hazard, where agents can affect the distribution of securities' payoffs via some unobservable action, should be now more evident. The crucial distinction between adverse selection economies on one side, and hidden information as well as moral hazard economies on the other, lies in the fact that in the first case the informational asymmetry arises before the contract is signed, while in the latter agents have no private information when their trades in securities are decided, asymmetric information only arises at a later date (see also Hart and Holmstrom [22]).

In the presence of (a) the message agent (h, n) will choose to send will not typically be a constant message (the same for all s^h) if the securities' payoff depends jointly on the messages sent by all agents with the same index n (i.e. by agents whose private information is correlated). The same is true under (b) if securities' payoffs depend on the commonly observed state σ as well as on the agents' messages and if these are sent by agents before learning the realization of $\tilde{\sigma}$. Both the joint dependence of security payoffs on σ and the message m sent by all agents with the same index n as well as the fact that agents' messages have to be sent before the realization of $\tilde{\sigma}$ is commonly observed are important then to ensure that we have non trivial message choices.

The fact that correlation may induce some discipline on the agents' opportunistic behavior and so enhance incentives is well-known in the moral hazard literature: (a) for instance can be viewed as an abstract representation of a situation of relative performance evaluation (see e.g. Lazear and Rosen [29]), while an application of the idea behind (b) can be found in Townsend [44]. As it will appear more clear later, in the present framework, where the contracts traded are standardized contracts and strong exclusivity conditions may not be (are not) enforceable, the only incentive compatible choice is a trivial one if neither (a) nor (b) hold (equivalently, the agents' optimal message will be constant if securities' payoff only depends on $\tilde{s}^{h,n}$).

Let us describe now the agents' choice problem and define competitive equilibria for economies with hidden information.

Each agent (h, n) faces here the following optimization problem. He has to choose (i) his date 0 consumption level $c_0^{h,n} \in \Re_+^L$ and portfolio holdings $(\theta^{h,n}, \theta_p^{h,n}) \in \Re^{2J}$; (ii) the message plan $m^{h,n} \equiv (m^{h,n}(s^h)_{s^h \in S^S}) \in (M^h)^{S^h}$, specifying the message to send at the beginning of date 1, for every possible realization of $\tilde{s}^{h,n}$; (iii) his date 1 consumption plan $c_1^{h,n} = (c_1^{h,n}(\sigma, m^{-h}, s^h)$, $\sigma \in \Sigma$, $m^{-h} \in M^{-h}$, $s^h \in S^h) \in \Re_+^{L\Sigma M^{-h} S^h}$, specifying the level of consumption for every possible realization s^h of his idiosyncratic uncertainty, σ of the aggregate shock, and every possible collection of messages $m^{-h} \equiv ((m^{h'})_{h' \neq h})$ sent by agents of other types.

The timing of an agent's choices is illustrated in Fig. 2.

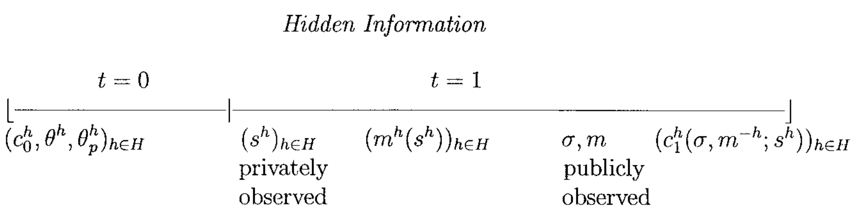

FIG. 2. Timing.

Formally agent (h, n) has then to solve the following program:[18]

$$(c^{h,n}, \theta^{h,n}, \theta_p^{h,n}, m^{h,n}) \in \arg\max \sum_{\sigma, m, s} \pi(\sigma, m^{-h}, s^h)\, u^h(c_0^{h,n}, c_1^{h,n}(\sigma, m^{-h}, s^h)) \qquad (P_{HI}^h)$$

subject to:

$$p_0 \cdot (c_0^{h,n} - w_0^h) + q \cdot (\theta^{h,n} - \theta_p^{h,n}) \leqslant 0$$

$$p_1(\sigma) \cdot (c_1^{h,n}(\sigma, m^{-h}, s^h) - w_1^h(\sigma, s^h))$$

$$\leqslant \sum_j \theta_j^{h,n} r_j(m^{h,n}(s^h), m^{-h}, \sigma) + \sum_j \theta_{p,j}^{h,n} r_j^p(\sigma); \quad \forall \sigma \in \Sigma, \quad m^{-h} \in M^{-h}, \quad s^h \in S^h$$

$$c^{h,n} \in \Re_+^{L(1+\Sigma M^{-h} S^h)}, \quad (\theta^{h,n}, \theta_p^{h,n}) \in \Re^{2J}, \quad m^{h,n} \in (M^h)^{S^h}$$

where $\pi(\sigma, m^{-h}, s^h) \equiv \sum_{s^{-h}} \pi(s, \sigma)\, \pi(m^{-h} \mid s^{-h})$ and $\pi(m^{-h} \mid s^{-h}) \equiv \prod_{h' \neq h} \pi(m^{h'} \mid s^{h'})$ describes the probability distribution over the messages sent by agents of types $h' \neq h$ for every possible realization of their idiosyncratic uncertainty.

The agent's optimization problem P_{HI}^h is now a non-convex problem: the agent's choice set is clearly not convex since M^h is a discrete set. But even if the agent were allowed to randomize in his choice of which message to send for every realization s^h, his problem would still be not convex (as in that case the objective function is not concave). The fact that agents can choose both the unit payoff (via their message) and the quantity traded of each "individual" security generates in fact an inherent non-convexity in their choice problem. We will show that the economy can be "convexified" by exploiting the large number of agents.[19] This requires that ex-ante iden-

[18] By requiring $c^{h,n} \in \Re_+^{L(1+\Sigma M^{-h} S^h)}$ we are implicitly imposing the condition that the level of consumption of agent (h, n) has to lie in the consumption set for every possible message of agents of type $h' \neq h$, even though some of these messages may be given zero probability by $\pi(m^{-h}/s^{-h})$. This may appear unduly restrictive, in particular when markets are incomplete, and is mainly motivated by reasons of technical convenience.

[19] The presence of non-convexities often characterizes agents' problems in the presence of moral hazard. Another route to overcome such difficulties is followed by Prescott and Townsend [38]; in their set-up the space of admissible individual choices is enlarged to allow for lotteries; the convexification is so introduced at the level of the individuals' demand. Kehoe, Levine and Prescott [26] have recently shown, for the economy considered by Prescott and Townsend, that essentially the same equilibria obtain if sunspot uncertainty is introduced instead; as shown by Shell and Wright [41] sunspots provide generally another way to "convexify" an economy, again at the level of the economy. See Garratt, Keister, Qin and Shell [16] for a more complete discussion of the relations among these various routes to deal with non-convexities.

tical agents behave differently at equilibrium, i.e. may end up choosing, at equilibrium, different levels of trades and different messages (this explains why we could no longer omit here the index n in the specification of the agents' problem P^h_{HI}). As a consequence, even though each agent chooses to send a single message in each state, the distribution over the possible messages sent by agents of each given type h as a function of their state, $\pi(m^h \mid s^h)$, will be non-degenerate.

In particular, we will show that it is enough to consider the case in which agents of the same type will make at most an arbitrarily large but finite[20] number V of different choices at equilibrium, denoting by $c^{h,v}$, $\theta^{h,v}$, $\theta^{h,v}_p$, $m^{h,v}$ the v-th choice of agents of type h and by $\gamma^{h,v}$ the fraction of agents of this type making such choice, $v = 1, ..., V$.

A *competitive equilibrium with hidden information* is defined by a specification of the payoff of "pool" securities R^p, an array of prices $(p_0, (p_1(\sigma))_{\sigma \in \Sigma}, q)$, a collection of consumption, portfolio, and message plans for agents of type h together with their relative frequency in the population of agents of that type $(c^{h,v}, \theta^{h,v}, \theta^{h,v}_p, m^{h,v}; \gamma^{h,v})_{v \in V}$, and probability distributions over other types' messages $(\pi(m^{-h} \mid s^{-h})_{s^{-h} \in S^{-h}})_{h \in H}$, for all h, such that:

- for every h all plans $(c^{h,v}, \theta^{h,v}, \theta^{h,v}_p, m^{h,v})_{v \in V}$ are solutions of (P^h_{HI}) at the prices $(p_0, (p_1(\sigma))_{\sigma \in \Sigma}, q)$, "pool" securities' payoff R^p, and distribution over other agents' message strategies $(\pi(m^{-h} \mid s^{-h}))_{s^{-h} \in S^{-h}}$;
- for all h $(\pi(m^{-h} \mid s^{-h}))_{s^{-h} \in S^{-h}}$ is consistent with the frequency of the strategies chosen by the population of individual agents of each type $h' \neq h$:

$$\pi(m^{-h} \mid s^{-h}) = \prod_{h' \neq h} \left(\sum_{v:\, m^{h',v}(s^{h'}) = m^{h'}} \gamma^{h',v} \right) \tag{4.6}$$

- commodity markets clear:

$$\sum_h \lambda^h \left(\sum_v \gamma^{h,v} c_0^{h,v} - w_0^h \right) \leqslant 0 \tag{4.7}$$

$$\sum_h \lambda^h \sum_s \pi(s \mid \sigma) \left(\sum_{v,\, m^{-h}} \gamma^{h,v} \pi(m^{-h} \mid s^{-h})\, c_1^{h,v}(\sigma, m^{-h}, s^h) - w_1^h(s^h, \sigma) \right) \leqslant 0, \quad \forall \sigma \tag{4.8}$$

- security markets clear: for all $j \in J$,

$$\sum_h \lambda^h \sum_v \gamma^{h,v} (\theta_j^{h,v} - \theta_{p,j}^{h,v}) = 0 \tag{4.9}$$

[20] In this case the Law of Large Numbers can still be exploited in the feasibility conditions. We will show that equilibria satisfying such condition always exist. However, there may also be other equilibria which violate it.

- the payoff of each "pool" security $j \in J$ is given by:

$$r_j^p(\sigma) = \left\{ \begin{array}{ll} -\dfrac{\sum_h \lambda^h \sum_s \pi(s \mid \sigma)(\sum_{v, m^{-h}} \pi(m^{-h} \mid s^{-h})\, \gamma^{h,v} r_j(m^{h,v}(s^h), m^{-h}, \sigma)\, \theta_j^{h,v})}{\sum_h \lambda^h \sum_v \gamma^{h,v} \theta_j^{h,v}}, & \\ \qquad \text{if } \sum_h \lambda^h \sum_v \gamma^{h,v} \theta_j^{h,v} \neq 0 & \\ \text{arbitrary}, \qquad \text{if } \sum_h \lambda^h \sum_v \gamma^{h,v} \theta_j^{h,v} = 0 & \end{array} \right\},$$

for all $\sigma \in \Sigma$ (4.10)

- $\gamma^{h,v} \geqslant 0$, $\sum_v \gamma^{h,v} = 1$[21].

Since the payoff of the "individual" securities traded by agent (h, n) may depend, as well as on his message, on the message sent by agents of different type, but with the same index n, a strategic element is introduced in the agent's choice problem. Condition (4.6) requires the consistency of what agent (h, n) considers to be the probability distribution over messages of agents of other types with the actual frequencies of these messages in the population. It ensures that, as a component of the above equilibrium notion, we have a Nash equilibrium in the agents' message game. The fact that it is the distribution over message choices in the whole population to be considered, follows from the anonymity property of this game (agents do not know the precise identity, and hence the message strategy, of the agents in the population characterized by their same index n, and can only base their behavior on the strategy of the population average).

The payoff of "pool" securities is also endogenously determined, as in the case of symmetric information and adverse selection economies, by the portfolio choices of all agents in the population. However, since the direct effect of each individual agent on the "pool's" payoff is negligible, this is taken as given by each agent.

4.2.1. *Why existence is a problem with hidden information.* The main problems posed by the presence of hidden information for the viability of markets for contingent contracts are the same as the ones we found under adverse selection:

[21] We allow $\gamma^h = (\gamma^{h,v})_{v \in V}$ to be any real vector in the simplex Δ^{V-1} even though, with countably many agents we should limit our attention to rational numbers. Since rational numbers are dense in the reals, the equilibrium we obtain is, strictly speaking, an approximate equilibrium. To overcome this fact we could have considered, without any change in the nature of the results, the case of a continuum of agents, as in Aumann [4], and made appeal to the results by Al-Najjar [2] and Sun [43] on the Law of Large Numbers in such framework.

1. *Feasibility*. The non-convexity in the agents' choice problem implies, as we noticed, that ex-ante identical agents may choose, at equilibrium, different portfolios and different messages. As a consequence, the payoff of a security will depend (non-linearly) on what is the portfolio choice of the agent. We face so again the problem that the fact that the market clearing condition for securities is satisfied does not imply that the aggregate payoff will also be 0.

Formally, if Eq. (4.9) holds and, in addition

$$\sum_h \lambda^h \sum_v \gamma^{h,v} \theta_j^{h,v} = 0,$$

it does not necessarily follow that, for all σ,

$$\sum_h \lambda^h \sum_s \pi(s \mid \sigma) \left(\sum_{v, m^{-h}} \pi(m^{-h} \mid s^{-h})\, \gamma^{h,v} r_j(m^{h,v}(s^h), m^{-h}, \sigma)\, \theta_j^{h,v} \right) = 0; \tag{4.11}$$

hence the total payoff may not be 0. We see in fact, from expression (4.11), that the message sent by an arbitrary agent of type h, $m^{h,v}$, depends on the specific level of trades of this agent, $(\theta^{h,v}, c^{h,v})$, and that the aggregate return is a non-linear function of the total level of trades, so that again aggregate portfolios and payoffs cannot be separated.[22]

The argument parallels exactly the one of the previous section for adverse selection economies. The intuition is also essentially the same: we can have agents who, having bought a security, send a message implying a high payoff, while agents who sold the same security send a message inducing a low payoff, so that on the whole total payoff is not 0, and feasibility is not satisfied.

2. *Arbitrage*. The fact that the agents can affect the payoff of the securities via the choice of their message (have the possibility to determine, to some extent, the support of securities' payoffs) gives them additional arbitrage opportunities.

More precisely, the set of prices of the J "individual" securities precluding arbitrage opportunities to agents of type h is given by:

$$K^h \equiv \left\{ q \in R^J : \exists \rho \in \Re_{++}^{M^{-h}\Sigma}, \text{ s.t. } q = \sum_{m^{-h}, \sigma} \rho_{m^{-h},\sigma} r(m^h, m^{-h}, \sigma)\ \forall m^h \right\}$$

[22] On the other hand, if all agents of the same type h make the same choice of portfolios and messages, i.e. if $\theta^{h,v}$, $m^{h,v}$ do not depend on v, it is immediate to see from the expression in (4.11) that total returns will be a linear function of trades.

Therefore, for no agent to have any arbitrage opportunity we need:

$$\bigcap_{h \in H} K^h \neq \varnothing \qquad (NA')$$

It is easy to see that, as for adverse selection economies, there is a trade-off between the aim of ensuring larger insurance opportunities, thus requiring that securities' payoff are non-trivially affected by the agents' message choices, and the need of preventing arbitrage opportunities.

On the other hand, no specific, additional problems are caused by the non-convexity of the agents' choice problem. In economies with hidden information, non-convexities are then a source of difficulties for existence only in the sense that they induce the same correlation of portfolios and returns which was at the root of the problems we have identified for adverse selection economies. We will show in the next section that existence for economies with hidden information can be established under essentially the same conditions as for adverse selection economies.

Remark 4. At a more abstract level we can view the main consequence of the presence of asymmetric information in markets for contingent contracts as the induced correlation of portfolios and returns, i.e. the fact that the effective return on a contract of a given type is not constant throughout the economy, and the quantity traded will typically be different for different levels of the return. This feature is indeed common both to adverse selection and hidden information economies (as well as moral hazard economies) and is the source of the existence problems we discussed. In particular, while with adverse selection these differences in the returns to agents trading a contract derive from the exogenously given dependence on s of the payoff (and the way agents' portfolios vary as a function of s is endogenously determined), with hidden information both the probability structure of portfolios and of returns (via the message choice), i.e. their dependence on v, are endogenously chosen. This explains the sense in which adverse selection economies can be viewed in our framework as a reduced form of hidden information economies.

5. A NON-EXISTENCE EXAMPLE

In the previous section we identified two classes of problems for the existence of competitive equilibria, concerning the aggregate feasibility of trades in "individual" contracts and the presence of arbitrage opportunities. We present here a robust example of an adverse selection economy for which no competitive equilibrium exists, and we argue that the reason for the non-existence is indeed the feasibility problem described above.[23]

Consider an economy with one commodity ($L=1$) and countably many agents all of the same type ($H=1$). Consumption only takes place at date 1. There is no aggregate uncertainty ($\Sigma=1$). The idiosyncratic shocks have two possible realizations, 1, 2, and each agent receives one out of two equiprobable signals at date 0: g or b. Let $\pi_g \equiv \pi(1 \mid g)$ and $\pi_b \equiv \pi(1 \mid b)$ be the probability of (idiosyncratic) state 1 conditional respectively on signal g and signal b.[24] We assume that $w(1) > w(2)$ and $\pi_g > \pi_b$; hence agents who receive signal g qualify as the "good risks" (i.e. have a higher probability of the good realization of their future income) and agents with signal b as the "bad risks". Agents have Von Neumann-Morgernstern preferences over consumption with utility function of the following form: $\ln c$.

After learning his signal but before knowing the realization of his idiosyncratic uncertainty, each agent can trade two securities, 1, 2. Security 1 pays one unit of the commodity when the agent's idiosyncratic state is 1. Similarly security 2 pays one unit of the commodity in idiosyncratic state 2. Let q and $1-q$ denote the (normalized) prices of, respectively, security 1 and 2.

The budget constraint of an agent who received signal g is then:[25]

$$\theta_1(g)\, q + \theta_2(g)(1-q) = 0.$$

Similarly for agents who received signal b.

The agents' utility maximization problem subject to the above constraint can be easily solved in this case and yields an explicit expression of the demand for consumption in the two idiosyncratic states (respectively for agents receiving signals g and b):

[23] It is easy to construct examples where equilibria do not exist because agents, as a consequence of their private information, have unbounded arbitrage opportunities.

[24] Though the structure here of the idiosyncratic uncertainty and of the agents' signals may appear slightly different from the one described in the previous section, the present economy could have also been written, at the cost of some extra notational burden, precisely in terms of that same structure.

[25] In the present set-up, since there is no aggregate uncertainty, the total return on the agents' positions in an "individual" security, and hence the payoff of the associated "pool" security, will always be deterministic. Agents will then always be able to replicate the "pool"'s payoff (or perfectly hedge it) by trading the two "individual" securities. "Pool" securities are then redundant here and need not be explicitly modelled, as long as we do not impose a separate market clearing condition for each individual security (see however the final section).

$$
\begin{aligned}
c(1; g) &= \pi_g \left(\frac{qw(1) + (1-q)\, w(2)}{q} \right) \\
c(2; g) &= (1-\pi_g) \left(\frac{qw(1) + (1-q)\, w(2)}{1-q} \right) \\
c(1; b) &= \pi_b \left(\frac{qw(1) + (1-q)\, w(2)}{q} \right) \\
c(2; b) &= (1-\pi_b) \left(\frac{qw(1) + (1-q)\, w(2)}{1-q} \right)
\end{aligned} \tag{5.1}
$$

The market-clearing condition for the commodity is[26]:

$$
\begin{aligned}
&c(1; g)\, \pi_g + c(1; b)\, \pi_b + (1-\pi_g)\, c(2; g) + (1-\pi_b)\, c(2; b) \\
&\quad = w(1)\, \pi_g + w(1)\, \pi_b + (1-\pi_g)\, w(2) + (1-\pi_b)\, w(2)
\end{aligned} \tag{5.2}
$$

A competitive equilibrium is then given by a price q and a consumption vector c such that (5.1), (5.2) hold.

For this economy the set of no-arbitrage prices is non-empty, and is given by all prices $q \in (0, 1)$. We will show that, nonetheless, for an open set of parameters, a competitive equilibrium does not exist.

The excess demand function (equivalently the overall profit function) we obtain from (5.1) is continuous, for all $q \in (0, 1)$. However, when $w(2)\, \pi_g / w(1)(1-\pi_g) > \pi_b/(1-\pi_b)$ this function has a negative value both when $q/(1-q) > \pi_g/(1-\pi_g)$ and when $q/(1-q) < w(2)\, \pi_b / w(1)(1-\pi_b)$. It is easy to see in fact, from the expressions of the agents' demand, that in the first case agents will be buying insurance, no matter what is the signal received, and will do this at more than fair terms, while the opposite happens in the second case, so that profits will be negative in both situations. For intermediate values of the relative price $((w(2)\, \pi_b / w(1)(1-\pi_b) < q/(1-q) < \pi_g/(1-\pi_g))$ the sign of aggregate demand cannot be unambiguously determined without further restrictions on the parameter values of the economy. In the next paragraph we present an open set of parameter values for which aggregate excess demand is negative also at all intermediate prices, so that no competitive equilibrium exists. Notice that this fact is a clear consequence of the feasibility problem we discussed in the previous section.

Consider then the following specification of the parameters of the economy: $w(1) = 0.8$, $w(2) = 0.2$, $\pi_b = 0.2$, and $\pi_g = 0.2 + \varepsilon$, $\varepsilon > 0$. For these values the condition $w(2)\, \pi_g / w(1)(1-\pi_g) > \pi_b/(1-\pi_b)$ reduces to $\varepsilon > 0.3$.

[26] This is also equivalent to the requirement that the sum of the total payoffs on the two existing securities equal zero, i.e. can be viewed as an overall zero-profit condition.

Solving equations (5.1) and (5.2) for equilibrium prices and allocations we find:

$$c(1;\,g) = -0.04\,\frac{\left(\begin{array}{c}-550\varepsilon^2 - 48 - 355\varepsilon + 125\varepsilon^3 + 480\varepsilon\rho + 128\rho\\ +500\rho\varepsilon^3 - 700\rho\varepsilon^2\end{array}\right)}{2 + 10\varepsilon + 25\varepsilon^2}$$

$$c(2;\,g) = \rho + 0.16 - 0.8\varepsilon\rho - 0.2\varepsilon$$

$$c(1;\,b) = -0.04\,\frac{-115\varepsilon + 25\varepsilon^2 - 48 + 128\rho - 160\varepsilon\rho + 100\rho\varepsilon^2}{2 + 10\varepsilon + 25\varepsilon^2}$$

$$c(2;\,b) = 0.64\rho + 0.16$$

$$\frac{q}{1-q} = \rho$$

where ρ takes one of the two following values:

$$\rho = \frac{(40 + 75\varepsilon - 125\varepsilon^2 \pm \sqrt{(576 + 2160\varepsilon - 11575\varepsilon^2 - 6750\varepsilon^3 + 5625\varepsilon^4)})}{2(128 - 160\varepsilon + 100\varepsilon^2)}$$

Straightforward computations reveal that, whenever $\varepsilon > 0.3$, no real-valued solution exists[27] for equilibrium prices and consumption, i.e. an equilibrium never exists in this region. It is then immediate to see that perturbing the values of the parameters does not restore existence, so equilibria fail to exist for an open set of parameter values.

On the other hand, when $w(2)\,\pi_g/w(1)(1-\pi_g) < \pi_b/(1-\pi_b)$, aggregate demand is always positive at the price $q/(1-q) = \pi_b/(1-\pi_b)$: agents receiving signal b buy insurance at fair terms, while agents with signal g also buy insurance but at less than fair terms, so that total profits (and hence excess demand) will be positive. By the continuity of the excess demand function therefore it follows that an equilibrium always exists in this region. In particular, for the same specification of the parameter values as above ($w(1) = 0.8$, $w(2) = 0.2$, $\pi_b = 0.2$, and $\pi_g = 0.2 + \varepsilon$) two admissible equilibrium solutions exist, as we already saw, when $\varepsilon < 0.3$. Moreover, it can be easily seen that these two competitive equilibria are always Pareto ranked.

To better understand the properties of the set of competitive equilibria we obtain in this economy, consider the equilibrium solutions when $\varepsilon = 0$ (in this case the signal received by the agents is totally uninformative, information is then symmetric):

(i) $c(1;\,g) = c(2;\,g) = c(1;\,b) = c(2;\,b) = 0.32;\ q/(1-q) = 0.25$

[27] The term $576 + 2160\varepsilon - 11575\varepsilon^2 - 6750\varepsilon^3 + 5625\varepsilon^4$ has in fact a negative value when $\varepsilon > 0.3$.

(ii) $c(1; g)=0.8$; $c(2; g)=0.2$; $c(1; b)=0.8$; $c(2; b)=0.2$; $q/(1-q)=0.0625$

The equilibrium in (i) is characterized by the presence of full insurance at fair prices (and is, evidently, Pareto efficient), while equilibrium (ii) has a zero level of trades for all agents.

Since the equilibrium values we obtained for consumption and prices are continuous functions of ε, the two equilibria we have with adverse selection when an equilibrium exists, i.e. when $0.3>\varepsilon>0$, arise by continuity from these two equilibria, the Pareto efficient and the no trade solution of the economy with symmetric information (for $\varepsilon=0$).

6. EXISTENCE RESULTS

The previous example shows that, with respect to the case in which information is symmetric, additional conditions are needed in economies with asymmetric information to overcome the problems discussed in Section 4 and guarantee the existence of competitive equilibria. In particular, some restrictions have to be imposed on the agents' trades, or on the structure of payoffs, or equivalently some form of non-linearity in prices must be introduced in markets characterized by the presence of hidden information or adverse selection. This, as well as the fact that securities' payoffs are partly determined by the agents' actions (thus reflecting their incentive compatibility constraints), implies that asymmetric information generates an endogenous limit on the set of insurance possibilities which can be attained via competitive markets.

In this section we focus our attention on "minimal" forms of non-linearity of prices of contracts (in the sense that they impose a minimal observability requirement) which are sufficient to guarantee existence of competitive equilibria in the class of economies studied; see Remark 5 below.

To overcome the "feasibility" problem we will impose the condition that agents are constrained to take only long positions in "individual" securities (e.g. that they can only buy, not sell short, insurance contracts):

$$\theta^h \in \Theta^h \subset \Re^J_+, \qquad \forall h \tag{C1}$$

where Θ^h denotes the set of admissible trades in "individual" securities. It is immediate to see that under (C1) the market clearing condition for securities always ensures feasibility of the trades in securities. In the adverse selection economy, if (C1) holds, the equality

$$\sum_h \lambda^h \sum_{s^h} \pi(s^h)\, \theta^h_j(s^h) = 0$$

implies

$$\sum_h \lambda^h \sum_s \pi(s)\, r_j(s, \sigma)\, \theta_j^h(s^h) = 0$$

thus ensuring feasibility also when the total position in "individual" securities is 0 (the same argument obviously holds in the case of hidden information).

To prevent the possibility of unlimited arbitrage opportunities arising from the agents' private information we will consider here the case where trades in "individual" securities cannot be unboundedly large. Alternatively, restrictions on the payoffs of existing securities could have been imposed, ensuring that condition (NA) (condition (NA')) is satisfied, i.e. that the set of no arbitrage prices is non-empty. The validity of all our results extends to such case.[28]

More precisely, the following condition will be imposed:[29]

$$\begin{aligned} &\text{(i)} \quad \theta^h \in \Theta^h, \text{ a compact and convex subset of } \mathfrak{R}^J, \text{ s.t. } 0 \in \Theta^h\ \forall h \\ &\text{(ii)} \quad Sp\left[\left(\sum_s \xi_s r_j(s, \sigma)\right)_{\sigma, j}\right] = \mathfrak{R}^\Sigma, \qquad \forall (\ldots, \xi_s, \ldots) \in \Delta^{S-1} \end{aligned} \tag{C2}$$

where $(\sum_s \xi_s r_j(s, \sigma))_{\sigma, j}$ is the matrix with generic element $(\sum_s \xi_s r_j(s, \sigma))$ and Δ^{S-1} is the $(S-1)$-dimensional simplex.

Condition (C2(i)) requires that agents' admissible trades in all "individual" securities are bounded both above and below. Evidently, if this condition holds no agent can ever have unbounded arbitrage opportunities arising from his private information. On the other hand, no restriction is imposed on the agents' trades in "pool" securities (for which there is no private information). Recalling that the price of each "pool" security equals the opposite of the price of the underlying security, by a standard argument we obtain that the non-empty open set

$$Q(R^p) = \{q \in \mathfrak{R}^J : \exists \rho \in \mathfrak{R}^\Sigma_{++}, \text{ s.t. } q = -(R^p)'\rho\} \tag{6.1}$$

characterizes the set of prices for which there are no arbitrage opportunities.

Since the payoff of "pool" securities is endogenously determined at equilibrium (and there are no restrictions on trades in these securities), the

[28] It should also be clear from our previous discussion that, if the agents' private information is not over the support of the securities' payoff but only over their probability distribution, neither of these conditions is needed.

[29] The condition as stated here applies to adverse selection economies. In the case of economies with hidden information, the only difference is that in (C2(ii)) s should be replaced with m.

agents' budget correspondence may fail to be continuous. Condition (C2(ii)) ensures that this never happens.[30] It says that any convex combination of the payoffs of the "individual" securities for the different values of s, for any given σ, has full rank.[31] It implies, when (C1) is also imposed, that whatever the level of agents' trades in "individual" securities, we always have $Sp[R^p]=\Re^{\Sigma}$. Hence when (C2(ii)) holds, agents are able to attain all possible payoffs contingent on σ by trading in "pool" securities, so that markets are always complete with respect to the aggregate uncertainty in the economy.

We show in the Appendix that under (C2) the agents' choice problem has always a solution and this is well-behaved, both with adverse selection and hidden information.

We will refer in what follows to the restrictions imposed by (C1), (C2) as *One-Side Constraints.*

As an alternative to (C1) we will also consider the case in which agents are allowed to go both long and short in "individual" securities but different prices are quoted for long and short positions (i.e. bid-ask spreads are allowed). More precisely, letting $q(\theta_j)$ denote the cost of trading θ_j units of security j, we have, for all $j\in J$:

$$\begin{aligned} \theta_j \geqslant 0 &\Rightarrow q(\theta_j)=q_j^b\theta_j \\ \theta_j \leqslant 0 &\Rightarrow q(\theta_j)=q_j^s\theta_j \end{aligned} \tag{C1'}$$

Thus q_j^b and q_j^s are respectively the unit buying and selling price of "individual" security j, $j\in J$, and (C1′) says that these prices may differ.

We will refer to conditions (C1′), (C2) as *Bid-Ask Spreads.*

This situation can be analyzed in our set-up by assuming that for each "individual" security there is another "individual" security with opposite, but otherwise identical, payoff (so that taking long positions in this claim corresponds to taking a short position in the "individual" security). We have then distinct "pool" securities, as well as distinct prices, associated with these securities, i.e. with the agents' long and short positions in the underlying claim. Therefore, the model with Bid-Ask Spreads can be formally reduced to a model with One-Side Constraints (and an expanded set of securities). By the same argument as above it follows that (C1′) also allows to overcome the feasibility problem.

[30] Evidently, the continuity of the agents' budget set is also ensured if, in alternative to (C2(ii)), we impose the condition that trades in all (not just the "individual") securities have to lie in a compact set.

[31] Sufficient condition for this property to hold is obviously the existence of a subset of "individual" securities with payoff only contingent on σ, spanning the whole aggregate uncertainty.

Remark 5. The introduction of any form of trading restriction, or non-linearity of prices, requires some observability of agents' trades in financial markets. We already commented in the Introduction on the very strong informational requirements needed to implement exclusivity conditions, or general non-linear price schedules. We intend to argue here that the implementation of one-side constraints as in condition (C1), or of bid-ask spreads as in condition (C1′), poses observability requirements which are, qualitatively, minimally demanding. Only the level of trades in the particular transaction being made has in fact to be observed to implement a constraint on the sign of the total level of trade in securities by an individual (as in the case of one-side constraints), or a variation of the unit price at a zero level of total trades (as in the case of bid-ask spreads). On the contrary, for constraints or price changes at any level of trades different from zero the whole set of transactions in one market (and possibly more) would need, in principle, to be observed.

On the other hand the imposition of bounds on trade requires essentially the observation of "large" portfolios, a stronger but natural requirement.

Remark 6. Condition (C1) implies that buyers and sellers are clearly separated in the markets for securities. In each market we have on one side the buyers of a given type of "individual" security, on the other the agents holding positions in the associated "pool" security. Therefore, no direct compensation of the positions in a given security in different villages is possible under this condition, and the only contingent trades among agents of different villages take place via their trades in "pool" securities. With bid-ask spreads buyers and sellers are also separated (as they face a different price), but some compensation of positions is possible in this case.

More generally, we can view the "feasibility" problem as arising from the fact that a direct compensation of the positions in a given security is not sufficient to ensure the feasibility of trades in that security. Hence the need to specify how, for all possible levels of trade, the losses arising in correspondence of the profits agents make by trading securities on the basis of their private information are distributed in the economy, and hence feasibility ensured.[32] Conditions (C1) and (C1′) imply some partially different mechanisms for distributing these losses.

We will show that, with the additional restrictions imposed by One-Side Constraints (or by Bid-Ask Spreads)[33], competitive equilibria always exist, both for economies with adverse selection and with hidden information. We consider then first the case in which conditions (C1), (C2) hold.

[32] Or, equivalently, so as to ensure the validity of a zero-profit condition for intermediaries.

[33] Our previous analysis also shows that these conditions are tight, i.e. existence is not ensured if they are relaxed.

By restricting agents to be all on one side of the market for "individual" securities, condition (C1) generate the possibility of a "trivial" solution to the existence problem. We can always find in fact a level of q sufficiently high (or low according to the sign of the security's payoff) such that no agent wants to buy any "individual" security, i.e. $\theta^h = 0$, $\forall h$. At such prices no trade takes place in markets characterized by the presence of asymmetric information, and the economy reduces so to a standard economy with incomplete security markets, where agents trade under conditions of symmetric information in all markets.

The following theorem however establishes a stronger result than the existence of competitive equilibria: the existence of equilibria where prices satisfy a "fairness" property. In particular, we will show that a competitive equilibrium always exists where the price of each "individual" security has the property of being (weakly) more than fair for some agent and (weakly) less than fair for some other agent. By fair for an agent we mean here that the idiosyncratic shock component of a security's payoff is evaluated fairly, i.e. its value is set equal to its expectation, conditionally on the private information of the agent.

In the case of economies with adverse selection the fairness property of prices is formally stated as follows:

$$q_j \in co\left\{\sum_{\sigma} \rho_\sigma \sum_{s^{-h}} \pi(s^{-h})\, r_j(\sigma, s^h, s^{-h});\ s^h \in S^h,\ h \in H\right\} \qquad (F_{AS})$$

where $co\{.\}$ denotes the convex hull of a set and $\sum_\sigma \rho_\sigma \sum_{s^{-h}} r_j(\sigma, s^h, s^{-h})$ constitutes a price of security j which is fair, in the above sense, for the agents of type h who observed s^h. Evidently, if equilibrium prices satisfy this property, at least some agent will choose a positive level of trade in each "individual" security, i.e. the equilibrium will typically not be "trivial".

Similarly with hidden information:

$$q_j \in co\left\{\sum_{\sigma, s} \rho_\sigma \pi(s \mid \sigma) \sum_{m^{-h}} \pi(m^{-h} \mid s^{-h})\, r_j(\sigma, m^h(s^h), m^{-h});\right.$$
$$\left. m^h \in (m^{h,v})_{v\in V},\ h \in H\right\} \qquad (F_{HI})$$

where again $\sum_{\sigma,s} \rho_\sigma \pi(s \mid \sigma) \sum_{m^{-h}} \pi(m^{-h} \mid s^{-h})\, r_j(\sigma, m^h(s^h), m^{-h})$ constitutes a fair price of security j for agents of type h who follow message strategy m^h. Since message strategies are endogenously chosen by agents, in (F_{HI}) prices are required to be fair only with respect to those message profiles actually chosen at equilibrium, i.e. $(m^{h,v})_{v\in V}$, typically a subset of $(M^h)^{S^h}$. However, if $\theta_h = 0$ agent h is indifferent among all the possible messages he can send; we will use then a condition in the spirit of "trembling hand perfection" to impose restrictions on the possible message strategies at equilibrium of agents who choose a zero level of trades, and hence further restrict equilibrium prices.[34]

The validity of such fairness property follows, as we will see more clearly in the Appendix, from the observation that, under (C1), the range of the map which determines the payoff of "pool" securities ((4.1) in the case of adverse selection and (4.10) with hidden information) lies in the same set for every nonzero level of trade in "individual" securities. This set is given in particular by the convex hull of the expectations of the payoff of "individual" securities, over the idiosyncratic uncertainty, conditionally on all possible signals received by agents. Prices are then directly related to the level of the payoff of "pool" securities'.

We can now state the main result (the proof is in the Appendix):

THEOREM 2. *Under Assumptions* 1–3, *and conditions* (C1′), (C2) (*i.e. with One-Side Constraints*), *a competitive equilibrium with fair prices* (*satisfying*, *respectively*, (F_{AS}), (F_{HI})) *always exists, both for economies with adverse selection and with hidden information.*

As argued above, the model with Bid-Ask Spreads can always be reduced, formally, to one with One-Side Constraints, so that existence of competitive equilibria with Bid-Ask Spreads obtains as a corollary of the previous result:

COROLLARY 2. *Under Assumptions* 1–3, *and conditions* (C1′), (C2) (*i.e. with Bid-Ask Spreads*), *a competitive equilibrium with fair prices exists*, *both for economies with adverse selection and with hidden information.*

Bid-ask spreads are endogenously determined at equilibrium as the difference between the price for long and short positions. It is immediate to see that in the present framework the equilibrium level of the bid-ask spread will always be non-negative, and typically positive, when information is asymmetric (while it is zero under symmetric information). The presence of a bid-ask spread is then to be imputed to the agents' private information over the payoff of securities, and the need to ensure feasibility in this case -or equivalently a zero-profit condition for intermediaries.[35]

With One-Side Constraints the losses arising from the fact agents' trades under private information are distributed to the buyers of "pool" securities:

[34] See the Appendix for a formal treatment of this argument; also Gale [14] and Dubey, Geanakoplos and Shubik [12] for discussions of refinements in similar environments.

[35] A similar role of bid-ask spreads has been earlier shown by Glosten and Milgrom [19]. These authors examine a specific intermediation model, with risk-neutral market-makers, and study the equilibrium of the market for one security, in the presence of adverse selection. It is interesting to notice that in this model market-makers play, effectively, the same role as "pool" securities in our framework.

the payoff of "pool" securities is typically lower than the expected value of the payoff of the associated "individual" securities[36] (where the expectation is taken over the idiosyncratic uncertainty component). On the other hand, under symmetric information the return on "pool" securities was always equal to this expected value (see (3.4)). In the presence of bid-ask spreads, the difference between the price faced by buyers and sellers constitutes then another way, in addition to the effects on the pool's payoff, in which losses can be distributed in the economy.

Remark 7. Various examples can be found of financial markets whose features resemble the properties of the economy we described and the ones implied by conditions (C2) and (C1), or (C1′). The securitization of the payoffs of standardized contracts (as is observed, e.g. for residential and commercial mortgages, loans, credit card receivables, and many others; see Kendall and Fishman [27]) can be viewed as an instance of the creation of "pool" securities. Credit markets usually have then borrowers on one side and, on the other side, suppliers of funds holding "pool" securities (depositors, or more generally holders of claims issued by intermediaries). Similarly, in insurance markets we observe standardized (insurance) contracts which agents can only buy, and "securitized" claims issued by insurance companies. The mortgage market is yet another example. A somewhat different situation characterizes the stock market and markets for derivative securities. In these cases, agents may often take both long and short positions, and market makers charge spreads to guarantee themselves zero profits in the presence of asymmetric information.

7. CONCLUSIONS

In the final section we discuss some important issues which arise from our analysis of existence of competitive equilibria for asymmetric information economies. We refer to the adverse selection economy just for the ease of the discussion, but without loss of generality.

The role of pool securities. We have assumed throughout the paper that associated to each "individual" security $j \in J$ there is a "pool" security, with return given by the opposite of the average total net amount due to agents who traded the security. With One-Side Constraints, if "pool" securities were not available, no trade would clearly be the only feasible allocation. Furthermore, in our set-up "pool" securities constitute the only way in which contingent trades among agents of different index n (across different "villages") take place: market clearing is obtained by compensating posi-

[36] Evidently, this will also be reflected on the equilibrium level of prices.

tions in "individual" securities with positions in the associated "pool" security.

However "pool" securities also play another, less evident role by allowing for the possibility of compensating positions in securities of different types. To see this, consider the case in which "pool" securities are not available and One-Side Constraints are not imposed, so that a nonzero level of trade is possible. Then, as we showed, a competitive equilibrium may not exist, but if it existed it would be characterized by the fact that long positions in each security are matched by short positions in the same security:

$$\sum_h \lambda^h \sum_{s^h} \pi(s^h)(\theta_j^h(s^h)) = 0 \tag{7.1}$$

This is unduly restrictive. Even though different "individual" securities may not exhibit collinearities in their returns at the level of an agent's trades, they may do so when their aggregate return is concerned[37]. On the other hand, in the market clearing condition with "pool" securities,

$$\sum_h \lambda^h \sum_{s^h} \pi(s^h)(\theta_j^h(s^h) - \theta_{p,j}^h(s^h)) = 0$$

since the distribution of the demand among collinear "pool" securities is indeterminate the compensation of positions in these different securities is allowed.

We should also stress that, as we already argued, if markets are sufficiently complete, the average total payoff to agents holding positions in "individual" securities can be perfectly hedged on the existing markets and the explicit presence of "pool" securities is no longer needed.

Restrictions on trades and prices. Another feature of the market structure we considered is the fact that every agent is not allowed to directly trade 'individual' securities with index n different from his own. Any such trade has to be mediated by positions in the associated "pool" securities. With symmetric information, since the "characteristics" of the contract being traded are the same in all "villages", this restriction never binds; hence "pool" securities properly summarize all what agents could do by trading in other "villages". In the presence of asymmetric information the same is true, as we already argued, as long as agents are unable to determine the precise index (the "village") of the partners of each of their transactions. The characteristics of the contracts are no longer the same in each "village" at the time in which markets open; thus if agents were able to

[37] This was clearly the case in the example considered in Section 5, where the aggregate return on both securities was deterministic.

obtain a portfolio with the same position in each village n, this would allow them to avoid the adverse selection problem and achieve a payoff which reflects the average "characteristics" in the population. This typically dominates the payoff of the "pool" security (which is, as we saw, below the population average); markets would then unravel.

The restriction that securities' price are identical across villages and equal to the opposite of the price of the pool securities, $q = -q_p$ would obtain then as a no arbitrage condition if agents were allowed to trade, under the above informational assumptions, in all markets for "individual" securities. Such restriction would also obtain as a zero profit condition if intermediaries, taking positions in "individual" securities and issuing, on that basis, "pool" securities were modelled.

A more explicit analysis of the informational assumptions behind the pricing structure considered here would clearly benefit from the examination of a model with strategic intermediaries and of its limit behavior. This is clearly an important open issue.

How general is the existence result? In this paper we have shown that, at the root of the difficulties for the "viability" of markets in economies with asymmetric information, is the fact that whenever agents' types cannot be separated, what are effectively different contracts are restricted to trade at the same price. As a consequence, our results apply to more general (and abstract) economies where several different goods are restricted to trade at the same price. Such a situation characterizes various other types of economies with asymmetric information, as Akerlof's "lemons" model, but also other circumstances where asymmetric information is not the source of the problem: For instance electricity prices are often restricted not to vary at different times of the day, many commodity prices are "sticky" over time (because, e.g., of menu costs; see Akerlof and Yellen [1]); segregation and local public goods are other examples.[38]

To illustrate this claim, consider a simple economy with 4 commodities and H consumers, and suppose that the first three commodities must trade at the same price: $p_1 = p_2 = p_3 = p$. Clearly this economy has in general no competitive equilibrium, as there are not enough prices to clear all markets. Consider then the following conditions:

(i) In the markets for commodities 1, 2, 3 agents can only buy, not sell, these goods (i.e. one-side constraints are imposed);

(ii) There also exists a market where fixed bundles composed of α units of good 1, β units of good 2, and $(1-\alpha-\beta)$ units of good 3 can be both bought and sold, at the price p (i.e. a "pool" of the commodities whose prices are restricted is also marketed);

[38] See Balasko [5] for a related analysis of a model with price restrictions.

(iii) the proportions of the various goods in this bundle (i.e. the terms α and β), are determined endogenously at equilibrium by: $\alpha/(1-\alpha-\beta) = (\sum_h x_1^h / \sum_h x_3^h)$, $\beta/(1-\alpha-\beta) = (\sum_h x_2^h / \sum_h x_3^h)$, where x_l^h is the amount of good l, $l = 1, 2, 3$, purchased by agent h in the market for this good.

By a fairly immediate reformulation of our earlier argument we can show that under the above conditions feasibility can be ensured and competitive equilibria always exist for this economy.

The one-side constraint (or bid-ask spread) conditions are tight, in the sense that they cannot be relaxed without generating problems for existence. However other conditions which allow to overcome the existence problems we identified, and in particular the feasibility problem, could be explored. For instance the introduction of entry fees, which agents are required to pay to be able to trade in markets for "individual" securities, and are endogenously determined at equilibrium, allows to prove the existence of competitive equilibria even with linear prices and no short-sale restrictions. In this case there is no separation between buyers and sellers and the entry fee operates as a mechanism, symmetric on the two sides of the market, to redistribute the losses arising from the presence of asymmetric information so as to ensure feasibility. In this respect, bankruptcy institutions, or sets of taxes and transfers, could also serve the same purpose and ensure existence.

An analysis of the different implications for the nature of markets under asymmetric information and in particular for the efficiency properties of competitive equilibria of these alternative conditions, as well as of the forms of non-linearities of prices which can be implemented when some information on agents' trades is easily available, constitutes an important objective of our future work.

APPENDIX

Proof of Theorem 2

We will show first that under (C2) the agents' optimization problem has a well-behaved solution. It is immediate to see that condition (C2(ii)) implies that, when agents are restricted to take only long positions in "individual" securities, the matrix R^p of the values of the payoff of "pool" securities obtained from (4.1) is such that we always have $Sp[R^p] = \Re^{\Sigma}$. This shows, since trades in "pool" securities are unrestricted, that agents can indeed obtain any payoff which is contingent only on the aggregate uncertainty, i.e. that markets are complete with regard to the aggregate risk. Agents' behavior is then unaffected by changes in R^p at equilibrium (so this term can be omitted from the arguments of the demand correspondence). In addition, we use (6.1) to replace q with $-(R^p)'\,\rho$ so that demand can be written as a function simply of (ρ, p_0, p_1) (and, with hidden information, of $\pi(m^{-h}/.)$).

LEMMA A.1. *Under Assumptions* 1–3 *and* (C2), *the individual choice problem* P^h_{AS} *has a solution for all* $(\rho, p_0, p_1) \in \Re^{\Sigma}_{++} \times \Re^{L(1+\Sigma)}_{++}$ *and all the values of* R^p *which can be generated by* (4.1). *The solution is described by the correspondence* $(c^h(s^h), \theta^h(s^h), \theta^h_p(s^h))\,(\rho, p_0, p_1)$, *non-empty, upper-hemi-continuous, convex-valued, and exhibiting the following boundary behavior,* $\forall s^h, h$: *for any sequence* $\{\rho^{(\tau)}, p_0^{(\tau)}, p_1^{(\tau)}\}_\tau \in (\Re^{\Sigma}_{++} \times \Re^{L(1+\Sigma)}_{++})$, *converging to* $(\rho, p_0, p_1) \in \partial(\Re^{\Sigma}_{++} \times \Re^{L(1+\Sigma)}_{++})$, $\inf\{\|c^h, \theta^h, \theta^h_p\|: (c^h, \theta^h, \theta^h_p) \in (c^h(s^h), \theta^h(s^h), \theta^h_p(s^h))(\rho^{(\tau)}, p_0^{(\tau)}, p_1^{(\tau)})\} \to \infty$.

The same properties, with the only exception of convex-valuedness, hold for the solutions of P^h_{HI}, *described by* $(c^h, \theta^h, \theta^h_p, (m^h(s^h))_{s^h})((\rho, p_0, p_1), \pi(m^{-h}/s^{-h}))$.

Proof. Consider first the agent's choice problem under adverse selection, P^h_{AS}. Under (C2), using (6.1) and the date 1 budget constraints to substitute for q, $\theta^h_p(s^h)$ in the expression of the agent's constraint at date 0, the feasible set of problem P^h_{AS} can be rewritten as follows:

$$
\begin{aligned}
B^h_{AS}(\rho, p_0, p_1; s^h) = \Big\{ & c^h(s^h) \in \Re^{L(1+\Sigma S^{-h})}_{+},\ \theta^h(s^h) \in \Theta^h: \\
& p_0 \cdot (c^h_0(s^h) - w^h_0) + (-(R^p)'\,\rho \cdot \theta^h(s^h) \\
& + \sum_\sigma \rho_\sigma \Big[\, p_1(\sigma) \cdot (c^h_1(s, \sigma; s^h) - w^h_1(s, \sigma)) \\
& - \sum_j \theta^h_j(s^h)\, r_j(s, \sigma) \Big] \leqslant 0;\ s \in S \Big\}
\end{aligned} \tag{A.1}
$$

Hence we see that the budget equations faced by the agent imply that his admissible consumption and portfolio plans must satisfy the following condition:

$$
\begin{aligned}
& \Big[\, p_1(\sigma) \cdot (c^h_1(s, \sigma; s^h) - w^h_1(s, \sigma)) - \sum_j \theta^h_j(s^h)\, r_j(s, \sigma) \Big] \\
& \quad = \Big[\, p_1(\sigma) \cdot (c^h_1(s', \sigma; s^h) - w^h_1(s', \sigma)) - \sum_j \theta^h_j(s^h)\, r_j(s', \sigma) \Big] \ \forall s' \neq s
\end{aligned}
$$

i.e. the value of excess demand less the return on "individual" securities has to be the same for all s. This condition describes the constraints on income transfers across states arising from the incompleteness of the market.[39]

Under Assumption 2, $B^h_{AS}(\rho, p_0, p_1; s^h)$ has clearly a non-empty interior, and is closed, convex and compact for all $(\rho, p_0, p_1) \in \Re^{\Sigma}_{++} \times \Re^{L(1+\Sigma)}_{++}$. Moreover, B^h_{AS} is defined by the intersection of budget hyperplanes, and the choice variables which appear in its expression, (c^h, θ^h), are all, by assumption bounded below. Therefore, by a standard argument, the correspondence defined by $B^h_{AS}(\rho, p_0, p_1; s^h)$ is also continuous. Upper-hemi-continuity and convex-valuedness of demand then follow from the continuity and concavity properties of the agents' utility function (stated in Assumption 3).

It is immediate to see that, under Assumption 2, $B^h_{AS}(.)$ has a non-empty interior also at prices $(\rho, p_0, p_1) \in \partial(\Re^{\Sigma}_{++} \times \Re^{L(1+\Sigma)}_{++})$, so that the boundary behavior property of demand holds.

Consider next the agent's problem in the economy with hidden information. A similar expression as above can be obtained for the admissible choice set $B^h_{HI}(\rho, p_0, p_1, \pi(m^{-h} \mid s^{-h}))$ of problem P^h_{HI}. We easily see that B^h_{HI} has the same properties as B^h_{AS}, with the only exception of convexity. Agents have in this case an additional choice variable, the message m^h, which affects the payoff of the securities they trade; as we already noticed since M^h is finite, the set B^h_{HI} is not convex. Since the other choice variables of P^h_{HI} are perfectly divisible, under Assumption 2, B^h_{HI} also has a non-empty interior and is defined by the intersection of budget hyperplanes, so that the continuity of the correspondence defined by $B^h_{HI}(.)$ is preserved. Hence the rest of the above argument still applies. ▮

We are now ready to prove that competitive equilibria exist. We will prove first the result for economies with adverse selection.

(AS) The level of aggregate excess demand is obtained as follows from individual demands:

$$z_0(\rho, p_0, p_1) = \sum_h \lambda^h \sum_{s^h} \pi(s^h)[c^h_0(s^h)(\rho, p_0, p_1) - w^h_0]$$

$$(\theta, \theta_p)(\rho, p_0, p_1) = \sum_h \lambda^h \sum_{s^h} \pi(s^h)(\theta^h(s^h), \theta^h_p(s^h))(\rho, p_0, p_1)$$

$$z_1(\sigma)(\rho, p_0, p_1) = \sum_h \lambda^h \sum_s \pi(s)[c^h_1(s^{-h}, \sigma; s^h)(\rho, p_0, p_1) - w^h_1(s, \sigma)], \quad \sigma \in \Sigma$$

[39] Under (C2), as we showed, "pool" securities allow agents to fully insure against σ, while "individual" securities offer only some partial insurance against the idiosyncratic shocks s.

By Lemma 1 it follows that the above expression inherits the same properties of individual demand: It is an upper-hemi-continuous, non-empty, convex-valued correspondence for all $(\rho, p_0, p_1) \in \Re^{\Sigma}_{++} \times \Re^{L(1+\Sigma)}_{++}$, and exhibits the appropriate boundary behavior. Moreover, it satisfies the following expressions defining Walras law at date 0 and date 1 in state σ: for all $(\rho, p_0, p_1) \in \Re^{\Sigma}_{++} \times \Re^{L(1+\Sigma)}_{++}$

$$p_0 \cdot z_0(\rho, p_0, p_1) + \rho \cdot (R^p(\rho, p_0, p_1)(\theta^p(\rho, p_0, p_1) - \theta(\rho, p_0, p_1)) = 0 \tag{A.2}$$

$$(..., p_1(\sigma) \cdot z_1(\sigma)(\rho, p_0, p_1), ...) + R^p(\rho, p_0, p_1)(\theta(\rho, p_0, p_1) - \theta^p(\rho, p_0, p_1)) = 0 \tag{A.3}$$

where $R^p(\rho, p_0, p_1)$ denotes the map obtained by substituting agents' demand correspondences for the level of their portfolio holdings in the expression of the payoff of "pool" securities (4.1). Equations (A.2), (A.3) are obtained by aggregating across agents the budget constraints, after replacing q with $-R^{p\prime}\rho$, and using the specification of $R^p(.)$ in (4.1).[40]

Normalize date 0 and date 1 prices in every aggregate state σ on the simplex. Consider then the following truncated price sets: $\Delta^{L+\Sigma-1}_{\delta} \equiv ((\rho, p_0) \in \Re^{L\Sigma}_{+}: \sum_l p_{0,l} + \sum_\sigma \rho_\sigma = 1;\ p_{0,l},\ \rho_\sigma \geqslant \delta)$, $\Delta^{L-1}_{\delta} \equiv ((p_1(\sigma) \in \Re^{L}_{+}: \sum_l p_{1,l}(\sigma) = 1;\ p_{1,l}(\sigma) \geqslant \delta)$, for δ sufficiently "small". Pick a convex, compact set $K_\delta \subset \Re^{L(1+\Sigma)} \times \Re^{2J}$ such that the image of the excess demand map $(z_0, (..., z_1(\sigma), ...), \theta, \theta_p)(\Delta^{L+\Sigma-1}_{\delta}, (\Delta^{L-1}_{\delta})^{\Sigma}) \subset K_\delta$.

Examine next the map $R^p(\rho, p_0, p_1)$. It is upper-hemicontinuous and convex-valued if such is agents' demand. It is then immediate to see from the expression of (4.1) that, under (C1), for all $(\theta^h_j(s^h)_{s^h, h})$ such that $\theta^h_j(s^h) \neq 0$ for some h and s^h, we have $r^p_j(\sigma) \in co\{r_j(\sigma, s^h);\ s^h \in S^h,\ h \in H\}$, where $r_j(\sigma, s^h) \equiv (\sum_{s^{-h}} r_j(\sigma, s)\, \pi(s^{-h}/s^h))$, i.e. $r_j(\sigma, s^h)$ equals the expected payoff of security j conditionally on σ, s^h. Therefore, if we require the payoff $r^p_j(\sigma)$ to lie in the set $co\{r_j(\sigma, s^h);\ s^h \in S^h,\ h \in H\}$ also when $\theta_j = \sum_h \times\ \lambda^h \sum_{s^h} \pi(s^h)\, \theta^h_j(s^h) = 0$, upper-hemicontinuity is preserved. We will impose in what follows this restriction on the payoff of the "pool" securities in the case of no trade, and show that it implies the validity of the fairness property (F_{AS}) of equilibrium prices.

Hence the range of the map $R^p(\rho, p_0, p_1)$ is given by $\bar{R}_{AS} \equiv \{R \in \Re^{\Sigma \times J}: r^p_j(\sigma) \in co[r_j(\sigma, s^h);\ s^h \in S^h, h \in H],\ j \in J, \sigma \in \Sigma\}$ for all ρ, p_0, p_1, i.e. by the convex hull of a finite set of points, and is thus a convex, compact set. Furthermore, the range of $-(R^p)'\rho$, when $R^p \in \bar{R}_{AS}$, $(\rho, p_0) \in \Delta^{L+\Sigma-1}$, is also compact and will be denoted by Q_{AS}.

[40] The validity of condition (C1) is crucial, as we already argued, for the aggregate payoff of "individual" securities to be 0, and hence for (A.3) to hold, also when $\theta = 0$.

Consider then the map:

$$(z_0, (..., z_1(\sigma), ...), \theta, \theta_p, R^p, \rho, p_0, p_1, q)_\delta: \\ K_\delta \times \bar{R}_{AS} \times \Delta_\delta^{L+\Sigma-1} \times (\Delta_\delta^{L-1})^\Sigma \times Q_{AS} \\ \to K_\delta \times \bar{R}_{AS} \times \Delta_\delta^{L+\Sigma-1}, (\Delta_\delta^{L-1})^\Sigma \times Q_{AS}$$

defined by:

$$(z_0, (..., z_1(\sigma), ...), \theta, \theta_p) = (z_0, (..., z_1(\sigma), ...), \theta, \theta_p)(\rho, p_0, p_1)$$

$$r_j^p(\sigma) = -\frac{\sum_h \lambda^h \sum_s \pi(s)\, r_j(s, \sigma)\, \theta_j^h(s^h)(\rho, p_0, p_1)}{\theta_j}, \qquad \forall \sigma, j$$

$$\rho, p_0 \in \arg\max\{p_0 \cdot z_0 + \rho \cdot (R^p(\theta^p - \theta))\}$$

$$p_1(\sigma) \in \arg\max\{p_1(\sigma) \cdot z_1(\sigma)\}, \qquad \forall \sigma$$

$$q = -(R^p)'\, \rho$$

Under the above assumptions this map is upper-hemicontinuous and convex-valued, and its domain is compact, convex. Therefore, by Kakutani's Theorem it has a fixed point $[z_0, (..., z_1(\sigma), ...), \theta, \theta_p, R^p, \rho, p_0, p_1, q]_\delta$.

Recalling the expression of Walras' law derived above it is immediate to see that if, at the fixed point $(\rho, p_0, p_1)_\delta \in int\{\Delta_\delta^{L+\Sigma-1} \times (\Delta_\delta^{L-1})^\Sigma\}$, we have $[(z_0), (R^p(\theta^p - \theta)), (..., z_1(\sigma), ...)]_\delta = 0$, i.e. an equilibrium[41] for the perturbed economy. If not, let $\delta \to 0$ and consider the associated sequence of fixed points. By a standard argument (see, e.g., Werner [45]) we can show that this sequence is convergent and, given the boundary behavior property of excess demand, the limit value $(\rho, p_0, p_1)^* \in int\{\Delta^{L+\Sigma-1} \times (\Delta^{L-1})^\Sigma\}$.

Furthermore, notice that at the equilibrium we obtained we have, for all j:

$$q_j^* \in co\left\{\sum_\sigma \rho_\sigma^* r_j(\sigma, s^h);\ s^h \in S^h, h \in H\right\} \tag{A.4}$$

Recalling the definition of $r_j(\sigma, s^h)$ it is immediate to see that (A.4) is equivalent to condition (F_{AS}). Therefore we have shown that at equilibrium the price of every security is always (weakly) more than "fair" for some agent and (weakly) less than "fair" for some other agent (where at least one of the two inequalities is strict).

(*HI*) Part of the proof for economies with hidden information is essentially the same as for economies with adverse selection. However, in this

[41] The equality $R^p(\theta^p - \theta) = 0$ implies that the values such that $\theta^p - \theta = 0$ also belong to the aggregate demand correspondence.

case we have also to show that the economy can be "convexified" by exploiting the large number of agents. Furthermore, to show existence of an equilibrium where, in addition to (F_{HI}), a restriction in the spirit of "trembling hand perfection" is imposed on the message strategies of agents who choose a zero level of trades (for whom the message choice is trivial). We will have to introduce a perturbation of the economy and proceed then by a limit argument. In what follows we will focus on the new parts of the argument, referring to the proof above for the common parts.

Let B^J_ε be an ε-ball in R^J. We will prove first the existence of competitive equilibria for the "perturbed" economy where agents' trades in "individual" securities are restricted to lie in the set $\Theta^h_\varepsilon \equiv \Theta^h \backslash B^J_\varepsilon$, for all h and for ε sufficiently "small". By taking the limit as $\varepsilon \to 0$ we obtain a sequence of "perturbed" economies which converges to the "original" economy where agents' behavior is subject to the "original" trading constraints $(\Theta^h)_{h \in H}$. In a "perturbed" economy agents have to trade some minimal nonzero amount of the "individual" securities; therefore the payoff of "pool" securities is always given by the "average" payoff of individual securities and the price is fair with respect to the message strategy optimally chosen by the agents. In the limit, the same property also holds.

Let $E^h \equiv [e^i \in \Re^{M^h}: e^i_i = 1, e^i_j = 0 \ \forall j \neq i;\ i \in M^h]$ be the collection of unit vectors in $\Re^{M^h}$. Evidently, there is a one-to-one correspondence between elements of E^h and of M^h, so that we can equivalently state the agents' message choice in terms of the choice of an element of E^h. Let $(c^h, \theta^h, \theta^h_p, (e^h(s^h))_{s^h})((\rho, p_0, p_1), \pi(e^{-h} \mid s^{-h})_{s^{-h}}; \Theta^h_\varepsilon)$ denote then the solution of P^h_{HI} when trades in "individual" securities are restricted to lie in the set Θ^h_ε, and m^h has been replaced by e^h.

Define the "convexified" choice correspondence as

$$
\begin{aligned}
&(\hat{c}^h, \hat{\theta}^h, \hat{\theta}^h_p, (\hat{e}^h(s^h))_{s^h},\\
&\qquad [\hat{r}_j(e^h(s^h), e^{-h}, \sigma)\, \theta^h_j]_{j, \sigma, s^h})((\rho, p_0, p_1), \pi(e^{-h} \mid s^{-h})_{s^{-h}}; \Theta^h_\varepsilon)\\
&\quad \equiv co\{(c^h, \theta^h, \theta^h_p, e^h)((\rho, p_0, p_1), \pi(e^{-h} \mid s^{-h})_{s^{-h}}; \Theta^h_\varepsilon);\\
&\qquad\qquad [r_j(e^h(s^h), e^{-h}, \sigma)\, \theta^h_j((\rho, p_0, p_1), \pi(e^{-h} \mid s^{-h})_{s^{-h}}; \Theta^h_\varepsilon)]_{j, \sigma, s^h}\}
\end{aligned}
$$

where $co\{\Phi(.)\}$ denotes, for any map $\Phi(.)$, the convex hull of the image of the map. By Lemma 1 it follows that the above expression is a upper-hemicontinuous, non-empty[42] correspondence for all $(\rho, p_0, p_1) \in \Re^{\Sigma}_{++} \times \Re^{L(1+\Sigma)}_{++}$, $\pi(e^{-h} \mid s^{-h})_{s^{-h} \in S^{-h}} \in \Pi_{h' \neq h}(\Delta^{M^{h'}})^{S^{h'}}$ and exhibits the appropriate boundary behavior; it is also, by construction, convex-valued.

Let $\Delta^{L+\Sigma-1}_\delta$, Δ^{L-1}_δ be defined as before.

[42] Under assumption 2 we can always find ε sufficiently small so that the agents' feasible set is nonempty also when they are restricted to trade at least ε of all 'individual' securities.

The range of the map $R^p((\rho, p_0, p_1), \pi(e^h \mid s^h)_{s\in S}; (\Theta^h_\varepsilon)_{h\in H})$ we obtain by substituting the expression of $\hat{\theta}^h((\rho, p_0, p_1), \pi(e^{-h}/s^{-h})_{s^{-h}\in S^{-h}}; \Theta^h_\varepsilon)$, $h\in H$, in the expression of (4.10) lies now in the set $\bar{R}_{HI} \equiv \{R \in \Re^{S\times J}\colon r^p_j(\sigma) \in co[\sum_s \pi(s\mid\sigma) \sum_{m^{-h}} \pi(m^{-h} \mid s^{-h})\ r_j(\sigma, m^h(s^h), m^{-h});\quad m^h \in (M^h)^{S^h}, \pi(e^{-h} \mid s^{-h})_{s^{-h}\in S^{-h}} \in \Pi_{h'\neq h}(\Delta^{M^{h'}})^{S^{h'}}, h\in H\}$, convex, compact. Similarly, let Q_{HI} be the range of $-(R^p)'\rho$, when $R^p \in \bar{R}_{HI}$, $(\rho, p_0) \in \Delta^{L+\Sigma-1}$, also compact.

Consider then the map:

$$(z_0, (..., z_1(\sigma), ...), \theta, \theta_p, R^p, \rho, p_0, p_1, q, \pi(e^h/s^h)_{s^h, h})_\delta\colon$$
$$K_\delta \times \bar{R}_{HI} \times \Delta^{L+\Sigma-1}_\delta \times (\Delta^{L-1}_\delta)^\Sigma \times Q_{HI} \times \Pi_h(\Delta^{M^h})^{S^h}$$
$$\to K_\delta \bar{R}_{HI} \times \Delta^{L+\Sigma-1}_\delta \times (\Delta^{L-1}_\delta)^\Sigma \times Q_{HI} \times \Pi_h(\Delta^{M^h})^{S^h}$$

defined by:

$$z_0 = \sum_h (\lambda^h \hat{c}^h_0(.) - w^h_0)$$

$$(..., z_1(\sigma), ...)) = \sum_{h,s} \lambda^h \pi(s|\sigma)[\Pi_{h'\neq h}(\pi^{h'}(e^{h'}/s^{h'}))\ \hat{c}^h_1(\sigma, e^{-h}, s^h)(.) - w^h_1(s^h, \sigma)], \quad \sigma\in\Sigma$$

$$(\theta, \theta_p) = \sum_h \lambda^h(\hat{\theta}^h, \hat{\theta}^h_p)(.)$$

$$r^p_j(\sigma) = -\frac{\begin{pmatrix}\sum_{h,s} \lambda^h \pi(s/\sigma) \sum_{e^{-h}} (\Pi_{h'\neq h} \pi^{h'}(e^{h'}/s^{h'})) \\ \times [\hat{r}_j(e^h(s^h), e^{-h}, \sigma)\ \theta^h_j](.)\end{pmatrix}}{\theta_j}, \qquad \forall\sigma, j$$

$$\pi(e^h \mid s^h) = \hat{e}^h(s^h)\ \forall h, s^h$$

$$q = -(R^p)'\rho$$

$$\rho, p_0 \in \arg\max\{p_0 \cdot z_0 + \rho \cdot (R^p(\theta^p - \theta))\}$$

$$p_1(\sigma) \in \arg\max\{p_1(\sigma) \cdot z_1(\sigma)\}, \forall\sigma$$

This map is upper-hemicontinuous and convex-valued, and its domain is compact, convex. Therefore Kakutani's theorem can again be applied, yielding the existence of a fixed point. Proceeding as above we can show that the same expression of Walras' laws hold, and that, by letting $\delta \to 0$ the associated sequence of fixed points converges to an equilibrium of the perturbed, "convexified" economy $(z_0, (..., z_1(\sigma), ...), \theta, \theta_p, R^p, \rho, p_0, p_1, q, \pi(e^h \mid s^h)_{s^h, h})^*_\varepsilon$. If we then let $\varepsilon \to 0$ we obtain another sequence of fixed points (each of which is an equilibrium of the associated perturbed, "convexified" economy) which converges to an equilibrium of the "convexified" economy, $(z_0, (..., z_1(\sigma), ...), \theta, \theta_p, R^p, \rho, p_0, p_1, q, \pi(e^h \mid s^h)_{s^h, h})^*$. At this equilibrium demand and messages are determined by $(\hat{c}^h, \hat{\theta}^h, \hat{\theta}^h_p, (\hat{e}^h(s^h))_{s^h}, [\hat{r}_j(e^h(s^h), e^{-h}, \sigma)\, \theta^h_j]_{j, \sigma, s^h})((\rho, p_0, p_1), \pi(e^{-h} \mid s^{-h})_{s^{-h} \in S^{-h}}; \Theta^h)^*$, i.e. by the "convexified" choice map at the "original" trading constraints $(\Theta^h)_h$, and are such that, at the prices $(\rho, p_0, p_1, q)^*$, commodity and securities' markets clear, the payoff of "pool" securities is consistent with agents' messages, and $\pi(e^h \mid s^h)^*_{s^h, h}$ is consistent with $(\hat{e}^h(s^h))^*_{s^h}$ (the Nash equilibrium component).

By Caratheodory's theorem, as long as $V \geqslant [(L(1 + \Sigma M^{-h} S^h) + 2J + M^h S^h + J \Sigma M]$, we can always find a set of weights $(\gamma^{h, v})^*_{h \in H, v \in V}$ and a set of points, all belonging to the original demand map, such that

$$(\hat{c}^h, \hat{\theta}^h, \hat{\theta}^h_p, (\hat{e}^h(s^h))_{s^h}, [\hat{r}_j(e^h(s^h), e^{-h}, \sigma)\, \theta^h_j]_{j, \sigma, s^h}) \times ((\rho, p_0, p_1), \pi(e^{-h} \mid s^{-h})_{s^{-h} \in S^{-h}}; \Theta^h)^*$$

$$= \sum_v \gamma^{h, v} (c^{h, v}, \theta^{h, v}, \theta^{h, v}_p, (e^{h, v}(s^h))_{s^h}, [r_j(e^{h, v}(s^h), e^{-h}, \sigma)\, \theta^{h, v}_j]_{j, \sigma, s^h})^* \ \forall h,$$

where $(c^{h, v}, \theta^{h, v}, \theta^{h, v}_p, (e^{h, v}(s^h))_{s^h})^* \in (c^h, \theta^h, \theta^h_p, (e^h(s^h))_{s^h})((\rho, p_0, p_1)^*, \pi(e^{-h} \mid s^{-h})^*_{s^{-h} \in S^{-h}}; \Theta^h)\ \forall v$. Hence $((c^{h, v}, \theta^{h, v}, \theta^{h, v}_p, (e^{h, v}(s^h))_{s^h})^*_{v, h}, (\gamma^{h, v})^*_{v, h}, (\rho, p_0, p_1)^*, \pi(e^{-h} \mid s^{-h})^*_{s^{-h} \in S^{-h}})$ constitutes a competitive equilibrium of the original economy.

It is then immediate from the inspection of the fixed point map and the limit argument, that at this equilibrium property (F_{HI}) holds. Moreover, by construction, the message strategies of agents who hold a zero amount of securities are consistent with (in the sense of being "close" to) their optimal strategy when they trade a small amount of securities.

This completes the proof. ∎

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