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Review of Economic Dynamics 6 (2003) 313–338
Review of Economic Dynamics
www.elsevier.com/locate/red

Competitive markets for non-exclusive contracts with adverse selection: the role of entry fees

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Received 5 April 2000; revised 10 January 2002

Abstract

This paper studies competitive equilibria in economies characterized by the presence of asymmetric information, where non-exclusive contracts are traded in competitive markets and agents may be privately informed over contracts' payoffs. For such economies competitive equilibria may not exist when contracts trade at linear prices. We show that (non-trivial) competitive equilibria exist, under general conditions, when prices exhibit a minimal form of non-linearity (or, equivalently, a minimal requirement on the observability of agents' trades): the presence of two-part tariffs suffices, where the cost of trading each contract consists of an entry fee and a linear component in the quantity traded. The entry fee is determined at equilibrium and represents a measure of adverse selection in the economy.

JEL classification: D50; D82

Keywords: General equilibrium; Asymmetric information; Two-part tariffs

1. Introduction

This paper studies competitive equilibria in economies characterized by the presence of asymmetric information. We consider pure exchange economies where standardized, non-exclusive contracts are traded in competitive markets.

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1094-2025/03/$ – see front matter
doi:10.1016/S1094-2025(02)00008-X

Contracts are standardized as their terms (price and payoff specification) are independent of the identity of the agents entering the contract, and the same type of contract is available to many agents; they are non-exclusive as the terms of each contract do not depend on the transactions made by an agent in other markets (or even in the same market).[1] Contracts differ from securities as their payoff depends on the realization of the individual uncertainty affecting the agent entering the contract (as, for instance, in the case of an individual insurance contract). Hence contracts of the same type, when entered by different agents, are in effect different contracts as their return is contingent on the realization of different sources of uncertainty. To the eyes of the outside investors trading such contracts, however, they are indistinguishable; thus there is a single market—and price—for all contracts of the same type. To make explicit the role and activity of outside investors we assume that, in addition to consumers, there are competitive intermediaries who 'make the market' for contracts by compensating the buying and selling positions of the agents trading the contract.

We will examine situations where agents have some private information over the realization of their own individual risk. Since this risk only affects the payoff of the specific contracts an agent enters, and all contracts of the same type, but entered by different agents, trade at the same price, the trades of any agent are negligible with respect to the size of the market; hence we can argue that in this set-up asymmetric information is not incompatible with price taking behavior (see also Bisin et al., 1999). On the other hand, while with symmetric information the payoff distribution of contracts of the same type, at the time agents enter the contract, is the same for all of them, this is no longer true when agents have some private information. In that case the payoff distribution will be different for agents with different information. If these contracts are traded together in a single market at linear prices, (uninformed) intermediaries who 'make the market' may as a consequence end up having negative profits, whatever the price level. To insure that this does not happen, and hence the viability of markets and the existence of competitive equilibria in the presence of asymmetric information and informed traders, some degree of non-linearity of prices is needed; this provides in fact a mechanism according to which the losses intermediaries (outside investors) make by trading with agents with superior information can be recovered.

We are interested here in characterizing a minimal form of non-linearity of the pricing of contracts, which guarantees existence and can be implemented *with minimal requirements on the observability*[2] *of agents' trades*.[3] In this paper we show that competitive equilibria

[1] See Hellwig (1983), Arnott and Stiglitz (1993), Dubey et al. (1995), Bisin and Guaitoli (1995), Bisin and Gottardi (1999) and Kahn and Mookherjee (1998) for other analyses of economies where non-exclusive contracts are traded.

[2] Though no explicit assumption is made in the paper over the observability of agents' trades, our analysis is particularly relevant in situations where the information available over agents' trades is very limited (neither trades in the other markets nor even trades in the same market can be monitored by intermediaries). Thus contracts must trade at prices which are, essentially, linear.

[3] When agents' trades can be fully monitored, and hence exclusive contracts are available, the existence (and the efficiency) of competitive equilibria with moral hazard was shown by Prescott and Townsend (1984) (see also Bennardo (1997), Bennardo and Chiappori (1998), Citanna and Villanacci (1997), Kehoe et al. (1998), Lisboa (1997), Magill and Quinzii (1997)) and, more recently, by Bisin and Gottardi (2000) for adverse selection economies.

exist, for general economies with asymmetric information, if the unit price of any contract is independent of the quantity traded, but agents also have to pay, to be able to transact in the market for the contract, a fixed entry fee (i.e., with a two-part tariff scheme). Obviously, with sufficiently high entry fees the existence of 'trivial' equilibria, with a zero level of trade in contracts, can be easily established. We show however that other, 'non-trivial' equilibria always exist, where the entry fee is set at zero in all non-active markets (in which no agent wishes to trade). Hence at these equilibria there is generically trade in all markets. In each active market the entry fee is set at a level which perfectly offsets the profits made by the informed traders when trading the contract with the uninformed intermediaries. At equilibrium the total profit of each intermediary 'making' a market is then zero; thus there is no cross-subsidization across markets for different contracts.

To better illustrate the nature of the markets we are considering, the role of entry fees and their relationship with asymmetric information, we will analyze more in detail the properties of the equilibria we obtain within the set-up of a simple economy, constituting our leading example. There we show that, in the absence of entry fees, competitive equilibria may not exist; also, that the level of the entry fee at equilibrium increases when agents' private information increases. Entry fees provide therefore a measure of the level of asymmetric information in the economy.

In our set-up, the role of intermediaries in ensuring the clearing of markets with asymmetric information is threefold:

(i) they pool together and compensate all buying and selling positions of the agents in a market (their own net position then only depends on aggregate risk);
(ii) they fully hedge their net position in financial markets;
(iii) they charge an entry fee to be able to fully fund their hedging portfolio.

Though our set-up is admittedly rather abstract, the market clearing mechanism we consider seems to capture some important features of the ways financial markets actually clear. Insurance companies do pool agents' positions to diversify agents' idiosyncratic shocks, so do exchanges with regard to traders' default risk, and banks with mortgage repayment risks. Also, intermediaries trade in financial markets to hedge the aggregate risks in their own position, for instance by securitizing their portfolios (see Kendall and Fishman (1998) for various examples). Finally, examples of two-part price schedules are relatively common, for instance in derivatives' exchanges or in over-the-counter markets. In these markets traders may be able to take both short and long positions at the same (unit) price, and dealers or exchanges compensate the traders' different positions in securities and guarantee against their default risks upon charging them an entry fee.

The introduction of an entry fee generates a non-convexity in the agents' choice problem. By considering a large economy, with infinitely many agents, of finitely many types, we will show that the aggregate excess demand is convex-valued. However, to show the existence of *non-trivial* competitive equilibria the standard existence arguments for large economies in the presence of non-convexities cannot be applied and our proof contains various elements of novelty. We need in fact to insure a consistency condition, requiring that only the agents actively trading in a market pay the fee, and that the fee's level is such that the intermediaries' profits are non-negative in each market. To this end, we show that the individual demand correspondence can be written as the union of finitely many single-valued maps. We then exploit this property of individual demand to construct a map which describes the agents' participation rate in the various markets; this allows us to show in a subsequent step that the correspondence yielding the equilibrium level of the entry fee is well-behaved over its whole domain.

Our result shows that price schedules characterized by two components, the linear unit price and the entry fee, are sufficient to clear markets in economies with asymmetric information, whatever the nature of the set of contracts available for trade, and independently of the 'dimension' of the sources of asymmetric information in the economy (i.e., of the cardinality of the set of possible unobservable types or actions of the agents trading the contracts). The information on agents' trades which is required to implement two-part price schedules is clearly minimal: one only needs to be able to identify the agents who trade in a market.

In a related paper (Bisin and Gottardi, 1999) we have shown that the existence of competitive equilibria with asymmetric information is also guaranteed, without an entry fee, if the buying and selling prices of contracts are possibly different (but are otherwise still a linear function of the quantity traded). The spread between bid and ask prices plays a similar role to the entry fee, by allowing to recover the losses made on average, per unit traded, by uninformed investors. The argument of the proof is however quite different and, in fact, simpler.

While to implement bid–ask spreads only a very limited information over agents' trades is also needed, the informational requirement of entry fees is arguably even smaller: for bid–ask spreads we simply have to be able to separate buyers and sellers (i.e., only the sign of each agent's transaction needs to be observed), while for entry fees it suffices to separate agents who trade and those who do not trade in a market.

Combining our existence results with bid–ask spreads and entry fees, we obtain that competitive equilibria with arbitrary piecewise linear price schedules (and hence, effectively, with any non-linear price schedule) also exist, under general conditions. General forms of non-linearity are clearly informationally more demanding than bid–ask spreads or entry fees; on the other hand, they may allow to enhance incentives and hence to obtain equilibria with better welfare properties.

Within the set-up of our leading example, we also compare the properties of competitive equilibria with entry fees and with bid–ask spreads. We show that equilibria with entry fees can be Pareto superior, for appropriate specifications of the two-part tariff schedule (in particular, when the linear component is sufficiently high).

The analysis is developed in the framework of a two-period, pure exchange economy with adverse selection.[4] The structure of the economy and the notion of competitive equilibrium with entry fees are presented in Section 2. The existence of 'non-trivial' equilibria is then shown in the following section where the properties of the equilibria in the economy constituting our leading example are also analyzed.

4 All our results extend to economies with moral hazard. As shown by Bisin and Gottardi (1999), the existence problem with linear prices and the conditions under which competitive equilibria exist are essentially the same with moral hazard and with adverse selection.

2. The economy

Our main result is derived for a general economy with heterogeneous agents where there is both aggregate and idiosyncratic uncertainty. In this set-up a rationale can be provided both for markets for individual contracts and for financial markets where securities are traded. It is convenient however to introduce first a simpler economy with no aggregate risk where the structure of the individual uncertainty and information as well as the nature of contracts traded can be more easily understood; this should also help the reader to gain some familiarity with the notation we later use. This economy constitutes our leading example.

The leading example. Consider a two-period pure exchange economy with countably many agents, all ex-ante identical. There is a single commodity, and consumption only takes place at date 1. Every agent has Von Neumann–Morgernstern preferences over consumption (logarithmic for simplicity). His endowment depends on the realization of an idiosyncratic shock, with two possible realizations, $\{1, 2\}$, i.i.d. across agents; let the respective endowment realizations be $w(1)$, $w(2)$.

The realization of the endowment becomes commonly known only at date 1. However, at the beginning of date 0 each agent privately observes the realization of a signal, correlated with his own endowment; there are two possible and equiprobable realizations of the signal, (g, b). Let $\pi_g \equiv \pi(1/g)$ and $\pi_b \equiv \pi(1/b)$ be, respectively, the probability of state 1 conditional on having received signals g and b. We assume that $w(1) > w(2)$ and $\pi_g > \pi_b$; hence agents who receive signal g constitute the 'good risks' (i.e., have a higher probability of the good realization of their future income) and agents with signal b the 'bad risks.'

At date 0 markets open, where each agent can trade two types of individual contracts. Contract 1 pays one unit of the commodity when the agent's idiosyncratic state is 1, and 0 otherwise. Similarly, contract 2 pays one unit of the commodity in his idiosyncratic state 2. A riskless bond is also available, with a constant payoff of one unit.

What we have described is then a simple economy with insurance markets characterized by the presence of adverse selection.

The general economy we study is the following. There are L commodities, labeled by $l \in L = \{1, \dots, L\}$, available for consumption both at date 0 and at date 1; commodity 1 is the designated numeraire in every spot. There are finitely many types of consumers in the economy, indexed by $h \in H = \{1, \dots, H\}$, and countably many agents of each type. A consumer is then identified by a pair (h, n), where $n \in \mathbb{N}$ and $\mathbb{N}$ is the set of natural numbers. Let λ^h be the fraction of the total population made of agents of type h. In addition to consumers, there are intermediaries, of J different types, who trade in the markets.

2.1. Uncertainty and information

In the economy there is both aggregate and idiosyncratic uncertainty and agents have some private information over the realization of their own idiosyncratic shock.

More precisely, the uncertainty affecting the agents is described by the collection of random variables $\tilde{\sigma}$, $(\tilde{s}^n)_{n\in\mathbb{N}}$, with support Σ and S (the same for all n), respectively. Both Σ and S are assumed to be finite sets, $\Sigma = \{1, \ldots, \Sigma\}$ and $S = \{1, \ldots, S\}$, with generic element σ and s, respectively.

The random variable $\tilde{\sigma}$ represents the economy's aggregate uncertainty, affecting all agents in the economy; $\tilde{\sigma}$ is realized at date 1, and is observed by all agents.

The random variables $(\tilde{s}^n)_{n\in\mathbb{N}}$ describe purely idiosyncratic sources of uncertainty: the variables $(\tilde{s}^n)_{n\in\mathbb{N}}$ are independent of $\tilde{\sigma}$, identically and independently distributed across n, and $\tilde{s}^n$ only affects the (finitely many) agents of index n.

The realization of $(\tilde{s}^n)_{n\in\mathbb{N}}$ also becomes commonly known at date 1. However, at the beginning of date 0 each agent (h, n) privately observes the realization of a signal $(\tilde{\xi}^{h,n})$, correlated with $(\tilde{s}^n)$. Let $\Xi^h = \{1, \ldots, \Xi^h\}$ be the (finite) support of $(\tilde{\xi}^{h,n})$, and ξ^h its generic element. We consider here the case in which the signal received $\tilde{\xi}^{h,n}$ carries information over the distribution, but not the support, of $(\tilde{s}^n)$.[5] Furthermore, we allow signals to be correlated across agents of same index n (so that agents' private information over $\tilde{s}^n$ may not be fully exclusive): $\tilde{\xi}^{h,n}$ may be correlated with $\tilde{\xi}^{h',n}$, for any n and $h' \neq h$. Let $\tilde{\xi}^n \equiv (\tilde{\xi}^{h,n})_{h\in H}$, $\Xi \equiv \prod_{h\in H} \Xi^h$.

Let π denote the probability distribution of $(\tilde{\sigma}, \tilde{s}^n, \tilde{\xi}^n)$ for all $n \in \mathbb{N}$, and $\pi(\sigma, s, \xi) = \pi(\tilde{\sigma} = \sigma, \tilde{s}^n = s, \tilde{\xi}^n = \xi)$. The above conditions are formally stated in the following assumption.

Assumption 1. The probability distribution of $(\tilde{\sigma}, \tilde{s}^n, \tilde{\xi}^n)$ satisfies:

(i) $\pi(\sigma, s, \xi) = \pi(\sigma)\pi(s, \xi)$, $\forall s, \sigma, \xi$;
(ii) $\pi(\tilde{s}^n = s, \tilde{\xi}^n = \xi) = \pi(\tilde{s}^{n'} = s, \tilde{\xi}^{n'} = \xi)$, $\forall n, n' \in \mathbb{N}$, $s \in S$, $\xi \in \Xi$;
(iii) $\pi(\tilde{s}^n = s, \tilde{\xi}^n = \xi, \tilde{s}^{n'} = s', \tilde{\xi}^{n'} = \xi') = \pi(\tilde{s}^n = s, \tilde{\xi}^n = \xi)\pi(\tilde{s}^{n'} = s', \tilde{\xi}^n = \xi')$, $\forall n \neq n' \in \mathbb{N}$, $h, h' \in H$, $s, s' \in S$, $\xi, \xi' \in \Xi$;
(iv) $\pi(s/\xi) > 0$, $\forall s \in S$, $\xi \in \Xi$.

2.2. Consumers

Uncertainty enters the economy via the level of the agents' date 1 endowment. Each agent $(h, n) \in H \times \mathbb{N}$ has an endowment w_0^h at date 0, and his date 1 endowment, $w_1^h(\tilde{\sigma}, \tilde{s}^n)$, depends upon the realization of his idiosyncratic shock $(\tilde{s}^n)$ and the aggregate shock $(\tilde{\sigma})$. We assume the following.

Assumption 2. $w_0^h \in \mathbb{R}^L_{++}$, $w_1^h \equiv (w_1^h(\sigma, s);\ \sigma \in \Sigma,\ s \in S) \in \mathbb{R}^{L(\Sigma S)}_{++}$.

A consumption plan for an arbitrary agent (h, n) specifies the level of his consumption of the L commodities at date 0 and for every possible realization of the uncertainty at

[5] Our analysis easily extends to the case in which agents have private information also over the support of $(\tilde{s}^n)$. For a description of the case in which the information revealed by $(\tilde{\xi}^{h,n})$ over the idiosyncratic shock $(\tilde{s}^n)$ has a partitional structure and, in addition, the collection of signals received by agents with same index n fully reveals $(\tilde{s}^n)$, see Bisin and Gottardi (1999).

date 1. The consumption set is the non-negative orthant of the Euclidean space. Agents are assumed to have Von Neumann–Morgenstern preferences over consumption plans and the utility index of agent (h, n) is given by a function $u^h : \mathbb{R}^{2L}_{+} \to \mathbb{R}$ satisfying the following assumption.

Assumption 3. $u^h(.)$ is continuous, strictly increasing, and strictly concave.

2.3. Markets and contracts

At date 0 the L commodities are traded on spot markets. At the same date, markets for securities and contracts also open, where agents trade to insure against their aggregate as well as their idiosyncratic shocks.

Contracts are claims contingent on the individuals' idiosyncratic uncertainty, they are thus agent-specific. We assume that for each n there are J contracts, with payoff contingent on the realization of $(\tilde{s}^n, \tilde{\sigma})$, which allow agents of index n to insure their individual uncertainty: each unit of contract $j \in J$ pays $r_j(s, \sigma)$ units of the numeraire commodity when $\tilde{s}^n = s$, $\tilde{\sigma} = \sigma$. These are standardized contracts as the terms of all contracts of a given type $j \in J$ are the same for all n, independent of the identity of the agent entering the contract to insure his individual uncertainty. Ex ante the payoff distribution of all contracts of the same type is then the same for all n; ex-post however their payoff will be different, as it will vary with the specific realization of $\tilde{s}^n$ across n.

In addition to individual contracts there are securities, claims whose payoff is contingent only on the aggregate uncertainty. Consumers and intermediaries trade such claims to hedge their aggregate risk. With no loss of generality, we assume that markets are complete with respect to the aggregate uncertainty: in particular, there are Σ 'Arrow' securities each agent can freely trade, with security $\sigma \in \Sigma$ paying one unit of numeraire if state σ is realized, and zero otherwise.

At date 1, after the realization of $((\tilde{s}^n)_{n \in \mathbb{N}}, \tilde{\sigma})$ becomes known to all agents, contracts and securities liquidate their payoff and the L commodities are again traded on spot markets.

Given the information structure of the economy, the spot markets as well as the markets for the Σ Arrow securities, whose payoff is only contingent on σ, operate under conditions of symmetric information. On the other hand, the markets for the J types of contracts, whose payoff is contingent on the agents' idiosyncratic uncertainty, are characterized by the presence of adverse selection: each agent (h, n) can choose in fact his level of trade in contracts with payoff contingent on $(\tilde{s}^n, \tilde{\sigma})$ after having privately observed the realization of a signal $\tilde{\xi}^{h,n}$, correlated with $\tilde{s}^n$.

We assume, without loss of generality, that the payoffs of individual contracts and securities are linearly independent, so that there are no redundant claims.

2.4. Intermediaries

Each intermediary of type j, $j \in J$, trades contracts of type j with the consumers. The intermediary has no resources of his own and acts so as to maximize profits: he balances the agents' different positions in the contract and, in addition, hedges his overall position by trading on the financial markets for Arrow securities (for this he can use the proceeds from the trades made with consumers). Thus the intermediary, in effect, 'makes the market' for contract j; he can also be viewed as an exchange or a dealer.

2.5. Prices

Markets are perfectly competitive, as consumers and intermediaries act as price-takers in all markets. Let $p_0 \in \mathbb{R}^L_+$ and $p_1(\sigma) \in \mathbb{R}^L_+$ be the vectors of commodity spot prices at date 0 and date 1, respectively, in the aggregate state σ; $p_1 \equiv (\ldots, p_1(\sigma), \ldots) \in \mathbb{R}^{L\Sigma}_+$. The price vector of the Arrow securities is then $\rho \equiv (\ldots, \rho_\sigma, \ldots)^{\mathrm{T}} \in \mathbb{R}^\Sigma_+$.

Moreover, all contracts of a given type j, which only differ for the index n trade at the same price; they are in fact all equivalent for the uninformed investors (as the intermediaries) and are then traded together within a single, large market. In such market, the level of trade of each agent has a negligible impact on aggregate trades, and these are only measurable with respect to the aggregate uncertainty;[6] thus no part of the information agents have over their idiosyncratic shocks is revealed so at a competitive equilibrium by the level of date 0 prices (differently from Radner (1979)). Therefore, agents retain some specific and exclusive private source of information, though remaining 'small' in terms of the level of their trades (so that their price-taking behavior is justified).

Our primary interest here is for situations where the information available over agents' trades is limited so that the exclusivity of contracts cannot be enforced and, more generally, non-linearities in the pricing schedule of contracts are hard to sustain. We will consider the case where for each contract $j \in J$ the price is given by a two-part tariff: there is a constant fee F_j each agent has to pay to be able to trade a non-zero amount in the market for this contract, and a second component which is linear in the quantity traded, with factor q_j. Thus the total cost $q_j(\theta_j)$ to trade θ_j units of contract j, is:

$$q_j(\theta_j) = \begin{cases} F_j + q_j\theta_j, & \text{if } \theta_j \neq 0, \\ 0, & \text{if } \theta_j = 0, \end{cases} \qquad j = 1, \ldots, J. \tag{TPP}$$

We will assume that agents face no trading restriction[7] and are then free to choose any level of trade at $q_j(\theta_j)$. Notice that the informational requirement of such a pricing schedule is indeed very minimal: it suffices to know whether an agent is trading or not in the market for type j contracts.

Let $q \equiv (q_j)_{j\in J}$ and $F \equiv (F_j)_{j\in J}$.

Under the above assumptions on the agents' information, consumer (h, n) chooses at date 0 his current consumption and portfolio after learning the realization ξ^h of $\tilde{\xi}^{h,n}$; his choice will then depend on ξ^h. Let $c_0^{h,n}(\xi^h) \in \mathbb{R}^L_+$ be the agent's date 0 consumption, $\zeta^{h,n}(\xi^h) = (\ldots, \zeta_\sigma^{h,n}(\xi^h), \ldots) \in \mathbb{R}^\Sigma$ his portfolio of Arrow securities, and $\theta^{h,n}(\xi^h) =$

[6] This follows from a standard application of the Law of large numbers; see also below.

[7] On the other hand, when the agents' private information is also over the support of $\tilde{s}^n$, there could be unlimited arbitrage opportunities at all prices and some bound may then have to be imposed on the level of trade in contracts (see Bisin and Gottardi (1999)).

$(\ldots, \theta_j^{h,n}(\xi^h), \ldots) \in \mathbb{R}^J$ his portfolio of standardized contracts.[8] Agent (h, n)'s date 1 consumption plan specifies then the level of consumption for every possible realization s of the idiosyncratic uncertainty and σ of the aggregate uncertainty; the agent's consumption plan still depends on the signal ξ^h received, as this affects the agent's portfolio choices at $t = 0$: $c_1^{h,n}(\xi^h) = (c_1^{h,n}(s, \sigma; \xi^h);\ s \in S,\ \sigma \in \Sigma) \in \mathbb{R}_+^{LS\Sigma}$.

Let $c^{h,n}(\xi^h) \equiv (c_0^{h,n}(\xi^h);\ c_1^{h,n}(\xi^h))$, $\pi(s/\xi^h) \equiv \pi(\tilde{s}^n = s/\xi^h)$. Formally, agent (h, n) has then to solve the following problem, for each $\xi^h \in \Xi^h$:

$$\max_{[c^{h,n}(\xi^h), \theta^{h,n}(\xi^h), \zeta^{h,n}(\xi^h)]} \sum_{s,\sigma} \pi(\sigma)\pi\left(s/\xi^h\right) u^h\left(c_0^{h,n}\left(\xi^h\right), c_1^{h,n}\left(\xi^h\right)\right) \qquad (P^h(\xi^h))$$

such that

$$p_0 \cdot \left(c_0^{h,n}\left(\xi^h\right) - w_0^h\right) + \sum_j q_j\left(\theta_j^{h,n}\left(\xi^h\right)\right) + \rho \cdot \zeta^{h,n}\left(\xi^h\right) \leqslant 0,$$

$$p_1(\sigma) \cdot \left(c_1^{h,n}\left(s, \sigma; \xi^h\right) - w_1^h(s, \sigma)\right) \leqslant \sum_j \theta_j^{h,n}\left(\xi^h\right) r_j(s, \sigma) + \zeta_\sigma^{h,n}\left(\xi^h\right),$$

$$(s, \sigma) \in S \times \Sigma, \qquad c^{h,n}\left(\xi^h\right) \in \mathbb{R}_+^{L(1+S\Sigma)}, \qquad \theta^{h,n}\left(\xi^h\right) \in \mathbb{R}^J, \qquad \zeta^{h,n}\left(\xi^h\right) \in \mathbb{R}^\Sigma,$$

where, for all j, $q_j(.)$ satisfies (TPP). Note that agents are free to go unlimitedly long and short both in contracts and in securities; thus, for a solution to their choice problem to exist, the linear components of the prices q and ρ must be suitably restricted so that no arbitrage opportunity exists.

Given the presence of the entry fee, the budget set in the consumers' problem $(P^h(\xi^h))$ fails to be convex. We will show that it is possible to overcome this difficulty by exploiting the large number of agents to 'convexify' the economy. This requires that, even though the choice problem is the same for all consumers of the same type h who received the same signal ξ^h, they may still make different choices at equilibrium. In particular, we will show that it is enough to consider the case in which these agents make at most a finite number V of different choices at equilibrium. Let $c^{h,v}(\xi^h)$, $\theta^{h,v}(\xi^h)$, $\zeta^{h,v}(\xi^h)$ denote the vth different choice of the agents of type h who observed signal ξ^h, and $\gamma^{h,v}(\xi^h)$ the fraction of agents of this type making such choice, for $v = 1, \ldots, V$. Let then $\gamma^h(\xi^h) \equiv (\gamma^{h,v}(\xi^h))_{v \in V}$ and Δ^{V-1} be the $(V-1)$-dimensional simplex.

Each intermediary of type j chooses the fraction k^j of the total population of consumers in the economy he will trade contracts of type j with. For these agents, the intermediary offsets their different positions in the contract, by compensating their payments both at date 0 and at date 1 for each possible realization of the agents' idiosyncratic uncertainty and of the aggregate uncertainty, and stands ready to meet any shortfall among these payments; he is also ready to pick up any difference between the purchases and sales of the agents he is trading with. As a result, the intermediary may have to make and/or receive some

[8] It should be clear that, in our set-up, agent (h, n) has no interest in trading other contracts than those with payoff contingent on his own individual uncertainty, $\tilde{s}^n$. Any portfolio of contracts with payoff contingent on $\tilde{s}^{n'}$, $n' \neq n$, is in fact dominated, in terms of its hedging possibilities of the aggregate uncertainty, by a portfolio of Arrow securities.

net payment at date 0 and in each state at date 1: let Q_j and $R_j(\sigma)$ be the intermediary's anticipated net revenue from this activity (on a per-capita basis), respectively, at date 0 and at date 1 in state σ, for $\sigma \in \Sigma$. Both Q_j and $R_j(\sigma)$ may be positive or negative. To hedge this position, the intermediary may then choose to trade also on the markets for Arrow securities, and we will denote by $\zeta^j = (\dots, \zeta^j_\sigma, \dots) \in \mathbb{R}^\Sigma$ the intermediary's portfolio of Arrow securities (again on a per-capita basis).

Formally, the problem of each intermediary j, $j \in J$,[9] consists then in choosing $k^j \in \mathbb{R}_+$ and $\zeta^j \in \mathbb{R}^\Sigma$ so as to maximize his profits at date 0, taking Q_j and $[R_j(\sigma)]_{\sigma \in \Sigma}$ as given:

$$\max_{[k_j \in \mathbb{R}_+, \zeta^j \in \mathbb{R}^\Sigma]} k^j Q_j - \rho \cdot \zeta^j \tag{P^j}$$

such that

$$0 \leqslant \zeta^j(\sigma) + k^j R_j(\sigma), \quad \sigma \in \Sigma,$$

where the latter expression describes the intermediary's resource constraint at date 1.

We require that, at equilibrium, the expectations of each intermediary $j \in J$ over his net revenue from trading in the market for contract j are consistent with, respectively, the sum of all the payments made and/or received by all consumers trading this contract at date 0, and the sum of all the payoffs due to—or owed by—these consumers at date 1:[10]

$$Q_j = \sum_h \lambda^h \sum_{\xi^h, v} \pi(\xi^h) \gamma^{h,v}(\xi^h) q_j(\theta_j^{h,v}(\xi^h)), \tag{2.1}$$

$$R_j(\sigma) = -\sum_h \lambda^h \sum_{s,v} \gamma^{h,v}(\xi^h) \pi(s, \xi^h) \theta_j^{h,v}(\xi^h) r_j(s,\sigma), \quad \sigma \in \Sigma, \tag{2.2}$$

where $q_j(.)$ satisfies (TPP).

Since the intermediation technology is characterized by constant returns to scale, the solutions of problem (P^j) have a simple form.[11] The intermediary will always choose to perfectly hedge his position, $\zeta^j_\sigma = -k^j R_j(\sigma)$. He will choose then a positive but finite market share $(0 < k^j < \infty)$ only if $Q_j = -\rho \cdot R_j$, a condition which insures that the profits from the intermediation activity are equal to zero. Using (TPP), (2.1), and (2.2), the condition $Q_j = -\rho \cdot R_j$ can also be written as:

$$\begin{aligned}
&\sum_h \lambda^h \sum_{\xi^h, v} \pi(\xi^h) \gamma^{h,v}(\xi^h) q_j \theta_j^{h,v}(\xi^h) + F_j \sum_{h,\xi^h} \lambda^h \pi(\xi^h) \sum_{v:\, \theta_j^{h,v}(\xi^h) \neq 0} \gamma^{h,v}(\xi^h) \\
&\quad = \sum_h \lambda^h \sum_{s,\xi^h,v} \gamma^{h,v}(\xi^h) \pi(s,\xi^h) \theta_j^{h,v}(\xi^h) \sum_\sigma \rho_\sigma r_j(s,\sigma),
\end{aligned} \tag{2.3}$$

[9] Since the intermediation technology described above is characterized by constant returns to scale, all intermediaries of the same type can be treated as a single firm.

[10] The nature of the agents' idiosyncratic uncertainty allows us to use here the Law of large numbers to simplify the expressions of the aggregate net payments by agents on contract j. In particular, it insures that these expressions are independent of the realization of the agents' idiosyncratic shocks.

[11] By the same reason, all intermediaries of the same type j can be treated as one.

i.e., the sum of all the payments made or received by consumers at date 0 when trading contract j has to equal the present value, at 0, of all the payoffs paid or received by consumers on this contract at date 1.

From condition (2.3) it follows that in all active markets (i.e., where

$$\sum_{h,\xi^h} \lambda^h \pi\left(\xi^h\right) \sum_{v:\ \theta_j^{h,v}(\xi^h)\neq 0} \gamma^{h,v}\left(\xi^h\right) \neq 0)$$

the level of the fixed entry fee F_j, paid by all agents who trade the contract, equals the difference between the present value of the total net payoff received by consumers at date 1 and the total net payment made at date 0 for trading contract j at the constant unit price q_j:

$$F_j = \left(\sum_{h,s,\xi^h,v} \lambda^h \gamma^{h,v}\left(\xi^h\right) \pi\left(s,\xi^h\right) \theta_j^{h,v}\left(\xi^h\right) \sum_{\sigma} \rho_\sigma r_j(s,\sigma) \right.$$
$$\left. - \sum_{h,\xi^h,v} \lambda^h \pi\left(\xi^h\right) \gamma^{h,v}\left(\xi^h\right) q_j \theta_j^{h,v}\left(\xi^h\right) \right) \Big/ \left(\sum_{h,\xi^h} \lambda^h \pi\left(\xi^h\right) \sum_{v:\ \theta_j^{h,v}(\xi^h)\neq 0} \gamma^{h,v}\left(\xi^h\right) \right). \tag{2.4}$$

Remark 1. Each intermediary of a given type j chooses how many consumers to trade contract j with, on the basis of the average expected profitability of trading the contract (expectation which, in equilibrium, has to be consistent with the average profitability of trading the contract with the consumers in the economy). Through this choice, the intermediary indirectly determines his volume of trade in the contract, as this is given by the level of trade chosen by the consumers he trades with. The role of the intermediary is to balance the different positions of the consumers in contract j. This may require the intermediary to take a net (long or short) position in the market, and to make or receive some payments. These payments can be perfectly hedged on the financial markets and condition (2.3) insures that, at equilibrium, the present value of all current and future payments is zero.

This role of intermediaries is meant to capture, though in a rather stylized way as argued in the introduction, the role played in markets for derivative securities by exchanges and dealers, who balance agents' trades, meet possible temporary shortfalls, and hedge their position in financial markets to diversify the aggregate risks.

Definition 2.1. A *competitive equilibrium with adverse selection and entry fees* is then defined by a collection of prices and entry fees (p_0, p_1, ρ, q, F), consumption and portfolio plans for each consumer and each possible signal received, together with their relative frequency in the population[12] $(c^{h,v}(\xi^h), \theta^{h,v}(\xi^h), \zeta^{h,v}(\xi^h); \gamma^{h,v}(\xi^h))_{v\in V}$ and, for

[12] By allowing $\gamma^h(s^h)$ to be any real—rather than a rational—vector in the simplex Δ^{V-1}, our definition characterizes what is, strictly speaking, with countably many agents, only an approximate equilibrium. With appropriate assumptions on the probability space ensuring the validity of the implications of the Law of large numbers we are using (see, e.g., Al-Najjar (1995) and Sun (1998)), our analysis could be extended to economies with a continuum of agents, in which case no approximation is required.

each intermediary $j \in J$, anticipated net revenue Q_j, $[R_j(\sigma)]_{\sigma \in \Sigma}$, and the chosen market share and hedging portfolio (k^j, ζ^j), such that:

- consumers optimize: for each h, ξ^h, all plans $(c^{h,v}(\xi^h), \theta^{h,v}(\xi^h), \zeta^{h,v}(\xi^h))_{v \in V}$ solve $(P^h(\xi^h))$ at the prices (p_0, p_1, ρ, q, F);
- intermediaries maximize profits: for all j, (k^j, ζ^j) solve (P^j) given Q_j, $[R_j(\sigma)]_{\sigma \in \Sigma}$;
- for all j, Q_j and $[R_j(\sigma)]_{\sigma \in \Sigma}$ satisfy the consistency conditions (2.1), (2.2);
- commodity markets clear:[13]

$$\sum_h \lambda^h \left(\sum_{\xi^h, v} \lambda^{h,v}(\xi^h) \pi(\xi^h) \left(c_0^{h,v}(\xi^h) - w_0^h\right) \right) \leqslant 0, \tag{2.5}$$

$$\sum_h \lambda^h \sum_{s,v,\xi^h} \gamma^{h,v}(\xi^h) \pi(\xi^h, s) \left(c_1^{h,v}(s, \sigma; \xi^h) - w_1^h(s, \sigma)\right) \leqslant 0, \quad \sigma \in \Sigma; \tag{2.6}$$

- financial markets (for the Arrow securities) clear:

$$\sum_h \lambda^h \sum_{\xi^h, v} \pi(\xi^h) \gamma^{h,v}(\xi^h) \zeta^{h,v}(\xi^h) + \sum_{j \in J} k^j \zeta^j = 0; \tag{2.7}$$

- markets for contracts clear:

$$k^j = 1, \quad j \in J. \tag{2.8}$$

Remark 2. The market clearing condition for contracts (2.8) simply says that, for each contract, intermediaries choose to serve the whole market, i.e., to trade the contract with all the consumers in the economy. As shown earlier, this requires that a zero profit condition holds for each contract. This condition, since (2.1), (2.2) also hold, has the form given in (2.3): the present value of all current and future net payments to consumers trading the contract has to equal zero. Thus no cross-subsidization across contracts takes place in equilibrium: markets clear security by security, and the profits of the intermediary 'making' each market are zero.

The leading example (continued). Consider the leading example introduced earlier in this section. In the example we have $H = 1$, $L = 1$, and $\Sigma = 1$; moreover, $S = (1, 2)$, $\Xi = (g, b)$, $\pi(g) = \pi(b) = 1/2$. The Von Neumann–Morgenstern utility index is $\ln(c)$.

The two types of individual contracts have payoff $r_1(1) = r_2(2) = 1$, and 0 otherwise (they span so the individual uncertainty). In this simple economy a single security, a riskless bond with a constant payoff of 1, insures market completeness with respect to the aggregate uncertainty.

In this set-up, Bisin and Gottardi (1999) have shown that, if contracts trade at linear prices, with no entry fees, a competitive equilibrium might not exist, and non-existence is robust. We briefly recall the key elements of this result here.

[13] The Law of large numbers is again used here as it insures that aggregate consumption only depends on the realization of the aggregate shocks, while the idiosyncratic shocks are 'averaged out' when summing across all agents.

The market-clearing condition for the commodity is, as in (2.6):

$$\begin{aligned} &c(1;g)\pi_g + c(1;b)\pi_b + (1-\pi_g)c(2;g) + (1-\pi_b)c(2;b) \\ &\quad - \bigl\{w(1)\pi_g + w(1)\pi_b + (1-\pi_g)w(2) + (1-\pi_b)w(2)\bigr\} = 0. \end{aligned} \tag{2.9}$$

It is immediate to see that this also constitutes an overall zero-profit condition for the intermediaries.

Let q and $1-q$ denote the (normalized) prices of, respectively, security 1 and 2. In the absence of entry fees, the budget constraint of an agent who received signal g is then:[14]

$$\theta_1(g)q + \theta_2(g)(1-q) = 0. \tag{2.10}$$

Similarly for agents with signal b. For this economy the set of no-arbitrage prices is non-empty, and is given by all prices $q \in (0,1)$. Solving the agents' choice problem and substituting in the expression on the left-hand side of (2.9), we obtain the excess demand function (equivalently, the opposite of the overall profit function), continuous for all $q \in (0,1)$. However, when

$$\frac{w(2)\pi_g}{w(1)(1-\pi_g)} > \frac{\pi_b}{(1-\pi_b)},$$

this function has a positive value both when

$$\frac{q}{(1-q)} > \frac{\pi_g}{(1-\pi_g)} \quad \text{and} \quad \frac{q}{(1-q)} < \frac{w(2)\pi_b}{w(1)(1-\pi_b)};$$

it is easy to see, in fact, from the expressions of the agents' demand, that in the first case agents will be buying insurance, no matter what is the signal received, and will do this at more than fair terms; while in the second case, agents will sell insurance, no matter what is the signal received, at less than fair prices (that is, less than fair prices for the buyer, the intermediary). In either case overall profits of the intermediaries will be negative, and excess demand will characterize the commodity market.

We can then show (see Bisin and Gottardi (1999)) that, for open sets of parameter values, the aggregate excess demand is also positive for all intermediate values of the relative price, i.e.,

$$\frac{w(2)\pi_b}{w(1)(1-\pi_b)} < \frac{q}{(1-q)} < \frac{\pi_g}{(1-\pi_g)}.$$

This is true, for instance, in a neighborhood of $w(1) = 0.8$, $w(2) = 0.2$, $\pi_b = 0.2$, and for any $\pi_g \geqslant 0.508$; hence an equilibrium never exists in this region.

In the presence of entry fees, i.e., when prices satisfy (TPP), the agents' budget constraint becomes:[15]

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[14] When contracts trade at linear prices, agents can replicate the bond simply by trading the two types of individual contracts; hence, we can safely omit the consideration of the market for the bond.

[15] In the presence of entry fees, the riskless bond is no longer redundant. Trading the two individual contracts allows to attain the same payoff as trading one of them and the bond. However, the latter is clearly preferable, as only one entry fee is paid. Moreover, to insure that agents are willing to trade both types of individual contracts (and hence that both markets are active), the entry fee should be the same in the two markets. This allows us to

$$\theta_1(g)q + \theta_2(g)(1-q) + \mathbf{1}_{\theta_1 \neq 0} F = 0, \tag{2.11}$$

where $\mathbf{1}_{\theta_1 \neq 0}$ denotes the indicator function for the event $\theta_1 \neq 0$. We will continue the analysis of competitive equilibria with entry fees in this example at the end of the next section.

3. Competitive equilibria with entry fees

With prices of contracts given by two-part tariffs, it is always possible to find an entry fee F_j sufficiently high so that no consumer wishes to trade contract j, for all j. The existence of such 'trivial' equilibria can then be easily shown.

In this section we will show the existence of competitive equilibria with entry fees which satisfy the following additional property:

The entry fee equals zero in all non-active markets, where no agent trades:

$$F_j = 0, \quad \text{if} \quad \sum_{h,\xi^h} \lambda^h \pi(\xi^h) \sum_{\upsilon:\, \theta_j^{h,\upsilon}(\xi^h) \neq 0} \gamma^{h,\upsilon}(\xi^h) = 0,\ j \in J. \tag{NT}$$

Condition (NT) insures that generically at equilibrium there will be a nonzero level of trade in all markets. This condition, together with (2.4), setting the entry fee in all active markets at a level such that profits are zero, uniquely determines the level of the entry fee.

As already said, the domain of admissible prices ρ of the securities and values q of the linear component of contracts' prices has to be suitably restricted to insure that there are no arbitrage opportunities. The set of no arbitrage prices is given by

$$\mathbb{Q} \equiv \left\{ (p,q) \in \mathbb{R}^{\Sigma}_{++} \times \mathbb{R}^J \colon \exists \upsilon \in \mathbb{R}^S_{++} \text{ such that } q_j = \sum_{\sigma} \rho_\sigma \sum_s \upsilon_s r_j(s,\sigma), \right. \\ \left. \text{for all } j \in J \right\},$$

where we see that for the (Arrow) securities, any strictly positive price clearly will do, while for contracts we have to take into account that the component of the return varying with the aggregate uncertainty can be fully hedged with the existing securities.

We will show that a competitive equilibrium exists for any given choice of the linear component of the price schedule q in $\mathbb{Q}$. As shown by (2.4), the other component of the schedule, the entry fee F_j, can always be set at a level such as to clear the market. Obviously, different choices of q lead to different properties of the equilibrium. We want to mention here two important cases:

(i) q satisfies the following 'fairness' property: for every $j \in J$, q_j equals the present value of the unconditional expectation, over the idiosyncratic component of the uncertainty, of the payoff of contract j:

simplify the expression of the budget set, avoiding an explicit consideration of the bond and specifying only one entry fee, F.

$$q_j = \sum_{\sigma} \rho_\sigma \sum_{s} \pi(s) r_j(s, \sigma), \quad j \in J. \tag{FA}$$

This value provides a rather natural benchmark. If we consider in fact the case where agents receive no signal over the realization of their idiosyncratic shocks, i.e., with symmetric information, the equilibrium we obtain, under (NT) and (FA), is characterized by a zero level of the entry fee in all markets and is Pareto efficient. Thus, under (FA) the level of the entry fee can be viewed as a measure of the 'cost' imposed by the presence of adverse selection in the markets.

(ii) q_j is set at a level such that the total net position of consumers in contract j is zero, or $\sum_h \lambda^h \sum_{\xi^h, v} \pi(\xi^h) \gamma^{h,v}(\xi^h) \theta_j^{h,v}(\xi^h) = 0$, for all j.
In this case the intermediary's net position in the market will be zero; however, the total net payment due to the agents at date 1 will still typically be nonzero, because of the presence of adverse selection, and so will be the level of the entry fee.

The implications of these alternative specifications of q for the properties of the equilibria will be examined within the set-up of the example.

Let us denote by $(c^h(\xi^h), \theta^h(\xi^h))(p_0, p_1, \rho, q, F)$ the correspondence describing the solution, for consumers of type h who received signal ξ^h, of their choice problem $(P^h(\xi^h))$ with respect to the level of consumption and trade of contracts, for all prices. The correspondence $(c^h(\xi^h), \theta^h(\xi^h))(.)$ is well-behaved and exhibits standard properties, with the only exception of convex-valuedness, as follows.

Lemma 3.1. *Under Assumptions* 1–3*, for all h, ξ^h, the individual demand correspondence $(c^h(\xi^h), \theta^h(\xi^h))(p_0, p_1, \rho, q, F)$ is non-empty and upper-semicontinuous for all $(p_0, p_1, \rho, q, F) \in \mathbb{R}_{++}^{L(1+\Sigma)} \times \mathbb{Q} \times \mathbb{R}^J$; in addition, it exhibits the following boundary behavior*:

for any sequence $\{p_0^{(\tau)}, p_1^{(\tau)}, \rho^{(\tau)}, q^{(\tau)}, F^{(\tau)}\} \in \mathbb{R}_{++}^{L(1+\Sigma)} \times \mathbb{Q} \times \mathbb{R}^J$, converging to $(p_0, p_1, \rho, q, F) \in \partial(\mathbb{R}_{++}^{L(1+\Sigma)} \times \mathbb{Q}) \times \mathbb{R}^J$ as $\tau \to \infty$,

$$\inf\Big\{ \big\| c^h(\xi^h), \theta^h(\xi^h) \big\| \colon \big(c^h(\xi^h), \theta^h(\xi^h)\big) \in \big(c^h(\xi^h), \theta^h(\xi^h)\big)\big(p_0^{(\tau)}, p_1^{(\tau)}, \rho^{(\tau)}, q^{(\tau)}, F^{(\tau)}\big) \Big\} \to \infty.$$

The argument of the proof is fairly standard and is only sketched below.

Proof. Consider problem $(P^h(\xi^h))$. Resolving the date 1 budget equations for every σ with respect to the portfolio of the associated Arrow security, $\zeta_\sigma^{h,n}(\xi^h)$, and substituting the expressions obtained into the date 0 budget equation, we find that the budget set of these agents can be rewritten as follows:

$$B^h\big(p_0, p_1, \rho, q, F; \xi^h\big) = \Big\{ c^h\big(\xi^h\big) \in \mathbb{R}_+^{L(1+\Sigma S)},\ \theta^h\big(\xi^h\big) \in \mathbb{R}^J \colon p_0 \cdot \big(c_0^h\big(\xi^h\big) - w_0^h\big) + \sum_j q_j \big(\theta_j^h\big(\xi^h\big)\big)$$

$$+\sum_{\sigma}\rho_{\sigma}\left[p_1(\sigma)\cdot\left(c_1^h\left(s,\sigma;\xi^h\right)-w_1^h(s,\sigma)\right)-\sum_j r_j(s,\sigma)\theta_j^h\left(\xi^h\right)\right]\leqslant 0,$$

$$s\in S\Bigg\},\tag{3.1}$$

where, for all j, $q_j(.)$ satisfies (TPP).

Under Assumption 2, $B^h(p_0, p_1, \rho, q, F; \xi^h)$ has clearly a non-empty interior, and is closed and compact for all $((p_0, p_1), \rho, (q, F)) \in \mathbb{R}_{++}^{L(1+\Sigma)} \times \mathbb{Q} \times \mathbb{R}^J$. Moreover, $B^h(p_0, p_1, \rho, q, F; \xi^h)$ is defined by the intersection of hyperplanes. Therefore, by a standard argument, the correspondence defined by $B^h(p_0, p_1, \rho, q, F; \xi^h)$ is also continuous. Upper-semicontinuity of demand then follows from the continuity of the agents' utility function (insured by Assumption 3).

It is immediate to see that, under Assumption 2, $B^h(p_0, p_1, \rho, q, F; \xi^h)$ has a non-empty interior also at prices $(p_0, p_1, \rho, q, F) \in \partial(\mathbb{R}_{++}^{L(1+\Sigma)} \times \mathbb{Q}) \times \mathbb{R}^J$, so that the boundary behavior property of demand holds. □

The following is then the main result of the paper.

Theorem 3.2. *Under Assumptions* 1–3, *a competitive equilibrium with adverse selection and entry fees satisfying* (NT) *always exists, for any choice of q in $\mathbb{Q}$.*

Remark 3. In Bisin and Gottardi (1999) the existence of competitive equilibria was established for pricing schedules of the following form:

$$q_j(\theta_j)=\begin{cases} q_j^b\theta_j, & \text{if } \theta_j>0,\\ q_j^s\theta_j, & \text{if } \theta_j<0,\end{cases}$$

i.e., with bid–ask spreads. Together with this earlier result, the one in this paper implies then the existence of competitive equilibria with pricing schedules which exhibit both discontinuities as well as non-linearities. The implementation of such, more general, pricing schedules clearly requires the availability of more information over the agents' trades. However, in the presence of asymmetric information, it may allow to achieve higher level of welfare. In particular, exclusive contracts—which, as we argued, allow the decentralization of (interim) incentive efficient allocations—require a very general form of non-linearity as the price of each contract has to depend on the trades in all markets.

Before presenting the proof of the theorem, we will analyze the properties of equilibria with entry fees within the set-up of the example introduced in the previous section, investigating the consequences of adverse selection and comparing the properties of the equilibria with entry fees to those with bid–ask spreads.

Leading example (concluded). For small enough levels of the entry fee F, so that both types choose to trade in the market for insurance contracts, the solution of the agents' optimization problem subject to the budget constraint (2.11) yields the following expressions for consumption demand:

$$c(1;g) = \pi_g \frac{qw(1) + (1-q)w(2) - F}{q},$$
$$c(2;g) = (1-\pi_g) \frac{qw(1) + (1-q)w(2) - F}{1-q},$$
$$c(1;b) = \pi_b \frac{qw(1) + (1-q)w(2) - F}{q},$$
$$c(2;b) = (1-\pi_b) \frac{qw(1) + (1-q)w(2) - F}{1-q}.$$

Substituting these terms in the market clearing condition (2.9), we obtain the equilibrium level of the entry fee (when both types trade):

$$F = \frac{\left(qw(1) + (1-q)w(2)\right)\left[\left(\pi_g^2 + \pi_b^2\right) - q(\pi_g + \pi_b)\right] + qw(1)(2q - (\pi_g + \pi_b))}{\left[(1-q)\left(\pi_g^2 + \pi_b^2\right) + q((1-\pi_g)^2 + (1-\pi_b)^2)\right]}. \tag{3.2}$$

From (3.2) it is immediate to verify that the entry fee is positive when the linear component of the price of contracts satisfies the fairness condition (FA), which in this environment is

$$q = \frac{\pi_g + \pi_b}{2}, \tag{3.3}$$

as well as for all $q > (\pi_g + \pi_b)/2$. This provides further support to our previous claim that under (FA) the entry fee is a measure of the costs imposed by adverse selection for the functioning of markets.

As further evidence of this, we show next that, when the degree of adverse selection in the economy (measured by the difference between π_g and π_b) increases, while aggregate resources remain constant, the equilibrium level of the entry fee also increases, whatever is q. Differentiating (3.2) with respect to π_g, while π_b is suitably adjusted so that $\{w(1)\pi_g + w(1)\pi_b + (1-\pi_g)w(2) + (1-\pi_b)w(2)\}$ stays constant,[16] we obtain:

$$\operatorname{sgn} \frac{\partial F}{\partial \pi_g} = \operatorname{sgn}(\pi_g - \pi_b) > 0;$$

this holds no matter what is the level of q and for all values of the parameters describing an economy such that F is sufficiently low. In this case we can say that the equilibrium level of the entry fee is positively related to the extent by which adverse selection is present in the markets.

We compare now equilibria with entry fees to equilibria with bid–ask spreads (as in Bisin and Gottardi (1999)) and show that entry fees may increase the agents' participation in the markets and yield Pareto superior outcomes. In particular, we will show that we can find economies such that:

(i) if contracts can be freely traded at linear prices, there is no equilibrium;
(ii) with bid–ask spreads there is a unique equilibrium in which agents of type g do not trade;

[16] Or, equivalently, so that $\pi_g + \pi_b$ stays constant.

(iii) with entry fee there is a unique equilibrium where, for an appropriately chosen value of q, both agents of type g and b trade in the insurance markets.

Consider the following specification of the parameters describing an economy: $w(1) = 0.8$, $w(2) = 0.2$, $\pi_b = 0.2$, $\pi_g = 0.508$. In our previous analysis of the example we argued that, for such parameter values, an equilibrium does not exist when contracts' prices are linear and there is no restriction on trade (the agents' budget set is given by (2.10)).

If buying and selling prices of contracts are allowed to differ, competitive equilibria always exist, as shown in Bisin and Gottardi (1999), for the class of economies studied in this paper; in the context of the example these equilibria are characterized by the fact that agents either buy insurance, i.e., $\theta_1(i) \leqslant 0$, $\theta_2(i) \geqslant 0$, $i = b, g$, or do not trade (they never choose to sell insurance). It can be easily seen that, for the above parameter values, there is a unique equilibrium with bid–ask spreads, where the relative price for buying insurance is $q^b/(1-q^b) = 0.25$. This is the fair price of insurance for the agents of type b; hence they choose to fully insure. The type g agents, on the other hand, choose not to trade, since at the price $q^b/(1-q^b) = 0.25$ they prefer to sell rather than to buy insurance (and the equilibrium price for selling insurance, $q^s/(1-q^s) = 1.03$, is such that they do not want to sell).

Turning now our attention to equilibria with entry fees, agents are free in this case to buy and sell insurance at the same relative price $q/(1-q)$ and this has clearly the effect of favoring trade. However, to trade in each market agents have to pay an entry fee and this has the opposite effect of discouraging trades. The equilibrium level of the entry fee F depends, as we saw, on the level at which the linear component q of the price is set; the lower is F, the higher the chance agents will want to trade. While it is impossible to say in general if entry fees favor trade more than bid–ask spreads, we show here that, for the parameter values specified above, we can find a level of q such that all types of agents choose to trade.

Set q at a level such that $q/(1-q) = 0.29$. We claim that there is an equilibrium where both agents trade. The equilibrium level of the entry fee F/q, normalized by the contingent price of individual state 1, is then obtained from (3.2), and is equal to 0.00047, small but positive. We can then verify that, at this level of F/q, both types enjoy a higher utility by trading than by staying out of the market, and consuming their endowments: the gains from trade are 0.00137 for the g types and 0.2397 for the b types, positive for both. This confirms that, at the prices $q/(1-q) = 0.29$, $F/q = 0.00137$, both types choose to trade and hence the one we found is indeed an equilibrium with entry fee.

Moreover, this equilibrium (interim) Pareto dominates the equilibrium with bid–ask spreads. Agents of type g do not trade with bid–ask spreads while they trade with entry fees (and choose $c(1; g) = 0.756$, $c(2; g) = 0.212$), therefore they are obviously better off in the second case. As for agents of type b, they fully insure at fair prices with bid–ask spreads, and therefore consume $c(1; b) = c(2; b) = 0.32$, while they choose $c(1; b) = 0.298$, $c(2; b) = 0.346$ with entry fees; comparing the associated utility levels, we find that the b types are also better off with entry fees, as they trade at better than fair prices and the level of the entry fee is sufficiently small so as to less than compensate this effect.[17]

[17] We have, in fact, $0.2\ln(0.29784) + 0.8\ln(0.34549) = -1.0925 > \ln(0.32) = -1.1394$.

We conclude that entry fees can have an advantage with respect to bid–ask spreads as a market clearing device for economies with adverse selection, in that a suitable choice of the linear component q of the price can facilitate trade and so improve efficiency.

We proceed now with the proof of our main result, Theorem 3.2.

Proof. The proof is organized in separate steps.

Step 1. We show first that there exists V finite such that, for every h and ξ^h, the image of the individual demand correspondence $(c^h(\xi^h), \theta^h(\xi^h))(p_0, p_1, \rho, q, F)$ has at most V values, for $((p_0, p_1), \rho, q, F)) \in \mathbb{R}^{L(1+\Sigma)}_{++} \times \mathbb{Q} \times \mathbb{R}^J$.

For any $\mathcal{J} \subseteq J$, define

$$w_0^h(\mathcal{J}) = \begin{bmatrix} w_{01}^h - \sum_{j\in\mathcal{J}} F_j \\ w_{02}^h \\ \vdots \\ w_{0L}^h \end{bmatrix}.$$

Given the expression of $B^h(p_0, p_1, \rho, q, F; \xi^h)$ in the proof of Lemma 3.1, it is immediate to see that

$$\begin{aligned} &B^h_{\mathcal{J}}\big(p_0, p_1, \rho, q, F; \xi^h\big) \\ &\quad \equiv \Bigg\{ c^h\big(\xi^h\big) \in \mathbb{R}_+^{L(1+\Sigma S)},\ \theta^h\big(\xi^h\big) \in \mathbb{R}^J\colon\ p_0 \cdot \big(c_0^h\big(\xi^h\big) - w_0^h(\mathcal{J})\big) + \sum_{j\in\mathcal{J}} q_j\theta_j^h\big(\xi^h\big) \\ &\qquad + \sum_\sigma \rho_\sigma \Bigg[p_1(\sigma) \cdot \big(c_1^h\big(s, \sigma; \xi^h\big) - w_1^h(s, \sigma)\big) - \sum_{j\in\mathcal{J}} r_j(s, \sigma)\theta_j^h\big(\xi^h\big)\Bigg] \leqslant 0, \\ &\qquad s \in S;\ \theta_j^h\big(\xi^h\big) = 0 \text{ for all } j \in J\backslash\mathcal{J} \Bigg\} \end{aligned}$$

describes the budget set of a consumer of type h with signal ξ^h who has chosen to trade (and pay the entry fee) in the subset $\mathcal{J} \subseteq J$ of the existing markets for contracts. It is easy to see that $B^h_{\mathcal{J}}(p_0, p_1, \rho, q, F; \xi^h)$ is closed, compact and, in addition, convex; this set may be empty for some, but not all, pairs $\mathcal{J}$, (p_0, p_1, ρ, q, F). Let then $(c^h(\xi^h), \theta^h(\xi^h))_{\mathcal{J}}(p_0, p_1, \rho, q, F)$ be the solution of the problem of maximizing the agent's utility, subject to $(c^h(\xi^h), \theta^h(\xi^h)) \in B^h_{\mathcal{J}}(p_0, p_1, \rho, q, F)$; this is now a convex problem. It follows that $(c^h(\xi^h), \theta^h(\xi^h))_{\mathcal{J}}(p_0, p_1, \rho, q, F)$ will be either empty or single-valued.

It is then immediate to see that

$$\big(c^h\big(\xi^h\big), \theta^h\big(\xi^h\big)\big)(p_0, p_1, \rho, q, F) \subset \Bigg\{ \bigcup_{\mathcal{J}\subset J} \big(c^h\big(\xi^h\big), \theta^h\big(\xi^h\big)\big)_{\mathcal{J}}(p_0, p_1, \rho, q, F) \Bigg\},$$

where $\bigcup_{\mathcal{J}\subset J}$ denotes the union over all the subsets of J. As there are finitely many possible subsets of J, and $(c^h(\xi^h), \theta^h(\xi^h))_{\mathcal{J}}(p_0, p_1, \rho, q, F)$ contains at most a single element, the stated result follows.

Step 2. We construct here V single-valued maps, obtained as selections from the individual demand correspondence, such that their union equals the individual demand correspondence. On this basis we can find a map which describes the agent's participation rate in the various markets for contracts; this map will allow us to determine the payments made by agents as entry fees. Furthermore, we derive a convenient representation of the correspondence describing the aggregate (per capita) demand and participation rate in the various markets for contracts of all agents of type h who observed signal ξ^h, for every h, ξ^h.

From the previous result, we know we can find finitely many (V) selections from the individual demand correspondence. We will construct here these selections according to the value of the norm of demand. Define the map $(c^{h,1}(\xi^h), \theta^{h,1}(\xi^h))(p_0, p_1, \rho, q, F)$ by associating to each (p_0, p_1, ρ, q, F) the vector (c^h, θ^h) in the set $(c^h(\xi^h), \theta^h(\xi^h))(p_0, p_1, \rho, q, F)$ which has minimal norm:[18]

$$\left\|\left(c^h, \theta^h\right)\right\| \leqslant \left\|\left(\bar{c}^h, \bar{\theta}^h\right)\right\| \quad \text{for all } \left(\bar{c}^h, \bar{\theta}^h\right) \in \left(c^h\left(\xi^h\right), \theta^h\left(\xi^h\right)\right)(p_0, p_1, \rho, q, F).$$

Proceed then iteratively to construct the other maps $c^{h,v}(\xi^h), \theta^{h,v}(\xi^h))(p_0, p_1, \rho, q, F)$, $v = 2, \ldots, V$, by associating for all prices the value in $(c^h(\xi^h), \theta^h(\xi^h))(p_0, p_1, \rho, q, F)$ with vth minimal norm. More precisely,

$$\begin{aligned}&\left(c^{h,v}\left(\xi^h\right), \theta^{h,v}\left(\xi^h\right)\right)(p_0, p_1, \rho, q, F)\\ &\quad = \begin{cases} \left(c^h, \theta^h\right) \in \left(c^h\left(\xi^h\right), \theta^h\left(\xi^h\right)\right)(p_0, p_1, \rho, q, F) \text{ satisfying (i)–(ii) below,} \\ \qquad\qquad\qquad\qquad\qquad\qquad \text{if such } (c^h, \theta^h) \text{ exists;} \\ \left(c^{h,v-1}\left(\xi^h\right), \theta^{h,v-1}\left(\xi^h\right)\right), \quad \text{otherwise;} \end{cases}\end{aligned}$$

(i) $(c^h, \theta^h) \neq (c^{h,i}(\xi^h), \theta^{h,i}(\xi^h))(p_0, p_1, \rho, q, F)$, for $i = 1, \ldots, v-1$;
(ii) $\|(c^h, \theta^h)\| \leqslant \|(\bar{c}^h, \bar{\theta}^h)\|$, for all $(\bar{c}^h, \bar{\theta}^h) \in (c^h(\xi^h), \theta^h(\xi^h))(p_0, p_1, \rho, q, F)$ such that $(\bar{c}^h, \bar{\theta}^h) \neq (c^{h,i}(\xi^h), \theta^{h,i}(\xi^h))$, $i = 1, \ldots, v-1$.

Evidently,

$$\left(c^h\left(\xi^h\right), \theta^h\left(\xi^h\right)\right)(p_0, p_1, \rho, q, F) = \bigcup_v \left(c^{h,v}\left(\xi^h\right), \theta^{h,v}\left(\xi^h\right)\right)(p_0, p_1, \rho, q, F).$$

For all h, ξ^h, and for each selection v we define then the map

$$I_j^{h,v}\left(\xi^h\right)(p_0, p_1, \rho, q, F) \equiv \begin{cases} 1, & \text{if } \theta_j^{h,v}\left(\xi^h\right)(p_0, p_1, \rho, q, F) \neq 0, \\ 0, & \text{if } \theta_j^{h,v}\left(\xi^h\right)(p_0, p_1, \rho, q, F) = 0, \end{cases}$$

an indicator function describing whether or not the agent is trading in market j, for all j; $I^{h,v}(\xi^h) \equiv (I_j^{h,v}(\xi^h))_{j \in J}$.

We can then use the fact that for every h, ξ^h, there are infinitely many consumers of type h who received signal ξ^h to write the correspondence describing the average (per capita)

[18] If there is more than one vector with minimal norm, any other criterion can be used to select one vector among them.

net demand and associated participation rate of these consumers as the convex hull of the maps defined above:

$$\begin{aligned}
&\big(\hat{c}^h(\xi^h), \hat{\theta}^h(\xi^h), \hat{I}^h(\xi^h)\big)(p_0, p_1, \rho, q, F) \\
&\quad = \mathrm{co}\Big[\bigcup_v \big(c^{h,v}(\xi^h), \theta^{h,v}(\xi^h), I^{h,v}(\xi^h)\big)(p_0, p_1, \rho, q, F)\Big] \\
&\quad = \Big\{\sum_v \gamma^{h,v}(\xi^h)\big(c^{h,v}(\xi^h), \theta^{h,v}(\xi^h), I^{h,v}(\xi^h)\big)(p_0, p_1, \rho, q, F),\ \forall \gamma^h(\xi^h) \in \Delta^{V-1}\Big\},
\end{aligned} \tag{3.4}$$

where co[.] denotes the convex hull of a set.

Step 3. The representation obtained in the previous step for the map describing the average participation rate in the various markets for contracts of the agents of type h, ξ^h allows us to show here that the correspondence yielding the equilibrium level of the entry fee is well-behaved for all (p_0, p_1, ρ, q, F).

Let $\gamma \equiv (\gamma^h(\xi^h))$, $\Delta \equiv \prod_{h\in H,\ \xi^h\in\xi^h} \Delta^{V-1}$. From (2.4) and the representation of agents' demand obtained above, we obtain the following expression for the correspondence describing the equilibrium level of the entry fee, for all (p_0, p_1, ρ, q, F) such that $\sum_{h,\xi^h} \lambda^h \pi(\xi^h) \sum_{v:\ \theta_j^{h,v}(\xi^h)\neq 0} \gamma^{h,v}(\xi^h) \neq 0$:

$$\begin{aligned}
&F_j(p_0, p_1, \rho, q, F) \\
&\quad \equiv \Bigg\{\bigg(\sum_h \lambda^h \sum_{s,\sigma,\xi^h} \pi(s, \xi^h)(\rho_\sigma r_j(s,\sigma) - q_j) \sum_v \gamma^{h,v}(\xi^h)\theta_j^{h,v}(\xi^h)(p_0, p_1, \rho, q, F)\bigg) \\
&\qquad\quad \times \bigg(\sum_{h,\xi^h} \lambda^h \pi(\xi^h) \sum_{v:\ \theta_j^{h,v}(\xi^h)(.)\neq 0} \gamma^{h,v}(\xi^h)\bigg)^{-1},\ \forall \gamma \in \Delta\Bigg\},
\end{aligned}$$

that can be equivalently written as:

$$\begin{aligned}
&= \Bigg\{\bigg(\sum_h \lambda^h \sum_{s,\sigma,\xi^h} \pi(s, \xi^h)(\rho_\sigma r_j(s,\sigma) - q_j) \sum_v \gamma^{h,v}(\xi^h) I_j^{h,v}(\xi^h)(.)\theta_j^{h,v}(\xi^h)(.)\bigg) \\
&\qquad \times \bigg(\sum_{h,\xi^h} \lambda^h \pi(\xi^h) \sum_v \gamma^{h,v}(\xi^h) I_j^{h,v}(\xi^h)(.)\bigg)^{-1},\ \forall \gamma \in \Delta\Bigg\} \\
&= \Bigg\{\sum_h \sum_{s,\sigma,\xi^h} \pi(s/\xi^h)\big(\rho_\sigma r_j(s,\sigma) - q_j\big) \\
&\qquad\qquad \times \left(\frac{\lambda^h \pi(\xi^h) \sum_v \gamma^{h,v}(\xi^h) I_j^{h,v}(\xi^h)(.)\theta_j^{h,v}(\xi^h)(.)}{\lambda^h \pi(\xi^h) \sum_v \gamma^{h,v}(\xi^h) I_j^{h,v}(\xi^h)(.)}\right)
\end{aligned}$$

$$\times \left(\frac{\lambda^h \pi(\xi^h) \sum_v \gamma^{h,v}(\xi^h) I_j^{h,v}(\xi^h)(.)}{\sum_{h,\xi^h} \lambda^h \pi(\xi^h) \sum_v \gamma^{h,v}(\xi^h) I_j^{h,v}(\xi^h)(.)} \right), \ \forall \gamma \in \Delta \Bigg\}$$

$$= \Bigg\{ \sum_h \sum_{s,\sigma,\xi^h} \pi\big(s/\xi^h\big)\big(\rho_\sigma r_j(s,\sigma) - q_j\big)\hat{\theta}_j^h\big(\xi^h\big)(.)$$

$$\times \left(\frac{\lambda^h \pi(\xi^h) \sum_v \gamma^{h,v}(\xi^h) I_j^{h,v}(\xi^h)(.)}{\sum_{h,\xi^h} \lambda^h \pi(\xi^h) \sum_v \gamma^{h,v}(\xi^h) I_j^{h,v}(\xi^h)(.)} \right), \ \forall \gamma \in \Delta \Bigg\},$$

where the last equality follows from (3.4), noting that the weights

$$\frac{\gamma^{h,v}(\xi^h) I_j^{h,v}(\xi^h)(.)}{\sum_v \gamma^{h,v}(\xi^h) I_j^{h,v}(\xi^h)(.)}$$

are non-negative and their sum over v equals 1 (so they have the same properties as the weights $\gamma^{h,v}(\xi^h)$).

Since the terms

$$\left(\frac{\lambda^h \pi(\xi^h) \sum_v \gamma^{h,v}(\xi^h) I_j^{h,v}(\xi^h)(.)}{\sum_{h,\xi^h} \lambda^h \pi(\xi^h) \sum_v \gamma^{h,v}(\xi^h) I_j^{h,v}(\xi^h)(.)} \right)$$

are also non-negative and add to 1 when summed over h, ξ^h, we obtain:

$$F_j(p_0, p_1, \rho, q, F) = \mathrm{co}\Bigg\{ \sum_{s,\sigma} \pi\big(s/\xi^h\big)\big(\rho_\sigma r_j(s,\sigma) - q_j\big)\hat{\theta}_j^h\big(\xi^h\big)(p_0, p_1, \rho, q, F);\ \forall \xi^h \in \xi^h,\ h \in H \Bigg\}, \tag{3.5}$$

for all (p_0, p_1, ρ, q, F) such that

$$\sum_{h,\xi^h,v} \lambda^h \pi\big(\xi^h\big) \gamma^{h,v}\big(\xi^h\big) I_j^{h,v}\big(\xi^h\big)(p_0, p_1, \rho, q, F) \neq 0.$$

Thus the map defining F, for (p_0, p_1, ρ, q, F) such that

$$\sum_{h,\xi^h} \lambda^h \pi\big(\xi^h\big) \sum_v \gamma^{h,v}\big(\xi^h\big) I_j^{h,v}\big(\xi^h\big)(p_0, p_1, \rho, q, F) \neq 0,$$

is a well-defined continuous function of upper-semicontinuous, convex-valued correspondences, and hence is also upper-semicontinuous and convex-valued.

It remains to show that upper-semicontinuity also holds at points where

$$\sum_{h,\xi^h} \lambda^h \pi\big(\xi^h\big) \sum_v \gamma^{h,v}\big(\xi^h\big) I_j^{h,v}\big(\xi^h\big)(.) = 0,$$

i.e., where no agent wishes to trade (given (NT), convex- valuedness is clearly satisfied at such points). For this, note that along any sequence $(p_0, p_1, \rho, q, F)^\tau$ such that

$$\sum_{h,\xi^h} \lambda^h \pi\big(\xi^h\big) \sum_v \gamma^{h,v}\big(\xi^h\big) I_j^{h,v}\big(\xi^h\big)\big((p_0, p_1, \rho, q, F)^\tau\big) \to 0, \quad \text{as } \tau \to \infty,$$

$F_j(p_0, p_1, \rho, q, F)$ also converges to 0. This follows from the fact that, as shown above, $F_j(p_0, p_1, \rho, q, F)$ can be equivalently written as:

$$\sum_h \sum_{s,\sigma,\xi^h} \pi\left(s/\xi^h\right)\left(\rho_\sigma r_j(s,\sigma) - q_j\right)\hat{\theta}_j^h\left(\xi^h\right)\left((p_0, p_1, \rho, q, F)^\tau\right)$$
$$\times \left(\frac{\lambda^h \pi(\xi^h) \sum_v \gamma^{h,v}(\xi^h) I_j^{h,v}(\xi^h)((p_0, p_1, \rho, q, F)^\tau)}{\sum_{h,\xi^h} \lambda^h \pi(\xi^h) \sum_v \gamma^{h,v}(\xi^h) I_j^{h,v}(\xi^h)((p_0, p_1, \rho, q, F)^\tau)}\right),$$

and $\hat{\theta}_j^h(\xi^h)((p_0, p_1, \rho, q, F)^\tau)$ converges to zero, while the terms

$$\left(\frac{\lambda^h \pi(\xi^h) \sum_v \gamma^{h,v}(\xi^h) I_j^{h,v}(\xi^h)(.)}{\sum_{h,\xi^h} \lambda^h \pi(\xi^h) \sum_v \gamma^{h,v}(\xi^h) I_j^{h,v}(\xi^h)(.)}\right),$$

as we argued, always lie in the simplex.

Step 4. In this final step we construct a map and show it exhibits all the needed properties so that a fixed point theorem can be applied and yield the existence of a competitive equilibrium. The argument is presented for the case in which q is defined by (FA); it should be clear though that it can be easily reformulated to apply to any other possible choice of q in $\mathbb{Q}$.

Normalize date 0 prices and date 1 prices in every aggregate state σ on the simplex. Consider then the following truncated price simplices:

$$\Delta_\delta^{L+\Sigma-1} \equiv \left\{(p_0, \rho) \in \mathbb{R}_+^{L+\Sigma}\colon \sum_l p_{0,l} + \sum_\sigma \rho_\sigma = 1;\ p_{0,l}, \rho_\sigma \geqslant \delta,\ \forall l, \sigma\right\},$$
$$\Delta_\delta^{L-1} \equiv \left\{p_1(\sigma) \in \mathbb{R}_+^{L}\colon \sum_l p_{1,l}(\sigma) = 1;\ p_{1,l}(\sigma) \geqslant \delta,\ \forall l\right\}.$$

Also, let $E \equiv (\sum_{\sigma\in\Sigma} \rho_\sigma \sum_s \pi(s) r_j(s,\sigma),\ j \in J$, for some $\rho \in \Delta^{\Sigma-1})$, a compact convex set.

The aggregate, per capita, excess demand for consumption is readily obtained by summing over h, ξ^h the expressions obtained above for the average demand of all agents with the same type and signal:

$$z_0(p_0, p_1, \rho, q, F) = \sum_h \lambda^h \left(\sum_{\xi^h, v} \gamma^{h,v}\left(\xi^h\right)\pi\left(\xi^h\right)\left(c_0^{h,v}\left(\xi^h\right)(.) - w_0^h\right)\right),$$
$$z_1(\sigma)(p_0, p_1, \rho, q, F)$$
$$= \sum_h \lambda^h \left(\sum_{\xi^h, s, v} \gamma^{h,v}\left(\xi^h\right)\pi\left(s, \xi^h\right)\left(c_1^{h,v}\left(s, \sigma; \xi^h\right)(.) - w_1^h(s,\sigma)\right)\right), \quad \sigma \in \Sigma.$$

Summing the budget constraints in (3.1) across agents and using the specification of the entry fee in (2.4) and (NT), we find that the following expression of Walras law holds for the economy under consideration:

$$
\begin{aligned}
& p_0 \cdot z_0(p_0, p_1, \rho, q, F) + \sum_\sigma \rho_\sigma \big(\ldots, p_1(\sigma) \cdot z_1(\sigma)(p_0, p_1, \rho, q, F), \ldots\big) \\
& \quad + \sum_j \bigg[F_j \bigg(\sum_{h,\xi^h} \lambda^h \pi\big(\xi^h\big) \sum_v \gamma^{h,v}\big(\xi^h\big) I_j^{h,v}\big(\xi^h\big)(.) \bigg) \\
& \qquad\qquad - \sum_h \lambda^h \sum_{s,\sigma,\xi^h} \pi\big(s, \xi^h\big)\big(\rho_\sigma r_j(s,\sigma) - q_j\big) \sum_v \gamma^{h,v}\big(\xi^h\big)\theta_j^{h,v}\big(\xi^h\big)(.) \bigg] \\
& = p_0 \cdot z_0(p_0, p_1, \rho, q, F) + \sum_\sigma \rho_\sigma \big(\ldots, p_1(\sigma) \cdot z_1(\sigma)(p_0, p_1, \rho, q, F), \ldots\big) \\
& = 0,
\end{aligned}
\tag{3.6}
$$

for all p_0, p_1, ρ, q, F.

The finitude of per-capita resources at date 1 and of the number of possible choices V made by agents at equilibrium, together with the fact that the payoff of contracts and securities are linearly independent, imply that we can find a compact set Θ where agents' demand for contracts must lie at equilibrium. From this it follows, given the expression obtained in Step 3 for the map describing the equilibrium level of the entry fee, that there is also a subset F_{EF} of $\mathbb{R}$, compact and convex, such that the range of the map $F_j(.)$ lies in F_{EF} when $\hat{\theta}_j^h(\xi^h)(.)$ is in Θ. Let then K_δ be a convex, compact set containing the image of the aggregate demand map at all prices (p_0, ρ, p_1, q, F) in $\Delta_\sigma^{L+\Sigma-1} \times (\Delta_\delta^{L-1})^\Sigma \times F_{\mathrm{EF}}$.

Consider then the map $(z_0, (\ldots, z_1(\sigma), \ldots), F, p_0, p_1, \rho, q)$ from the set $K_\delta \times F_{\mathrm{EF}} \times \Delta_\delta^{L+\Sigma-1} \times (\Delta_\delta^{L-1})^\Sigma \times E$ into itself, defined by:

$$
\text{(i)} \begin{bmatrix} z_0 \\ (\ldots, z_1(\sigma), \ldots) \\ F \end{bmatrix}
= \left\{
\begin{array}{l}
\sum_h \lambda^h \Big(\sum_{\xi^h, v} \gamma^{h,v}\big(\xi^h\big)\pi\big(\xi^h\big)\big(c_0^{h,v}\big(\xi^h\big)(p_0, p_1, \rho, q, F) - w_0^h\big)\Big) \\
\vdots \\
\sum_h \lambda^h \Big(\sum_{s,\xi^h,v} \gamma^{h,v}\big(\xi^h\big)\pi\big(s,\xi^h\big)\big(c_1^{h,v}\big(s,\sigma;\xi^h\big)(p_0, p_1, \rho, q, F) - w_1^h(s,\sigma)\big)\Big) \\
\vdots \\
\begin{cases}
\sum_h \sum_{s,\sigma} \pi(s/\xi^h)(\rho_\sigma r_j(s,\sigma) - q_j) \\
\qquad \times \left(\dfrac{\lambda^h \pi(\xi^h) \sum_v \gamma^{h,v}(\xi^h) \theta_j^{h,v}(\xi^h)(p_0, p_1, \rho, q, F)}{\sum_{h,\xi^h} \lambda^h \pi(\xi^h) \sum_v \gamma^{h,v}(\xi^h) I_j^{h,v}(\xi^h)(p_0, p_1, \rho, q, F)} \right), \\
\qquad \text{if } \sum_{h,\xi^h} \lambda^h \pi(\xi^h) \sum_v \gamma^{h,v}(\xi^h) I_j^{h,v}(\xi^h)(p_0, p_1, \rho, q, F) \neq 0, \\
0, \quad \text{otherwise},
\end{cases} \\
\forall \gamma \in \Delta.
\end{array}
\right\};
$$

(ii) $(p_0, p_1, \rho) \in \arg\max\{p_0 \cdot z_0 + \rho \cdot (\ldots, p_1(\sigma) \cdot z_1(\sigma), \ldots)\}$;

(iii) $q = \sum_\sigma \rho_\sigma \sum_s \pi(s) r_j(s, \sigma)$.

The domain of the above map is clearly compact, convex. We will now show that it is also upper-semicontinuous and convex-valued.

The aggregate excess demand correspondence above is obtained by taking the sum of correspondences which by Lemma 3.1 are upper-semicontinuous and, as established in Step 2, also convex-valued. Upper-semicontinuity and convex-valuedness of the maps defining (p_0, p_1, ρ, q) then follows by a standard argument.

Kakutani's theorem can then be applied to show that the map has a fixed point. Recalling the expression of Walras law we obtained (see (3.6)), it is immediate to see that if, at the fixed point, $(\rho, p_0, p_1)_\delta \in \operatorname{int}\{\Delta_\delta^{L+\Sigma-1} \times (\Delta_\delta^{L-1})^\Sigma\}$, we have $[(z_0), (\ldots, z_1(\sigma), \ldots)]_\delta = 0$, i.e., an equilibrium. If not, let $\delta \to 0$ and consider the associated sequence of fixed points. By a standard argument (see, e.g., Werner, 1985) we can show that this sequence is convergent and, given the boundary behavior property of excess demand, the limit value $(\rho, p_0, p_1)^* \in \operatorname{int}\{\Delta^{L+\Sigma-1} \times (\Delta^{L-1})^\Sigma\}$. $\square$

Acknowledgments

The project of this paper was begun when the first author was visiting Delta and was continued while the second author was visiting NYU and Yale; we wish to thank the three institutions for their very kind hospitality. We are grateful to Fernando Alvarez, an anonymous referee, and David Levine (the Co-editor) for very helpful comments. The first author gratefully acknowledges the financial and institutional support of the C.V. Starr Center for Applied Economics. The second author is grateful to CNR and MURST for financial support.

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