General Equilibrium with Endogenously Incomplete Financial Markets

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The present paper studies a class of general equilibrium economies with imperfectly competitive financial intermediaries and price-taking consumers. Intermediaries optimally choose the securities they issue and the bid–ask spread they charge. Financial intermediation is costly, and hence markets are endogenously incomplete. An appropriate equilibrium concept is developed, and existence is proved.

Competitive equilibria for this class of economies display full indexation of securities payoffs and monetary neutrality even if intermediaries are restricted to issue "nominal" securities and financial markets turn out to be incomplete. This is in sharp contrast with the indeterminacy and non-neutrality results established in the literature for incomplete markets economies with exogenously given "nominal" securities. *Journal of Economic Literature* Classification Numbers: D52, G20.
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1. INTRODUCTION

Financial markets in general equilibrium economies are traditionally modelled, following Arrow [4], by exogenously specifying the set of securities consumers are allowed to trade. The qualitative properties of equilibria depend then on the set of securities being (exogenously postulated) complete vs. incomplete, and on the denomination of securities' payoffs. In particular, incomplete market models with securities' payoffs denominated in units of account ("nominal" securities) have received large attention in the literature, because they are characterized by indeterminate (and/or sunspot) real equilibrium allocations; moreover the monetary versions of these economies display monetary non-neutrality due to pure portfolio effects *á la Tobin* (cf. Tobin [35]).[1]

[1] Cf. Duffie [16], Geanakoplos [20], and Magill–Shafer [26] for surveys on the Incomplete Markets literature. Cf. Cass [10] and Magill–Quinzii [19] for surveys of incomplete markets models with "nominal" securities.

The present paper studies instead a class of general equilibrium economies in which the properties of the securities issued (i.e., their payoffs and their denomination) arise endogenously as choices of optimizing intermediaries.[2] As a consequence, financial markets are endogenously complete or incomplete depending on the parameters of the economy (e.g., agents' endowments, and intermediation costs).

The main objective of the paper is to compare the qualitative properties of real equilibrium allocations in economies with optimizing intermediaries with those of economies in which the set of tradable securities is exogenously specified. In particular, we will be concerned with the robustness of the equilibrium properties of incomplete market models with "nominal" securities. The paper also attempts to develop a theory of incomplete markets and financial innovation based on imperfect competition and intermediation costs. An appropriate equilibrium concept is developed, and an existence result is proved.

Economies characterized by an exogenously given incomplete set of "nominal" securities generically have the property that real competitive equilibrium allocations depend continuously on price levels, i.e., equilibria display *real indeterminacy*. (This indeterminacy result is due to Cass [9], and has then been generalized by Balasko–Cass [5] and by Geanakoplos–Mas Colell [21]; but cf. also Werner [37].) This result has been exploited to construct examples of "sunspot" equilibria (cf. Cass [8] and the following literature surveyed in Cass [10]), of partially revealing rational expectations equilibria (cf. Polemarchakis–Siconolfi [31] and Rahi [32]), and of monetary non-neutrality (cf. Gottardi [22] and Magill–Quinzii [27]).

The postulation of exogenously given securities' payoffs is crucial for this *real indeterminacy* result: Indeterminacy occurs because perturbing the price levels perturbs the payoff in real terms paid by given "nominal" securities, and hence real equilibrium allocations depend on the price levels. But if the microeconomics of financial markets is explicitly modelled, and securities are issued by financial intermediaries, securities' payoffs could be indexed in equilibrium to the price levels (and hence the independence of equilibrium allocations of price levels would be re-established). This paper proves that full indexation is an equilibrium property, under mild restrictions on costs, for a general class of economies with imperfectly competitive intermediaries. As a consequence: (i) real equilibrium allocations are independent of the level of prices (the *real indeterminacy* result as formulated in the literature does not hold), and (ii) money is neutral if monetary injections have no distributional effects; also, (iii) sunspot

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[2] Cf. Allen-Gale [3] and Duffie–Rahi [17] for recent surveys on the "security design" literature.

equilibria or non-revealing equilibrium prices might exist but are independent of the denomination of securities' payoffs.

The class of economies studied in this paper is more specifically characterized as follows. Intermediaries are imperfectly competitive. They design the payoff structure of the securities they issue to maximize profits, and choose the spread they charge on each security. A fixed cost and a proportional transaction cost are required to intermediate each security, and hence financial markets are endogenously incomplete. Consumers are price takers. Once the securities are issued and the spread chosen, intermediaries "make the market," and trading occurs at prices quoted by a Walrasian auctioneer to clear the spot and securities' markets. Equilibrium prices and demands are rationally anticipated by intermediaries when evaluating their profits. This class of economies accounts for several extensive form games played by the intermediaries, proxying for different institutional and technological settings: e.g., sunk vs fixed operating costs per security issued, and simultaneous vs sequential entry in financial markets.

The choice of modelling imperfectly competitive intermediaries complicates substantially the analysis, and departs from most of the literature on optimal security design in general equilibrium (but cf. Allen–Gale [2] for a related imperfectly competitive approach). On the other hand, economies with competitive intermediaries would restrict the scope of the analysis because perfect competition (i) is incompatible with non-convex intermediation technologies, while the few existing empirical studies suggest the relevance of fixed (possibly sunk) costs of issuing securities (cf., e.g., Tufano [36]); and moreover (ii) it might require restrictions on the intermediaries' strategy space, e.g., to short-sales-constrained securities (as in Allen–Gale [1]) or to derivative securities (as in Pesendorfer [30]), which are not suitable for modelling markets for pure intermediation, and blur the neutrality properties of these economies; cf. the discussion of Pesendorfer [30] in Section 4.

An appropriate equilibrium concept for this class of economies is developed and existence is proved. Technical difficulties in proving existence derive from possible discontinuities in intermediaries' profit function (because equilibrium prices are in general not continuous in spreads and securities' payoffs, and intermediaries are assumed to be able to form rational expectations about equilibrium prices).

2. ECONOMIES WITH IMPERFECTLY COMPETITIVE INTERMEDIARIES

The economy, lasting two periods, is populated by a finite number of (types of) agents, and intermediaries are characterized by given preferences

and uncertain endowments. Prior to the beginning of time the intermediaries strategically choose the set of securities they issue, and the spread between the bid and ask prices of securities they charge. Intermediaries have rational expectations, hence they correctly anticipate the demand and the equilibrium price for each security they issue. The technology for intermediating securities involves both fixed and variable transaction costs. In the first period consumers are then allowed to trade on spot markets for consumption goods and on (financial) markets for the securities issued by the intermediaries. Consumers are price-takers, and trading occurs at prices quoted by a Walrasian auctioneer to clear markets; in financial markets, in particular, the auctioneer quotes bid and ask prices for given spread chosen by the intermediaries. In the second period uncertainty resolves, securities pay dividends, and agents trade on spot markets for consumption goods.

2.1. The Class of Economies

The economy lasts for 2 periods, $t = 0, 1$. States of the uncertainty in period 1 are indexed by $s = 1, ..., S$. Commodities in each period and state are indexed by $l = 1, ..., L$ (S and L are assumed finite). The number of state and time contingent goods of the economy is then $n = (S + 1) L$.

Preferences and Endowments. Each agent i, for $i = 1, ..., I$, is defined by an utility function $u^i: \Re_+^n \to \Re$, and by a vector of commodity endowments $w^i = (w_0^i, w_1^i, ..., w_S^i) \in \Re_+^n$. Let $u = (u^1, ..., u^I)$ and $w = (w^1, ..., w^I)$.

The following assumptions about preferences and endowments are strong but quite standard in the incomplete markets literature.

Assumption 1. For every i the function u^i satisfies the following properties:

   a. Regularity of preferences:

      a.1. $u^i: \Re_+^n \to \Re$ is continuous on $\Re_+^n$ and $\mathscr{C}^2$ on $\Re_{++}^n$;

      a.2. if $U^i(\zeta) = \{x \in \Re_+^n : u^i(x) \geqslant u^i(\zeta)\}$, then $U^i(\zeta) \subset \Re_{++}^n$, $\forall \zeta \in \Re_{++}^n$;

      a.3. the derivative $du^i(x) \in \Re_{++}^n$, $\forall x \in \Re_{++}^n$; and $v' d^2 u^i(x) v < 0$, $\forall v \neq 0$ s.t. $du^i(x) v = 0$, where $d^2 u^i(x)$ is the Hessian matrix;

   b. Interiority of endowments:

      b.1. $w^i \in \Re_{++}^n$ for every i.

Financial Markets and Intermediation Costs. The economy is also populated by "financial intermediaries." A generic intermediary h, for $h = 1, ..., H$, is an agent (with given preferences and endowments). We simplify his life by assuming that he only cares about (and has endowment of) good 1 in $t = 0$. As a consequence any intermediary h can effectively be thought of as maximizing profits in units of good 1 (this is without loss of generality as far as existence and indeterminacy results are concerned; cf.

Section 4 for a discussion of this point). We denote the endowment of intermediary h by $w^h \in \Re_{++}$.

Intermediaries are endowed with a simple technology for issuing and trading financial securities. A security is represented by a vector $a = (a_1, ..., a_S) \in \Re^S$ of payoffs in units of account (a_s is the security's payoff in state s). Let $v = (p_{10}, p_{11}, ..., p_{1S}) \in \Re^{S+1}_{++}$ denote the vector of prices of the numeraire good (good 1) in period 0 and in all states of period 1. For any $a \in \Re^S$ and $v \in \Re^{S+1}_{++}$ the functions $\varepsilon(a, v)$, $c(a, v)$ denote respectively variable costs (proportional to trading volume) and fixed costs associated to the trading of a security with payoff vector a. Costs are denominated in units of numeraire in $t = 0$. Let $a/v = (a_1/p_{11}, ..., a_S/p_{1S})$ denote the vector of payoffs in units of numeraire.

Assumption 2.   Transaction costs satisfy the following restrictions:

   (a) $\varepsilon(a, v)$, $c(a, v) \geqslant 0$; $= 0$ iff $a = 0$, $\forall v \in \Re^{S+1}_{++}$;

   (b) $\varepsilon(a, v)$, $c(a, v)$ are, respectively, homogeneous of degree 1 and 0 in a, $\forall v \in \Re^{S+1}_{++}$;

   (c) $\varepsilon(a, \lambda v) = (1/\lambda)\, \varepsilon(a, v)$ and $c(a, \lambda v) = c(a, v)$,   for $\lambda \in \Re_+$, $\forall a \in \Re^S_{++}$;

   (d) $\varepsilon(a, v)$, $c(a, v)$ are $\mathscr{C}^2$ for $a/v \neq 0$ and $\forall v \in \Re^{S+1}_{++}$.

Condition (a) imposes non-negativity of costs (it is required for existence; cf. Hara [23]); also, it guarantees that issuing a security $a = 0$ is equivalent to not issuing any security. Conditions (b) and (c) are not strictly required for existence but impose "natural" and weak forms of independence of costs from the denomination of payoffs (cf., e.g., also Pesendorfer [30]). More precisely, condition (b) assures that, in terms of both variable and fixed costs, issuing two identical securities with payoff a is equivalent to issuing one security with payoff $2a$; while condition (c) implies that changes in the price levels in the second period do not affect transaction costs (i.e., if all payoffs were to be measured in cents instead of dollars, then variable costs would decrease by a factor of a hundred, while fixed costs would remain unchanged). In Section 4 a stronger form of independence of costs from payoffs denomination is introduced (Assumption 3), which is required for the neutrality result (Proposition 1). Condition (d), finally, is a technical requirement which can be substantially relaxed.

It is important to note that Assumption 2 implies that

   - securities payoffs can be normalized to

$$\mathscr{A}^0 = \left\{ a \in \Re^S \mid \text{either } \max_s |a_s/p_{1s}| = 1; \text{ or } a_s = 0, \forall s \right\},$$

   - fixed costs $c(a, v)$ are bounded above, and we call the bound $\bar{c}$.

The focal example of costs satisfying Assumption 2 is the case of constant costs (modulo a normalization):

$$\forall a, a' \in \mathscr{A}^0 \text{ such that } a, a' \neq 0, \; c(a, v) = c(a', v) \qquad \text{and} \qquad \varepsilon(a, v) = \varepsilon(a', v). \tag{1}$$

Intermediaries play a game which determines both the securities which are issued (and hence are tradable in $t = 0$), and the bid-ask spread they charge on each of them to recoup costs. Intermediary h issues in equilibrium J^h securities, and the generic security he issues is denoted by $j = 1, ..., J^h$. Security j is then characterized by payoff a_j and bid-ask spread γ_j. The bid-ask spread is denominated in units of account. The vector of spreads in units of numeraire is then denoted $\gamma/v = (\gamma_1/p_{10}, ..., \gamma_J/p_{10})$.[3]

The generic game played by intermediaries, denoted by G, includes the following elements.

(i)   The set of players is $\{1, ..., H\}$.

(ii) The action space of intermediary h is $\Gamma_{J^h} \times \mathscr{A}^0_{J^h}$, where

$$\Gamma_{J^h} = \{\gamma \in \mathfrak{R}^{J^h}_+ \mid \gamma_j/p_{10} \geq \varepsilon(a_j, v), \; \forall j = 1, ..., J^h\}; \tag{2}$$

$$\mathscr{A}^0_{J^h} = [\mathscr{A}^0]^{J^h} \text{ for given } J^h \text{ such that } J^h \bar{c} < w^h. \tag{3}$$

(iii) Player h's payoff is defined by

$$\pi^h = \sum_{j=1}^{J^h} [(\gamma_j/p_{10} - \varepsilon(a_j, v)) z_{+j} - c(a_j, v)];$$

where z_{+j} denotes the aggregate long positions on security j.

Several points are worth noticing. Intermediary h chooses $(\gamma^h, A^h) \in \Gamma_{J^h} \times \mathscr{A}^0_{J^h}$. While restricting $A^h \in \mathscr{A}^0_{J^h}$ is a normalization (due to Assumption 2), the definition of agents' strategy space (equations 2 and 3) also requires $\gamma_j/p_{10} \geq \varepsilon(a_j, v)$ and $J^h \bar{c} < w^h$. These are quite strong restrictions. But they are stronger than needed and are imposed to avoid considering bankruptcy of the intermediaries, a problem which could be dealt with but would take us afar from the central point of this paper.

The description of the game G played by intermediaries requires that J^h, the number of securities which each intermediary h can issue, be exogenously given (of course each intermediary is free to design the payoff structure of each security he issues). But since we allow intermediaries to issue securities with payoff $a = 0$ at zero costs (Assumption 2a) this obviously imposes no lower bound on the number of securities that each intermediary h can issue. Theorem 2 moreover shows that J^h can be chosen

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[3] Since securities' payoffs are not constrained to be non-negative, and hence non-trivial securities can have zero price in equilibrium, spreads are not quoted in percentage terms. Note also that intermediaries' choice variables, securities' payoffs and spreads, are denominated in units of account, while other variables like costs are denominated in units of numeraire. This notation is instrumental to the analysis of neutrality in Section 4.

large enough (though finite) so that effectively also no upper bound is imposed on the number of securities.

We restrict here ourselves for simplicity to the analysis of the following two games played by intermediaries:[4]

A simultaneous move game: intermediaries play simultaneously, and also choose the spreads γ^h and the securities' payoffs A^h simultaneously; fixed costs $c(a, v)$ are paid after the choice; this game structure captures economies in which intermediation costs have a simple "fixed cost" component. Consequently, we denote this game by G_{fc}.

A sequential game: intermediaries still play simultaneously, but they first choose the securities' payoffs A^h, and pay the associated fixed costs $c(a, v)$; and only then, after observing the securities issued by competing intermediaries, choose the spreads γ^h. This game structure captures economies in which intermediation costs have a "sunk cost" component. Consequently, we denote this game by G_{sc}.

We say that the generic game G belongs to $\{G_{fc}, G_{sc}\}$. Let $J = \sum_{h=1}^{H} J^h$ (abusing notation we let the index j run also from 1 to J, and hence index both the set of securities issued by each intermediary h as well as the set of securities tradable in the economy). Let also

$$A = (A^1 \vdots \cdots \vdots A^H), \quad \gamma = \begin{pmatrix} \gamma^1 \\ \vdots \\ \gamma^H \end{pmatrix}, \quad \mathscr{A}_J^0 = \mathscr{A}_{\gamma^1}^0 \times \cdots \times \mathscr{A}_{\gamma^h}^0.$$

Finally, we will in the paper refer to $\langle u, w, G \rangle$ as an "economy."

2.2. The Equilibrium Concept

This section introduces the equilibrium concept used in the paper for any economy $\langle u, w, G \rangle$.

Let p_{l0} (resp. p_{ls}) denote the spot price of good l at time 0 (resp. at time 1 in state s), and $p = (p_0, p_1) \in \mathfrak{R}_{++}^L \times \mathfrak{R}_{++}^{LS}$ the spot price vector; also let $q = (q_1, ..., q_J) \in \mathfrak{R}^J$ denote the ask price vector for securities.

Let $x^i = (x_0^i, x_1^i, ..., x_S^i) \in \mathfrak{R}_+^n$ and $x^h \in \mathfrak{R}_+$ denote respectively consumer i's and intermediary h's consumption vectors (x_{l0}, x_{ls} denote consumption of good l respectively at time 0 and at time 1 in state s). Finally, let

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[4] Both the existence and the neutrality results (respectively in Sections 3 and 4) can be proved for a larger class of games (including for instance sequential games). What is in fact needed for the results of the paper is just that (i–iii) characterize the normal form representation of the game played by intermediaries.

$z^i_+ = (z^i_{+1}, ..., z^i_{+J}) \in \Re^J_+$ denote consumer i's asset portfolio vector, $z^i_- \in \Re^J_+$ his liability portfolio vector, and let $z^i = z^i_+ - z^i_-$.

Roughly, (p, q, γ, A) represents an equilibrium if,

- (γ, A) is a Nash equilibrium for G_{fc} (a perfect Nash equilibrium if game G_{sc} is played):

- intermediaries' expectations about competitive equilibrium prices are rational;

- (p, q) is a competitive equilibrium given (γ, A).

A formal definition involves several technical details, though. Before going into the intricacies of the definition and existence of equilibrium, we then study a simple parametric example: quasi-linear preferences, a single intermediary, and two consumers (this specification of preferences violates Assumption 1.a, but this is immaterial for the specific example).

2.2.1. A Simple Example

The example is constructed to set aside the existence problems so that the gist of the securities' design problem solved by intermediaries is clearly exposed.

Let $I = 2$, $S = 2$, $L = 1$, $H = 1$, $J^h = J = 1$. Normalize $p_{10} = p_{1s} = 1$, $\forall s$. Moreover take $\varepsilon(a, v) = 0$, and $c(a, v) = c$ arbitrarily small, $\forall a \neq 0$. Normalize the intermediary's security's payoff choice to either $a = (0, 0)$ or $a = (a_1, 1)$, $a_1 \in \Re$ (since $\varepsilon(a, v) = 0$ this financial structure is equivalent to a special case of the focal example with constant costs for any $a \in \mathscr{A}^0_1$ as in 1.) Preferences are quasi-linear

$$u^i(x) = x_0 - 1/2 \sum_{s=1}^{2} (B - x_s)^2.$$

If $a \neq 0$, agent i chooses $(x^i_0, x^i_1, x^i_2, z^i_+, z^i_-) \in \Re^5_+$ to maximize $u^i(x^i)$ subject to:

$$(x^i_0 - w^i_0) + (\gamma + q) z^i_+ - q z^i_- = 0;$$

$$(x^i_s - w^i_s) = a_s(z^i_+ z^i_-), \qquad s = 1, 2; \quad a_2 = 1.$$

Finally assume that $\tilde{w}_s = w^2_s - w^1_s > 0$ for $s = 1, 2$, and $\tilde{w}_1 < \tilde{w}_2$. At a competitive equilibrium for given $(\gamma, a \neq 0)$ agents have portfolios

$$z^1 = -z^2 = \frac{a_1 \tilde{w}_1 + \tilde{w}_2 - \gamma}{2((a_1)^2 + 1)}.$$

The intermediary maximizes profits perfectly anticipating competitive equilibrium prices and quantities associated to any of his choice of (γ, a):

$$\max_{\gamma, a_1} \gamma \frac{a_1 \tilde{w}_1 + \tilde{w}_2 - \gamma}{2((a_1)^2 + 1)};$$

whose solution is

$$\gamma = 1/2\tilde{w}_1 \left(\frac{\tilde{w}_1}{\tilde{w}_2} + 1 \right); \qquad a_1 = \frac{\tilde{w}_1}{\tilde{w}_2}.$$

2.2.2. The Equilibrium Set

Proceeding by backward induction, the *competitive equilibrium corre-spondence*, mapping (γ, A) into market clearing pricing, is first defined (Definition 1). Intermediaries' profit functions are then introduced, and it is shown how they depend on intermediaries' beliefs about equilibrium prices, for any (γ, A). (We implicitly impose a rational expectations assumption on beliefs.) The game played by intermediaries can at this point be formally defined. The definition of the *equilibrium set* requires (γ, A) to constitute a Nash equilibrium for G_{fc} (Definition 2), and a perfect Nash equilibrium for game G_{sc} (Definition 3; in fact we use a somewhat weaker notion: Details are later in the section).

Competitive Equilibrium Correspondence. Agent i, given the securities' payoff matrix A, the bid-ask spreads γ, and prices (p, q) solves:

$$\max_{x \in \mathfrak{R}^n_+, (z_+, z_-) \in \mathfrak{R}^{2J}_+} u^i(x) \tag{4}$$

$$s.t. \ p_0(x_0 - w_0^i) + (\gamma + q) z_+ - qz_- = 0 \tag{5}$$

$$p_s(x_s - w_s^i) = \sum_{j=1}^{J} a_{sj}(z_{+j} - z_{-j}), \qquad \forall s. \tag{6}$$

Intermediary h, given v, consumes:

$$x^h = \sum_{j=1}^{J^h} \sum_{i=1}^{I} \left[(\gamma_j/p_{10} - \varepsilon(a_j, v)) z_{+j}^i - c(a_j, v) \right] + w^h. \tag{7}$$

DEFINITION 1. *Let $w \in \mathfrak{R}^{nI}_{++}$ be given. Let E denote the correspondence mapping $(\gamma, A) \in \Gamma_J \times \mathscr{A}_J^0$ into $(p, q) \in \mathfrak{R}^n_{++} \times \mathfrak{R}^J$ such that:*

(a) $(x^i, z^i_+, z^i_-)_i$ *solve 4–6;*

(b) $(x^h)_h$ *satisfies 7;*

(c)   *prices* $(p, q) \in \Re^n_{++} \times \Re^J$ *are such that markets clear*:

$$\sum_{i=1}^I (x^i_{10} - w^i_{10}) + \sum_{h=1}^H (x^h - w^h) - \sum_{j=1}^J c(a_j, v) - \varepsilon(a_j v) \left[\sum_{i=1}^I z^i_{+j} \right] = 0$$

$$\forall l \neq 1, \sum_{i=1}^I (x^i_{l0} - w^i_{l0}) = 0$$

$$\forall s, \sum_{i=1}^I (x^i_s - w^i_s) = 0$$

$$\sum_{i=1}^I (z^i_+ - z^i_-) = 0.$$

E is called *competitive equilibrium correspondence.*[5]

We can now construct intermediaries' profit functions. Let $z^i_{-j}(p, q, \gamma, A)$ and $z^i_{+j}(p, q, \gamma, A)$ denote, respectively, the demand of liability and asset j by agent i. Intermediary h faces the following profit correspondence:

$$\pi^h(E(\gamma, A), \gamma, A)$$

$$= \sum_{j=1}^{J^h} \sum_{i=1}^I [(\gamma_j/p_{10} - \varepsilon(a_j, v)) z^i_{+j}(E(\gamma, A), \gamma, A) - c(a_j, v)]. \quad (8)$$

Let then $\pi(E(\gamma, A), \gamma, A) = [\pi^h(E(\gamma, A), \gamma, A)]_{h=1, \dots, H}$. We say that *admissible intermediaries' profit functions* are all vector valued functions mapping $(\gamma, A) \in \Gamma_J \times \mathscr{A}^0_J$ into $\Re^H$ constructed as selections of the convex hull of $\pi(E(\gamma, A), \gamma, A)$.

Remark 1.   What does it mean to take the convex hull of profit correspondences? This construction is illustrated by explicitly relating it to intermediaries' beliefs about equilibrium prices for any (γ, A). *Admissible intermediaries' beliefs* about competitive equilibrium prices are all collections of maps from (γ, A) to probability distributions over (p, q) (for any set K call Δ_K the set of probability distributions over K), denoted $\{b^h_{p, q}\}_{h=1, \dots, H}$, which satisfy:

(a) $b^h_{p, q} : \Gamma_J \times \mathscr{A}^0_J \to \Delta_{\Re^n_{++} \times \Re^J}, \forall h$;

(b) $b^h_{p, q} = b_{p, q}, \forall h$;

(c) $\text{supp}(b_{p, q}(\gamma, A)) \subseteq E(\gamma, A); \quad \forall (\gamma, A) \in \Gamma_J \times \mathscr{A}^0_J$; where $\text{supp}(\cdot)$ denotes the support.

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[5] Definition 1 imposes market clearing security-by-security (point (c)). This limits the analysis if A^h has not maximum rank in equilibrium for some h. Weaker market clearing concepts (e.g., firm-by-firm) can be allowed at some notational costs, without changing the results of the paper. On the other hand weakening market clearing can be important in more detailed characterizations of equilibria.

Restriction (a) allows the belief function to be random, mapping each $(\gamma, A) \in \Gamma_J \times \mathscr{A}_J^0$ into a probability distribution over equilibrium prices. Restriction (b) is commonality of beliefs about equilibrium prices across intermediaries: It is not strictly required but natural. Restriction (c) finally imposes rational expectations: Beliefs are a selection of the competitive equilibrium correspondence.

Admissible intermediaries' profit functions can then be equivalently written as:

$$\int \sum_{j=1}^{J^h} \sum_{i=1}^{I} [(\gamma_j/p_{10} - \varepsilon(a_j, v)) z^i_{+j}(p, q, \gamma, A) \, db_{p,q}(\gamma, A) - c(a_j, v)]$$

for any admissible belief $b_{p,q}$.[6]

We can now define formally the *equilibrium set* relative to the games G_{fc} and G_{sc}. The equilibrium concept we use is Nash equilibrium for the simultaneous game G_{fc}, and a weaker notion of perfect Nash equilibrium for the sequential game G_{sc}.

Let $NE(G_{fc})$ collect all Nash equilibria of game G_{fc} with respect to all *admissible intermediaries' profit functions*; and denote by $\sigma_{\gamma, A} \in \Delta_{\Gamma_J \times \mathscr{A}_J^0}$ its generic element. Note that, by the construction in Remark 1, any element of $NE(G_{fc})$ is associated to a particular selection of $\pi(E(\gamma, A), \gamma, A)$, and hence to a particular admissible belief $b_{p,q}$.

DEFINITION 2. The *equilibrium* set of game G_{fc}, denoted by $\mathscr{E}_{fc}$, contains all distributions of prices, spreads and securities' payoffs induced by elements of $NE(G_{fc})$. Formally, $\mathscr{E}_{fc}$ contains all $e_{fc} \in \Delta_{\mathfrak{R}^n_{++} \times \mathfrak{R}^J \times \Gamma_J \times \mathscr{A}_J^0}$ such that[7] $e_{fc} = b_{p,q} \circ \sigma_{\gamma, A}$, where $\sigma_{\gamma, A} \in NE(G_{fc})$, and $b_{p,q}$ is associated to the Nash equilibrium $\sigma_{\gamma, A}$.

For any A, let $NE(G_{sc} \mid A)$ collect all Nash equilibria of the last subgame of G_{sc}, with respect to all *admissible intermediaries profit functions*. Its generic element is denoted $\sigma_\gamma(A)$. Clearly, $\sigma_\gamma(A)$ is a mapping from $\mathscr{A}_J^0$ into Δ_{Γ_J}. Denote by $\pi(\sigma_\gamma(A), A)$ the correspondence $\pi(E(\gamma, A), \gamma, A)$ evaluated at the (mixed strategies) Nash equilibrium spreads $\sigma_\gamma(A)$. Let $NE(G_{sc} \mid \sigma_\gamma(A))$ collect all Nash equilibria of the first sub-game with respect to intermediaries' profit functions constructed as selection of the convex hull of $\pi(\sigma_\gamma(A), A)$. Its generic element is denoted $\sigma_A \in \Delta_{\mathscr{A}_J^0}$. We say that a

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[6] Since in general $z^i_{+j}(\cdot)$ is a correspondence, in fact intermediaries profit functions also implicitly select sharing rules of agents' long positions. Integrability conditions are taken care of in the proof of existence; see the Appendix where we refer to the main theorem in Simon–Zame [34].

[7] Given a map $f: K \to \Delta_X$ and a $g \in \Delta_K$, we denote with $f \circ g$ the distribution in $\Delta_{K \times K}$ defined, for any measurable subset $k \times x \in K \times X$, by $f \circ g(k \times x) = (g(k), g(f^{-1}(x) \cap k))$.

probability distribution $\sigma_{\gamma,A}$ is in $P^*NE(G_{sc})$ (the set of *perfect* Nash
equilibria* of game G_{sc}) if it is constructed as $\sigma_\gamma(A) \circ \sigma_A$ for $\sigma_\gamma(A) \in$
$NE(G_{sc}\,|\,A)$ and $\sigma_A \in NE(G_{sc}\,|\,\sigma_\gamma(A))$. Again an admissible belief $b_{p,q}$ is
associated to any element of $P^*NE(G_{sc})$.

DEFINITION 3. The *equilibrium set of game* G_{sc}, denoted by $\mathscr{E}_{sc}$,
contains all distributions of prices, spreads and securities' payoffs induced
by elements of $P^*NE(G_{sc})$. Formally, $\mathscr{E}_{sc}$ contains all $e_{sc} \in \Delta_{\mathfrak{R}^n_{++} \times \mathfrak{R}^J \times \Gamma_J \times \mathscr{A}^0_J}$
such that $e_{sc} = b_{p,q} \circ \sigma_{\gamma,A}$, where $\sigma_{\gamma,A} \in P^*NE(G_{fc})$, and $b_{p,q}$ is associated
to the *perfect* Nash equilibrium* $\sigma_{\gamma,A}$.

Note that this construction involves convexifying over second stage Nash
equilibrium payoffs (since intermediaries' profit functions in the first
subgame are chosen as selections of the *convex hull* of $\pi(\sigma_\gamma(A), A)$). This is
necessary in general to guarantee existence, and implies that perfect* Nash
equilibrium is a weaker concept than perfect Nash equilibrium (in the spirit
of correlated equilibrium); cf. also Harris *et al.* [24]. If $x^i(p, q, \gamma, A)$
denotes agent i's demand function and $x(p, q, \gamma, A) = [x^i(p, q, \gamma, A)]_{i=1, \dots, I}$,
then the set of *real equilibrium allocations* is defined by:[8] $\mathscr{X}(G_{fc}) =$
$\{x_{fc} \in \Delta_{\mathfrak{R}^{nI}_+} \,|\, x_{fc} = x e_{fc}, \text{ and } e_{fc} \in \mathscr{E}_{fc}\}$, and similarly for $\mathscr{X}(G_{sc})$.

3. EXISTENCE

The objective of this section is to show that the equilibrium concept used
in this paper is logically consistent, in the sense that the *equilibrium set* is
non-empty for both the fixed costs and the sunk costs games.

The existence problem is complicated by the following reason. Inter-
mediaries' profit functions may not be continuous, as required by standard
Nash equilibrium existence theorems, because they are constructed as
selections of profit correspondences which in general have no continuous
selections (this property of profit correspondences is in turn derived by the
fact that competitive equilibrium correspondences have in general no
continuous selections). Cf. Fig. 1 for an intuitive characterization of these
complications.

While a Nash equilibrium for any admissible profit function of inter-
mediaries might not exist in the class of games studied in this paper,[9]

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[8] Given a map $f: K \to \Delta_X$ and a $g \in \Delta_K$, we denote with fg the distribution in Δ_X defined,
for any measurable subset $x \in X$, by $fg(x) = g(f^{-1}(x))$.

[9] Examples of economies $\langle u, w, G_{fc} \rangle$ such that, for particular admissible profit functions, a
Nash equilibrium of the game does not exist can be constructed as a reformulation, for the
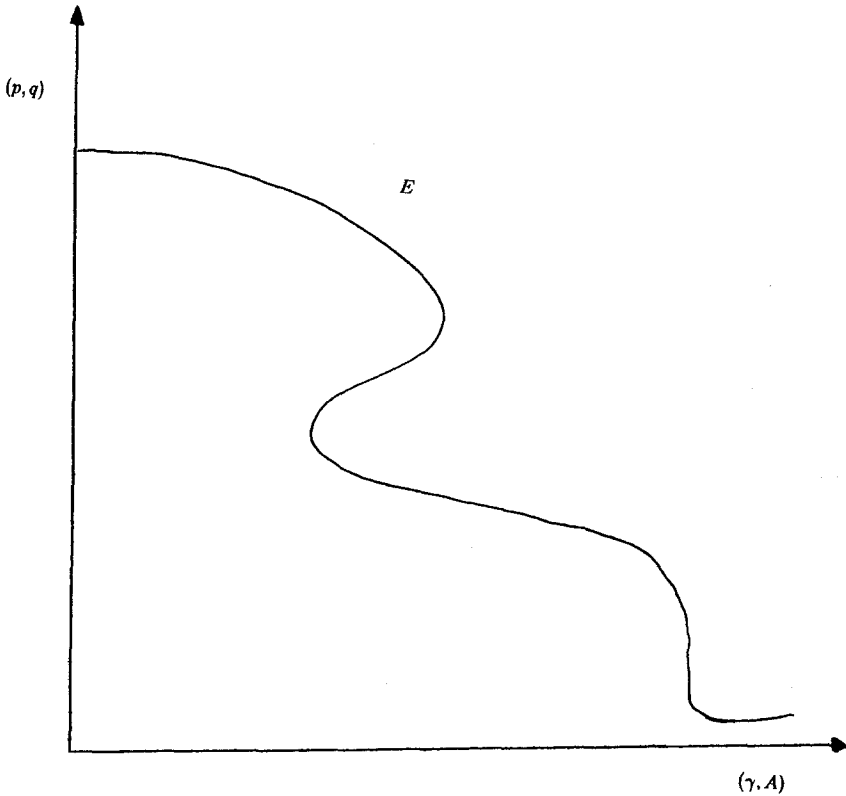class of economies studied in this paper, of an example in Dierker–Grodal [14].

FIG. 1. Discontinuous intermediaries' profit. The *competitive equilibrium correspondence E* does not contain a continuous selection, and not even a continuous random selection. This is a property of E which is robust to perturbations of $w \in \mathfrak{R}^{nI}_{++}$. As a consequence, intermediary h's profits, $\pi^h(E, \gamma, A)$, are not in general a continuous function, for any h.

the equilibrium concept constructed in the previous section only requires the existence of some admissible intermediaries' profit functions (or equivalently of some belief system, $b_{p,q}$) for which a Nash equilibrium of the game exists. In other words, the equilibrium concept requires a common knowledge assumption on equilibrium selection mechanisms to guarantee existence in the intermediaries' game (this is perfectly in the spirit of rational expectations equilibrium concepts).

The definition of *equilibrium set* allows for random intermediaries' beliefs over competitive equilibrium prices (in general we let $b_{p,q} \in \Delta_{\mathfrak{R}^n_{++} \times \mathfrak{R}^J}$); and in the case of the sunk cost game, G_{sc}, we have to allow for some form of correlation over Nash equilibria of the sub-games to be able to prove existence. This weakens the requirements of the equilibrium concept.

The existence proof (relegated in the Appendix) exploits and extends Nash equilibrium existence results for games with upper-hemicontinuous profit correspondences due to Simon–Zame [34].[10]

THEOREM 1. *For every economy* $\langle u, w, G \rangle$, *with* $G \in \{G_{fc}, G_{sc}\}$, *which satisfies Assumptions* 1 *and* 2, *the* equilibrium set *is non-empty.*

The number of securities each intermediary h can issue in equilibrium, J^h, is exogenous. Since the set of possible securities in intermediaries' choice set include securities with zero payoffs (to which zero fixed costs are associated), in fact J^h only imposes an upper bound on the number of securities. We turn next to show that in fact there is an upper bound on the number of securities each intermediary would wish to issue in equilibrium. In other words the constraint imposed by fixing J^h exogenously is not binding for J^h large enough (though finite).

THEOREM 2. *For every economy* $\langle u, w, G \rangle$, *with* $G \in \{G_{fc}, G_{sc}\}$, *which satisfies Assumptions* 1 *and* 2, $\exists$ *a finite* $\bar{J}$ *such that if the number of securities that intermediary* h *can trade is* $J^h > \bar{J}$, $\forall h$, *then each intermediary will not issue more than* $\bar{J}$ *securities. Moreover,* $\bar{J}$ *can be taken independent of the specification of the economy,* $\langle u, w, G \rangle$, *and independent of* $w^h, \bar{c}$.

An upper bound for the number of securities issued in equilibrium immediately derives from the fact that a positive fixed costs is associated to issuing any security, and intermediaries' endowments are bounded. Theorem 2 though proves the much stronger result that the upper bound on the number of securities can be taken to be independent of the specification of the economy and especially of intermediaries' endowments and costs. This result holds because each agent does not need more than S securities to span his endowment risk. As a consequence, if more than SI securities are issued by the same intermediary in equilibrium, some of them are redundant. But since issuing a security requires a fixed cost and intermediaries perfectly anticipate equilibrium allocations, they never issue redundant securities (see the proof in the Appendix for details).

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[10] Note also that excess demand systems are not smooth for the class of economies studied in this paper for $\gamma > 0$. Smoothness is not required for existence via fixed points, and local uniqueness can be studied e.g. using non-smooth analysis (cf. Clarke [13]). This is outside the scope of the present paper, but cf. Bisin [7].

4. NOMINAL SECURITIES AND INDETERMINACY

Balasko–Cass [5] and Geanakoplos–Mas Colell [21] have proved that competitive equilibria display *real indeterminacy* for the same class of economies studied in this paper, except that in their economies financial markets are characterized by an incomplete set of securities whose payoffs are exogenously given in units of account ("nominal" securities). These real indeterminacy results have been extended in various directions (see, e.g., Cass [11] for a survey) and has been exploited to show e.g. (i) monetary non-neutrality (Gottardi [22], Magill–Quinzii [27]); (ii) examples of sunspot equilibria (see Cass [10] for a survey); (iii) non-revealing rational expectation equilibria (Polemarchakis–Siconolfi [31], Rahi [32]).

The real indeterminacy result for economies with exogenous "nominal" securities is easily understood by examining the budget constraints 5–6). If (γ, A) are exogenously given, the budget sets are not homogeneous in prices: Perturbing v perturbs (i) the spreads in "numeraire" terms, γ/v; (ii) the span of the "securities'" payoff matrix in "numeraire" terms, $A/v = [a_1/v, ..., a_J/v]$. This in turn generically perturbs real equilibrium allocations as well when markets are incomplete. As a consequence real equilibrium allocations can be parametrized by v. In other words the real indeterminacy for economies with given "nominal" securities consists in the dependence of real equilibrium allocation of price levels v.

On the other hand, when securities' payoffs and spreads (γ, A) are chosen by financial intermediaries, then they generally in equilibrium will depend on v. Suppose moreover that securities are implicitly fully indexed in equilibrium, i.e., intermediaries choose (γ, A) so that spreads and securities' payoffs in "numeraire" terms $(\gamma/v, A/v)$ are independent of v. In this case the economy is effectively one in which securities payoffs are denominated in units of numeraire, homogeneity of agents' budget constraints is re-established, and real allocations are independent of price levels. As a consequence the dependence of real equilibrium allocations on v breaks down when (γ, A) are perfectly indexed in equilibrium.

But under which conditions do optimizing intermediaries choose to issue fully indexed securities? (Note that full indexation can occur in the intermediaries' mind; no explicit indexation clauses on securities' payoffs are needed for these arguments.) It turns out that full indexation, as an equilibrium property, only requires the following form of "no-money-illusion" on the part of intermediaries (the argument does not depend on which game $G \in \{G_{fc}, G_{sc}\}$ intermediaries play):

1. intermediaries' profit function must be defined in "real" terms;

2. intermediaries' strategy set must be independent of price levels; and also

3. intermediation costs must depend on real securities' payoffs.

Why is it never optimal for intermediaries to only partially index securities, if (1–3) are satisfied? This is because partial indexation implies that $(\gamma^h/v, A^h/v)$ for some h depends on v; hence intermediary h's equilibrium profit depend on v. But this is never optimal (intermediary h could guarantee himself the maximal level of profits for any v) if intermediary h only cares about "real" profits (Condition 1), and his profit function only depends on v through $(\gamma^h/v, A^h/v)$ (this is guaranteed by Conditions (2) and (3)).

But how restrictive are requirements (1–3)? Are they satisfied in our set-up? Requirement (1) is a basic no-money illusion condition, and it is satisfied because intermediaries maximize profits in units of numeraire (but equivalently we could have modelled intermediaries as agents with well-defined preferences, or as firms owned by agents; Dierker–Grodal [14] study the indeterminacy due to requirement (1) not being satisfied). Also requirement (2) is satisfied because intermediaries can effectively issue any security with payoff $A^h \in \mathfrak{R}^{J^h}$ ($A^h \in \mathscr{A}^0_{J^h}$ is just a normalization given Assumption 2). Whenever requirement (2) is not satisfied, variations of the price levels may expand the strategy set of the intermediaries. Obviously then full indexation might not be optimal. In this sense, for instance, exogenous restrictions to the intermediaries' choice set are at the root several results on optimality of partial indexation (cf. e.g., the early contributions of Fisher [18] and the collection of papers in Dornbusch–Simonsen [15]; and, more recently, Benassy [6], Freeman–Tabellini [19], and Magill–Quinzii [29]). Finally requirement (3) is satisfied if costs satisfy the following Assumption.

Assumption 3. Intermediation costs satisfy

$$\varepsilon(a, v) = \varepsilon(a/v), \qquad c(a, v) = c(a/v), \qquad \forall a \neq 0, \, v \in \mathfrak{R}^{S+1}_{++}.$$

The assumption restricts transaction costs to depend on securities' payoffs in "numeraire terms." It generally encompasses the homogeneity condition in Assumption 2. (But the reader can immediately verify that these conditions are equivalent in the focal example of constant transaction costs.) It has the important property that, together with Assumption 2, it

implies that $\varepsilon(a/v)$, $c(a/v)$ are respectively homogeneous of degree 1 and 0 in a/v.[11]

We are now ready to state our neutrality result, Proposition 1.[12]

PROPOSITION 1. *For every economy* $\langle u, w, G \rangle$, *with* $G \in \{G_{fc}, G_{sc}\}$, *which satisfies Assumptions* 1–3, *the set of real equilibrium allocations,* $\mathfrak{X}(G)$, *is independent of* v.

Remark 2. As argued above, Proposition 1 can be interpreted as a full indexation result. But Proposition 1 is easily interpreted as well as a monetary neutrality result. Suppose in fact that Quantity Theory equations are added to determine the price level (this can be formally justified in a Cash in Advance environment, cf. Magill–Quinzii [27]):

$$v \propto m, \qquad m \in \mathfrak{R}^{S+1}_{++}$$

where m denotes the vector of money supply at all times and states. In this case then Proposition 1 implies that real equilibrium allocations are independent of money supply m.

Proposition 1 also shows that sunspot equilibria and non-revealing rational expectation equilibria cannot be constructed by perturbing the price levels v (as done in the literature referred to in the beginning of this section), when securities' payoff are chosen by profit maximizing intermediaries.[13]

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[11] An example of a cost structure which does not satisfy Assumptions 2 and 3 is the following. Suppose intermediation costs increase with the "informational complexity" of the security, e.g. costs are higher if securities payoffs are conditional to S:

$$\varepsilon(a, v) = \begin{cases} 0 & \text{for} \quad a = 0, \\ \varepsilon_1 & \text{for} \quad a_s = a, \forall s, \\ \varepsilon_2 & \text{otherwise,} \end{cases}$$

where $\varepsilon_1 < \varepsilon_2$. This cost structure captures an economy in which it is costly for intermediaries to observe ex-post (i.e., once it is realized) the state s. If price levels are correlated with s (and are freely observable) it might be optimal for intermediaries to condition securities' payoffs to the price levels v (i.e., not to fully index). In this case though neutrality will be generically recovered if intermediaries were allowed to condition securities' payoffs to relative prices.

[12] Proposition 1 is in fact more general than stated. It holds for a larger class of games played by intermediaries (including, e.g., games with sequential entry), for more general structures of intermediation costs (e.g., depending on relative prices of goods), and is robust to limitations on the rationality of intermediaries' beliefs (e.g., in the spirit of "temporary equilibria").

[13] However the result has obviously no direct implications in general for sunspot equilibria as constructed in the literature started by Shell [33] and Cass–Shell [12].

Remark 3. Proposition 1 proves that, for economies with optimizing intermediaries, the set of equilibrium allocations is independent of v. Even though the real indeterminacy result for economies with given "nominal" securities in the literature is due to the dependence of the set of equilibrium allocations on v, Proposition 1 does not imply that equilibrium allocations for economies with optimizing intermediaries are locally unique. In other words, we cannot claim that equilibria are determinate when security design is introduced. The reason is the following: the equilibrium concept, as constructed in Section 2, requires intermediaries to form beliefs which are consistent with competitive equilibrium prices for any possible choice of securities' spreads and payoffs (i.e., as selections of $E(\gamma, A)$). But since in general competitive equilibria are not unique ($E(\gamma, A)$ is a correspondence) many systems of beliefs satisfy the rational expectations consistency requirement (are "admissible" in our terminology). This multiplicity of admissible beliefs could potentially generate an indeterminacy of equilibrium allocations. This is only natural in economies in which a subset of agents are not price-takers and form rational expectations about competitive prices.

But it is important to note that the nature of the possible indeterminacy is nonetheless quite different with and without security design. In the first case it is only due to the multiplicity of beliefs of intermediaries about equilibrium relative prices, while in the second case it is due to the (assumed) inability of financial markets to react to differences in the level of prices.

Also, our neutrality result implies that indeterminacy is not a generic property of the equilibrium set (while it is when securities' payoffs are exogenous). This point can be easily illustrated. Robust economies which have unique competitive equilibria for a subset of the securities payoffs can be easily constructed. And cost structures such that only as subset of the securities can effectively be issued by optimizing intermediaries can similarly be constructed. This is in fact enough to generate robust economies with determinate equilibrium allocations.

The only paper to my knowledge which studies directly the *real indeterminacy* result for economies in which intermediaries design securities is Pesendorfer [30]. Pesendorfer [30] models perfectly competitive intermediaries who choose the payoffs of derivative securities on a set of basic securities whose payoffs are exogenously given in "nominal" terms, and which can be costlessly traded. In this set-up Pesendorfer [30] shows that real allocations are indeterminate in equilibrium, but the indeterminacy shrinks (in a precise way) whenever costs tend to zero, since then financial markets approach completeness. In other words neutrality holds only in the limit when innovation requires no costs, financial markets are complete, and competitive equilibrium allocations are Pareto efficient.

Proposition 1 on the contrary proves neutrality even in the case in which markets are incomplete (and intermediation costs arbitrarily high).

The reason why Pesendorfer [30] cannot eliminate the dependence of equilibrium allocations on price levels for positive costs is that in his set-up some of the securities traded have "nominal" payoffs exogenously given; this is in turn assumed to justify the competitive equilibrium concept used (since then derivative securities issued have no effect on the span of financial markets and hence on equilibrium prices). We take this to partially motivate our complex construction of a class of economies with imperfectly competitive intermediaries.

5. CONCLUSIONS

This paper has developed a model of financial innovation, based on intermediation costs and strategic interaction across intermediaries, in a general equilibrium framework. The model delivers interesting implications about "neutrality" of equilibrium allocations.

While in our opinion the model is also well suited as a foundation for detailed analysis of financial markets, much more structure has to be imposed, e.g. on the institutional aspects of the organization of intermediaries and on the structure of costs, to derive implications about the properties of optimally designed securities and about the endogenous structure of incompleteness of financial markets. Concepts and methods of the industrial organization literature appear easily and profitably applicable to this end.

APPENDIX: PROOFS

Proof of Theorem 1. Fix an arbitrary $w \in \mathfrak{R}^{nI}_{++}$. Let int$[X]$ denote the interior of a set X. The proof requires four basic steps. Step 1 proves a closed graph property for $E(\gamma, A)$; Step 2 proves that $E(\gamma, A)$ is non-empty valued. Step 3 proves non-emptiness of $\mathscr{E}_{fc}$. Finally Step 4 shows non-emptiness of $\mathscr{E}_{sc}$.

It is convenient to study the properties of $E(\gamma, A)$ for $A \in \mathscr{A}_J = \{A \in \mathfrak{R}^{SJ} \mid \max_s \mid a_{sj}/p_{1s} \mid = 1; \; \forall j \in J\}$, and then to exploit the fact that $\mathscr{A}^0_J = \mathscr{A}_J \cup [\mathscr{A}_{J-1} \cup \{0\}] \cup [\mathscr{A}_{J-2} \cup \{0, 0\}] \cdots \cup \{0, ..., 0\}$.

Step 1. Normalize $p_{10} = p_{1s} = 1$, $\forall s$. The *competitive equilibrium correspondence* $E(\gamma, A)$ has closed graph, for (γ, A) in a compact set.

The proof requires some preparatory lemmas.

LEMMA 1. *Fix* $(\gamma, A) \in \Gamma_J \times \mathscr{A}_J$. *Assumption* 2 *implies that* $\exists$ *convex, compact sets* L_-, L_+ *such that in equilibrium*

$$z^i_- \in int[L_-], \qquad z^i_+ \in int[L_+], \qquad \forall i.$$

Proof. Both z^i_+ and z^i_- are non-negative, for all i. Also in equilibrium $\sum_i z^i_+ = \sum_i z^i_-$. We then just need to construct an upper bound on z^i_+. But, given γ, clearly in equilibrium $\gamma z^i_+ \leqslant \sum_i w^i_{10} + \sum_{h \in H} w^h$. The construction of L_-, L_+ is now trivial. ∎

Define the aggregate endowments $\bar{w}_{10} = \sum_i w^i_{10} + \sum_h w^h$, $\bar{w}_{l0} = \sum_i w^i_{l0}$, $\overline{w_{ls}} = \sum_i w^i_{ls}$, $\forall l$, $\forall s$. Also,

$$\bar{w}_0 = \begin{bmatrix} \bar{w}_{10} \\ \vdots \\ \bar{w}_{L0} \end{bmatrix}, \qquad \bar{w}_s = \begin{bmatrix} \bar{w}_{1s} \\ \vdots \\ \bar{w}_{Ls} \end{bmatrix}.$$

Construct the set $K \subset \mathfrak{R}^{nI}_+$ as a compact set containing

$$\begin{bmatrix} 0, 2 \begin{bmatrix} \bar{w}_{10} \\ \vdots \\ \bar{w}_{LS} \end{bmatrix} \end{bmatrix}.$$

Let $x^i(p, q, \gamma, A)$, $z^i_{+j}(p, q, \gamma, A)$, $z^i_{-j}(p, q, \gamma, A)$, $\forall j$, denote agent i's demands derived from the maximization of 4 subject to:

$$\{(x^i, z^i_+, z^i_-) \in K \times L_+ \times L_- \mid 5, 6 \text{ hold}\}.$$

(Truncating demands this way is obviously without loss of generality in equilibrium.) Let

$$d^i(\cdot) = \begin{bmatrix} x^i(\cdot) \\ z^i_+(\cdot) \\ z^i_-(\cdot) \end{bmatrix},$$

and finally denote with

$$f(p, q, \gamma, A) = \begin{bmatrix} \sum_i x^i_{10}(\cdot) + \sum_h x^h - \bar{w}_{10} \\ \vdots \\ \sum_i x^i_{LS} - \bar{w}_{LS} \\ \sum_i (z^i_+(\cdot) - z^i_-(\cdot)) \end{bmatrix}$$

the aggregate excess demand.

LEMMA 2. *For the class of economies satisfying Assumptions* 1 *and* 2, $d^i: \mathfrak{R}^n_+ \times \mathfrak{R}^J \times \Gamma_J \times \mathscr{A}_J \to K \times L_+ \times L_-$ *and* $f: \mathfrak{R}^n_+ \times \mathfrak{R}^J \times \Gamma_J \times \mathscr{A}_J \to K \times \mathfrak{R}^J$ *are upper-hemicontinuous convex valued correspondences.*

Proof. It trivially suffices to prove upper-hemicontinuity for d^i, mapping (p, q, γ, A) into (x^i, z^i_+, z^i_-), $\forall i$. The same properties then follow for $f(\cdot)$. Budget feasibility restricts agent i's allocations to:

$$\mathscr{B}(p, q, \gamma, A) = \{x^i \in K \mid \exists (z^i_+, z^i_t) \in \times L_+ \times L_t \text{ s.t. 5, 6 hold}\}$$

which is easily shown to be a continuous, non-empty, compact, and convex valued correspondence. Assumption 1 then guarantees that $x^i(\cdot)$ is a continuous function, and that $z^i_+(\cdot)$ and $z^i_-(\cdot)$ are upper-hemicontinuous convex valued correspondences (not continuous functions since A has not necessarily full rank). This proves the lemma. ∎

LEMMA 3. *Pick arbitrary compact subsets of* Γ_J, $\mathscr{A}_J$, *denoted respectively* G, C. *Then the set*

$$\{(p, q, \gamma, A) \in [\mathfrak{R}^n_+ \times \mathfrak{R}^J \times G \times C] \cap [f^{-1}(0)] \mid p_{10} = p_{1s} = 1, \forall s\}$$

is also compact.

Proof. Fix[14] $(\gamma, A) \in G \times C$. Define with $\Pi: f^{-1}(0) \to \Gamma_J \times \mathscr{A}_J$ the projection from (p, q, γ, A) into (γ, A) with $p_{10} = p_{1s} = 1$, $\forall s$. We must show $\Pi^{-1}(G \times C)$ is compact. Note that $(x^i_0, x^i_s) < (\hat{w}^i_0, \hat{w}^i_s) = \hat{w}$ for some upper bound $\hat{w}$, $\forall \gamma \in G$, $\forall A \in C$. By optimization and Assumption 1, $u^i(\underline{w}^i) \leqslant u^i(x^i)$ for some $\underline{w}^i \leqslant w^i$ bounded away from 0, $\forall \gamma \in G$, $\forall A \in C$. By Lemma 1, L_+ and L_- are compact. Then $R^i = \{(x^i, z^i_+, z^i_-) \in \mathfrak{R}^n_+ \times L_+ \times L_- \mid u^i(\underline{w}^i) \leqslant u^i(x^i); x^i \leqslant \hat{w}\}$ is compact.

Consider the correspondence $m: \mathfrak{R}^n_+ \times L_+ \times L_- \times G \times C \to \mathfrak{R}^n_+ \times \mathfrak{R}^J$, mapping $(x^i, z^i_+, z^i_-, \gamma, A)$ into (p, q), defined by: $p_{10} = p_{1s} = 1$; $\forall s \in S$; $p_{l0} = (\partial/\partial x^i_{l0}) u^i(x^i)/(\partial/\partial x^i_{10}) u^i(x^i)$, $\forall l$; $p_{ls} = (\partial/\partial x^i_{ls}) u^i(x^i)/(\partial/\partial x^i_{1s}) u^i(x^i)$, $\forall l$; $\forall s$; $\sum_s a_{sj}(\partial/\partial x^i_{1s}) u^i(x^i)/(\partial/\partial x^i_{10}) u^i(x^i) - \gamma_j \leqslant q_j \leqslant \sum_s a_{sj}(\partial/\partial x^i_{1s}) u^i(x^i)/(\partial/\partial x^i_{10}) u^i(x^i)$. $m(\cdot)$ is then a continuous correspondence; hence $m(R^i)$ is compact.

Since $\sum_s a_{sj}(\partial/\partial x^i_{1s}) u^i(x^i)/(\partial/\partial x^i_{10}) u^i(x^i) - \gamma_j \leqslant q_j \leqslant \sum_s a_{sj}(\partial/\partial x^i_{1s}) u^i(x^i)/(\partial/\partial x^i_{10}) u^i(x^i)$ is a no-arbitrage restriction (cf. Jouini–Kallal [25]), clearly $\Pi^{-1}(G \times C) \subseteq (\cap_i m(R^i)) \times G \times C$, a compact set. Since Π is continuous and $(G \times C)$ is closed, $\Pi^{-1}(G \times C)$ is a closed subset of $E(G \times C)$; since $f(\cdot)$ is upper-hemicontinuous (by Lemma 2), and $\{0\}$ is closed, $E(G \times C)$ is a closed subset of $\mathfrak{R}^n_+ \times \mathfrak{R}^J \times \Gamma_J \times \mathscr{A}_J$.

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[14] Cf. Woodford [38] for a similar argument in a different context.

As a consequence $\Pi^{-1}(G \times C)$ is a closed subset of $m(R^i) \times G \times C$, hence compact. ∎

This proves that, once prices are normalized and for $(\gamma, A) \in \Gamma_J \times \mathscr{A}_J$, the graph of $E(\gamma, A)$ is closed.

Noting that $\mathscr{A}_J^0 = \mathscr{A}_J \cup [\mathscr{A}_{J-1} \cup \{0\}] \cup [\mathscr{A}_{J-2} \cup \{0, 0\}] \cdots \cup \{0, ..., 0\}$, it is trivial to show that extending the domain of $E(\cdot)$ to $\Gamma_J \times \mathscr{A}_J^0$ preserves the closed graph property of E. This proves Step 1. ∎

Note that Lemma 3 allows us to construct a continuous correspondence which maps p_{10} into a set $Q(p_{10}) \subseteq \Re^J$ to which securities prices are restricted in equilibrium without loss of generality. Also, the correspondence is compact valued for any $p_{10} > 0$.

Step 2.   The *competitive equilibrium correspondence* $E(\gamma, A)$ is non-empty valued.

Fix an arbitrary $(\gamma, A) \in \Gamma_J \times \mathscr{A}_J$. We construct truncated price domains as follows. By Lemma 3, if $p_{10} \geq r > 0$, securities' prices are restricted in equilibrium without loss of generality to the compact set $Q_r = Q(r)$. We can then define the set

$$T_{0r} = \left\{ (p_0, q) \in \Re_+^L \times \Re^J \;\middle|\; \sum_{l=1}^{L} p_{l0} + \sum_{j=1}^{J} q_j = 1; \, p_{l0} \geq r, \, \forall l; \, q \in Q_r \right\}.$$

Also define

$$T_{sr} = \left\{ p_s \in \Re_+^L \;\middle|\; \sum_{l=1}^{L} p_{ls} = 1; \, p_{ls} \geq r, \, \forall l \right\}, \quad \forall s$$

and $T_r = T_{0r} \times T_{1r} \times \cdots T_{Sr}$.

Let $x(p, q)$, $z_+(p, q)$, $z_-(p, q)$ denote the aggregate demand and portfolios correspondences, respectively (for fixed $(\gamma, A) \in \Gamma_J \times \mathscr{A}_J$).

Consider now the correspondence

$$\phi: K \times L_+ \times L_- \times T_r \to K \times L_+ \times L_- \times T_r$$

mapping (x, z_+, z_-, p, q) into itself, and constructed as follows

$$\phi_1(x, z_+, z_-, p, q) = x(p, q)$$

$$\phi_2(x, z_+, z_-, p, q) = z_+(p, q)$$

$$\phi_3(x, z_+, z_-, p, q) = z_-(p, q)$$

$$\phi_4(x, z_+, z_-, p, q) = \arg\max_{(p, q) \in T_{0r}} p_0(x_0 - \bar{w}_0) + q(z_+ - z_-)$$

$$\phi_5(x, z_+, z_-, p, q) = \arg\max_{p_s \in T_{sr}} p_s(x_s - \bar{w}_s) - a_s(z_+ - z_-), \quad \forall s.$$

Using Lemmata 1–3, it is now straightforward to show that ϕ is upper-hemicontinuous, non-empty, compact, convex valued. By Kakutani Fixed Point Theorem, then ϕ has a fixed point, $(x^*, z^*_+, z^*_-, p^*, q^*)$.

The argument which shows that the fixed point is a competitive equilibrium, for r small enough, given $(\gamma, A) \in \Gamma_J \times A_J$, is now standard if we note that (i) Walras' Law implies that

$$p_0^*(x_0^* - \bar{w}_0) + q^*(z^*_+ - z^*_-) = 0; \qquad p_s^*(x_s^* - \bar{w}_s) - a_s(z^*_+ - z^*_-) = 0$$

and that (ii) the correspondence Q, constructed by Lemma 3 and in turn used to construct Q_r, is compact valued for any $p_{10} > 0$ (which is in turn guaranteed by a standard boundary behavior argument). ∎

Non-emptiness of $E(\gamma, A)$ for all $(\gamma, A) \in \Gamma_J \times \mathscr{A}_J$ obviously implies non-emptiness for all $(\gamma, A) \in \dot{\Gamma}_J \times \mathscr{A}_J^0$. Also, Steps 1 and 2 imply that $E(\gamma, A)$ is a upper-hemicontinuous correspondence.

Step 3. For any economy $\langle u, w, G_{fc} \rangle$, which satisfies Assumptions 1 and 2, the equilibrium set $\mathscr{E}_{fc}$ is non-empty.

It is clearly enough to show that $NE(G_{fc})$ is non-empty. Fix $v = 1$ (without loss of generality, cf. Proposition 1), and J^h, $\forall h$. Define the payoff vector correspondence, mapping $(\gamma, A) \in \Gamma_J \times \mathscr{A}_J^0$ into $\mathfrak{R}^H$, as follows: $\pi(\gamma, A)$ is the convex hull of $\pi(E(\gamma, A), \gamma, A)$, whose h entry is $\pi^h(E(\gamma, A), \gamma, A) = \sum_{j \in J^h} (\gamma_j - \varepsilon(a_j, 1)) z_{+j}(E(\gamma, A), \gamma, A) - c(a_j, 1)$.

The main theorem in Simon–Zame [34] can be directly used to show that the set of Nash Equilibria of the game G_{fc} is non-empty if (i) the strategy space $\Gamma_{J^h} \times \mathscr{A}_{J^h}^0$ is compact, $\forall h$, (ii) the profit correspondence $\pi(\gamma, A)$ is upper-hemicontinuous and compact, convex valued. Step 3 is then proved if (i) and (ii) are satisfied.

Proof of (i). $\mathscr{A}_{J^h}^0$ is compact $\forall h$; and γ^h can be restricted without loss of generality to $\Gamma_h = \{\gamma^h \in \Gamma_{J^h} \mid \gamma^h \leqslant \bar{\gamma}^h\}$, for some $\bar{\gamma}^h$ such that $z_+^i(E(\gamma, A), \gamma, A) = z_-^i(E(\gamma, A), \gamma, A) = 0$, for any i and any

$$\gamma > \bar{\gamma} = \begin{bmatrix} \bar{\gamma}^1 \\ \vdots \\ \bar{\gamma}^H \end{bmatrix}.$$

Such a $\bar{\gamma}$ can be constructed as follows:

$$\bar{\gamma}_j > \max_i \sum_{s=1}^S a_{sj} MRS_s^i(x^{ia}) - \min_i \sum_{s=1}^S a_{sj} MRS_s^i(x^{ia})$$

where $MRS_s^i(x^{ia}) = (\partial/\partial x_{1s}^{ia}) u^i(x^{ia})/(\partial/\partial x_{10}^{ia}) u^i(x^{ia})$, and x^{ia} denotes i's allocation at the "autarchic" competitive equilibrium, in which no trade in

securities is allowed (if more than one equilibrium exists just pick the one to which the maximum $\bar{y}$ is associated). Since Assumption 1 guarantees that $MRS^i_s(x^{ia})$ is finite, for any s at any autarchic equilibrium, the construction of Γ_h is done. This proves condition (i).

   Proof of (ii). $\pi^h(\gamma, A)$ is upper-hemicontinuous because compositions of upper-hemicontinuous correspondences are upper-hemicontinuous; it is convex valued because is defined as a convex hull; and finally it is compact valued trivially using Lemma 1, Lemma 3 and (i) above. This proves (ii) and Step 3. ∎

   Step 4. For any economy $\langle u, w, G_{sc} \rangle$, which satisfies Assumptions 1 and 2, the equilibrium set $\mathscr{E}_{sc}$ is non-empty.

   It is enough to show that $P^*NE(G_{sc})$ is non-empty. We proceed by back-ward induction. By the same main result in Simon–Zame [34] used in Step 3, there exists a mapping $\sigma_\gamma(A)$ from $\mathscr{A}^0_J$ to the set of Nash Equilibria (possibly in mixed strategies) of the last sub-game of G_{sc}, which we denote $NE(G_{sc}\,|\,A)$. The mapping $\sigma_\gamma(A)$ takes values in Δ_{μ_J}. Define $\pi(A)$ as the convex hull of $\pi(\sigma_\gamma(A), A)$, whose h entry is $\pi^h(\sigma_\gamma(A), A) = \int \sum_{j=1}^{J^h} [(\gamma_j - \varepsilon(a_j, 1)) z_{+j}(E(\gamma, A), \gamma, A) - c(a_j, 1)] \, d\sigma_\gamma(A)$. It clearly is sufficient to show that $\pi^h(A)$ is an upper-hemicontinuous correspondence to exploit again the theorem in Simon–Zame [34] and prove Step 4 ($(\pi(A)$ is trivially convex valued by construction and compact valued since $\pi(\gamma, A)$ is).

   Proof of Upper-Hemicontinuity of $\pi(A)$. Endow Δ_{μ_J} with the topology of weak convergence. Let $[\tilde{\gamma}^h, \gamma^{-h}]$ denote the vector γ in which the h entry is substituted by $\tilde{\gamma}^h$. Pick arbitrary sequences (going to sub-sequences when necessary) $A^k \to A$, and $\sigma^k_\gamma(A^k) \in NE(G_{sc}\,|\,A^k)$ such that $\sigma^k_\gamma(A^k) \to \sigma_\gamma(A)$. We need to show that $\sigma_\gamma(A) \in NE(G_{sc}\,|\,A)$. Suppose not. Then $\exists h$ and $\tilde{\gamma}^h \in \mu_{J^h}$ such that $\pi([\tilde{\gamma}^h, \gamma^{-h}], A) \geqslant \pi(\gamma, A)$ for γ in the support of $\sigma_\gamma(A)$. Also, either

$$\nexists \tilde{\gamma}^{h,k} \to \tilde{\gamma}^h \quad \text{and} \quad \gamma^{-h,k} \to \gamma^{-h} \text{ such that}$$

$$\pi([\tilde{\gamma}^{h,k}, \gamma^{-h,k}], A^k) \to \pi([\tilde{\gamma}^h, \gamma^{-h}], A) \text{ or else}$$

$$\nexists \gamma^{h,k} \to \gamma^h \quad \text{and} \quad \gamma^{-h,k} \to \gamma^{-h} \text{ such that}$$

$$\pi([\gamma^{h,k}, \gamma^{-h,k}], A^k) \to \pi([\gamma^h, \gamma^{-h}], A)$$

both of which contradict upper-hemicontinuity of $\pi(\gamma, A)$. ∎

   Proof of Theorem 2. Fix $v = 1$ without loss of generality; cf. Proposition 1. Some notation is necessary. We say a portfolio $r(j) \in L_+ \times L_-$ *reproduces* security j if $Ar(j) = a_j$. Call $R(j)$ the set of portfolios in $L_+ \times L_-$ *reproducing* security j. Call $y_+(r(j))$ (respectively $y_-(r(j))$) the

buying (resp. selling) price of portfolio $r(j) \in R(j)$. We can then construct, for any securities' payoff matrix A, the set of bid and ask prices such that any security j is not dominated by some other portfolio (let this set be denoted by $O(A)$):

$$O(A) = \{ (\gamma, q) \in [\Gamma_1 \times \cdots \Gamma_H] \times \Re^J \mid \gamma_j + q_j \leqslant \min_{r(j) \in R(j)} y_+(r(j)),$$

$$q_j \geqslant \max_{r(j) \in R(j)} y_-(r(j)) \}.$$

We first show that by choosing γ intermediaries can effectively control if at equilibrium (γ, q) turns out in the interior of $O(A)$. More precisely, suppose q is a competitive equilibrium securities' price vector for an economy $\langle u, w, G \rangle$, and for given A. Then if $(\gamma, q) \notin \text{int}[O(A)]$, there does not exist a $(\gamma, \hat{q})$ such that $\hat{q}$ are competitive equilibrium prices for the same economy and $(\gamma, \hat{q}) \in \text{int}[O(A)]$. To check that this is so, just note that $\gamma_j + q_j \geqslant \min_{r(j) \in R(j)} y_+(r(j))$ and $q_j \leqslant \max_{r(j) \in R(j)} y_-(r(j))$ imply $\gamma_j \geqslant \min_{r(j) \in R(j)} y_+(r(j)) - \max_{r(j) \in R(j)} y_-(r(j))$ (the right-hand-side is independent from q). Hence $\gamma_j + \hat{q}_j < \min_{r(j) \in R(j)} y_+(r(j))$ requires $\hat{q}_j < \max_{r(j) \in R(j)} y_-(r(j))$, which is in contradiction with $\hat{q}$ being a competitive equilibrium securities' price vector.

Also, it is easy to see that, if $(\gamma_j, q_j)_{j=1, ..., J^h} \in \text{int}[O(A^h)]$, the subset of securities $j = 1, ..., J^h$ such that $z^i_{j+} > 0$ is $\leqslant S$, for any i (agents are indifferent on trading securities j characterized by either $\gamma_j + q_j = \min_{r(j) \in R(j)} y_+(r(j))$, or $q_j = \max_{r(j) \in R(j)} y_-(r(j))$).

Take then any (γ, A) in the support of the Nash equilibria of game G: If the dimensionality of the non-zero column vectors in A (the number of the non-trivial securities issued by intermediaries) is $> SI$, there exists a security j such that either $(\gamma_j, q_j) \notin \text{int}[O(A)]$ or $z^i_{j+} = 0$, $\forall i$. This is a contradiction with (γ, A) being in the support of the Nash equilibria of game G.

Moreover, the argument is clearly independent from $\langle u, w, G \rangle$ and from $w^h, \bar{c}$. ∎

Proof of Proposition 1. It is convenient to explicitly index variables, correspondences and sets by v: $x^i(p, q, \gamma, A; v)$, $z^i_+(p, q, \gamma, A; v)$, $\pi^h(\gamma, A; v)$, $E(\gamma, A; v)$.

The proof simply consists in showing that (i) $x^i(p, q, \gamma, A; v)$, $\forall i$, and $\pi^h(\gamma, A; v)$, $\forall h$, depend on v only via $(\gamma/v, A/v)$; (ii) $\Gamma_h \times \mathscr{A}^0_{J^h}$ is independent of v.

Clearly $x^i(p, q, \gamma, A; v)$, $\forall i$, depends on v only via $(\gamma/v, A/v)$ by the homogeneity properties of agents' budget constraints (Eq. 5, 6). Similarly for $z^i_+(p, q, \gamma, A; v)$. As a consequence $E(\gamma, A; v)$ also depends on v only via $(\gamma/v, A/v)$. And so it is then for $z^i_+(E(\gamma, A, v), \gamma, A; v)$. Finally then, using

Assumption 3, $\pi^h(\gamma, A; v)$ depends on v only via $(\gamma/v, A/v)$. This proves (i). Moreover (ii) is clear by the definition of $\Gamma_h \times \mathscr{A}^0_{jh}$. ∎

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